Lexical Analysis

What is Lexical Analysis

- What do we want to do?

  Example:

  ```
  if (i==j)
      z = 0;
  else
      z = 1;
  ```

- The input is just a string of characters

  "$if(i == j)\backslash n\backslash t\backslash tz = 0; \backslash telse\backslash n\backslash tz = 1; \backslash n$"

- Goal: partition input string into substrings
  - where these substrings are **tokens**

What is a Token ?

- A syntactic category of entities over alphabet
  - In English:

    noun, verb, adjective, ...
  - In a programming language:

    identifier, integer, keyword, whitespace, ...

- Tokens correspond to sets of strings
  - Identifier: strings of letters and digits, starting with a letter
  - Integer: a non-empty string of digits
  - Keyword: “else”, “if”, “while”, ...
  - Whitespace: a non-empty sequence of blanks, newlines, and tabs

What are Tokens For ?

- Classify program substrings according to role
- Output of lexical analysis is a stream of tokens
- Tokens are the input to the Parser
  - Parser relies on token distinctions
    a keyword is treated differently than an identifier

Designing a Lexical Analyzer

- Step 1:
  - Define a finite set of tokens
    - Describe all items of interest
    - Depend on language, design of parser

recall "$if(i == j)\backslash n\backslash t\backslash tz = 0;\backslash telse\backslash n\backslash tz = 1;\backslash n$"

- identifier, integer, keyword, whitespace
- "==" should be one token? or two tokens?

- Step 2:
  - Describe which string belongs to which token

Lexical Analyzer: Implementation

- An implementation must do two things
  1. Recognize substrings corresponding to tokens
  2. Return the value or lexeme of the token
- A token is a tuple `(type, lexeme)`
  "$if(i = j)\backslash n \backslash t \backslash tz = 0; \backslash telse \backslash n \backslash tz = 1; \backslash n$"
  - identifier: i, j, z
  - keywords: "if", "else"
  - integer: 0, 1
  - "(", ")", "=", ";": single character of the same name
- The lexer usually discards "non-interesting" tokens that don't contribute to parsing, e.g., whitespace, comments

- Lexical analysis looks easy but there are problems

Lexical Analyzer in FORTRAN

- FORTRAN compilation rule: whitespace is insignificant
  - rule was motivated from the inaccuracy of card punching by operators
- Consider
  - DO 5I=1,25
  - DO 5I=1.25
- We have
  - The first: a loop iterates from 1 to 25 with step 5
  - The second: an assignment
- Reading left-to-right, cannot tell if DO5I is a variable or DO statement; Have to continue until "," or "." is reached.

Lesson Learned

- Two important observations:
  - The goal is to partition the string. This is implemented by reading left-to-right, recognizing one token at a time.
  - "lookahead" may be required to decide where one token ends and the next one begins.

Lexical Analysis in C++

- Unfortunately, the problems continue today
- C++ template syntax

  FOO<Bar>
- C++ stream syntax

  cin> >var

- Now, the problem

  FOO<Bar<Bazz> >

Regular Languages

- Unfortunately, the problems continue today
- To describe tokens, we adopt **Regular Languages** formalism
  - Simple and useful theory
  - Easy to understand
  - Efficient implementations

Languages

Definition

Let $\sum$ be a set of characters, a **language** over $\sum$ is a set of strings of the characters drawn from $\sum$

Examples of Languages

- Alphabet = English characters
  Language = English sentences
- Alphabet = ASCII
  Language = C programs

  - Not every string on English characters is an English sentence
  - Note all ASCII strings are valid C programs

Notation

- Languages are **sets of strings**
- Need some notation for specifying which set we want to designate a language
  - Regular languages are those with some special properties
  - The standard notation for regular language is **regular expression**

Atomic Regular Expressions

- Single character denotes a set of one string
  'c' = { "c" }

- *Epsilon* or ϵ character denotes a set of non-zero length string ε = { "" }

- Empty set is { } = ϕ, not the same as ϵ
  size(ϕ) = 0
  size(ε) = 1
  length(ε) = 0

Compound Regular Expressions

- Union: if A and B are REs, then
  $A + B = \{ s \mid s \in A \text{ or } s \in B \}$
- Concatenation of sets/strings
  $AB = \{ ab \mid a \in A \text{ and } b \in B \}$
- Iteration (Kleene closure)
  $A^* = \cup_{i \geq 0} A^i$ where $A^i = A...A$ (i times)

  in particular
  $A^* = \{\varepsilon\} + A + AA + AAA + \ldots$
  $A+ = A + AA + AAA + \ldots = A\,A^*$

Regular Expressions

Definition

The **regular expressions (REs)** over $\sum$ are the smallest set of expressions including

- ε
- 'c' where c$\in \sum$
- A + B where A, B are **RE** over $\sum$
- AB where A, B are **RE** over $\sum$
- A* where A is a **RE** over $\sum$

Notation

- This notation means
  - $L(\varepsilon) = \{$ “” $\}$
  - L(‘c’) = { “c” }
  - $L(A+B) = L(A) \cup L(B)$
  - $L(AB) = \{ ab \mid a \in L(A) \text{ and } b \in L(B) \}$
  - $L(A^*) = \cup_{i \geq 0} L(A^i)$

Examples

- Keywords: “else” or “if” or “while” or ...
  - ‘else’ + ‘if’ + ‘while’ + ...
  - ‘else’ abbreviates
    ‘e’ (concatenate) ‘l’ (concatenate) ‘s’ (concatenate) ‘e’
  - keywords = { ‘else’, ‘if’, ‘then’, ‘while’, ... }

- Integer
  - digit = ‘0’ + ‘1’ + ‘2’ + ‘3’ + ‘4’ + ‘5’ + ‘6’ + ‘7’ + ‘8’ + ‘9’
  - integer = digit digit*
    - **Q:** is ‘000’ an integer?
    - **Q:** how to define another integer RE that excludes sequences with leading 0s?

More Examples

- Identifier: strings of letters or digits, starting with a letter
  - letter = 'A' + ... + 'Z' + 'a' + ... + 'z'
  - Identifier = letter (letter + digit)*
    - **Q:** is (letter* + digit*) the same?

- Whitespace: a non-empty sequence of blanks, newlines and tabs
  - whitespace = (' ' + '\n' + '\t') $^+$

More Examples

- Phones number: consider (412) 624-0000
  - $\sum$ = digit $\cup$ { -, (,) }
  - area = digit ^{3}
  - exchange = digit ^{3}
  - phone = digit ^{4}

  - phoneNumber = '(' area ')' exchange '-' phone

- Email address: student @ pitt.edu
  - $\sum$ = letter $\cup$ { ., @ }
  - name = letter $^+$

  - emailAddress = name '@' name '.' name

More Examples in Practice

- RE used in languages
  - By itself, it is a string, but semantically gets interpreted as a RE
  - RE in PERL,

    if ($str =∼ /(\d+)/) ...

    here,
    - $str denotes a variable
    - =∼ denotes RE matching
    - (\d+) defines a RE pattern
  - RE in C#,

    Match m = Regex.Match(“abrabceaab”, “(a|b|r)+”);

Some Common REs in Programming Languages

	Meaning		Meaning		Meaning
\d	Digits	\w	Any word char	\s	Space char
\D	Non-digits	\W	Non-word char	\S	Non-space char
[a-f]	Char range	[^a-f]	Exclude range	^	Matching string start
?	Optional	{n,m}	Appear n-m times	$	Matching string end
.	Any char	(…)	Capture matches	\(,\{	Matching (, { …
\.	Matching "."	+	Appear >=1 times	*	Appear 0 or many times

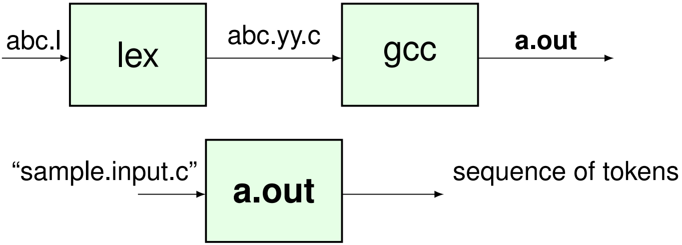
Implementation of Lexical Analysis

Implementation of Lexical Analysis

- We have learnt the formalism for lexical analysis
  — Regular expression (RE)

- How to actually get the lexical analyzer?
  - **Solution 1:** to implement using a tool — Lex (for C), Flex (for C++), Jlex (for java)
    - Programmer specifies the interesting tokens using REs
    - The tool generates the source code from the given REs
  - **Solution 2:** to write the code starting from scratch
    - This is also the code that the tool generates
    - The code is table-driven and is based on finite state automaton

Lex: a Tool for Lexical Analysis



- Big difference from your previous coding experience
  - Writing REs instead of the code itself
  - Writing actions associated with each RE
- A sepcification file has well-defined structures
- The detailed implementation will be discussed later

Lex Specifications

```
%{ /* include, extern, etc. */
extern int yytext, yylineno;
#include “token.h”
%}
/* declarations : declare variables, constants & regular definitions, */
digit       [0-9]
number      [0-9]+
%%
/* transition rules: regular expressions and actions */
/* R1 action where actions are program fragments written in C. */
number      { printf(“Token: int const %s and %d”, yytext, yyline); }
%%
/* auxiliary procedures */
myTableInsert()
{ ... }
```

Implementation Notes

- Write regular expressions for all/some of tokens
- Comments: keep track of nesting level
- String table written for you
- *yyline*, *yycolumn* maintained by yourself
- *yytext*, *yyleng* maintained by lex
- Special characters
  - ‘\n’ — newline
  - ‘\t’ — tab
  - ‘\‘’ — single quote
  - ‘\\’ — backslash

Discussion of RE and Lexical Analysis

- We use RE to assist lexical analysis

- Regular Expressions describe many useful languages
- Regular Expressions is a language specification
  - An implementation is still needed

- The problem we face is

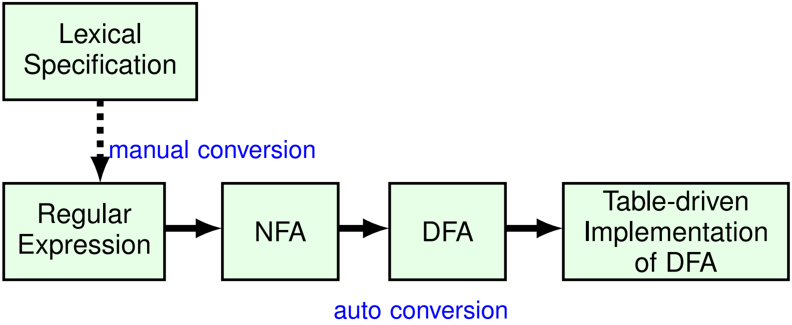
  Given a string **s** and a regular expression **RE**, is

  $\mathbf{s} \in \mathbf{L(RE)}$ **?**

Implementing Lexical Analysis with Finite Automata

An Overview of RE to FA

- Our implementation sketch



Implementation Outline

- RE ➟ NFA ➟ DFA ➟ Table-driven Implementation
  - Specifying lexical structure using regular expressions
  - Finite automata
    - Deterministic Finite Automata (DFAs)
    - Non-deterministic Finite Automata (NFAs)
  - Table implementations

Notations

- In the following discussion, we use some alternative notations

Union:	A $\mid$ B	$\equiv$ A + B
Option:	A ε	$\equiv$ A
Range:	‘a‘ + ‘b‘ + ... + ‘z‘	$\equiv$ [a-z]
Excluded range:		
	complement of [a-z]	$\equiv$ [^a-z]

Finite Automata

- A finite automata consists of 5 components $(\Sigma, S, n, F, \delta)$
  (1). An input alphabet Σ
  (2). A set of states S
  (3). A start state $n \in S$
  (4). A set of accepting states $F \subseteq S$
  (5). A set of transitions δ: $S_a \xrightarrow{input} S_b$

- For lexical analysis
  - Specification — Regular expression
  - Implementation — Finite automata

More About Transition

- Transition δ: $S_a \xrightarrow{input} S_b$

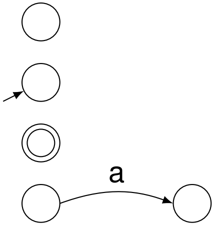
  read as

  in state S1 on input "a" go to state S2

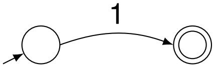
- At the end of input (or no transition possible), if current state X
  - $X \in$ accepting set F, then $\Rightarrow$ accept
  - otherwise, $\Rightarrow$ reject

State Graph

- Sometimes we use **state graph** to represent a FA
- A **state graph** includes
  - A set of states
  - A start state
  - A set of accepting states
  - A set of transition

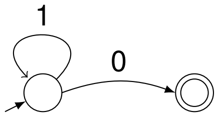


- Example: a finite state automata that accepts only “1”



More Examples

- A finite automata accepting any number of **1**s followed by a single **0**. Here we have Alphabet = {0,1}



- Example: What language does the following state graph recognize? Here we have Alphabet = {0,1}

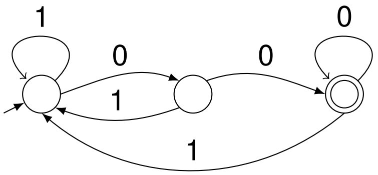
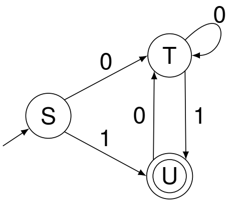


Table Implementation of a DFA

- Given the state graph of a DFA,



→ input characters

state ↓

	0	1
S	T	U
T	T	U
U	T	x

Table-driven Code:

```
DFA() {
    state = "S";
    while (!done) {
        ch = fetch_input();
        state = Table[state][ch];
        if (state == "x")
            perror("error");
    }
    if (state ∈ F)
        printf("accept");
    else
        printf("reject");
}
```

Discussion

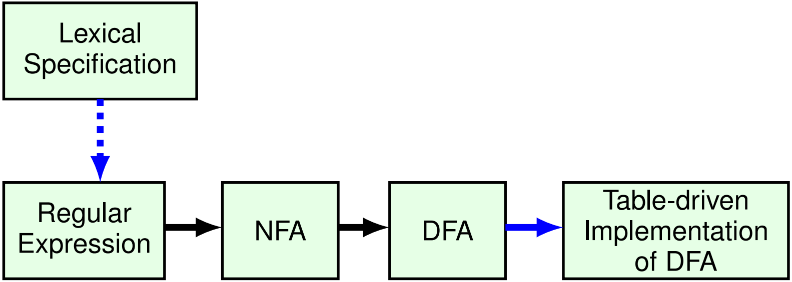
- Each RE has a different DFA / state graph
- For different REs,
  - their tables are different
  - their DFA recognition code is the same

- Revisit our implementation outline

RE ➟ NFA ➟ **DFA ➟ Table-driven Implementation**

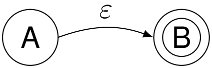
From RE to FA

- Our implementation sketch



Epsilon Moves

- Another kind of transition: ε-moves
  - Machine can move from state A to state B without reading any input

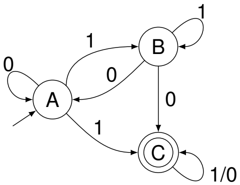


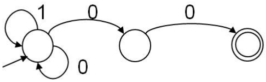
Deterministic and Nondeterministic Automata

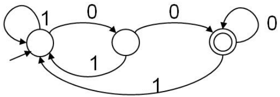
- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Non-deterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves

- Finite automata have finite memory
  - Need only to encode the current state

Examples

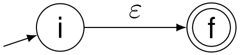




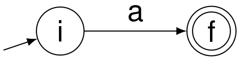


Converting RE to NFA

- McNaughton-Yamada-Thompson Algorithm
- Step 1: processing automic REs
  - ➢ ε expression

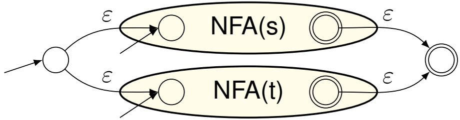


  - ➢ single character RE a

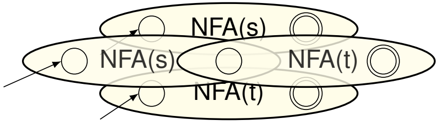


Converting RE to NFA (cont.)

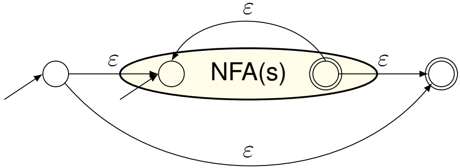
- Step 2: processing compound REs
  - r = s | t



  - r = s t

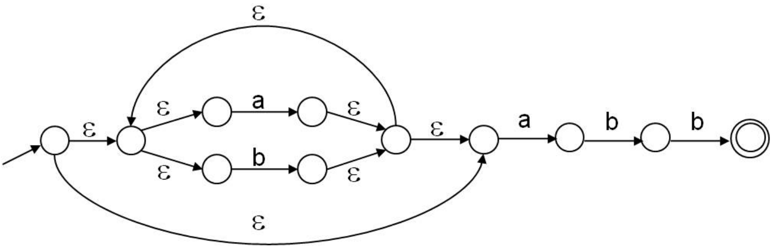


  - r = s *



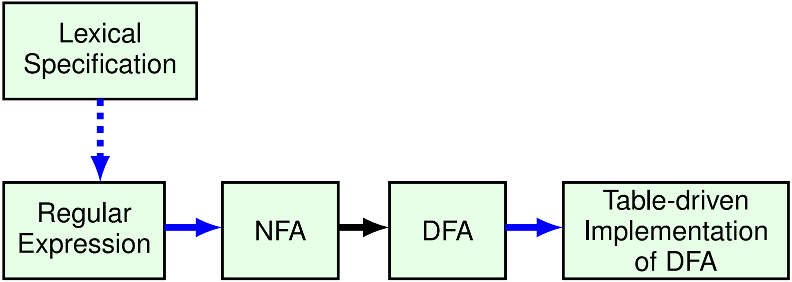
In-class Practice

- Convert “**(a|b)*****a b b**” to NFA



From RE to FA

- Our implementation sketch



Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- A NFA can take
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take
  - Acceptance of NFAs
    - An NFA can get into multiple states
    - **Rule**: the NFA accepts it if can get in a final state
- **Question:** which one is more powerful?

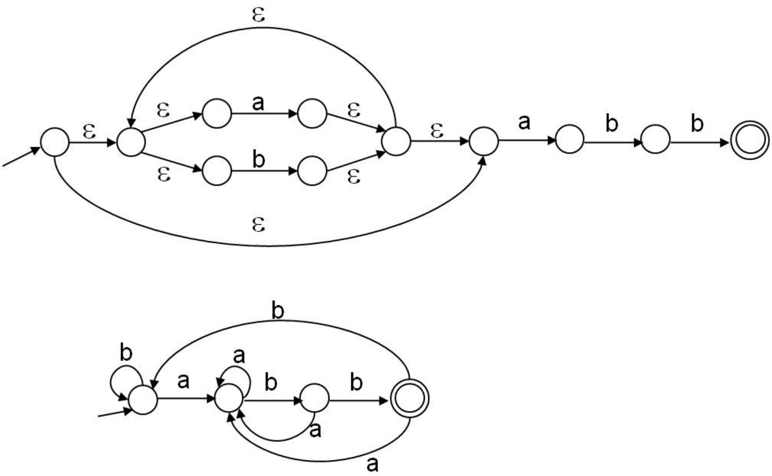
Comparing NFA and DFA

- **Theorem:** NFAs and DFAs recognize the same set of languages

- Both recognize regular languages
- DFAs are faster to execute
  - There are no choices to consider
- For a given language, NFA can be simple than DFA
- DFA can be exponentially larger than NFA
  - Example: DFA and NFA that accept "**(a|b)*****a b b**"

NFA and DFA

- Both accept "**(a|b)*****a b b**"



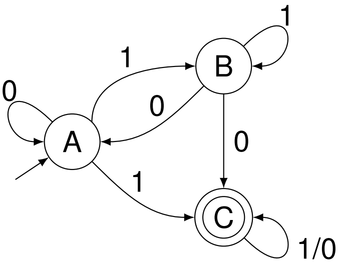
How to Convert NFA to DFA

- Basic idea: Given a NFA, simulate its execution using a DFA
  - At step n, the NFA may be in any of multiple possible states
- The new DFA is constructed as follows,
  - A state of DFA $\equiv$ a non-empty subset of states of the NFA
  - Start state $\equiv$ the set of NFA states reachable through ε-moves from NFA start state
  - A transition $S_a \xrightarrow{c} S_b$ is added **iff**

    S_b is the set of NFA states reachable from any state in S_a after seeing the input c, considering ε-moves as well

Example NFA to DFA

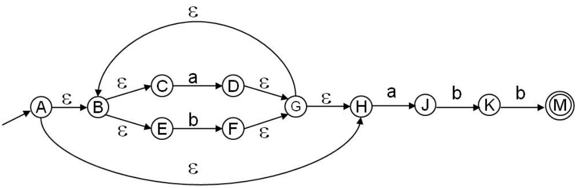
- What is the Equivalent DFA ?



state ↓ → input characters

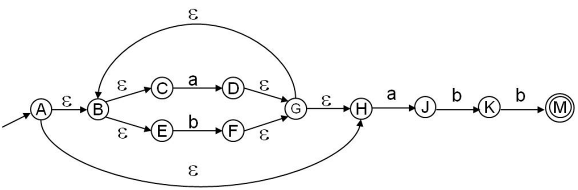
	0	1
A	A	BC
B	AC	B
C	C	C
AC	AC	BC
BC	AC	BC
AB	x	x
ABC	x	x

Algorithm Illustrated: Converting NFA to DFA



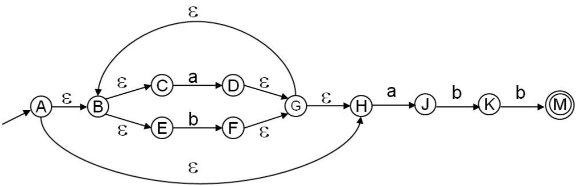
	ε	a	b
A			
B			
C			
D			
E			
F			
G			
H			
J			
K			
M			

Step 1: Construct the Table



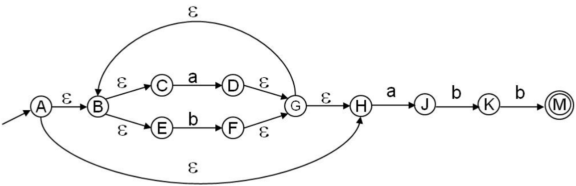
	ε	a	b
A	BH		
B	CE		
C		D	
D	G		
E			F
F	G		
G	BH		
H		J	
J			K
K			M
M			

Step 2: Construct ε-closure



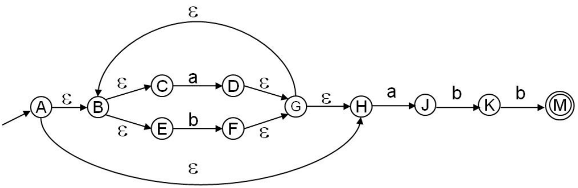
	ε	a	b
A	BHCE		
B	CE		
C		D	
D	GBHCE		
E			F
F	GBHCE		
G	BHCE		
H		J	
J			K
K			M
M			

Step 3: Update Other Columns



	ε	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

Step 4: Construct a New Table



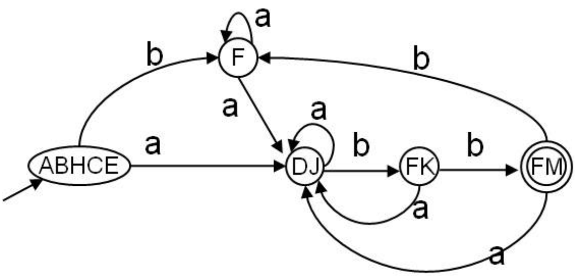
	ε	a	b
A	BHCE	DJ	F
B	CE	D	F
C		D	
D	GBHCE	DJ	F
E			F
F	GBHCE	DJ	F
G	BHCE	DJ	F
H		J	
J			K
K			M
M			

	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F
FK	DJ	FM
FM	DJ	F

Step 5: Generate the DFA

	a	b
ABHCE	DJ	F
DJ	DJ	FK
F	DJ	F
FK	DJ	FM
FM	DJ	F

- Note: the number of states is not minimized



NFA to DFA. Remarks

- An NFA may be in many states at any time

- How many different possible states?
  - If there are N states, the NFA must be in some subset of those N states
  - How many non-empty subsets are there ?
    - $2^N - 1$ many states

Implementation of DFA Revisited

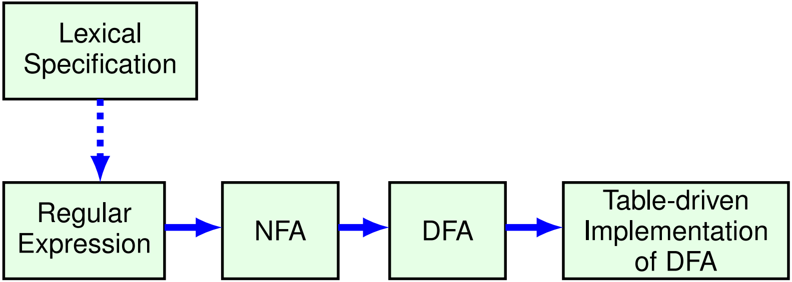
- A DFA can be implemented by a 2D table T
  - One dimension is “states”, the other dimension is “input characters”
  - For $S_a \xrightarrow{c} S_b$, we have T[S_a,c] = S_b
- DFA execution
  - If the current state is S_a and input is c, then read T[S_a,c]
  - Update the current state to S_b, assuming S_b = T[S_a,c]
  - It is efficient

Implementation Discussion

- NFA to DFA conversion is the heart of automated tools such as **lex**
- DFA could be very big
- In practice, lex-like tools trade off speed for space in the choice of NFA and DFA representations

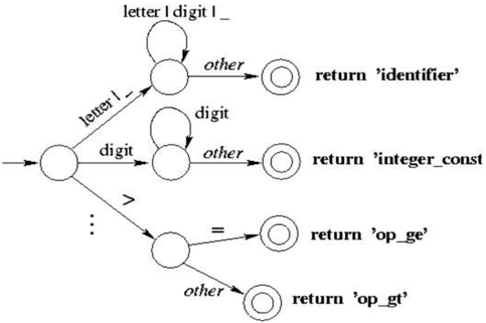
From RE to FA

- Our implementation sketch



Structure of a Scanner Automaton

- A scanner recognize multiple REs



How much should we match?

- In general, find the longest match possible

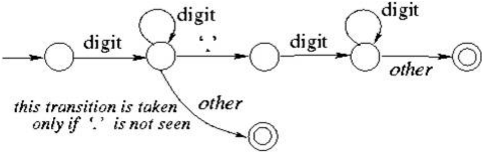
  Example:

  on input **123.45**, we match it as

  (numConst, 123.45)

  rather than

  (numConst, 123), (dot, "."), (numConst, 45)



How to Match Keywords?

- Approach 1: Hardcode the keywords
- Approach 2: When the token is identified, check a special table

Example: to recognize the following tokens

Identifiers: letter(letter|digit)*
Keywords: if, then, else

Beyond Regular Languages

- Regular language is powerful, accomplish our lexical analysis task
- Regular language can describe email addresses, phone numbers, ... , etc.

- However, it is the weakest formal language
  - Many languages are not regular
    - C programming language is not
    - "(((...)))" is also not
  - Finite automata cannot remember # of times
- We need more powerful languages for describing these structures
  - In the next lecture, we will introduce **context-free language**