The Effects of a Primordial Black Hole on the Surface of a Neutron Star

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Black Holes

- A black hole is a region of space where enough mass is concentrated such that nothing, including light, can escape its gravitational pull.
- Typical black holes form from the collapse of a massive star, and have a minimum mass on the order of a few solar masses.
- In the early universe while it was very hot and dense, small perturbations could be enough to create small black holes called primordial black holes.
- The mass of a primordial black hole is probably much less than one solar mass $(10^{-18} - 10^{-6} M_{\odot})$, and microscopic in size.

Primordial Black Holes as Dark Matter

Dark matter is matter that does not emit/absorb light, and is chargeless. Dark matter accounts for about a quarter of the energy in the universe, but we don't know what it is.

Primordial black holes may be a good candidate for dark matter since: they are chargeless, do not emit light, have a very small radius, and are non-relativistic. Also, most other proposed explanations of dark matter involve creating a new particle not part of the Standard Model.

Flat Star Model

Investigate the collision of a primordial black hole with a neutron star with the following assumptions:

- Neutron stars are flat and infinite
- Primordial black holes are point masses
- Neutron stars are incompressible fluids
- Gravitational interactions are Newtonian
- Primordial black hole has a constant velocity

Eigenfunctions of Laplacian

To find the velocity potential we need the eigenfunctions of the Laplacian. First, assume a product solution and solve the Laplacian.

$$
\begin{aligned}
\varphi &= R(r)Z(z)\Theta(\theta)T(t) \\
\nabla^2\varphi &= 0 \\
\implies \varphi &\propto \begin{Bmatrix} J_\mu(kr) \\ Y_\mu(kr) \end{Bmatrix} \begin{Bmatrix} e^{-kz} \\ e^{kz} \end{Bmatrix} \begin{Bmatrix} \sin(\mu\theta) \\ \cos(\mu\theta) \end{Bmatrix} T(t) \\
\implies \varphi &\propto J_0(kr)e^{kz}T(t)
\end{aligned}
$$

$T(t)$ comes from boundary conditions.

Hankel Transform

The Hankel transform is needed to solve the time component of the velocity potential, and the calculation of the energy.

Definition

The Hankel transform of a function $f(s)$ is given by

$$(\mathscr{H}_\nu f)(\sigma) = \int_0^\infty f(s) J_\nu(s\sigma) s \, ds,$$

where J_ν is the Bessel function of the first kind, of order $\nu \geq -\frac{1}{2}$, and σ is a non-negative real variable.

Corollary

The Hankel transform is self-reciprocal, that is, it is its own inverse.

Solving for φ

From the pressure condition

$$\left(\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial z} + \frac{\partial\Phi}{\partial t}\right)\bigg|_{z=0} = 0.$$

We can write the potential as a Hankel transform

$$\frac{\partial\Phi}{\partial t}\bigg|_{z=0} = \int_0^\infty \left(\mathscr{H}\,\frac{\partial\Phi}{\partial t}\bigg|_{z=0}\right)(k) J_0(kr) k\, dk.$$

Then,

$$\varphi = \frac{Gmv}{g}\int_0^\infty \frac{J_0(kr)e^{kz}}{1+kv^2/g}\Big(-\operatorname{sgn}(t)e^{-kv|t|} + 2\,\mathrm{H}(t)\cos(\omega_k t)\Big)\, dk,$$

with $\omega_k^2 = gk$

Deformation of Surface

We can easily solve for the shape of the surface now that we have the velocity potential, using

$$\left.\frac{\partial \varphi}{\partial z}\right|_{z=0} = \frac{\partial \eta}{\partial t}.$$

$$\eta = \frac{Gm}{g} \int_0^\infty \frac{J_0(kr)}{1 + kv^2/g} \left(e^{-kv|t|} + 2\,\mathrm{H}(t) v \sqrt{\frac{k}{g}} \sin(\omega_k t) \right) dk$$

Analytic Surface Waves

Figure: Waves!

Energy Transferred

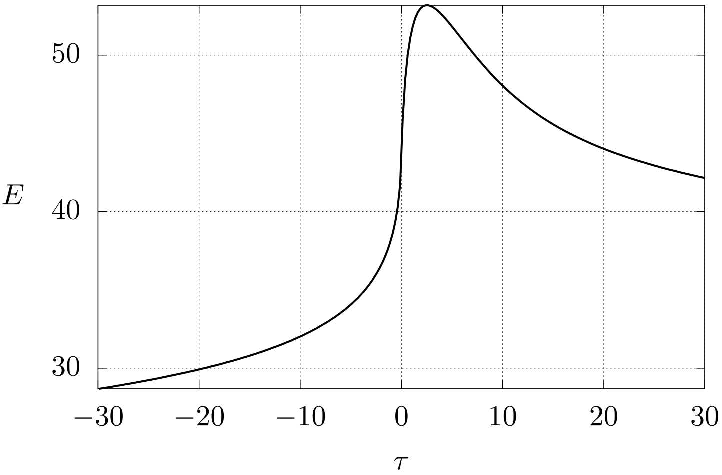
We can calculate the energy by taking the sum of the kinetic, and potential energies,

$$
\begin{aligned}
E(t) &= \frac{1}{2}\rho \int_{-\infty}^{0} \int_{0}^{\infty} |\nabla \varphi|^2 \, r dr dz \int_0^{2\pi} d\theta \\
&\quad + \rho g \int_0^{\infty} \int_0^{\eta} z dz \, r dr \int_0^{2\pi} d\theta
\end{aligned}
$$

$$
\lim_{t\to\infty} E(t) = 4\pi\rho \frac{G^2 m^2}{g}.
$$

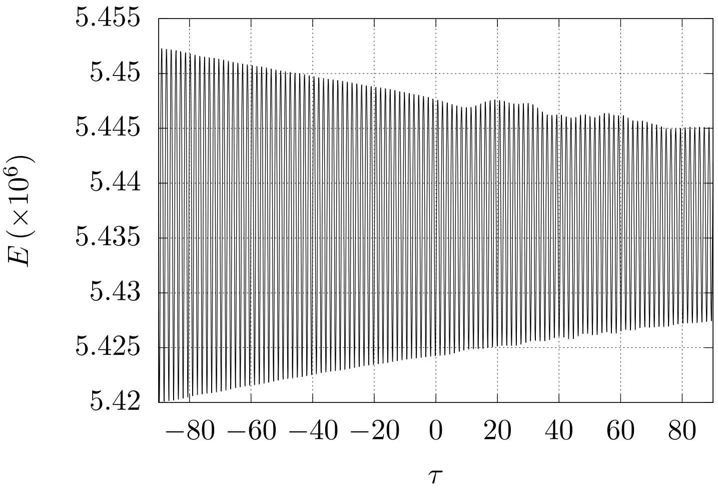
We take the limit as t approaches infinity, because we are interested in the total energy transferred.

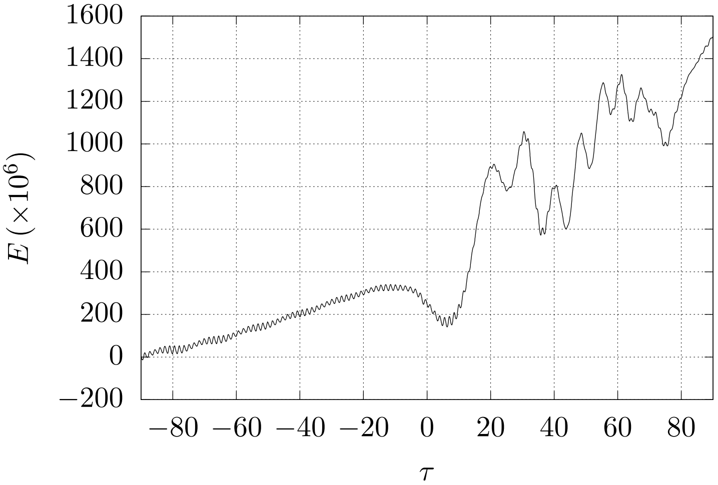
Analytic Energy



Analytic Surface Waves

Figure: Waves!


5.455
5.45
5.445
5.44
5.435
5.43
5.425
5.42
$E\ (\times 10^6)$
−80
−60
−40
−20
0
20
40
60
80
τ


1600
1400
1200
1000
800
600
400
200
0
−200
$E\ (\times 10^6)$
−80
−60
−40
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0
20
40
60
80
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