Collisions of Primordial Black Holes and Neutron Stars

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Introduction

A primordial black hole is a black hole that was hypothetically created in the early universe.

Flat Star Model

- Neutron stars are flat and infinite
- Primordial black holes are point masses
- Neutron stars are incompressible fluids
- Gravitational interactions are Newtonian
- Constant velocity

Eigenfunctions of Laplacian

Assume a product solution for the velocity potential and solve the Laplacian.

$$\begin{aligned}
\varphi &= R(r)Z(z)\Theta(\theta)T(t) \\
\nabla^2\varphi &= 0 \\
\implies \varphi &\propto \begin{Bmatrix} J_\mu(kr) \\ Y_\mu(kr) \end{Bmatrix} \begin{Bmatrix} e^{-kz} \\ e^{kz} \end{Bmatrix} \begin{Bmatrix} \sin(\mu\theta) \\ \cos(\mu\theta) \end{Bmatrix} T(t) \\
\implies \varphi &\propto J_0(kr)e^{kz}T(t)
\end{aligned}$$

$T(t)$ comes from boundary conditions.

Solving for φ

$$\Phi = \frac{-Gm}{\sqrt{r^2 + (z+vt)^2}}$$

$$\left(\frac{\partial^2 \varphi}{\partial t^2} + g\frac{\partial \varphi}{\partial z}\right)\Bigg|_{z=0} = -\frac{\partial \Phi}{\partial t}\Bigg|_{z=0}$$

$$\varphi = \frac{Gmv}{g}\int_0^\infty \frac{J_0(kr)e^{kz}}{1+kv^2/g}\left(-\operatorname{sgn}(t)e^{-kv|t|} + 2H(t)\cos(\omega_k t)\right)dk$$

with $\omega_k^2 = gk$

Deformation of Surface

$$\left.\frac{\partial \varphi}{\partial z}\right|_{z=0} = \frac{\partial \eta}{\partial t}$$

$$\eta = \frac{Gm}{g} \int_0^\infty \frac{J_0(kr)}{1 + kv^2/g} \left(e^{-kv|t|} + 2H(t)v\sqrt{\frac{k}{g}} \sin(\omega_k t) \right) dk$$

Plotting η

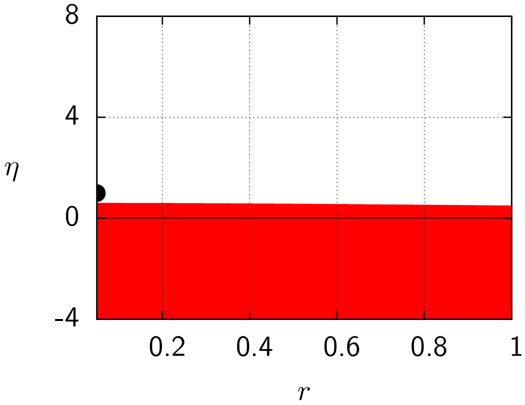

Figure : $t = -1$ s.

Plotting η

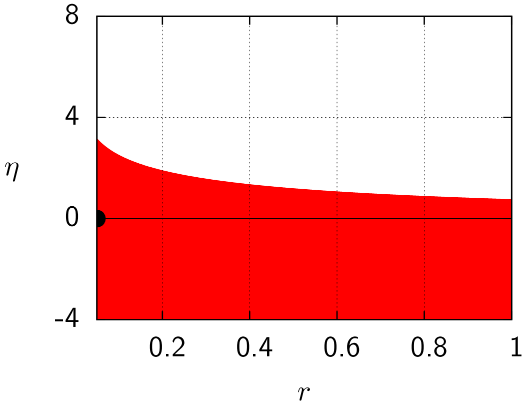

Figure : $t = 0$ s.

Plotting η

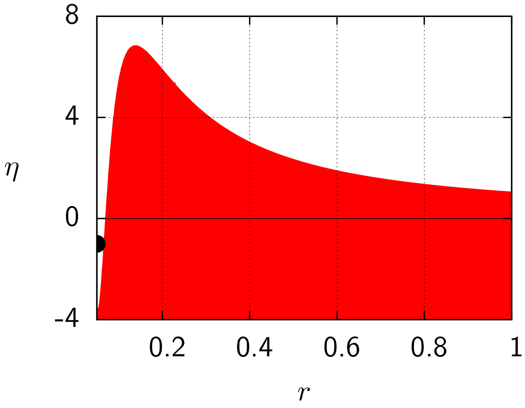

Figure : $t = 1$ s.

Plotting η

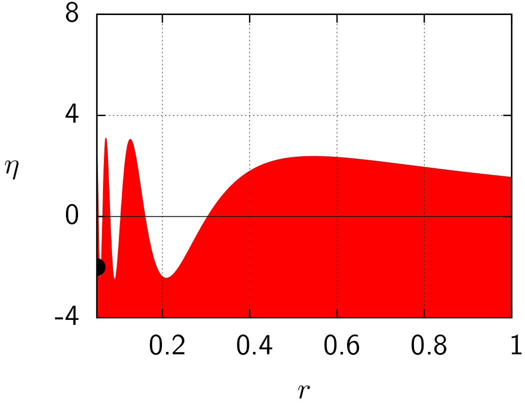

Figure : $t = 2$ s.

Plotting η

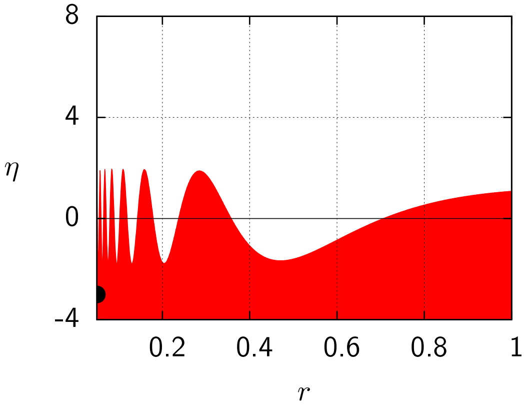

Figure : $t = 3$ s.

Plotting η

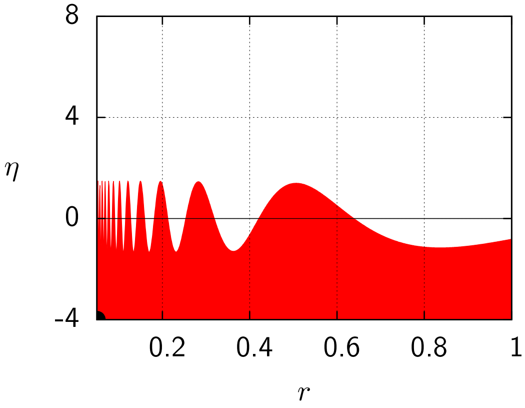

Figure : $t = 4$ s.

Energy Transferred

$$
\begin{aligned}
E &= \lim_{t\to\infty} \frac{1}{2}\rho \int_{-\infty}^{0} \int_{0}^{\infty} |\nabla\varphi|^2 \, r dr dz \int_0^{2\pi} d\theta \\
&\quad + \rho g \int_0^{\infty} \int_0^{\eta} z dz \, r dr \int_0^{2\pi} d\theta \\
&= 4\pi\rho \frac{G^2 m^2}{g}
\end{aligned}
$$