Chapter 1

Simulation Solution

1.1 Smoothed Particle Hydrodynamics

1.1.1 PySPH

The simulations were conducted with PySPH ***, open source SPH code written in Python and compiled in Cython. This allows a majority of the code to be written in pure Python, but is then converted to high performance Cython code to run at speeds closer to C and FORTRAN. For further optimizations, PySPH can run in parallel with OpenMP and MPI to take advantage of multiple cores / threads. Also, one of the main reasons for choosing PySPH is that it allows user defined classes, and equations which was needed to incorporate the primordial black hole.

1.2 Code

In order to simulate a primordial black hole collision with a neutron star we must first write a class to include the acceleration. The acceleration of course is just

$$\vec{a} = -\frac{1}{m_{part}}\vec{\nabla}\Phi.$$

In this simulation and coordinates

$$\vec{a} = \frac{-m}{\left(x^2 + s^2 + (y + v\tau)^2\right)^{3/2}} \begin{pmatrix} x \\ y + v\tau \end{pmatrix}$$

with $G = 1$, $\tau = t - t\_hit$, and s a softening length so the solution does not diverge at $\tau = 0$ at the origin. The mass of the particle is also omitted in the simulation because PySPH uses acceleration per unit mass. In the class function (Figure 1.1) the velocity is hard coded to be 1, and the parameter *t_hit* is used to offset the collision of the primordial black hole since the simulation starts at $t = 0$. Now that the class function has been written, it can be called within the equations block of the main file, *blackhole.py*. Figure 1.2 shows the important sections of *blackhole.py*; first the acceleration equation is imported, and the values for the simulation are specified in the preamble. Then, the black hole can be added to the equations group in the main acceleration block. The example *hydrostatic_tank.py* was used as the base for *blackhole.py*, it was the most useful starting point since it is a $2D$ tank filled with fluid, and included the force of gravity.

The parameters of the simulation were mostly the same as what were used to generate the images of the analytic solution, everything set equal to unity, with the exceptions of *t_hit* = 200, and $s = 0.01$. The reason *t_hit* is so large is due how the particles are initially placed. They are simply placed on a grid with spacing dx; once the simulation starts gravity pulls

```python
from pysph.sph.equation import Equation

class BlackHole2D(Equation):
    def __init__(self, dest, sources, soft=0.05, t_hit=5.0, M=1.0):
        self.soft = soft # softening length to not divide by zero
        self.t_hit = t_hit # time when the black hole crosses the origin
        self.M = M # mass of black hole
        super(BlackHole2D, self).__init__(dest, sources)

    def initialize(self, d_idx, d_au, d_av):
        d_au[d_idx] = 0.0
        d_av[d_idx] = 0.0

    # calculate the force due to the black hole
    def loop(self, d_x, d_y, d_idx, d_au, d_av, t):
        d_au[d_idx] += -self.M * d_x[d_idx] / pow((d_x[d_idx]**2 + self.soft**2 + (d_y[d_idx] + t - self.t_hit)**2),3.0/2.0)
        d_av[d_idx] += -self.M * (d_y[d_idx] + t - self.t_hit) / pow((d_x[d_idx]**2 + self.soft**2 + (d_y[d_idx] + t - self.t_hit)**2),3.0/2.0)
```

Figure 1.1: Class file for adding the acceleration due to the primordial black hole, *BlackHoleEquation.py*.

```
# Import the equations
from pysph.sph.equation import Group
from pysph.sph.BlackHoleEquation import BlackHole2D

# Domain and reference values
Lx = 120.0; H = 15.0; Ly = 1.5*H
gy = -1.0
Vmax = np.sqrt(abs(gy) * H)
c0 = 10 * Vmax; rho0 = 1.0
p0 = c0*c0*rho0
gamma = 1.0

soft = 0.01
t_hit = 200.0
Mass = 1.0
tf = 300.0

# Reynolds number and kinematic viscosity
Re = 0; nu = 0.01 # Ideal fluid

# Numerical setup
nx = 1600; dx = Lx/nx
ghost_extent = 5.5 * dx
hdx = 1.2

class BlackHole(Application):

    def create_equations(self):
        # Formulation for REF1
        equations1 = [

            # Main acceleration block
            Group(equations=[

                # Add the black hole
                BlackHole2D(dest='fluid', sources=None, soft=soft, t_hit=t_hit, M=Mass)

            ]),
        ]
```

Figure 1.2: Modifications of the hydrostatic tank example, *blackhole.py*.

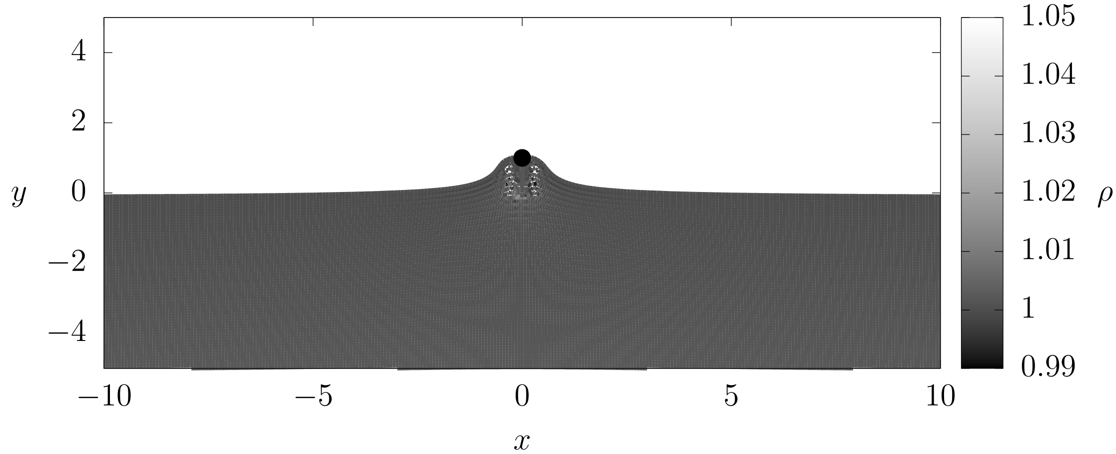
the surface down attempting to equilibrate to a linear pressure gradient. However, since the fluid is not at equilibrium, the surface undergoes harmonic motion and is slowly dampened to equilibrium, and so the long t_hit is to dampen out a majority of the oscillations.

Determining the dimensions of the tank required a considerable amount of care. If the tank was too narrow, the walls at the edges would reflect the initial waves which would then start to interfere with the secondary waves. This caused erroneous results in the energy calculations during testing. Similarly, if the tank was too shallow, the waves resembled shallow water waves instead of deep, as in our model. And of course, if the tank was needlessly large, it greatly effected the computation time of the simulation. In the end, $Lx = 120$, $Ly = 15$, and $dx = 0.075$ were decided on.

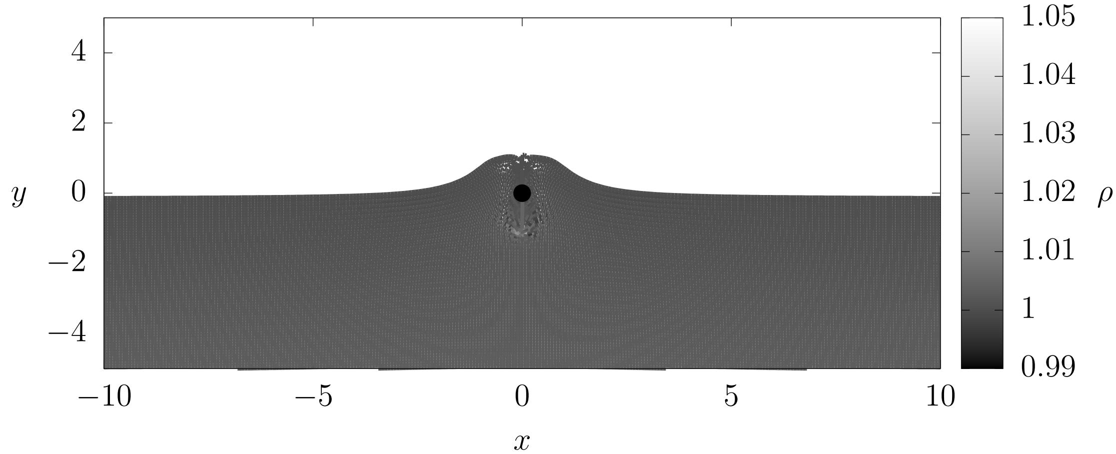
1.3 Simulation Results

In total the simulation took about 330 CPU hours over the course of three days. The resulting surface waves from the collision can be seen in Figure 1.3. As with the analytic results, we see that the surface of the neutron star is pulled upwards due to the gravitational force of the primordial black hole. Then, after the collision, the initial wave propagates outwards. It is fairly difficult to notice, however, a second much smaller amplitude wave is created as well. Unfortunately, because of the resolution of the simulation, waves smaller than dx cannot be seen, unlike in the analytic solution.

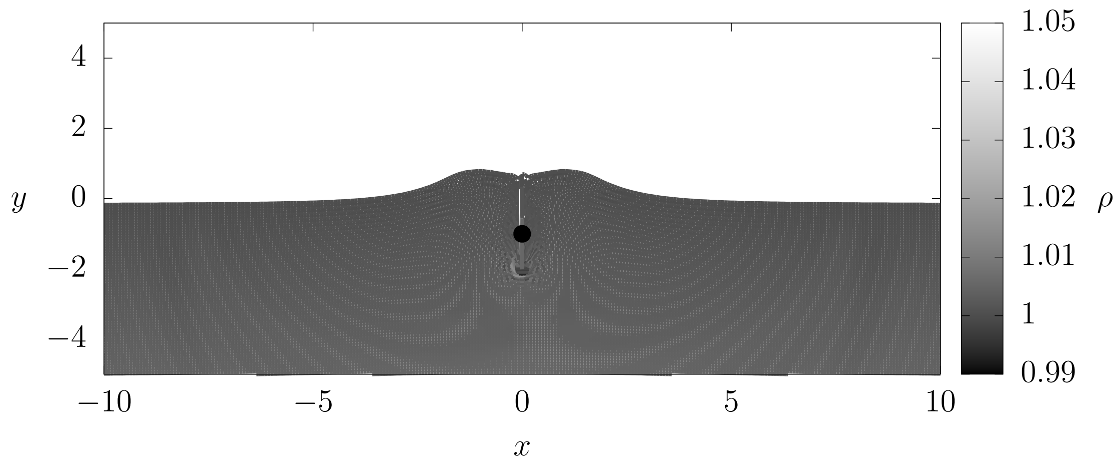
In order to calculate the energy we do so in the traditional way, by taking the sum of the



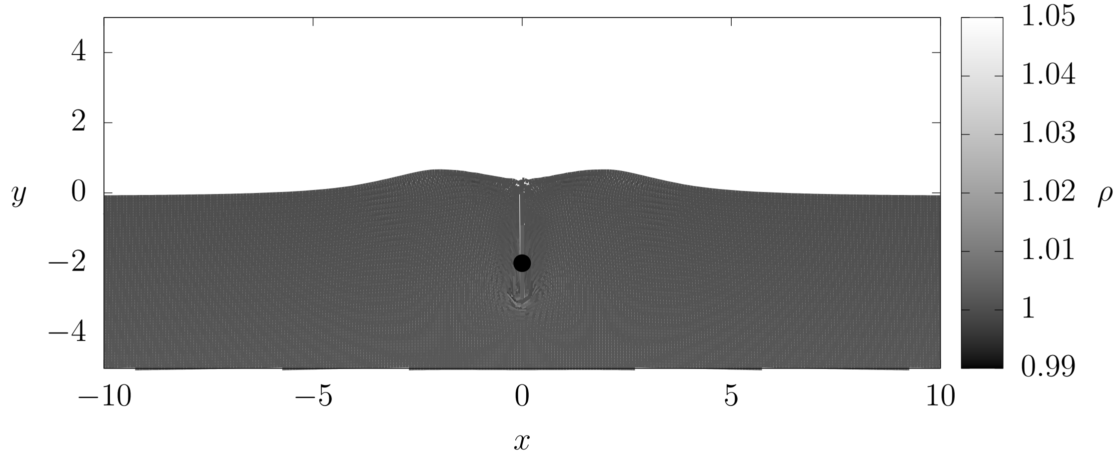
(a) $\tau = -1$



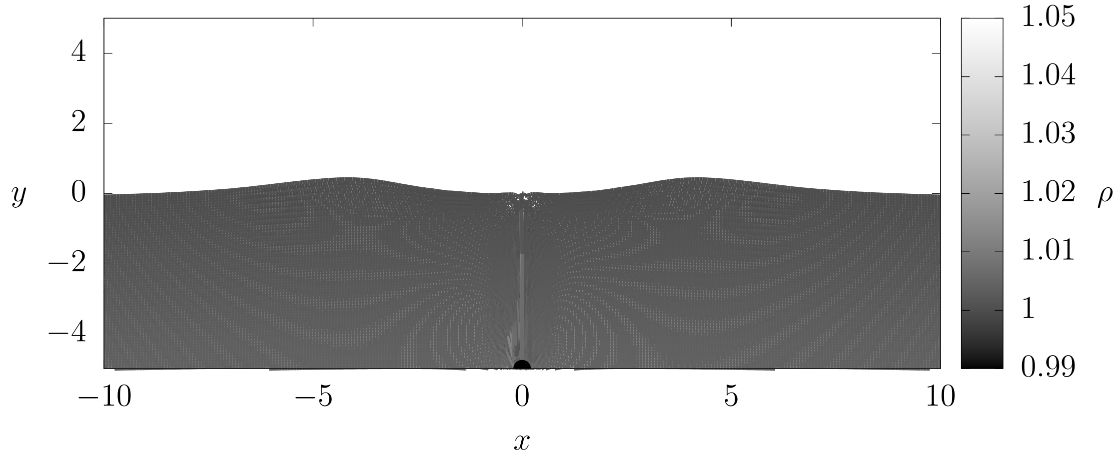
(b) $\tau = 0$



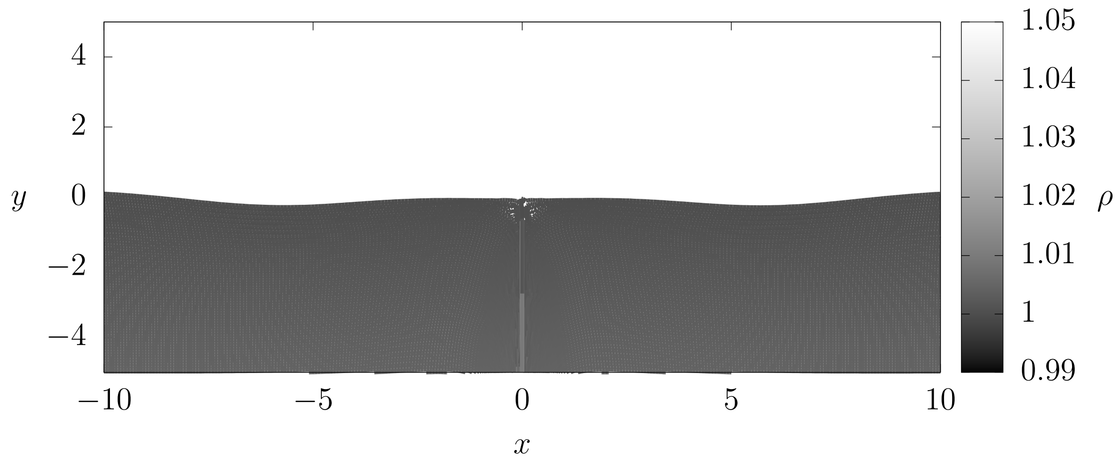
(c) $\tau = 1$



(d) $\tau = 2$



(e) $\tau = 5$



(f) $\tau = 10$

Figure 1.3: Simulated results. ***

potential and kinetic energies of each particle,

$$
\begin{aligned}
E &= \sum_i T_i + U_i, \\
&= \sum_i \frac{1}{2} m_i \left(u_i^2 + v_i^2\right) + m_i g y_i.
\end{aligned}
$$

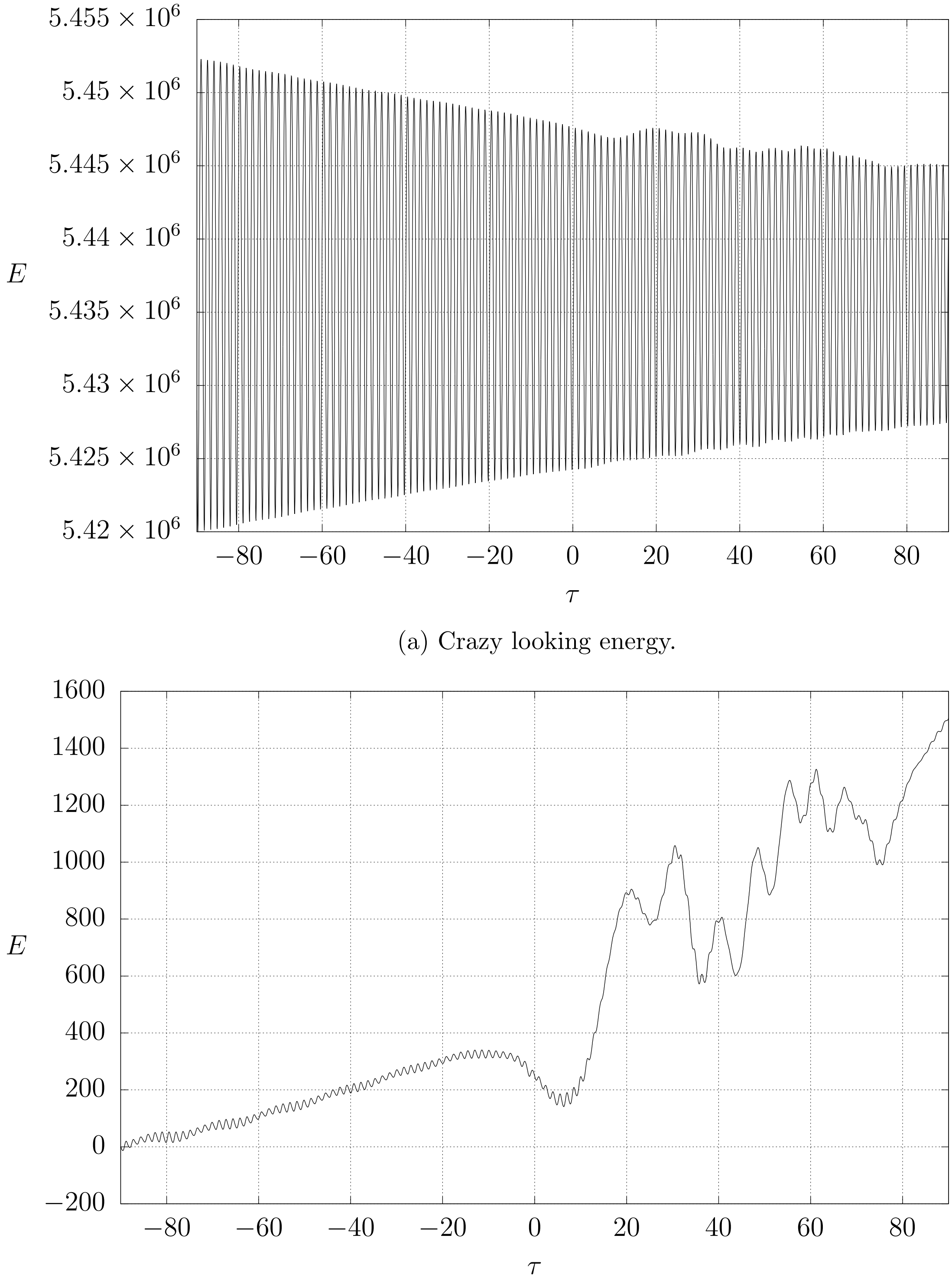
However, to compare to the analytic result, we must transform this into three dimensions by revolving this about the y axis. This can be done by weighting each particle by $\pi|x_i|/dx$, then,

$$
E = \frac{\pi}{dx} \sum_i \left(\frac{1}{2} \left(u_i^2 + v_i^2\right) + g y_i \right) m_i |x_i|.
$$

This is essentially the same as a shell integration, however, we only integrate through π, and the absolute value of the x coordinate is needed. The energy is calculated in this fashion for all times steps and is plotted in Figure 1.4a. Clearly, this is of little use since the energy is highly oscillatory. As mentioned before, this is also the result of how the particles are initially placed within the simulation. As the surface oscillates up and down while attempting to equilibrate, so too does the potential energy. The oscillations have a period of 39 time steps, and therefore, by taking running averages of 39 time steps, most of the noise is removed. Also, for clarity the damping is removed, and the energy has been shifted to start at 0, the resulting plot is much cleaner and is in Figure 1.4b.

This cleaned up energy has a very similar shape to the analytic calculation; initially, the neutron star gains energy as the primordial black hole approaches ***idk about that dip at 0***

At approximately $\tau = 40$ the initial wave hits the boundaries which causes the energy of the system to increase as the wave climbs the wall. After this the wave is reflected back towards the centre of the tank. This also causes the first wave to interfere with the smaller waves



(a) Crazy looking energy.

(b) Smoothed energy without noise.

Figure 1.4: Energy transfer from the simulation.

trailing it, which again causes the energy to increase.

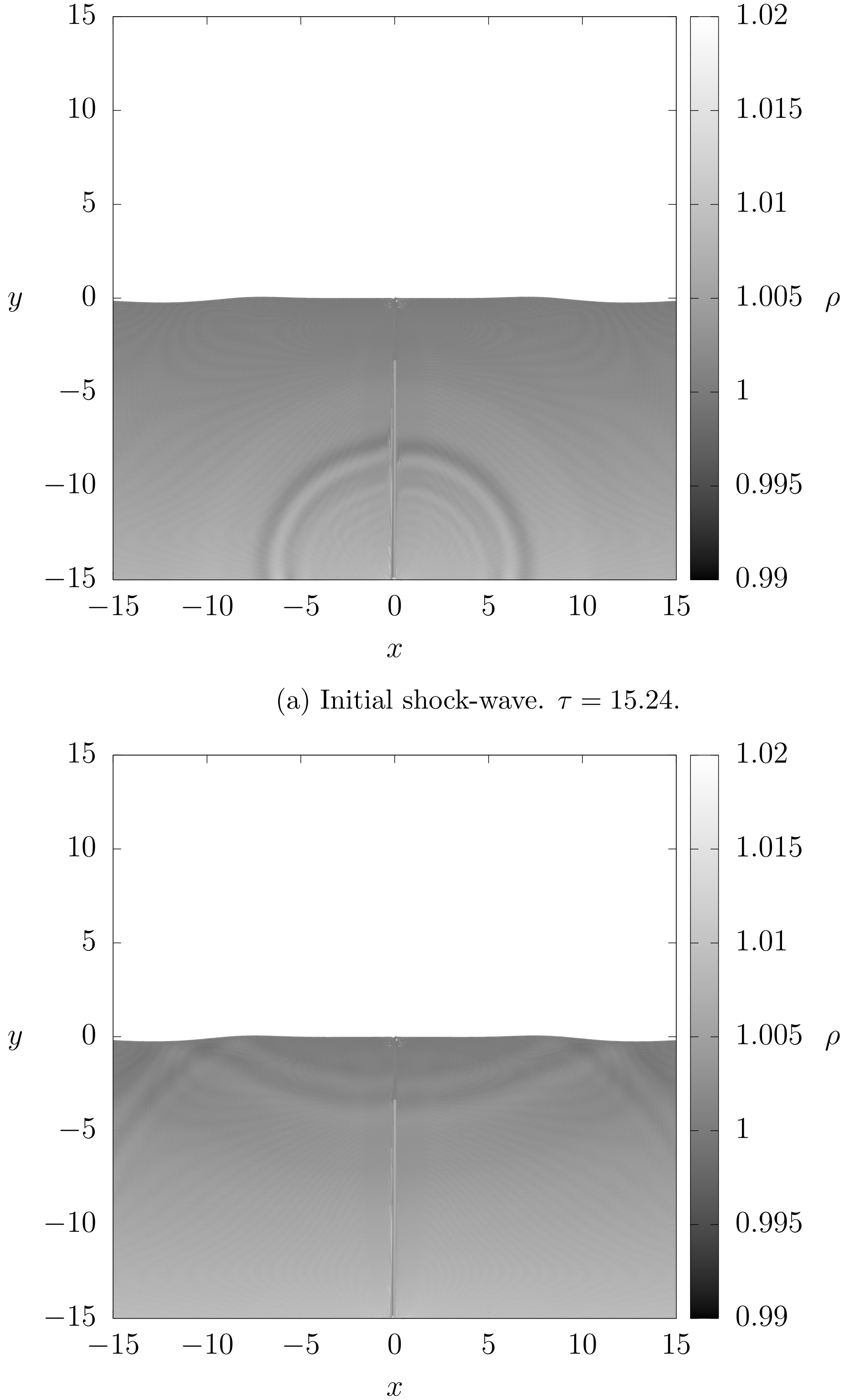
Another point of interest is that since the tank has a depth of 15 at $\tau = 15$ the primordial black hole exits the tank. During the lower resolution testing this did not appear to effect the results of the simulation. However, it seems to have an impact on the energy.

plots of waves

energy: calculation, 2d to 3d, noise, damping, weird increase

shock wave: didn't see in testing, PBH exiting tank, line in the middle is an artifact of the plot

waves crashing into walls 240s



(a) Initial shock-wave. $\tau = 15.24$.

(b) Reflected shock-wave. $\tau = 15.56$.

Figure 1.5: Shock-wave created by PBH leaving tank.