The Effects of a Primordial Black Hole on the Surface of a Neutron Star

Brady Metherall

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Primordial Black Holes

- Normal black holes have a minimum mass on the order of a few solar masses.
- In the early universe while it was very hot and dense, small perturbations could be enough to create a small black hole.
- The mass of a primordial black hole is much less than one solar mass $(10^{-18} - 10^{-6} M_{\odot})$, and microscopic in size.

Primordial Black Holes as Dark Matter

Dark matter is matter that does not emit/absorb light, and is chargeless. Dark matter accounts for about a quarter of the energy in the universe, but we don't know what it is.

Primordial black holes may be a good candidate for dark matter since: they are chargeless, do not emit light, have a very small radius, and are non-relativistic. Also, most other proposed explanations of dark matter involve creating a new particle not part of the standard model.

Flat Star Model

- Neutron stars are flat and infinite
- Primordial black holes are point masses
- Neutron stars are incompressible fluids
- Gravitational interactions are Newtonian
- Constant velocity

Eigenfunctions of Laplacian

Assume a product solution for the velocity potential and solve the Laplacian.

$$\begin{aligned}
\varphi &= R(r)Z(z)\Theta(\theta)T(t) \\
\nabla^2\varphi &= 0 \\
\implies \varphi &\propto \begin{Bmatrix} J_\mu(kr) \\ Y_\mu(kr) \end{Bmatrix} \begin{Bmatrix} e^{-kz} \\ e^{kz} \end{Bmatrix} \begin{Bmatrix} \sin(\mu\theta) \\ \cos(\mu\theta) \end{Bmatrix} T(t) \\
\implies \varphi &\propto J_0(kr)e^{kz}T(t)
\end{aligned}$$

$T(t)$ comes from boundary conditions.

Hankel Transform

Definition

The Hankel transform of a function $f(s)$ is given by

$$\mathscr{H}_\nu\{f\}(\sigma) = \int_0^\infty f(s) J_\nu(s\sigma) s \, ds,$$

where J_ν is the Bessel function of the first kind, of order $\nu \geq -\frac{1}{2}$, and σ is a non-negative real variable.

Corollary

The Hankel transform is self-reciprocal, that is, the inverse Hankel transform is also given by Definition 1.

Solving for φ

From the pressure condition:

$$\left(\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial z} + \frac{\partial\Phi}{\partial t}\right)\bigg|_{z=0} = 0$$

$$\text{where } \Phi = \frac{-Gm}{\sqrt{r^2 + (z+vt)^2}} = \int_0^\infty a(k) J_0(kr) k \, dk$$

$$\varphi = \frac{Gmv}{g}\int_0^\infty \frac{J_0(kr)e^{kz}}{1 + kv^2/g}\left(-\operatorname{sgn}(t)e^{-kv|t|} + 2\,\mathrm{H}(t)\cos(\omega_k t)\right) dk$$

$$\text{with } \omega_k^2 = gk$$

Deformation of Surface

We can easily solve for the shape of the surface now that we have the velocity potential,

$$\left.\frac{\partial \varphi}{\partial z}\right|_{z=0} = \frac{\partial \eta}{\partial t}$$

$$\eta = \frac{Gm}{g}\int_0^\infty \frac{J_0(kr)}{1+kv^2/g}\left(e^{-kv|t|} + 2\,\mathrm{H}(t)v\sqrt{\frac{k}{g}}\sin(\omega_k t)\right)dk.$$

Maple was used to perform numerical integration and plot the surface waves.

Analytic Surface Waves

Figure: Waves!

Energy Transferred

We can calculate the energy by taking the sum of the kinetic, and potential energies,

$$
\begin{aligned}
E &= \lim_{t\to\infty} \frac{1}{2}\rho \int_{-\infty}^{0} \int_{0}^{\infty} |\nabla\varphi|^2 \, r dr dz \int_0^{2\pi} d\theta \\
&\quad + \rho g \int_0^{\infty} \int_0^{\eta} z dz \, r dr \int_0^{2\pi} d\theta \\
&= 4\pi\rho \frac{G^2 m^2}{g}.
\end{aligned}
$$

We take the limit as t approaches infinity, because we are interested in the total energy transferred.

Future Work

My focus next semester will be on simulating the collision with smooth particle hydrodynamics. Smooth particle hydrodynamics is a computational fluid dynamics method that uses N-Body methods, and is typically used for free surface simulations.