# The Effects of a Primordial Black Hole on the Surface of a Neutron Star

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April 13, 2017

# Black Holes

- A black hole is a region of space where enough mass is concentrated such that nothing, including light, can escape its gravitational pull.
- Typical black holes form from the collapse of a massive star, and have a minimum mass on the order of a few solar masses.
- In the early universe, while it was very hot and dense, small perturbations could be enough to create small black holes called primordial black holes.
- The mass of a primordial black hole is probably much less than one solar mass $(10^{-18} - 10^{-6} M_\odot)$, and microscopic in size.

# Primordial Black Holes as Dark Matter



Dark matter is matter that does not emit/absorb light, and is chargeless. Dark matter accounts for about a quarter of the energy in the universe, but we don't know what it is.

Primordial black holes may be a good candidate for dark matter since: they are chargeless, do not emit light, have a very small radius, and are non-relativistic. Also, most other proposed explanations of dark matter involve creating a new particle not part of the Standard Model.

# Flat Star Model

Investigate the collision of a primordial black hole with a neutron star with the following assumptions:

- Neutron stars are flat and infinite
- Primordial black holes are point masses
- Neutron stars are incompressible fluids
- Gravitational interactions are Newtonian
- Primordial black hole has a constant velocity

# Eigenfunctions of Laplacian



To find the velocity potential we need the eigenfunctions of the Laplacian. First, assume a product solution and solve the Laplacian.

$$
\begin{aligned}
\varphi &= R(r)Z(z)\Theta(\theta)T(t) \\
\nabla^2\varphi &= 0 \\
\implies \varphi &\propto \begin{Bmatrix} J_\mu(kr) \\ Y_\mu(kr) \end{Bmatrix} \begin{Bmatrix} e^{-kz} \\ e^{kz} \end{Bmatrix} \begin{Bmatrix} \sin(\mu\theta) \\ \cos(\mu\theta) \end{Bmatrix} T(t) \\
\implies \varphi &\propto J_0(kr)e^{kz}T(t)
\end{aligned}
$$

$T(t)$ comes from boundary conditions.

# Hankel Transform



The Hankel transform is needed to solve the time component of the velocity potential, and the calculation of the energy.

## Definition

The Hankel transform of a function $f(s)$ is given by

$$(\mathscr{H}_\nu f)(\sigma) = \int_0^\infty f(s) J_\nu(s\sigma) s \, ds,$$

where $J_\nu$ is the Bessel function of the first kind, of order $\nu \geq -\frac{1}{2}$, and $\sigma$ is a non-negative real variable.

## Corollary

*The Hankel transform is self-reciprocal, that is, it is its own inverse.*

# Solving for $\varphi$



From the pressure condition

$$\left(\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial z} + \frac{\partial\Phi}{\partial t}\right)\bigg|_{z=0} = 0.$$

We can write the potential as a Hankel transform

$$\frac{\partial\Phi}{\partial t}\bigg|_{z=0} = \int_0^\infty \left(\mathscr{H}\,\frac{\partial\Phi}{\partial t}\bigg|_{z=0}\right)(k)J_0(kr)k\,dk.$$

Then,

$$\varphi = \frac{Gmv}{g}\int_0^\infty \frac{J_0(kr)e^{kz}}{1+kv^2/g}\left(-\operatorname{sgn}(t)e^{-kv|t|} + 2\,\mathrm{H}(t)\cos(\omega_k t)\right)dk,$$

with $\omega_k^2 = gk$.

# Deformation of Surface



We can easily solve for the shape of the surface now that we have the velocity potential, using

$$\left.\frac{\partial \varphi}{\partial z}\right|_{z=0} = \frac{\partial \eta}{\partial t}.$$

$$\eta = \frac{Gm}{g} \int_0^\infty \frac{J_0(kr)}{1 + kv^2/g} \left( e^{-kv|t|} + 2\,\mathrm{H}(t) v \sqrt{\frac{k}{g}} \sin(\omega_k t) \right) dk.$$

# Analytic Surface Waves

Figure: Analytic waves

# Energy Transferred

We can calculate the energy by taking the sum of the kinetic, and potential energies,

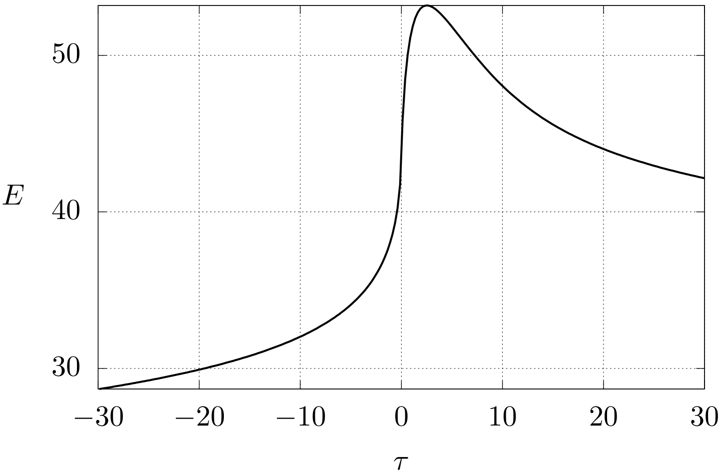
$$E(t) = \frac{1}{2}\rho \int_{-\infty}^{0} \int_{0}^{\infty} |\nabla\varphi|^2 \, r dr dz \int_0^{2\pi} d\theta + \rho g \int_0^{\infty} \int_0^{\eta} z dz \, r dr \int_0^{2\pi} d\theta$$

$$\lim_{t\to\infty} E(t) = 4\pi\rho \frac{G^2 m^2}{g}.$$

We take the limit as $t$ approaches infinity because we are interested in the total energy transferred.

# Analytic Energy





# Smoothed Particle Hydrodynamics



In order to simulate the collision smoothed particle hydrodynamics (SPH) was used.

- Developed in 1977 for astrophysics simulations
- Mesh-free, $N$-body methods
- Uses nearest neighbours

The neutron star is treated as a $120 \times 15$ tank with a particle spacing of $0.075$. The simulation was run in parallel over three days with PySPH, a SPH framework.

# Analytic Surface Waves



Figure: Simulation waves

# Energy Calculation

The energy can be calculated in the same fashion as before:

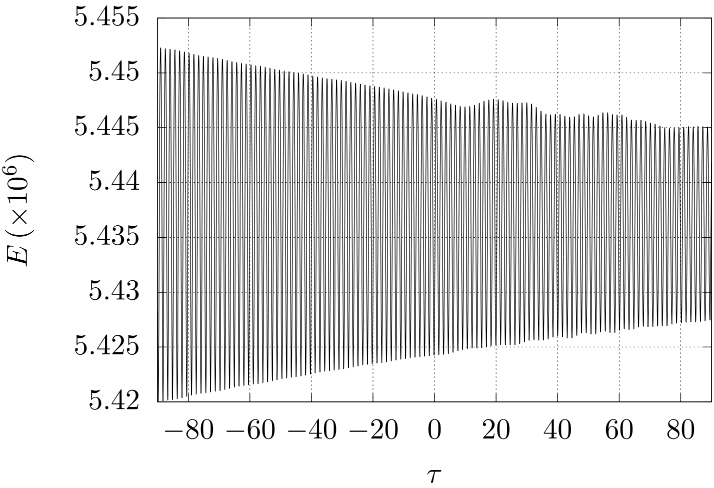
$$E = \sum_i \frac{1}{2} m_i \left(u_i^2 + v_i^2\right) + m_i g y_i.$$

However, this must be transformed to three dimensions:

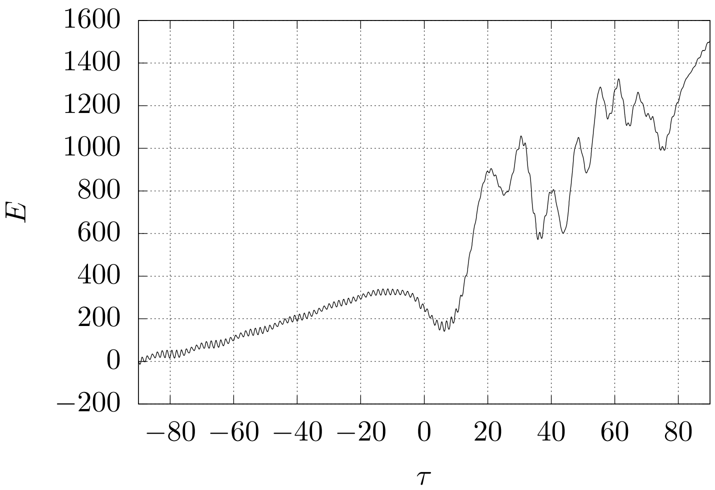
$$E = \frac{\pi}{dx} \sum_i \left( \frac{1}{2} \left(u_i^2 + v_i^2\right) + g y_i \right) m_i |x_i|.$$

# Raw Noisy Energy





# Cleaned Energy



# Conclusion

If this project were continued some modifications would be

- Better initial conditions for the simulation
- Use a spherical model instead of planar
- Use general relativity instead of Newtonian gravity