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Grey-level thresholding of images using a correlation criterion

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Abstract: A new threshold selection technique using a correlation criterion is presented here. An optimum threshold is selected by maximizing the correlation between the original halftone image and the thresholded bilevel image. Representative experimental results are presented.

Key words: Image processing, threshold selection, segmentation, contrast enhancement, correlation.

1. Introduction

Image processing often requires the selection of a suitable grey-level threshold for extracting objects from their background. Many different techniques for the automatic selection of such a threshold have been proposed (Weszka, 1978; Haralick and Shapiro, 1985). Most of these methods base the choice of threshold on the shape of the image's grey-level histogram, which is ideally bimodal with a distinct valley between the modes. For most real pictures, the valley bottom is often difficult to detect precisely. Further, the histogram may be unimodal with no detectable valley, or multimodal with several valleys. Various methods have been developed which base the threshold selection on other parameters of the image or the histogram. One such method was proposed by Kittler and Illingworth (1986). In this method the error involved in reclassifying each pixel into either 'black' or 'white' is evaluated and the cumulative error is minimized iteratively.

Relatively few methods base the segmentation on a comparison between the original image and the resulting bilevel image. Otsu (1979) has proposed one such technique. This involves minimizing the sum of the squares of the differences between the original and bilevel images.

The method presented here uses a similar approach. The correlation between the original and thresholded images is evaluated and maximized by iteration.

2. The coefficient of correlation

The coefficient of correlation (Freund, 1979; Oppenheim and Schafer, 1975), ρ_{xy}, for two sets of data $X = \{x_1, x_2, \ldots, x_s\}$ and $Y = \{y_1, y_2, \ldots, y_s\}$ is given by

$$\rho_{xy} = \frac{E_{xy} - E_x E_y}{(V_x V_y)^{1/2}} \tag{1}$$

where

$$E_x = \sum_{i=1}^{s} x_i p(x_i),$$

$$E_y = \sum_{i=1}^{s} y_i p(y_i)$$

are the expected values of the respective sets of data and

$$E_{xy} = \sum_{i=1}^{s} x_i y_i p(x_i y_i)$$

is the expected value of their product $XY = \{x_1y_1, x_2y_2, \ldots, x_sy_s\}$. In these equations x_i and y_i represent data values, $p(x_i)$ and $p(y_i)$ their probabilities and $p(x_iy_i)$ the probability associated with their product.

V_x and V_y are the variances of X and Y, given by

$$V_x = E_{xx} - [E_x]^2,$$

$$V_y = E_{yy} - [E_y]^2$$

where

$$E_{xx} = \sum_{i=1}^{s} x_i^2\, p(x_i),$$

$$E_{yy} = \sum_{i=1}^{s} y_i^2\, p(y_i).$$

The correlation coefficient takes on values from -1 to $+1$, depending on the type and extent of correlation between the sets of data.

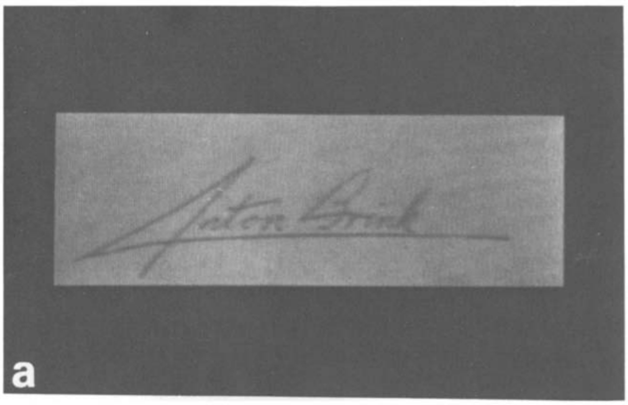

Figure 1. (a) Original image of handwriting.

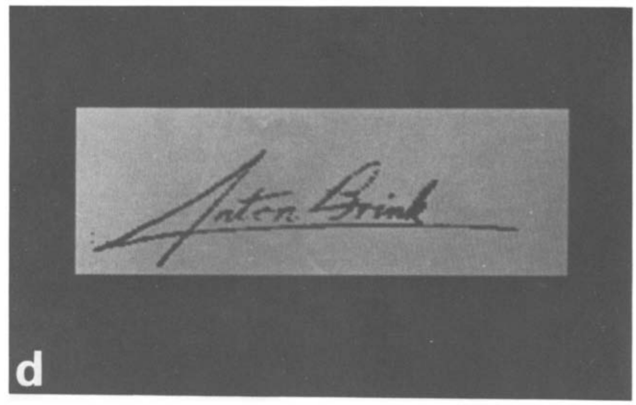

Figure 1. (d) The resulting bilevel image.

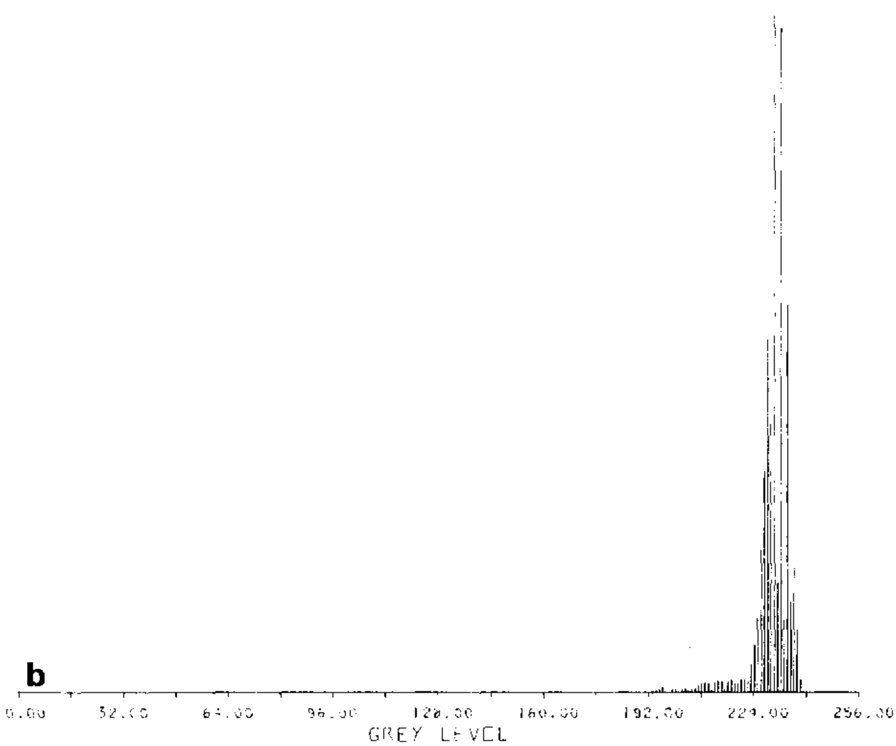

Figure 1. (b) The grey-level histogram.

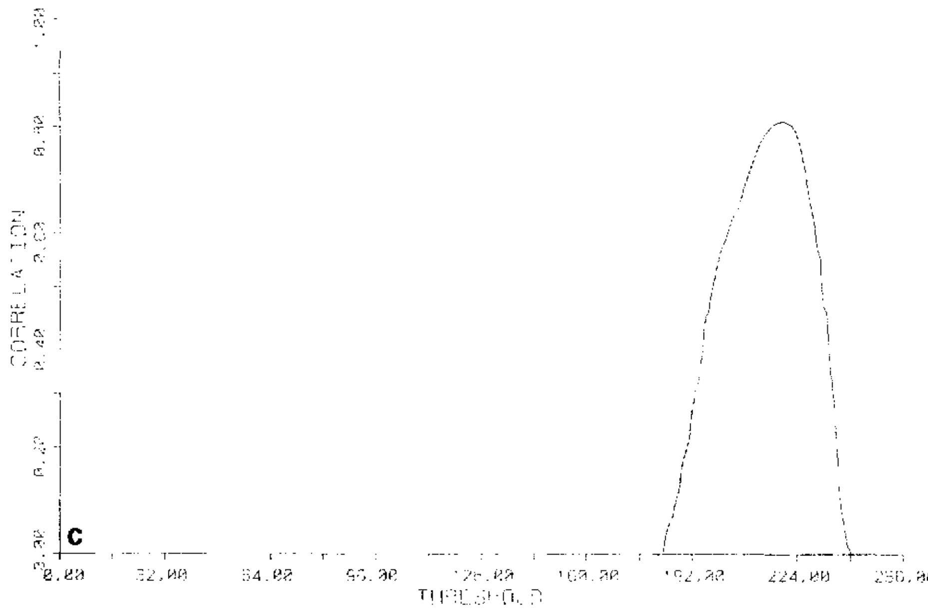

Figure 1. (c) The correlation as a function of threshold level (optimum threshold $T = 219$).

3. The thresholding algorithm

For application to the thresholding problem, let X represent the set of possible grey-levels g of the original image. Y represents the corresponding new levels ('black' or 'white') of the bilevel image. These levels are determined by the below- and above-threshold means $\mu_0(T)$ and $\mu_1(T)$ of the original image, given by

$$\mu_0(T) = \left[\sum_{g=0}^{T} g\, p_g\right] \Big/ \sum_{g=0}^{T} p_g, \tag{2}$$

$$\mu_1(T) = \left[\sum_{g=T+1}^{n} g\, p_g\right] \Big/ \sum_{g=T+1}^{n} p_g \tag{3}$$

where $g = 0, 1, \ldots, n$ are grey-levels, $T\ (0 \leq T < n)$ is the threshold level and p_g is the probability of grey-level g, obtained from the histogram as follows:

$$p_g = f_g / N$$

where N is the total number of pixels making up the image and f_g is the number of pixels having grey-level g.

The expected values become

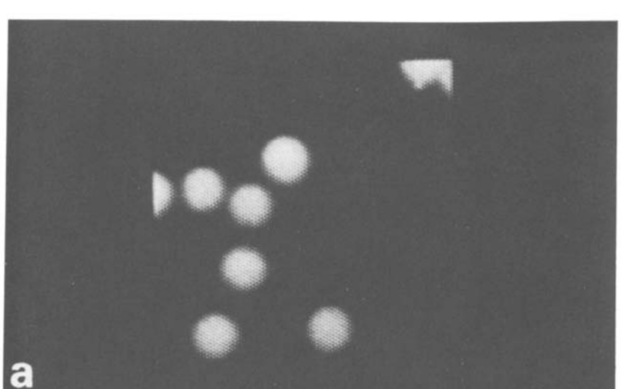

Figure 2. (a) Original image of colonies of bacteria.

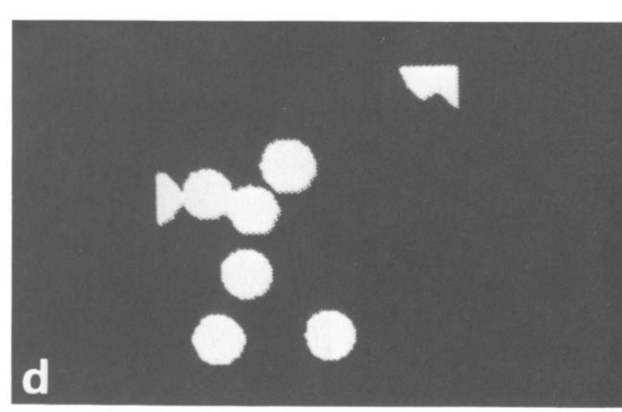

Figure 2. (d) The resulting bilevel image.

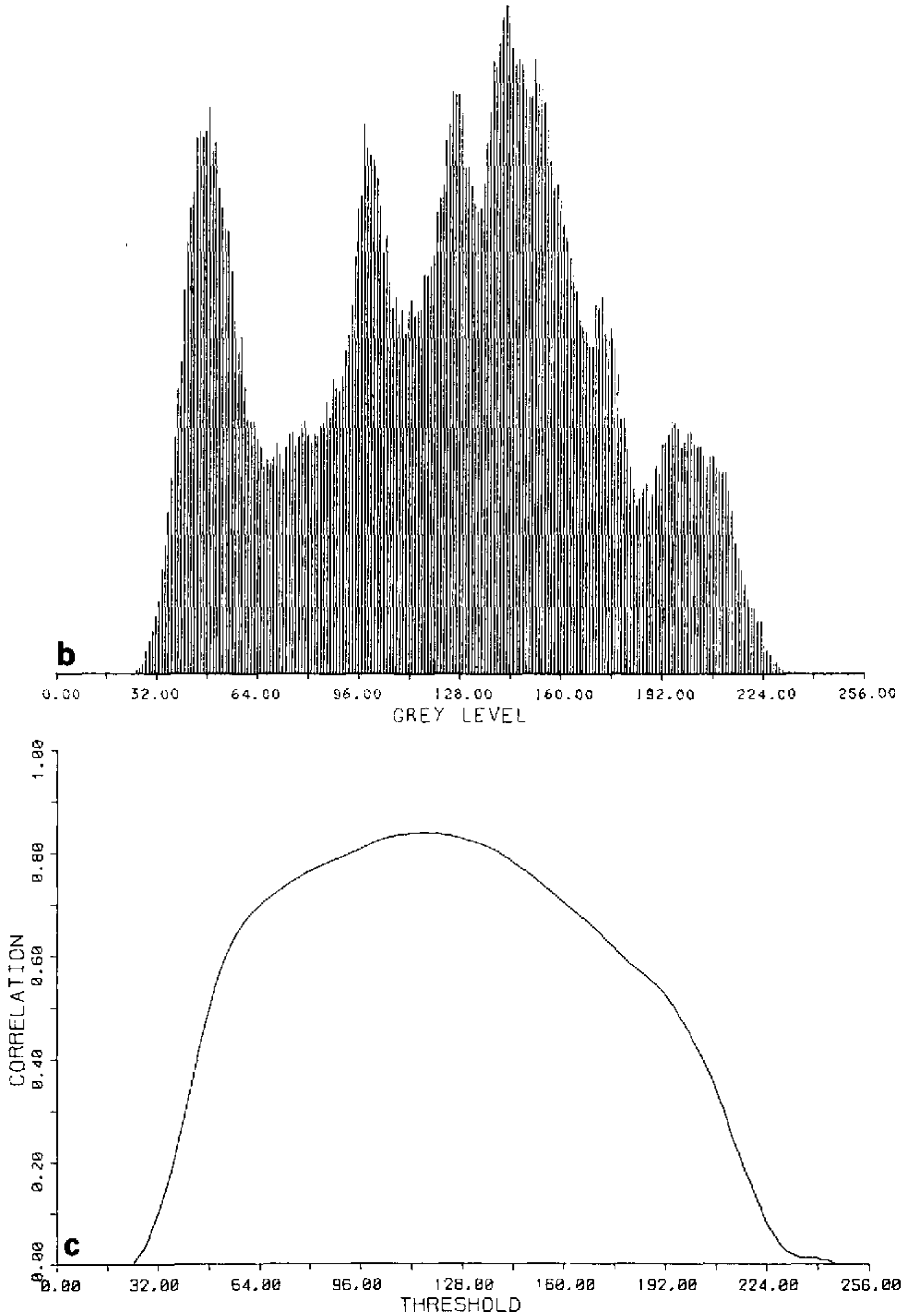

Figure 2. (b) The grey-level histogram, (c) the correlation as a function of threshold level (optimum threshold $T = 134$).

$$E_x = \sum_{g=0}^{n} g\, p_g,$$

$$E_y(T) = \sum_{g=0}^{T} \mu_0(T)\, p_g + \sum_{g=T+1}^{n} \mu_1(T)\, p_g,$$

$$E_{xy}(T) = \sum_{g=0}^{T} g\, \mu_0(T)\, p_g + \sum_{g=T+1}^{n} g\, \mu_1(T)\, p_g,$$

$$E_{xx} = \sum_{g=0}^{n} g^2\, p_g,$$

Figure 3. (a) Original standard test image of a girl.

Figure 3. (d) The resulting bilevel image.

$$E_{yy}(T) = \sum_{g=0}^{T} \mu_0^2(T)\, p_g + \sum_{g=T+1}^{n} \mu_1^2(T)\, p_g,$$

and the variances are given by

$$V_x = E_{xx} - [E_x]^2,$$

$$V_y(T) = E_{yy}(T) - [E_y(T)]^2.$$

E_x, E_{xx} and V_x are independent of the threshold T as they are obtained from the original image. The correlation coefficient is now a function of the threshold level, given by

$$\rho_{xy}(T) = \frac{E_{xy}(T) - E_x E_y(T)}{(V_x V_y(T))^{1/2}}, \quad 0 \le T \le n \qquad (4)$$

Representative results of the implementation of the algorithm follow.

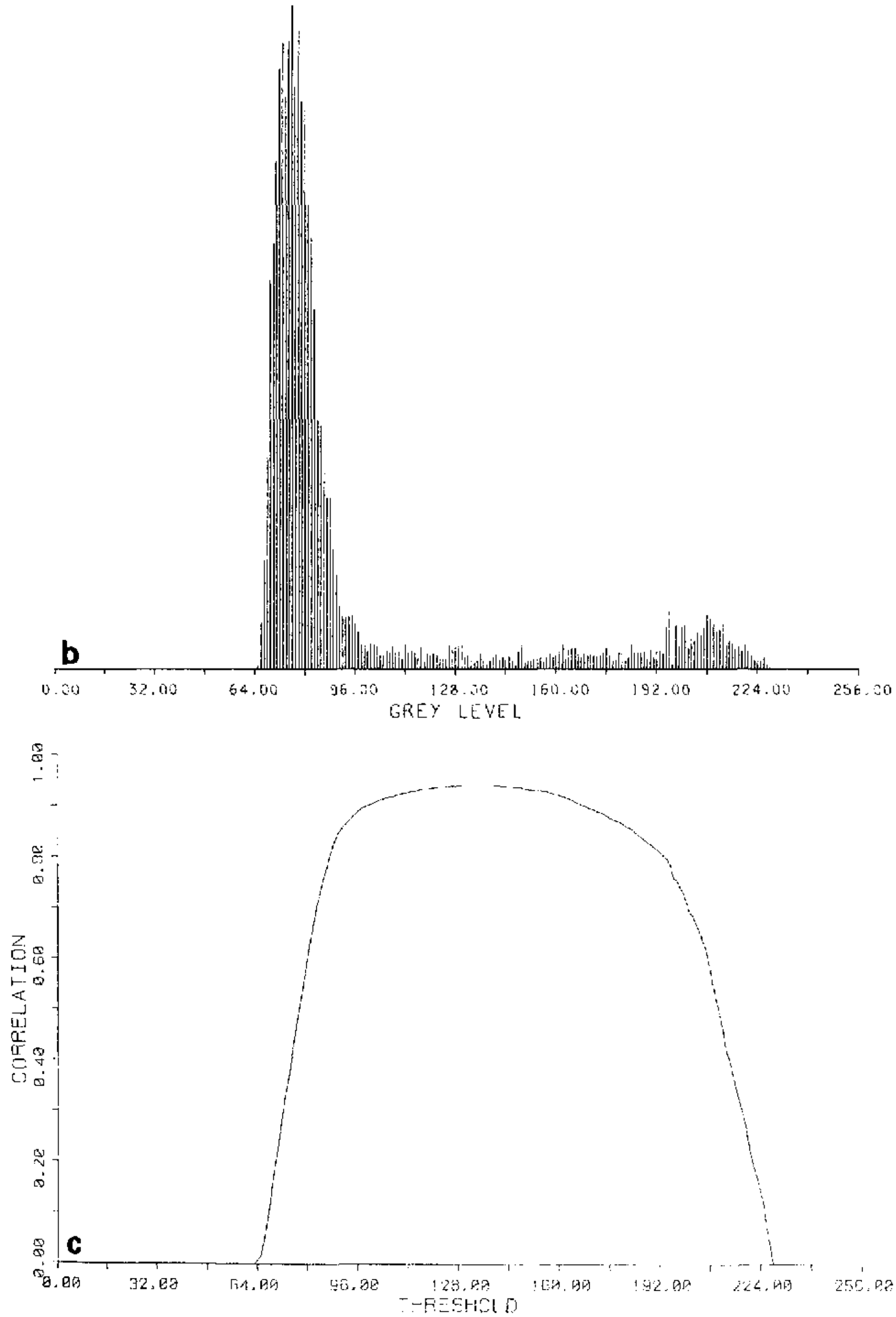

Figure 3. (b) The grey-level histogram, (c) the correlation as a function of threshold level (optimum threshold $T = 116$).

4. Results

The thresholding algorithm was implemented on a DEC VAX 11/730 minicomputer. It was tested using three different images having 256 grey-levels. These images had unimodal, bimodal and multimodal histograms respectively. The results obtained appear in Figures 1 to 3.

Figure 1 shows the result of applying the technique to an image of a sample of handwriting. The histogram in this case is near-unimodal.

Figure 2(a) is an image of colonies of bacteria

(Klebsiella pneumoniae) having a bimodal histogram.

Figure 3 uses a standard test image of a girl. This has a multimodal histogram and would normally be used to test multiple threshold selection methods.

5. Conclusion

The correlation method selected useful thresholds for the unimodal and bimodal distributions on which it was tested. A reasonable binary threshold was also selected for the multimodal distribution. The results also indicate some improvement over those obtained for the same images using the methods of Kittler and Illingworth (1986), Kapur et al. (1985) and Tsai (1985). The comparative results are given by Brink (1987).

The technique is relatively simple to implement and computational cost is low. It is also easily extended for the selection of multiple thresholds, as would normally be used for multimodal distributions.

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