Aufgabe 4

Betrachten Sie die folgenden Probleme:

> **SAT**
>
> **Gegeben:** Aussagenlogische Formel F in KNF
>
> **Frage:** Gibt es mindestens eine erfüllende Belegung für F?

> **DOPPELSAT**
>
> **Gegeben:** Aussagenlogische Formel F' in KNF
>
> **Frage:** Gibt es mindestens eine erfüllende Belegung für F, in der mindestens zwei Literale pro Klausel wahr sind?

(a) Führen Sie eine polynomielle Reduktion von SAT auf DOPPELSAT durch.

https://courses.cs.washington.edu/courses/csep531/09wi/handouts/sol4.pdf

DOUBLE-SAT is in NP. The polynomial size certificate consists of two assignments f1 and f2. First, the verifier verifies if f1 6= f2. Then, it verifies if both assignments satisfy φ by subtituting the values for the variables and evaluate the clauses of φ. Both checks can be done in linear time. DOUBLE-SAT is NP-hard. We give a reduction from SAT. Given an instance φ of SAT which is a CNF formula of n variables x1,x2,...xn, we construct a new variable xn+1 and let ψ = φ ∧(xn+1 ∨¬xn+1) be the corresponding instance of DOUBLE-SAT. We claim that φ has a satisfying assignment iff ψ has at least two satisfying assignments. On one hand, if φ has a satisfying assignment f, we can obtain two distinct satisfying assignments of ψ by extending f with xn+1 = T and xn+1 = F respectively. On the other hand, if ψ has at least two sastisyfing assignments then the restriction of any of them to the set x1,x2,...xn is a satisfying assignment for φ. Thus, DOUBLE-SAT is NP-complete.

https://cs.stackexchange.com/questions/6371/proving-double-sat-is-np-complete

Here is one solution:

Clearly Double-SAT belongs to NP, since a NTM can decide Double-SAT as follows: On a Boolean input formula $\phi(x_1, \ldots, x_n)$, nondeterministically guess 2 assignments and verify whether both satisfy ϕ.

To show that Double-SAT is NP-Complete, we give a reduction from SAT to Double-SAT, as follows:

On input $\phi(x_1, \ldots, x_n)$:

1. Introduce a new variable y. 2. Output formula $\phi'(x_1, \ldots, x_n, y) = \phi(x_1, \ldots, x_n) \wedge (y \vee \bar{y})$.

If $\phi(x_1, \ldots, x_n)$ belongs to SAT, then ϕ has at least 1 satisfying assignment, and therefore $\phi'(x_1, \ldots, x_n, y)$ has at least 2 satisfying assignments as we can satisfy the new clause $(y \vee \bar{y})$ by assigning either $y = 1$ or $y = 0$ to the new variable y, so $\phi'(x_1, ... ,x_n, y) \in$ Double-SAT.

On the other hand, if $\phi(x_1, \ldots, x_n) \notin$ SAT, then clearly $\phi'(x_1, \ldots, x_n, y) = \phi(x_1, \ldots, x_n) \wedge (y \vee \bar{y})$ has no satisfying assignment either, so $\phi'(x_1, \ldots, x_n, y) \notin$ Double-SAT.

Therefore, SAT $\leq_p$ Double-SAT, and hence Double-SAT is NP-Complete.

(b) Zeigen Sie, dass DOPPELSAT NP-vollständig ist.