causets

version 1.5

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(in development)

— supported with the PrOSET editor online —

$$\left\{ \bullet \bullet \bullet, \uparrow \bullet, \land, \bigvee, \downarrow \right\}, \quad \left\{ \begin{smallmatrix} 1 \bullet & 2 \bullet & 3 \bullet, \begin{smallmatrix} 2 \uparrow & 3 \bullet, \begin{smallmatrix} 3 \uparrow \\ 1 \bullet & 2 \bullet \end{smallmatrix}, \begin{smallmatrix} 2 \uparrow & 3 \bullet, \begin{smallmatrix} 2 \uparrow & 3 \bullet \\ 1 \bullet & 2 \bullet \end{smallmatrix}, \begin{smallmatrix} 3 \uparrow & 2 \bullet \\ 2 \bullet & 3 \bullet \end{smallmatrix}, \begin{smallmatrix} 2 \uparrow & 3 \bullet \\ 1 \bullet & 2 \bullet \end{smallmatrix}, \begin{smallmatrix} 3 \uparrow & 2 \bullet \\ 2 \bullet & 3 \bullet \end{smallmatrix}, \begin{smallmatrix} 2 \uparrow & 3 \bullet \\ 2 \bullet & 2 \bullet \end{smallmatrix} \right\},$$

The main purpose of this package is to typeset partial ordered sets (posets for short) as Hasse diagrams in a style that is particularly suited for causal set theory (a framework for quantum gravity). Causal sets (causets for short) are locally finite, partially ordered sets, where the partial order is the causal relation between elements. This relation forms a cone structure (light-cones) such that all past and future related elements of an element fall within a lower and upper cone, respectively, opening in 90 degree angles.

The drawing algorithm (based on TikZ) makes use of the fact that every finite causet that embeds in 1+1 dimensional Minkowski spacetime is a product of two total orders (2-order), where one order is taken to be the standard total order of integers labelling all causet elements, while the other is a permutation of the labels¹. Such a permutation is the main input for the macros. For any higher dimensional poset, there exists a 2-order such that the higher dimensional diagram is obtained by removing links from the 2-order. The online tool "PrOSET editor" helps to generate the macros starting from a visual representation.

¹See https://arxiv.org/abs/0706.0375 and https://arxiv.org/abs/2011.02965 for more details on 2-orders and causal sets.

1 The three main commands

This package is built on the package tikz to draw the diagrams. To import the causets package into a LATEX file, write

```
1 \usepackage{causets}
```

in the preamble of the document (this will automatically load the tikz package as well).

For the command introduction in Table 1, I use the package options black (only black and white diagrams) and permutation (also shows the permutation),

```
1 \usepackage[black,permutation]{causets}
```

At first, let us take a brief look at the three main commands of the package, before we look into the (physical) ideas behind these three commands.

The three main commands of this package are

```
1 \pcauset{...,i,...,k,...,j,...}
2 \rcauset{...}{...,i/j,...}
3 \causet{...}{...,i/j,...}
```

The first command takes a permutation of the integers from an interval [a,b] (where $N=b-a+1\geq 0$ is the cardinality of the causet). The permutation determines the event positions and causal structure such that events i and j are linked if i< j and there is no k>j between i and j in the permutation. The second command does the same, but removes (restyles) links between event pairs i/j as given in the second argument. The third command only uses the permutation to position the events, the links have to be explicitly specified by event pairs i/j in the second argument.

Any causet that can be embedded in 1+1 dimensional Minkowski spacetime can also be described by the product of two total orders (a 2D-order). Let the causet elements be labeled by an increasing null coordinate u. In the first diagram of Table 1, the u-coordinate axis points upwards on the right hand side. The causal relation (partial order) implies a label permutation along the null coordinate v (pointing upwards along the left axis). This permutation is the input of the \peauset command that computes the causal relation from the permutation. See example 1 in Table 1.

Every causet that does not embed in 1 + 1 dimensional Minkowski spacetime can be made embeddable by adding links, for which the \rcauset command is defined. Find an embedding causet by adding a minimal number of causal relations to the given causet. The so obtained, embedding causet gives a permutation as discussed before for the \pcauset causet. The links that have been added to obtain this permutation need to be removed from the generated causet, which is specified by link pairs i/j in the second argument. See example 2 in Table 1.

In some situations, it might be easier to specify the list of link pairs that exist, instead of removing links. For this purpose, use the \causet command. As an example, we rewrite the example 2 by specifying all existing links as pairs, see example 3 in Table 1. Note that the \rangle causet command was actually much shorter for this particular example. However,

#	command	inline output
1	\pcauset{4,7,2,6,1,5,3}	
2	\rcauset{4,7,2,6,1,5,3}{2/5}	
3	\causet{4,7,2,6,1,5,3}{1/3,1/5,2/3,2/6,4/5,4/6,4/7}	

Table 1: Examples of the three main commands.

if the list of link pairs in the \rcauset command is very long, specifying all existing link pairs instead with the \causet yields a faster performance.

You may use any of the three main commands to add a causet diagram in-line with text or within a mathematical expression. As an example for the use in a mathematical equation, here is the code for the first set of causets on the title page:

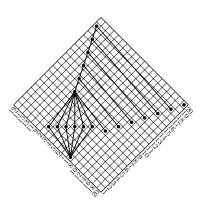
$$S = \left\{ \cdot \cdot \cdot, \uparrow, \Lambda, \bigvee, \dot{\uparrow} \right\}. \tag{1}$$

Of course, a given causet might have more than one representing permutation, so that any of those permutations may be used. However, you may consider it a good practice to choose the permutation that starts with the larger integer, so that disjoint elements appear to the right in the graphs. For example, you may write \pcauset{3,1,2}, \cdot\, e, as opposed to \pcauset{2,3,1}, \cdot\\.

The arguments of the commands are lists that are iterated over by a \foreach loop of PGF/TikZ. So you may use dots ... to complete ranges of integers automatically. You may use this feature to draw chains like (with \pcauset{1,...,5}) or antichains like ••••• (with \pcauset{5,...,1}). As a more complex example for the dots notation, consider \pcauset{20,18,...,8, 1,7,6,...,2, 9,11,...,19}

which produces the same diagram as

```
1 \pcauset{20,18,16,14,12,10,8, 1,7,6,5,4,3,2, 9,11,13,15,17,19}
```



In an optional argument, the main commands accepts any causets or TikZ keys to change the style of the causet. For example, the blue causet \bigwedge is obtained with

```
1 \pcauset[blue colors]{2,4,5,1,3}
```

Example diagrams together with their LaTeX macros are included on the project website https://c-minz.github.io/projects/poset_diagrams. To help create and modify poset diagrams visually, I developed the PrOSET editor as an online tool, which exports the result as LaTeX macros (and other data formats):

```
https://c-minz.github.io/assets/html/proset-editor.html
```

Whenever you use this package to compose a publication or a document that may be converted to XML or HTML, this package supports LaTeXML. Especially for a conversion to HTML documents and for screen readers, the diagrams can be complemented with an alternative text string. Please use $alt=\{\langle value\rangle\}$ in the optional argument to add $\langle value\rangle$ as a very short description of the diagram if possible. Here are some examples:

```
1 \pcauset[alt={N}]{2,4,1,3},
2 \pcauset[alt={N and 2-chain}]{5,6,2,4,1,3}
```

provides "N" as alternative text for \bigwedge , and \bigwedge ! with alternative text "N and 2-chain". Diagrams without the alternative text attribute get the default value that is stored in \causetDefaultAltText (initially "causet").

2 Package options

Each of the package options can either be specified in square brackets when declaring the package (option applies globally) or be changed with a macro at any point in the document (option applies to the local scope).

If no options are used, an output like $\nearrow \searrow$ is obtained with the code

```
1 \usepackage{causets}
2 ... \begin{document} ...
3 \rcauset[alt={3-crown with doubled top right}]{4,2,7,6,1,5,3}{2/5}
```

The options – given in the following table – are passed to the causets package as optional parameter and can be combined. The package options are listed together with example outputs created with the commands

1 \pcauset {4,2,7,6,1,5,3} 2 \rcauset {4,2,7,6,1,5,3}{2/5}

If not specified otherwise, any package option also works with the command \causet.

Package option	\pcauset	\rcauset
\usepackage[permutation]{causets} shows permutations (\causetsDrawPermutationtrue activates this option) (\causetsDrawPermutationfalse deactivates this option)		
\usepackage[permutation, uunlabeled]{causets} shows permutations without u-labels (\causetsDrawULabelstrue activates u-labels) (\causetsDrawULabelsfalse deactivates u-labels)		
\usepackage[permutation, vunlabeled]{causets} shows permutations without v-labels (\causetsDrawVLabelstrue activates v-labels) (\causetsDrawVLabelsfalse deactivates v-labels)		
\usepackage[labeled]{causets} shows labeled events (\causetsDrawLabelstrue activates labels) (\causetsDrawLabelsfalse deactivates labels)	3 4 4 1	3 N 4
\usepackage[continuous]{causets} allows links to cross over events. Link interruptions (default) only affect the \causet command and "restyled" links of \rcauset, but not the examples of shown on the right. Details on interrupted (broken) links are given below this table.		
\usepackage[caps]{causets} sets caps as link tips	i i	KW.
\usepackage[arrows]{causets} sets arrows (rounded stealth) as link tips		KW
\usepackage[unlinked]{causets} does not draw links (\causetsDrawLinkstrue actives links) (\causetsDrawLinksfalse deactivates links)	• • •	• • •

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Package option	\pcauset	\rcauset
\usepackage[tiny]{causets} for tiny diagrams (\tikzcausetsset{tiny} sets tiny sizes)	κζw	K)W
<pre>\usepackage[smaller]{causets} for very small diagrams (\tikzcausetsset{very small} sets very small sizes)</pre>	K/W	KW
\usepackage[small]{causets} for small diagrams (\tikzcausetsset{small} sets small sizes)	KW.	KW .
For comparison, here are the normal sized causets. (\tikzcausetsset{normal} sets normal sizes)	M	KW .
\usepackage[large]{causets} for large diagrams (\tikzcausetsset{large} sets large sizes)		
<pre>\usepackage[larger]{causets} for very large diagrams (\tikzcausetsset{very large} sets very large sizes)</pre>		
\usepackage[huge]{causets} for huge diagrams (\tikzcausetsset{huge} sets huge sizes) Details on size options are given in Sec. 5.		
\usepackage[black]{causets} for black and white causets (\tikzcausetsset{black colors} activates the option)	NW	₩
<pre>\usepackage[gray]{causets} for grey scale causets (de- fault) (\tikzcausetsset{gray colors} activates the option)</pre>	NW .	K)X
<pre>\usepackage[blue]{causets} for blue causets (\tikzcausetsset{blue colors} activates the option)</pre>		KW
\usepackage[neon]{causets} for brightly colored causets (\tikzcausetsset{neon colors} activates the option) Details on color options are given in Sec. 6.	N.	K)

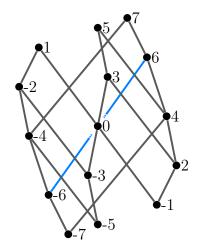
continued on the next page

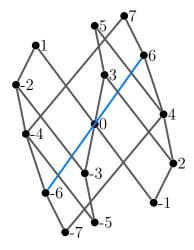
continued from the previous page

Package option	\pcauset	\rcauset
\usepackage[spatial]{causets} shows dashed lines between spatially separated events (which is a secondary partial order for the causets drawn with \pcauset). This option only affects the \pcauset and \rcauset commands, but not the \causet command. (\causetsDrawSpatialLinkstrue activates the option) (\causetsDrawSpatialLinksfalse deactivates the option)		
\usepackage[external]{causets} loads the external library, sets the external file name prefix to "causets/", and enables the use of causet names for the external file names. (This feature has to be loaded as option in the preamble not later in the document.) For details on externalization of causets, see Sec. 8.		₩

By default, the links in a diagram are interrupted if they cross over another event (exactly over the event centre). Any such crossing of links over events can only occur for the links explicitly specified for the \causet command or the locally restyled links of the \rcauset command, but not for the links drawn by \pcauset or automatically drawn links by \rcauset. This feature is activated by default (\causetsBreakLinkstrue) and can be deactivated with \causetsBreakLinksfalse, or with the option continuously linked.

For example, here is a causet that embeds in 3-dimensional but not in 2-dimensional Minkowski spacetime. This causet has one event (0) in the centre that is linked to events -3 and 3 but not to events -6 and 6, while a link (blue) from -6 to 6 crosses over it.





Note that the links are drawn in the order given by the second argument. Similar results are obtained with the following \rcauset commands (since version 1.4)

```
\rcauset[brokenly linked]{-1,2,-5,-7,4,-3,-6,0,6,3,-4,7,5,-2,1}

\{-7/-3,-6/0,-5/4,-4/5,0/6,3/7,-6/6/blue!50!cyan}

\rcauset[continuously linked]{-1,2,-5,-7,4,-3,-6,0,6,3,-4,7,5,-2,1}

\{-7/-3,-6/0,-5/4,-4/5,0/6,3/7,-6/6/blue!50!cyan}
```

where all pairs of integers in the second argument are links to be removed (by default, get the local style draw=none). All links with a local style specified (here the last link only) are drawn in the order they appear after all automatic links have been drawn. Local styles are explained in Sec. 7 in more detail.

All main commands, the short hand macros (see Sec. 3), and the command \causetfile (see Sec. 8) add a small padding to the left and right of the diagram for a better spacing within text and mathematical expressions. The padding is adjustable with padding=\(\nabla value \rangle\) and stored in the macro \causets@Padding (by default, it is \thinspace).

3 Short hand macros

The permutations and labels for causets can be switched on globally with the package options permutation and labeled, or at any point in the document with the switches \causetsDrawPermutationtrue (same as the command \tikzcausetsset{show permutation}) and \causetsDrawLabelstrue (or \tikzcausetsset{labeled}), respectively.

To show only single causets with either of these options, there exist short hand macros for all three main commands. Each main command can be followed by a capital letter P (to show the permutation only), L (to show event labels only), or X (to show both).

```
Causets with permutations (and labels) \pcausetP{4,1,2,3}, \pcausetX{1,2,3,4}, \pcausetL{1,2,4,3}, and without any \pcauset{2,4,1,3}.
```

These results are also obtained by using the optional parameter, for example:

```
Causets with permutations (and labels) \pcauset[permutation]{4,1,2,3}, \pcauset[permutation, labeled]{1,2,3,4}, \pcauset[labeled]{1,2,4,3}, and without any \pcauset{2,4,1,3}.
```

The short hand macros \causetFence and \causetCrown have been removed in version 1.5, because these commands may be unnecessary for most users and the Proset editor provides a better way to get the LaTeX code for more complex causets including fences, crowns and polygons.

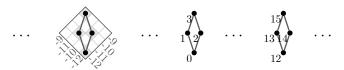
4 Labeling and replacing events (also to represent infinite causets)

Since version v1.3 of the package, it is possible to relabel and replace causet events. These features are useful for labelling events arbitrarily, and drawing diagrams of infinite causets using ellipsis.

By default, all events get named and labeled by the integers given in the arguments of the commands. For example, $\colon colon col$

To offset the names of all causet events by an integer amount, use the key-value pair $offset=\langle value \rangle$ to adjust the naming and labelling of events and get distinct event names, while the same integers are used in the command arguments. Consider the following example.

```
1 \begin{align*}
2 \tikzcausetsset{large}
3 \cdots\quad
4 \pcausetP[offset=-12]{0,2,1,3}\quad\cdots\quad
5 \pcausetL{0,2,1,3}\quad\cdots\quad
6 \pcausetL[offset=12]{0,2,1,3}\quad\cdots
7 \end{align*}
```



Here, the three commands have the same integer argument {0,2,1,3}, but the events are named with the specified offsets for the sub-causets, and the labels are adjusted accordingly. In this way, it will be possible to address the event (and label) nodes of the sub-causets distinctively within a larger TikZ picture. For more details on how to draw causets within a TikZ picture, see Sec. 10.

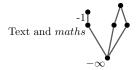
When you (only) want to change the labelling of an event to $\langle value \rangle$, add a slash and label= $\langle value \rangle$ directly after the event integer in the first command argument. For example, the causet Λ has six different labellings by (distinct) integers from the set $\{0, 1, 2\}$,

```
1 \begin{align}
```

```
\text{order-preserving:} &&
    L_{\mathrm{op}} \ &=
    \left\{
      \protect{1,0,2},
      \pcausetL{1/label=0,0/label=1,2}
6
    \right\}
    , \\
8
    \text{not order-preserving:} &&
9
    L_{\mathrm{nop}} \&=
    \left\{ \right.
      \pcauset {1/label=2,0/label=1,2/label=0},
12
      \pcausetL{1,0/label=2,2/label=0},
      \pcausetL{1/label=2,0,2/label=1},
      \pcauset {1/label=0,0/label=2,2/label=1}
    \right\}
17
18 \end{align}
```

order-preserving:
$$L_{\text{op}} = \left\{ \sqrt[3]{1}, \sqrt[3]{2} \right\}, \qquad (2)$$
not order-preserving:
$$L_{\text{nop}} = \left\{ \sqrt[3]{2}, \sqrt[3]{2}, \sqrt[3]{2}, \sqrt[3]{2} \right\}. \qquad (3)$$

The replacing labels can be any text. Longer text labels should be written in a T_EX group $\{...\}$. Here is an example

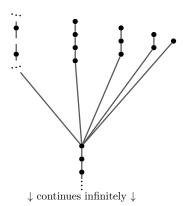


Notice that replaced labels will always be visible, even if labels are deactivated (\causetsDrawLabelsfalse) as in this example. To change the style of the replaced labels, add the key-value pair replaced labels=\(\nabels=\lambda value\rangle\), which extends the style of standard labels.

Similarly to the replacement of labels, you can replace event nodes by using a slash followed by $event=\langle value \rangle$. This feature may have its main application for the representation of infinite causets, for example, the code

```
% Define ellipsis rotated by #1 degrees:
2 \newcommand{\rdots}[1][0]{\rotatebox{#1}{...}}
3 ...
4 % Draw infinite causet with ellipsis
5 \pcauset[large, replaced labels={below, at=south}, replaced
        events={text scale=1}]
6 -2/{label={$\downarrow$ continues infinitely $\downarrow$},
        event={\rdots[90]}}, -1,0,1,16,14,15,11,12,13,7,8,9,10,
        2/event={\rdots[10]},3,4/event={^}},5,6/event={\rdots[-10]}}
```

yields the diagram



Here, three events are replaced by ellipsis with different rotations, and one event is replaced by an empty node ($\tilde{}$). The style of the replaced events is specified via the key replaced labels= $\langle value \rangle$ – where $\langle value \rangle$ is {text scale=1} in this example. This example also shows how to relabel and replace simultaneously (see the bottom-most event).

The style and the position of all event labels is set with $labels=\langle value \rangle$, while replaced $labels=\langle value \rangle$ extends this style. As shown in the previous example, the position of labels is defined with a key word like left, right, above, below, etc. (see also the TikZ manual) relative to the position given by the key $at=\langle value \rangle$. The at key holds a coordinate of the event note. The default positioning is below left, at=south west.

5 Changing sizes and fonts (for mathematical expressions)

The pre-defined size styles have the following scaling with respect to the normal size:

- tiny is $\frac{1}{3}$ times the normal size
- very small is $\frac{4}{9}$ times the normal size
- small is $\frac{2}{3}$ times the normal size
- large is $\frac{3}{2}$ times the normal size
- very large is $\frac{9}{4}$ times the normal size
- huge is 3 times the normal size

Apart from the pre-defined sizes, a causet can be scaled to any size using size unit= $\langle value \rangle$ (which is equal to the tile size, tile size= $\langle value \rangle$). The normal size unit has the value 0.9ex.

I recommend to change the size of a causet when used in indices or limits for mathematical expression. For a (first level) sub-script in a summation, you may use the small size,

```
1 \begin{equation}
2    \sum_{C \in \tikzcausetsset{small}}
3     \Bigl\{ \pcauset{1,2,3}, \pcauset{1,3,2,4}, \pcauset{1,4,3,2,5}, \pcauset{1,5,4,3,2,6} \Bigr\}} | C |
4    = \left| \pcauset{1,2,3} \right|
5    + \left| \pcauset{1,3,2,4} \right|
6    + \left| \pcauset{1,4,3,2,5} \right|
```

$$\sum_{C \in \left\{ \mathbf{i}, \mathbf{0}, \mathbf{0}, \mathbf{0} \right\}} |C| = \left| \mathbf{i} \right| + \left| \mathbf{0} \right| + \left| \mathbf{0} \right| + \left| \mathbf{0} \right|$$

$$(4)$$

Like any other options, sizes can also be specified in the optional argument of a single causet diagram or with \tikzcausetsset{...} for the current scope.

If you need to set the sizes of individual components of a causet, use the keys

- tile size= $\langle value \rangle$ to store $\langle value \rangle$ in the macro \causetTileSize as the size of the tiles in the permutation (this is equivalent to setting the size unit),
- region line width=\(\langle value \rangle\) to set the thickness for the line surrounding the permutation region,
- grid line width=\(\nabla value \rangle\) to set the line thickness of the permutation grid,
- event size=\(\nabla value \rangle\) to set the diameter of the events,
- event outline=\(\nabla value \rangle\) to set the thickness of the transparent line around the events,
- link width=\(\langle value \rangle\) to set the link thickness, and
- broken link gap=\(\nabla value \rangle\) to set the link gap before and after an event that interrupts a link.

The text of all labels is sized independently to the other components of the causet diagrams. To adjust the text typesetting, use

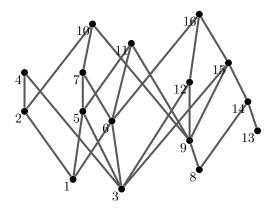
- text font=\(\nabla value\)\) to set the font used for all the text labels for example, text font=\fontsize{13}{15.6}\selectfont sets the font size to 13pt (and the baseline skip to 1.2 times this font size),
- text scale=\(\nabla value \rangle\) to set the scaling factor of all the text labels (relative to the font size).

The pre-defined size styles use the following font sizes,

- tiny is typeset with \tiny (scaled by 0.30)
- very small is typeset with \t iny (scaled by 0.40)
- small is typeset with \scriptsize (scaled by 0.43)
- normal is typeset with \footnotesize (scaled by 0.56)
- large is typeset with \small (scaled by 0.75)
- very large is typeset with \normalsize (scaled by 1.02)
- huge is typeset with \Large (scaled by 0.97)

Below is an example for a diagram drawn with some sizes defined manually:

```
1 \pcausetL[tile size=2ex, event size=1ex, link width=0.3ex, text scale=1] {13,8,14,9,3,15,12,1,6,16,5,11,7,2,10,4}
```



6 Changing colors

Colors can be set with the following keys:

- region color=\(\nabla value \rangle\) (or region color=\(\nabla value \rangle\)) for the frame of the permutation grid
- grid color=\(\nabla value \rangle\) (or grid color=\(\nabla value \rangle\)) for the lines of the permutation grid
- tile color=\(\nabla value \rangle\) (or tile color=\(\nabla value \rangle\)) for the occupied tiles
- ulabel color= $\langle value \rangle$ (or ulabel color= $\langle value \rangle$) for the u labels
- vlabel color= $\langle value \rangle$ (or vlabel color= $\langle value \rangle$) for the v labels
- event color= $\langle value \rangle$ (or event color= $\langle value \rangle$) for the events
- label color= $\langle value \rangle$ (or label color= $\langle value \rangle$) for the event labels
- link color=\(\nabla value \rangle\) (or link color=\(\nabla value \rangle\)) for the links
- spatial link color= $\langle value \rangle$ (or spatial link color= $\langle value \rangle$) for the spatial links.

When using any of these key-value pairs in an optional arguments of the causet macros, they apply to that diagram. To change colors within a scope or the entire document, use the \tikzcausetsset macro, for example

```
1 \tikzcausetsset{event color=purple, link color=cyan}
```

changes the event color to purple and the link color to cyan.

For the pre-defined color schemes, use the keys gray colors (or gray colors, default), black colors (or black colors, only black and white), blue colors (or blue colors, blue color tones), or neon colors (or neon colors, bright colors). These keys set all the colors listed above to pre-defined values.

7 Changing the styles of individual events and links

The full supported formats of the three main commands are

and similarly for all short hand macros as described in Sec. 3. The style values are all optional, where $\langle causet\ style \rangle$ is applied to that causet only, $\langle event\ style \rangle$ is applied to a single event, and $\langle link\ style \rangle$ is applied to a single link.

The feature to apply styles only to individual events and links is to highlight or modify subsets of a causet, for example, by using different colors:

```
rcausetL[large, subset A/.style={red, dotted}, subset B/.style=blue,
replaced labels={text scale=1, at=east, right}]{2/{subset A,
    label=event $a$}, 5/subset B, 1, 3/{subset A, label=$b$, replaced
    labels=orange}, 4/subset B, 6/subset B}{2/3/subset A, 4/6/subset B,
    5/6/subset B}
```



Note that label=event a and label=b adds (replaced) labels a and b, respectively, but their style is not affected by subset A. To change the style of the labels, we have to use the replaced labels key, see also Sec. 4.

8 Externalisation and standalone causets

For any document that contains many (large) causet diagrams or the same causets that are used repeatedly, the compilation time can be reduced by moving the graphics to external files using the externalisation library of TikZ or standalone files.

The externalisation feature of TikZ provides the tools to automatically externalise all causet diagrams and only recompile them when they are changed in the document. To enable this feature, use the external option when loading the package and make sure that the document is compiled with the flag

```
--shell-escape
```

The external option loads the external library, sets the prefix to add "causets/" before each external file, and calls \causetsNameExternaltrue to use the manually specified names for external file names. Any causet can be manually named by providing the name key-value pair name= $\langle value \rangle$ in the optional parameter of a causet command. As an example, the inline code

```
1 \pcauset[name=Vee, alt={Vee}]{1,3,2}
```

yields the causet $\$ and creates an external pdf file with the name causets/Vee.pdf relative to the main tex file. The alternative text alt={ $\langle value \rangle$ } may be the same as name for short names (like here), but the file names are usually shorter.

I recommend to always name causets, even if externalisation is not used. In this way, it is easier to read the source code and switch to externalising at a later point more easily. Note that the name of a causet does not have to be unique (especially if it is used repeatedly throughout the document), but you may give different names

to causets with different style options to avoid unwanted overwrites. For a repeated causet in the document, either use the same code or the \causetfile macro. For example, \pcauset[name=N]{2,4,1,3} can be repeated with \causetfile{N} to print the N causet \N . The macro \causetfile[#1]{#2} also accepts one optional argument [#1] and expands to

```
1 \causets@Padding%
2 \includegraphics[#1]{\causets@NamePrefix#2}%
3 \causets@Padding
```

To redefine the prefix stored in \causets@NamePrefix and also set the external library prefix to the same value, use \tikzcausetsset{set prefix=causets/}. However, to set \causets@NamePrefix only for causets but not for other TikZ externalisation, use name prefix=\(\lambda value \rangle\).

If you are already loading the external library for other uses, you do not want to use the external option, but add the following to the document's preamble:

```
1 \usetikzlibrary{external}
2 ...
3 \usepackage{causets}
4 \tikzcausetsset{set prefix=causets/, name external=true}
```

For externalising only those causets that are named manually, set

```
1 \tikzset{external/only named=true}
```

For switching externalisation on/off all together, the library provides the macros

```
1 \tikzexternaldisable
2 \tikzexternalenable
```

Please find further information on the external library in the TikZ manual.

Graphics may also be moved to standalone tex files manually and included with \causetfile or \includegraphics. For example, create a file named M.tex in the directory that was set to name prefix (by default, this is "./") with the following code:

```
1 \documentclass[tikz]{standalone}
2 \usepackage{causets}
3 \begin{document}
4 \pcauset{4,2,5,1,3}
5 \end{document}
```

The compiled diagram is then included with the macro \causetfile[alt={M}]{M} (where alt is directly passed to \includegraphics to be processed with LATEXML).

More examples on larger standalone graphics are given in Sec. 10.

9 Style changes in detail

In the optional argument of the causet macros, you may use other TikZ options to change the output of the macros, for example:

```
1 \usepackage{causets}
```

```
2 ... \begin{document} ...
3 A labeled causet with red-framed semi-transparent labels,
   \pcausetL[labels={draw=red, fill=white, opacity=0.8, inner sep=1pt,
        scale=1.5}]{2,4,5,1,3}.
```

A labeled causet with red-framed semi-transparent labels,

In the same way, it is possible to modify the existing styles of all parts of the diagrams, which means that the default drawing styles are extended by the options passed as value to these keys:

- 1. region=\langle value \rangle shows the permutation (show permutation) and appends to the style of the permutation frame,
- 2. grid=\(\frac{value}\) shows the permutation (show permutation) and appends to the style of the permutation grid,
- 3. tiles=\langle value \rangle shows the permutation (show permutation) and appends to the style of the occupied permutation tiles,
- 4. events=\(\nabla value \rangle\) appends to the style of the event nodes (graph vertices),
- 5. links=\langle value \rangle shows the links (show links) and appends to the style of the links (main graph edges),
- 6. link starts=\(\nabla value \rangle \) sets the starting line tip of each link,
- 7. link ends=\(\nabla value \rangle \) sets the ending line tip of each link,
- 8. link pauses=\(\nabla value \rangle \) sets the ending line tip of each link at an interruption,
- 9. link resumes=\langle value \rangle sets the starting line tip of each link at an interruption,
- 10. cap links sets the starting and ending line tip of each link to a (reversed) cap,
- 11. arrow links sets the ending line tip of each link to a rounded, stealth arrow,
- 12. spatial links=\langle value \rangle shows the spatial links (show spatial links) and appends to the style of the spatial links (secondary graph edges),
- 13. all labels=\langle value \rangle appends to the general style of all text labels (applies to the following three label types),
- 14. labels= $\langle value \rangle$ shows the event labels (show labels) and appends to the style that applies only to the event labels,
- 15. ulabels= $\langle value \rangle$ shows the u labels (show ulabels) and appends to the style that applies only to the u labels, and
- 16. vlabels= $\langle value \rangle$ shows the v labels (show vlabels) and appends to the style that applies only to the v labels.

In order to manually define the full style of the parts in the diagram – not extending, but overwriting all drawing options – set the style of the following keys:

- 1. every causet for the general style of the causet diagram (like baseline, see below),
- 2. every tile for the occupied permutation tiles,
- 3. every event for the event nodes (graph vertices),
- 4. every grid for the permutation grid,

- 5. every region for the permutation region,
- 6. every label base for all text labels,
- 7. every ulabel for the permutation u labels,
- 8. every vlabel for the permutation v labels,
- 9. every link for the links (graph edges),
- 10. every spatial link for the spacelike separations, and
- 11. every label for the event labels.

With the macro \tikzcausetsset{...}, any of these styles can also be set in the preamble of the document. So, for example,

```
1 \usepackage{causets}
2 \tikzcausetsset{events={rectangle}, links={-Triangle Cap[]}}
3 ... \begin{document} ...
4 \pcauset[large]{2,4,5,1,3}
```

will generate all causets with square nodes as events and triangle caps on the future (upper) link ends, like . .

By default, causets are placed such that a pre-defined coordinate point falls on the text baseline, baseline=(baseline point). The baseline point is tile size/ $\sqrt{2}$ below the centre of the diagram. You may adjust the baseline as for any TikZ graphic, for example,

```
1 \pcauset[baseline=2mm]\{2,4,5,1,3\}
```

places the diagram centre 2mm below the text base,



10 Using this package in TikZ pictures

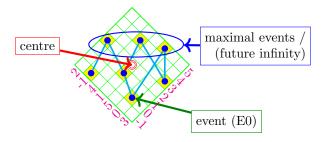
Since the package is TikZ based, using a causet within a TikZ picture is simple. Each of the three main commands is based on a draw command in TikZ that has the same name but starts with \draw.... The \draw macros, however, do not support an optional argument (since this can be specified directly in a TikZ picture) and they also do not have any short hand macros ending in P, L or X. The \draw... do not add extra padding (\causets@Padding) on the left and right of the graphic either.

The two following examples are given by external (standalone) TikZ graphics and included with \includegraphics{...}. More details on using causets in standalone files are given in Sec. 8.

Note that the output of the \draw macros has the point (0,0) in the centre, so in order to shift it, we may enclose it in a TikZ scope, for example:

```
1 \documentclass[tikz]{standalone}
2 \usepackage[permutation,larger,neon]{causets}
3 \usetikzlibrary{fit,shapes.geometric}
4 \begin{document}
5 \begin{tikzpicture}
```

```
% Create causet, shifted in x- and y-direction:
    \begin{scope}[xshift=3cm, yshift=-1cm]
      \tikzcausetsset{offset=-2}
      \drawpcauset \{5, 2, 7, 3, 6, 1, 4\}
9
      \draw[red, double] (0, 0) circle[radius=0.1];
    \end{scope}
    % Mark the maximal elements (future infinity):
    \node[draw=blue, inner sep=1pt, thick, ellipse, fit=(E2) (E4) (E5)]
     (Finf) {};
    % Add labels on top:
14
    \node[draw=blue, right, align=right] (FinfLabel) at (4.8, -0.5)
     {maximal events / \\ (future infinity)};
    \node[draw=red] (centerLabel) at (0.5, -0.5) {centre};
    \node[draw=green!50!black] (myEventLabel) at (5.5, -2.5) {event (E0)};
    \% Draw arrows from the labels to the references:
18
    \draw[ultra thick, blue, ->] (FinfLabel) -- (Finf);
    \draw[ultra thick, red, ->] (centerLabel) -- (3, -1);
     shifted by the scope
    \draw[ultra thick, green!50!black, ->] (myEventLabel) -- (E0);
22 \end{tikzpicture}
23 \end{document}
```



Note that this example also shows how to use the event nodes that are generated by the \drawpcauset macro. The green arrow points to the node of the second causet event (E5). The causet events, event labels, u-axis labels, and v-axis lables are the nodes (Ei), (Elli), (Elli), and (EVLi), respectively, where $i \in [-1, 5]$.

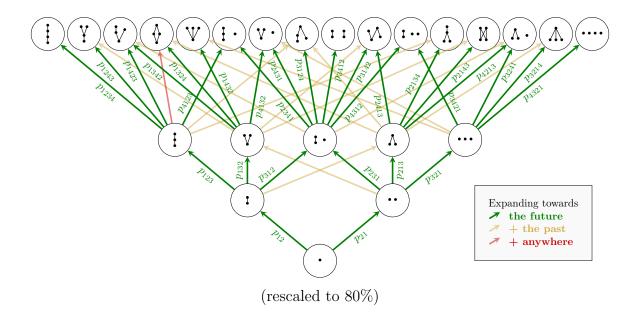
It is also possible to use multiple causets in more complicated graphics, like a tree for example, but note that whenever a causet is actually a text of a node within a TikZ picture (like in the following), we need to use the \pcauset (\rcauset or \causet) commands without the draw prefix again:

```
1 \documentclass[tikz]{standalone}
2 \usepackage[black]{causets}
3 \usetikzlibrary{fit,shapes.geometric}
4 % Set colours and style:
5 \colorlet{future colour}{green!50!black}
6 \colorlet{past colour}{yellow!50!orange!80!black}
7 \colorlet{anywhere colour}{red!80!black}
8 \tikzset{prob arrow/.style={line width=1.5pt}}
9 % Define some helper functions:
```

```
10 \def\def\cevents #1#2#3{\xdef\Ea{#1}\xdef\Eb{#2}\xdef\Ec{#3}}
11 \def\defDevents#1#2#3#4{\expandafter\defCevents#1#2#3\xdef\Ed{#4}}
12 \newcommand*{\semiopaque}[1]{% Adds a transparency group.
13 \begin{scope}[transparency group, opacity=0.5]
    #1
14
15 \end{scope}
16 }
17 \newcommand*{\drawprobarrow}[4][]{% Draws a (probability) arrow.
18 \draw[prob arrow] (#2) -- node[sloped, midway, below, #1] {$#3$} (#4);
19 }
20 \newcommand{\drawlegendsymbol}{% Draws the symbol for a legend entry.
21 \draw[prob arrow] (0,0) -- +(10pt, 6pt);
22 }
23 \newcommand{\legendsymbol}[1]{% Creates the legend symbol.
24 \begin{tikzpicture}
25
    \int ifnum#1=0
      \drawlegendsymbol
26
27
    \else
      \semiopaque {\drawlegendsymbol}
28
30 \end{tikzpicture}
31 }
32 \begin{document}
33 \begin{tikzpicture}[-stealth]
    \def\ystep{2.0cm}
    \def \xstep {0.60cm}
    % Create nodes of all causets with 1 to 4 events:
36
    \begin{scope}[nodes={draw, thin, circle, minimum size=1.1cm}]
      \node (C1) at (0, 0) {\pcauset{1}};
      \node (C12) at (-4*\xstep, 1*\ystep) {\pcauset{1,2}};
      \node (C21) at (4*\xstep, 1*\ystep) {\pcauset{2,1}};
40
      \foreach \perm [count=\i from 0] in \{123, 132, 312, 213, 321\}{%
41
42
        \expandafter\defCevents\perm
        \node (C\perm) at (4*\i*\xstep-8*\xstep, 2*\ystep)
      {\pcauset{\Ea,\Eb,\Ec}};
44
      }
      \foreach \perm [count=\i from 0] in {1234, 1243, 1423, 1324, 1432,
     4123, 4132, 3124, 3412, 3142, 4312, 2134, 2143, 4213, 3214, 4321}{%
46
        \expandafter\defDevents\perm
        \node (C\perm) at (2*\i*\xstep-15*\xstep, 3.75*\ystep)
47
     {\pcauset{\Ea,\Eb,\Ec,\Ed}};
      }
    \end{scope}
49
    % Add graph edges for expandings ...
    % ... to the past (additionally):
    \begin{scope}[past colour]
      \foreach \perma/\permb in \{12/213, 123/3124, 123/2134, 132/1243, 
     132/3142, 132/2143, 312/1423, 312/4213, 213/1324, 213/3124,
```

```
213/3214, 321/1432, 321/4132, 21/132}
         \semiopaque {\drawprobarrow {C\perma}{}{C\permb}};
    \end{scope}
    % ... to anywhere (additionally, further):
56
    \begin{scope}[anywhere colour]
       \sim {C123}{}{C1324}};
    \end{scope}
    % ... to the future:
    \begin{scope}[prob arrow, future colour]
61
       \drawprobarrow{C1}{p_{12}}{C12}
       \drawprobarrow{C12}{p_{123}}{C123}
       \drawprobarrow{C123}{p_{1234}}{C1234}
       \drawprobarrow[near end]{C123}{p_{1243}}{C1243}
65
       \drawprobarrow[near end]{C123}{p_{1423}}{C1423}
       \drawprobarrow[near start, above]{C123}{p_{4123}}{C4123}
67
       \drawprobarrow[above]{C12}{p_{132}}{C132}
       \drawprobarrow[near end]{C132}{p_{1342}}{C1423}
69
       \drawprobarrow[near end]{C132}{p_{1324}}{C1324}
       \drawprobarrow[midway, above]{C132}{p_{1432}}{C1432}
       \drawprobarrow[near start]{C132}{p_{4132}}{C4132}
72
       \drawprobarrow[near start, above]{C12}{p_{312}}{C312}
       \drawprobarrow[near end]{C312}{p_{3124}}{C3124}
74
       \drawprobarrow[near end]{C312}{p_{3412}}{C3412}
75
       \drawprobarrow[near end]{C312}{p_{3142}}{C3142}
       \drawprobarrow[near start]{C312}{p_{4312}}{C4312}
       \drawprobarrow{C1}{p_{21}}{C21}
       \drawprobarrow[near start, above]{C21}{p_{231}}{C312}
79
       \label{lem:condition} $$ \operatorname{cond}_{C312}_{p_{2341}}_{C4123} $$
       \drawprobarrow[near end]{C312}{p_{2431}}{C4132}
81
       \drawprobarrow{C21}{p_{213}}{C213}
82
       \drawprobarrow[above]{C213}{p_{2134}}{C2134}
83
       \drawprobarrow[near start]{C213}{p_{2413}}{C3142}
84
       \drawprobarrow[near end]{C213}{p_{2143}}{C2143}
85
       \drawprobarrow[near end]{C213}{p_{4213}}{C4213}
86
       \drawprobarrow{C21}{p_{321}}{C321}
       \drawprobarrow[near start, above]{C321}{p_{3421}}{C4312}
88
       \drawprobarrow[near end]{C321}{p_{3241}}{C4213}
       \drawprobarrow[near end]{C321}{p_{3214}}{C3214}
90
       \drawprobarrow{C321}{p_{4321}}{C4321}
91
    \end{scope}
92
    % Add legend:
93
    \node[above left, align=left, draw, inner sep=3ex, fill=gray!05]
       at (15*\xstep, 0)
       {Expanding towards \\
96
       \bfseries\color{future colour}\legendsymbol{0}\ the future\\
97
        \bfseries\color{past colour}\legendsymbol\{1\}\ + the past\\
        \bfseries\color{anywhere colour}\legendsymbol{1}\ + anywhere};
99
100 \end{tikzpicture}
```

101 \end{document}



11 Release notes

v1.5, (in development)



New features

• Adding support for LATEXML including the key alt={\langle value \rangle} to define an alternative text representing a diagram for screen readers.

Modifications and fixes

- Removed the short hand macros \causetFence and \causetCrown.
- Corrected typos in the manual and added reference to the PrOSET editor.

v1.4. 2023-03-06



New features

- Second values in the sequence of the first argument: now modifies the style of individual events, the events are replaced with the key event=\(\langle value \rangle \), and the labels are set with label=\(\langle value \rangle \).
- Third values in the sequence of the second argument of \causet and \rcauset: to modify the style of individual links.
- Broken links for restyled links of \rcauset: restyled links can also be drawn between events that would have not been automatically linked, hence these links might now be broken up like the links of \causet.
- New key name=\(\name \) to define a name of a causet for easier readability of the tex source code (and to be used as external file name when the corresponding package option has been activated).
- New option external: to load the external library of TikZ for automatic externalisation of all diagrams.
- New key set prefix=\(\langle value \rangle\): to pass a file name prefix that is combined with a causet name for the external files when using external.
- New key name external=\langle bool\rangle: to activate/deactivate using the causet names for external files.
- Main commands: padded by a thin space (adjustable with padding=\(\nabla value \rangle\)) for a better spacing in text and math mode.

Modifications and fixes

- Third values in the sequence of the first argument: no longer supported.
- Spatial links: are now drawn before (so on a lower layer than) links.
- Style arrow links: the arrow heads are now filled.
- Macro \causetspath: has been replaced by name prefix= $\langle value \rangle$.
- Short-hand macro \causetCrown: changed to take a single argument (and the optional style argument) and returns the same output as \causetClosedFence used to, which has been removed.

v1.3.1, 2022-07-27



Modifications and fixes

• Automatic event labels: now correctly printed when using offset=\(\nabla value \rangle \).

v1.3, 2022-05-23

New features

- Integer sequences in the arguments: use a permutation of an interval [a, b], where a and b do no longer have to be natural numbers but can be any integers $(a \le b)$ including 0 and negative numbers. These numbers name and label the events of the causet if the respective options are chosen.
- Second values in the sequence of the first argument: to change the name and label of events.
- New key offset=\(\nabla value \rangle \) (0 by default): to define an integer shift for all events before they are named and labeled.
- New style text font= $\langle value \rangle$ (additionally to text scale= $\langle value \rangle$): to adjust the font style of text (labels), for example, to set the font size to 20pt (and a baseline skip of 24pt), use text font=\fontsize{20}{24}\selectfont

Modifications and fixes

- Size options: all sizes are now scaled with respect to the normal size.
- Event style: line thickness of the event outline has been corrected.
- Size macros removed: \causetRegionLine, \causetGridLine, \causetEventSize, \causetLinkWidth, and \causetBrokenLinkGap. The respective values are now stored in PGF keys and are measured with respect to the size unit (stored in \causetTileSize).
- All package options: reduced to single words.
- Permutation for *causetL...: now deactivated (if activated previously), by default.
- Labels for *causetP...: now deactivated (if activated previously), by default.

v1.2, 2022-02-07



New features

- New style at=\langle value \rangle: to specify the relative point of events (at=\langle value \rangle, by default) and position relative to this point using TikZ keys left=\langle value \rangle, right=\langle value \rangle, etc.
- New styles cap links and arrow links: add different arrow tips to the start and end
 of links.
- New styles link starts= $\langle value \rangle$ and link ends= $\langle value \rangle$: to modify arrow tips to the start and end of links.
- Broken links for \causet: links crossing over unlinked events are broken up now.
- New styles link pauses= $\langle value \rangle$ and link resumes= $\langle value \rangle$: to set the link ending style (arrows) at breaking points.
- Spatial links for \rcauset: spatial links for spacelike separated events are now supported by \rcauset, too.

Modifications and fixes

• Color option blue: blue colors are now different shades of blue, gray and black only.

v1.1, 2020-12-15

New features

Ī

• All colors, size and style options have been moved to PGF keys in the path

1 /tikz/causets/

v1.0, 2020-11-08

• First release of the package causets on CTAN, https://ctan.org/pkg/causets, including all the main commands.

All previous package versions are at

https://github.com/c-minz/LaTeX-causets/tree/main/previous versions

12 Bug reports and package requests

To see reported problems, go to https://github.com/c-minz/LaTeX-causets/issues

If you have a problem when using the package or you would like to have another

feature to be implemented, please write a message to

christoph(dot)minz(AT)gmail(dot)com

For reporting a bug, I much appreciated if you do the following:

- Check the issues reported previously if your problem is already listed.
- Describe your problem including the errors and warning messages, information about the drivers and programs versions.
- Provide a minimal, working test file (only a single TEX file) that demonstrates the problem.