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A fast method for forward computation of multiple-shell spherical head models

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Summary Using a combination of 3 suitably located dipoles in a homogeneous sphere, the scalp potential due to a dipole source in a 4-shell spherical head model can be approximated with a high degree of precision and a more than 30-fold increase in computing speed. Magnitudes and locations of the 3 equivalent dipoles can be fitted in a homogeneous sphere to data generated from a source at one location in a 4-shell head model. The resulting parameters are used to compute scalp potentials for sources at other locations and orientations. Residual variance measures showed close agreement between the new approximation and a standard 4-shell computation method. Further tests of the method used scalp data from 500 randomly selected pairs of sources generated by the standard 4-shell computation and fitted using, for forward computations, the new approximation and the single-shell Ary-corrected head model. Errors with the new approximation were marginally larger than with the standard computation, but sources were located within 0.5 mm and 0.6° of the original position in 99% of the fits. 99% error limits for the Ary model were up to 18 mm and 25° and depended on the head model parameters.

Key words: Dipole source localization; Forward problem; Four-shell model; Ary correction; Model misspecification

When estimating the voltage distribution on the scalp resulting from dipolar or distributed sources inside the head, a model is required which describes the electrical conductivities and the dimensions of the conducting media. Since the computational cost of realistic head models is extremely high the conductivity properties are often approximated by spherical models in which the head is divided into concentric shells of varying conductivity. In the 4-shell model, the shells consist of the brain, the cerebrospinal fluid (CSF), the skull and the scalp. In the 3-shell model, the CSF is assumed to have the same conductivity as that of the brain. A more extreme approximation arises from the results of Ary et al. (1981), who noted that the voltage distribution due to a source in a homogeneous sphere was similar to that of a source in a 3-shell sphere, apart from a correction in eccentricity (distance from the center of the sphere) of the source. Due to the low conductivity of the skull, the potential distribution on the scalp due to a given source is spread out. The equivalent source in a homogeneous medium therefore appears further away from the scalp.

The simplicity of the calculations of the homogeneous model (Brody et al. 1973) makes its use, together with the Ary transformation of eccentricities, a fast and convenient method for modelling electrical sources in the brain, and it has been used in commercial packages such as BESA (Scherg 1990). Recently, Zhang and Jewett (1993) showed that using the homogeneous model to approximate the 3-shell model leads to errors which they call *model mismatch errors*. These occurred when attempting to fit pairs of sources using the homogeneous model to data that were simulated using the 3-shell model. Errors in location, orientation and magnitude of the sources occurred that were not present when fits were carried out using the same model as that used for generating the data. The errors were larger for sources at larger eccentricities and for sources that were close together. These errors were not evident when fitting single sources instead of pairs. Clearly the errors arise because of differences in the scalp potential distributions between that generated by a source in the multiple-shell model and by the equivalent source in the Ary approximation.

Zhang and Jewett's results suggest that it can be dangerous to use oversimplified conductivity models. Previously published methods of computing 3- and 4-shell models are slow and cumbersome to use on

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personal computers and many workstations. We have discovered an accurate approximation to these models which offers a more than 30-fold increase in computing speed. In the following, the approximation is described and then evaluated in comparison with (a) the standard 4-shell computation method and (b) the Ary-transformed homogeneous approximation. Residual variance and model mismatch errors were used as the criteria of accuracy.

A new approximation to 3- and 4-shell head models

One rationale underlying the approximation was the analogy with the Fourier series representation of a wave form: as a wave form is represented by the sum of a series of frequency components, so the scalp voltage distribution due to a source in a multiple-shell model might be represented by the sum of a series of source distributions in the homogeneous sphere. The approximation consists of computing the voltage due to a dipole source in a 3- or 4-shell head model by summing the contributions of 3 suitably selected sources in a homogeneous model. These sources lie on the same radial vector as the original and have identical orientations. The characteristics of the multiple-shell head model determine the relative magnitudes and eccentricities of the sources in the homogeneous model. It turned out that the underlying relationship is very simple and requires only 6 parameters, 3 magnitude and 3 eccentricity factors, for a given set of head dimensions (i.e., scalp, skull and CSF thicknesses, scalp, skull, CSF and brain conductivities). The relative magnitudes of the 3 equivalent sources in the homogeneous sphere are independent of eccentricity, and the eccentricities of the equivalent sources are linear functions of the original source eccentricity. The parameters are independent of source orientation. Thus, in order to obtain the magnitude and eccentricity factors for a given set of head dimensions, it is only necessary to determine the relationship for surface data generated from a source at one eccentricity and one orientation in the multiple-shell sphere.

Let e_1, e_2, e_3 be the relative eccentricities for the 3 equivalent sources. m_1, m_2, m_3 are the magnitude factors. The potential $V^4(\mathbf{r}, \mathbf{o}, \mathbf{p})$ at position $\mathbf{p}$ on the scalp, for a unit source located at $\mathbf{r}$, with orientation $\mathbf{o}$ in a 4- (or 3-) shell sphere is given by:

$$V^4(\mathbf{r}, \mathbf{o}, \mathbf{p}) = m_1 \cdot V^1(\mathbf{r} \cdot e_1, \mathbf{o}, \mathbf{p}) + m_2 \cdot V^1(\mathbf{r} \cdot e_2, \mathbf{o}, \mathbf{p}) + m_3 \mathrm{V}^1(\mathbf{r} \cdot e_3, \mathbf{o}, \mathbf{p}) \quad (1)$$

where $V^1(\mathbf{r}, \mathbf{o}, \mathbf{p})$ is a function giving the potential due to a unit source in the (1-shell) homogeneous sphere. Thus the 3 homogeneous sources have the same orientation ($\mathbf{o}$) as the original source and lie along the axis linking the original source to the center of the sphere. Their eccentricities are proportional to the original source eccentricity.

We have not discovered an analytical means of computing the 6 parameters of the approximation. Instead, we have obtained the magnitude and eccentricity factors of the equivalent sources empirically, by generating scalp data for a tangential source at 80% eccentricity using a standard 4-shell forward computation method (Stok 1986), and fitting the eccentricities of 3 parallel tangential sources that lie along the same radius as the original source. By symmetry, it is only necessary to match the scalp voltages within one plane in order to apply the results to the entire sphere. This allowed for a dense spacing of electrodes in order to make an accurate comparison between the multiple-shell data and the estimate of the new approximation. Scalp data were generated at 72 hypothetical equispaced electrode positions located in a plane through the center of the head, and separated by 5°. For the inverse calculation (i.e., fitting) we used the simplex algorithm (Nelder and Mead 1969) with a cut-off when changes in the residual variance within the simplex were smaller than 10^{-7}. Fig. 1 illustrates the equivalence of the 3 sources in a homogeneous sphere to the single source in the 4-shell sphere and shows the 72 electrode positions.

Equivalent sources at other locations and orientations are obtained using Eqn. 1. Fig. 2 illustrates the eccentricity relationships for the 3 head models which we have used to evaluate the method in this paper.

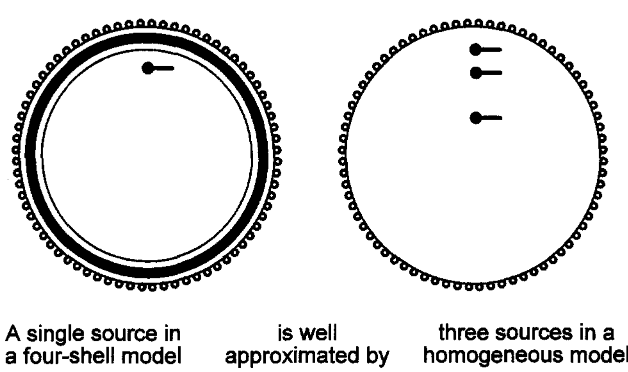

Fig. 1. Schematic illustration of the new approximation to a 4-shell spherical head model. In the 4-shell head (left), the skull layer is black. The equivalent 3 sources in the homogeneous medium are shown on the right. The 72 electrodes at which voltages were simulated are marked as small circles surrounding the outer shells. The locations and magnitudes of the equivalent sources are computed by fitting 3 sources in the homogeneous model to the scalp data generated at 80% eccentricity in the 4-shell model. The resulting magnitude and eccentricity factors may then be used in Eqn. 1 to generate scalp potentials due to a source in the 4-shell head by summing the activity of the 3 equivalent sources in the homogeneous model.

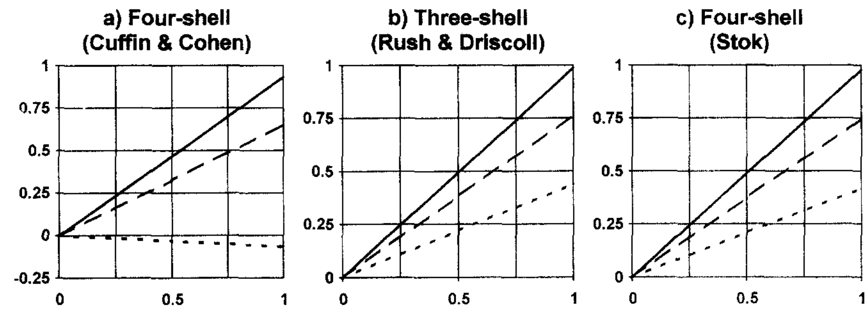

Fig. 2. Eccentricities of the 3 sources in the homogeneous model (y-axis) that generate surface distributions equivalent to a single source (x-axis) in (a) the Cuffin and Cohen 4-shell model, (b) the Rush and Driscoll 3-shell model, and (c) the Stok 4-shell model. The negative-going eccentricity of the innermost source in the Cuffin and Cohen approximation appears to be typical of head models with a relatively thin scalp and skull.

Evaluation of the approximation

Two measures were used to evaluate the approximation. In the first, the residual variance, i.e., the proportion of the variance of the original 4-shell data that was unexplained by the new approximation, was computed over all 72 electrode positions used for the model generation. In the second evaluation, using a more "realistic" set of electrode positions, the same head dimensions were applied in a test for model mismatch, comparing the goodness of fit for pairs of sources generated using the standard 4-shell computation method, and fitted using the standard computation, the new approximation and the Ary-transformed homogeneous model.

Method

Three head models were used in the evaluation. These were two 4-shell models with the dimensions and conductivities used by Cuffin and Cohen (1979) and by Stok (1986), and the 3-shell model originating from Rush and Driscoll (1968) and used by Ary et al. (1981) and Zhang and Jewett (1993). The parameters of the head models are given in Table I. Conductivities are essentially the same in each model and originate from Geddes and Baker (1967).

The forward computation of the 4-shell model is described in Stok (1986) and consisted of a translation into C of Stok's original Fortran program. The forward computation of the 3-shell model used the same program, setting the CSF and brain conductivities to be identical.

For each head model, the parameters for the new approximation were first generated by fitting the 3 equivalent homogeneous sources as described above. The residual variance (RV) was computed at 80% and 85% eccentricity (84% for the Stok parameters).

Secondly, for each head model the Ary approximation was optimized as follows by generating a table linking eccentricities in the 4- or 3-shell model to those in the homogeneous model. Using the same circle of 72 electrodes as for the new approximation, a tangential source was simulated using the multiple-shell model. Using the homogeneous model, a tangential source was fitted to obtain the equivalent eccentricity in the Ary approximation. This was repeated for eccentricities of the source in the multiple-shell model from 0% in steps of 1% to the radius of the brain. During subsequent fits using the Ary approximation, the equivalent eccentricity was obtained by linear interpolation within the table.

For the tests of model mismatch, the 21 electrode positions of the 10–20 system (including A1 and A2) were used. The simulated data were average referenced. Random pairs of sources were generated with the following constraints: (a) the sources were at least 20 mm apart, (b) all sources were within the upper half of the head (i.e., all sources were beneath the electrodes), and (c) all sources were within 84% eccentricity, i.e., within the brain. Each source of the pair was

TABLE I

Head model parameters used in the simulations.

	Conductivities ($1/\Omega$m)				Radii (prop. of head radius)		
	Scalp	Skull	CSF	Brain	Skull	CSF	Brain
Cuffin and Cohen (1979)	0.33	0.0042	1.0	0.33	0.9659	0.9205	0.8977
Rush and Driscoll (1968)	0.33	0.0041		0.33	0.9200		0.8700
Stok (1986)	0.33	0.0042	1.0	0.33	0.9467	0.8667	0.8400

TABLE II

Parameters of the new approximation to the multiple-shell computation method, as applied in Eqn. 1. Residual variances (RV) are given near the edge of the brain. RV was always smaller at lower eccentricities.

	Eccentricity factors			Magnitude factors			RV% at	
	e_1	e_2	e_3	m_1	m_2	m_3	0.8	0.85
Cuffin and Cohen (1979)	−0.0729	0.6521	0.9322	−0.0241	0.7939	0.1935	1×10^{-4}	1×10^{-3}
Rush and Driscoll (1968)	0.4407	0.7677	0.9895	0.4280	0.2872	0.0797	5×10^{-6}	1×10^{-5}
Stok (1986)	0.4191	0.7479	0.9791	0.4127	0.2669	0.0578	4×10^{-6}	2×10^{-5} *

* The residual variance was computed at radius 0.84 (edge of the brain) for the Stok head model.

given the same (unit) magnitude, and the corresponding potentials at the 21 electrodes were generated using the multiple-shell forward computation.

For each computation method, the data were then fitted using the same number of sources, with the original location and orientation used as the starting position. Two measures of error were recorded: (a) the distance between the original and the new location,

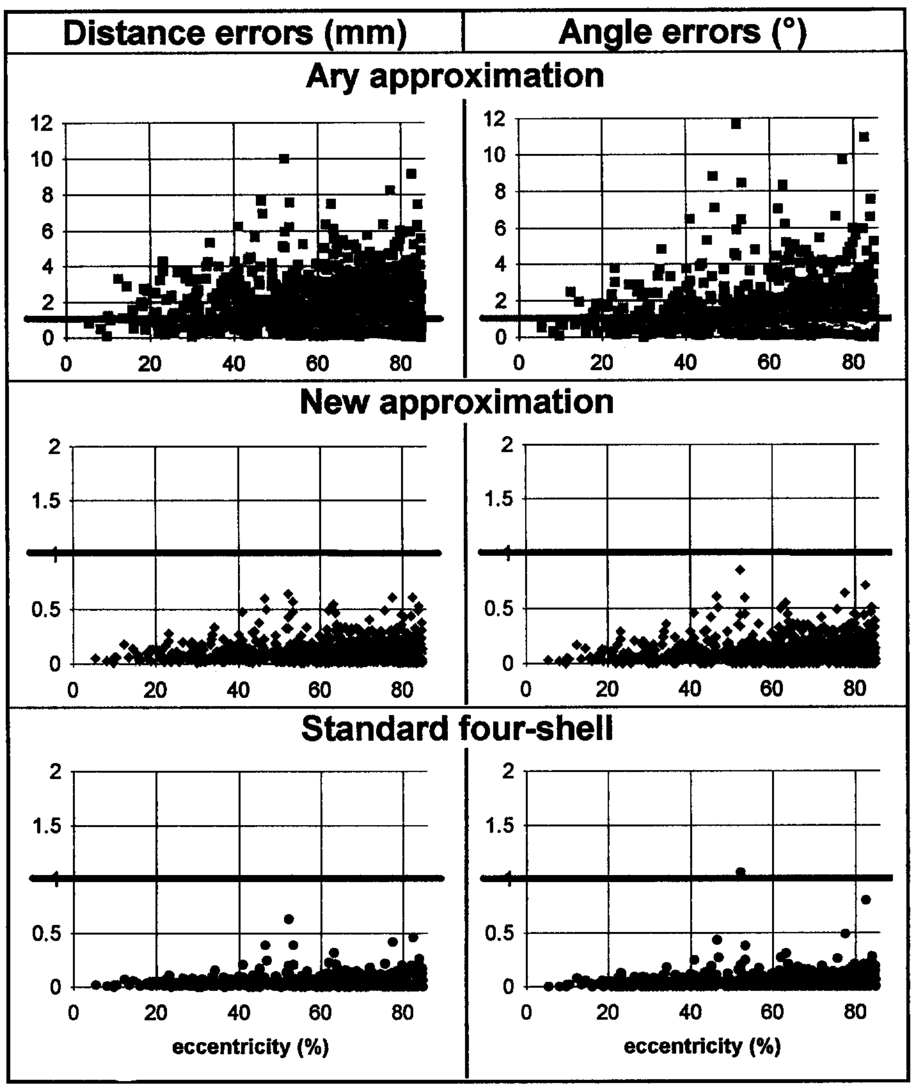

Fig. 3. Distance errors in mm (left) and angle errors in degrees (right) for 500 randomly selected pairs of sources modelled with the Cuffin and Cohen 4-shell model and fitted with the Ary approximation (top, squares), the new approximation (middle, diamonds), and the standard 4-shell computation method (bottom, circles). Note the different scale used for the Ary approximation. To aid comparison, the thick horizontal lines show the 1 mm/1° level on each diagram.

and (b) the angle between the original and the new orientation. Magnitude errors were not recorded, because in preliminary tests we observed that they provided little additional information: the size of magnitude errors correlated highly with distance and angle errors.

Each test was repeated for 500 randomly selected pairs of sources, yielding 1000 measures of distance and angle errors for each multiple-shell head model. The same sources were used for all 3 sets of head model parameters.

Timing

Computing speed was measured for a given computation method by generating surface data at 21 electrodes for 5000 randomly selected sources. In order to remove the overhead due to random generation of the sources, the time required to generate the sources was subtracted from the total. All calculations and modelling were performed on an IBM-compatible Compaq Deskpro 486/33L PC using a version of the BESA program (Scherg 1990), modified to incorporate the various computation methods.

Timing is influenced by cut-off values. In computations of the 4-shell data, series expansion was terminated by criteria based on the sum over the last 5 terms in the series. The expansion was terminated when the sum was smaller than 10^{-8}, or when the ratio between the sum and the total value of the expansion was smaller than 10^{-8} (Stok 1986, p. 105).

Results

Parameters of the new approximation

Table II shows the parameters obtained for the new approximation. The residual variance was higher at 85% than at 80% and depended on the head model. In all cases, the residual variance decreased monotonically with decreasing eccentricity. The values shown here are therefore upper limits for each head model, because they are near the outer edge of the brain. By the criterion of residual variance, the new approximation follows the original models very closely, since RV never exceeds 0.001%. In comparison, RVs at 85% eccentricity using the Ary model were 0.56% for the Cuffin and Cohen model, 1.55% for the Stok model, and 2.29% for the Rush and Driscoll model.

Fig. 2 shows the functions relating eccentricity of the 4-shell model to the eccentricity of the 3 equivalent sources in the homogeneous model.

Model mismatch errors

Distance and angle errors are plotted for all 1000 sources for the Cuffin and Cohen head model in Fig. 3

TABLE III

Statistics of model mismatch and fitting errors. Errors are given in terms of the distance and angle between source positions before and after fitting. Errors for the new approximation and multiple-shell fits are much lower than for the Ary approximation. Errors for the Ary approximation are lowest for the Cuffin and Cohen head dimensions.

Inverse method	Generating model	Distance errors (mm)			Angle errors (°)		
		Cuffin and Cohen	Stok	Rush and Driscoll	Cuffin and Cohen	Stok	Rush and Driscoll
Ary approx.	maximum	10.0	24.8	21.8	11.7	45.2	38.9
	99% limit	6.4	18.0	16.6	6.6	24.9	21.0
	95% limit	4.5	11.4	10.6	4.0	15.0	11.3
	median	1.48	3.96	3.88	0.93	3.47	3.45
New approx.	maximum	0.64	0.47	1.11	0.85	1.31	0.89
	99% limit	0.49	0.29	0.36	0.49	0.33	0.51
	95% limit	0.29	0.16	0.19	0.29	0.18	0.20
	median	0.07	0.04	0.04	0.06	0.03	0.04
Multiple-shell	maximum	0.63	0.54	0.90	0.80	1.56	1.40
	99% limit	0.22	0.22	0.25	0.25	0.25	0.32
	95% limit	0.11	0.12	0.12	0.13	0.13	0.14
	median	0.03	0.02	0.03	0.02	0.02	0.03

as a function of source eccentricity. Errors after fitting using the same head model led to the smallest errors. Errors using the new approximation were only slightly larger. In both cases, errors were mostly well below 1 mm and 1°. In contrast, errors using the Ary approximation were up to 10 mm and 11.7°.

Patterns of results for the other head models were similar, with one exception, that the mismatch errors were considerably (approximately 3 times) larger for the Ary approximation. A summary of the error distribution for all 3 multiple-shell head models is given in Table III.

Table III gives maxima, 99% and 95% error limits, and medians of the error measures. Model mismatch errors were greatest for the Ary approximation and were largest at large eccentricities. For the new approximation, 99% of the errors were below 0.49 mm or 0.51°, and all errors were below 1.2 mm and 1.4°. Errors when fitting with the original 4-shell computation method were the smallest, with 99% of the errors below 0.25 mm and 0.32°.

Timing

The times required for computing 5000 randomly selected sources was 1435 sec for the standard 4-shell computation, 44 sec for the new approximation, and 10.5 sec for the Ary approximation. The potential due to each source was computed at 21 scalp positions. The mean times per forward computation were therefore 13.7 msec for the standard computation, 0.42 msec for the new approximation, and 0.10 msec for the Ary approximation. The new approximation thus computed $13.7/0.42 = 32.6$ times faster than the standard method.

Discussion

The main aim of this study was to evaluate the new method for the forward computation of 3- or 4-shell head models. Residual variances when comparing data from a single source generated with a multiple-shell model and with the new approximation indicated a close match between the two methods (RV $< 0.005\%$). Model mismatch errors when fitting pairs of sources were extremely small and only slightly larger than the error produced when using the same starting and fitting model. Considering that other sources of error such as noise and electrode placement are likely to produce location errors well over 1 mm and orientation errors well over 1°, for all practical purposes the new approximation can be considered to be equivalent to the original computation. On the other hand, the new approximation computed 32 times faster than the standard method. Clearly, the exact speed relationship will depend on parameters of the implementation of the computer routines, but the implementation of the new approximation using the method of Brody et al. (1973) to compute sources in a homogeneous medium is much simpler than the standard method.

The approximation to the 3- or 4-shell model using 3 sources in a homogeneous sphere was an empirical discovery. To obtain the parameters of the approximation for a new set of head dimensions, the 3 sources need to be fitted to the data generated using a standard multiple-shell computation for a source at only one eccentricity and orientation. The simple relationships between eccentricity and magnitude of the source in the multiple-shell model and those of the equivalent 3 homogeneous sources suggests that an analytical solution exists that would allow the parameters of the approximation to be computed directly. De Munck et al. (1991) have shown that solutions exist which converge much more rapidly than the standard series expression for a source in a 3-shell spherical medium. However, their formula is not given in terms of a series of homogeneous sources, and it is not clear from their paper how economical computation would be in comparison to the method presented here.

The range of head model parameters over which the new approximation can be applied is not clear, although a number of tests lead us to believe that it remains accurate well beyond the range that would apply to a reasonably dimensioned head. Since so many parameters can be varied, it is difficult to test all the possible combinations. We have tried a number of extreme values and, in all cases, the residual variance at the edge of the brain remained below 0.01% and was mostly below 0.001%. The highest RV (0.008%) was obtained with a very thin scalp and skull, with skull, CSF and brain diameters of 0.99, 0.98 and 0.97 respectively. The approximation remains accurate when varying the skull conductivity by a factor of 10 in either direction.

More accuracy at cost of speed may be obtained by increasing the number of equivalent sources in the homogeneous sphere to four or more, although we did not attempt this. The use of two equivalent sources can be expected to provide results intermediate between the Ary approximation and the 3-equivalent-source approximation. The curves in Fig. 2 and the parameters in Table II suggest that two sources might be sufficient to match the Cuffin and Cohen head model well (one source has small relative magnitude and low dependency on eccentricity), but the other two head models clearly make use of all 3 equivalent sources.

In contrast to the new approximation, the Ary approximation resulted in much larger mismatch errors, and the residual variance of the fit between data generated by the multiple-shell computation and by the Ary approximation was at least 100 times higher than that for the new approximation. Although the nature of the mismatch errors for the Ary approximation is not the main topic of this paper, a number of points can be made that confirm and extend those made by Zhang and Jewett (1993). In general, our results replicate those of Zhang and Jewett, although some direct comparisons are difficult because they used a small number (47) of specific source pairs, whereas here a more statistical approach with a larger number (500) of randomly selected pairs was used. We used 21 as opposed to their 65 electrode locations. In addition, we restricted pairs to a separation greater than 20 mm. Zhang and Jewett obtained the greatest mismatch errors when the sources were close together. On average, mismatch errors increased with increasing eccentricity (cf. Fig. 3). Errors were smaller when both members of a pair had low eccentricity. Errors were larger when the orientations of the sources were co-planar, or when sources were oriented such that the maximum voltage variation on the scalp lay between electrodes. In general, these error patterns are consistent with the fact that the voltage distribution over the scalp due to a source differs between the (Ary-approximated) homogeneous and the multiple-shell models, particularly at large eccentricities. The residual arising from the relatively bad fit between homogeneous and multiple-shell scalp distributions for sources at large eccentricities acts like the addition of noise that leads to errors, particularly when fitting more than one source.

The magnitudes of the mismatch errors using the Cuffin and Cohen (1979) head model were smaller than those for the Stok (1986) and Rush and Driscoll (1968) models. This is probably due to the relative scalp and skull thicknesses used in the models. The Cuffin and Cohen model has a relatively thin scalp and skull compared to the others. For an 85 mm radius head, the scalp thickness in the Cuffin and Cohen model is 2.9 mm, compared with 4.5 mm for Stok and 6.8 mm for Rush and Driscoll. Relative skull thicknesses are 3.9, 6.8 and 4.2 mm respectively. If the conductivity of the CSF is ignored, in the limit that the scalp or skull thickness tends to zero, the multiple-shell and homogeneous models become identical. Therefore the Cuffin and Cohen model, being closer to the homogeneous model, should give rise to smaller mismatch errors with the Ary approximation.

The errors when fitting with the original model were small. The magnitude of these errors appears to be mainly a function of the numerical accuracy of the computations and the cut-off criterion employed in the simplex fitting algorithm. Since we used the original starting locations for fitting the source pairs, there were no cases in which sources moved to incorrect local minima.

In conclusion, the new approximation to multiple-shell spherical head models provides a fast alternative to the conventional computing methods with negligible loss of accuracy. In contrast, model mismatch errors arising when using the Ary approximation on data generated using a multiple-shell model are an order of magnitude larger. The models applied in this paper were all spherical. Model mismatch errors arising from the use of the 4-shell spherical model on data from the realistic head remain to be investigated.

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