Practical adaptive quantum tomography

Christopher Granade

www.cgranade.com/research/talks/aip-2016

Joint work with:

Christopher Ferrie   Steven T. Flammia

Centre of Excellence for Engineered Quantum Systems
University of Sydney, NSW

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The Tomographic Problem

$$\text{States: } \rho \in \mathbb{C}^{d\times d}$$

$$\text{Measurements: } P_k \in \mathbb{C}^{d\times d}$$

$$\Pr(\text{click}|\rho; P_k) = \text{Tr}(P_k\rho)$$

Given data record $D = \{d_1, \dots, d_N\}$, what should we report as our estimate $\hat{\rho}$?

"Conventional" Approach

- Expand ρ, P_k in a basis $\{B_1, \ldots, B_{d^2-1}\}$ of operators.

$$\rho = \frac{\mathbb{1}}{d} + \sum_{j=1}^{d^2-1} \theta_j B_j$$

$$P_k = \frac{\mathbb{1}}{d} + \sum_{j=1}^{d^2-1} x_{kj} B_j.$$

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- Solve.

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}\right)^{-1} \boldsymbol{X}^{\mathrm{T}} \left(\hat{\boldsymbol{f}} - \frac{1}{d}\right)$$

Oops.

Problems w/ linear inversion estimator:

- Doesn't enforce $\rho \geq 0$.
- Doesn't weight according to variance of $\hat{\boldsymbol{f}}$ about $\boldsymbol{f}$.
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Alternative: maximum likelihood estimator.

$$\hat{\rho}_{\text{MLE}} := \arg\max_{\rho} \prod_k \Pr(d_k | \rho; P_k)$$

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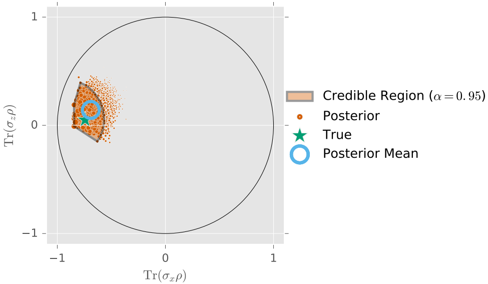
$$\hat{\rho}_{\mathrm{MLE}} := \arg\max_{\rho} \prod_{k} \Pr(d_k | \rho; P_k)$$

- Still tricky to get error bars.
- Reports rank-deficient estimates.
  Blume-Kohout DOI 10/cn772j
- Can't include prior information.

Bayesian Approach

Use to find $\Pr(\rho|\text{data})$ from $\Pr(\text{data}|\rho)$.

- Particle filtering: easy to use numerical impl.
  qinfer.org • 1610.00336
- Can include prior information.
- Built-in error bars.



Huszár and Houlsby DOI 10/s86, Granade *et al.* DOI 10/bhdw

Tomography as Optimization

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Ferrie DOI 10/bchr

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Express tomography as optimization problem:

$$|\hat{\psi}\rangle = \underset{|\phi\rangle}{\arg\max}\, |\langle\phi|\psi\rangle|^2$$

Overlap can be est. by measuring *test states* $|\phi\rangle$. Thus, optimization algorithms choose test states when they evaluate objective function f:

$$\begin{aligned} f(\phi) &:= |\langle\phi|\psi\rangle|^2 \\ d_k &\sim \operatorname{Bin}(N, f(\phi_k)). \end{aligned}$$

Ferrie DOI 10/bchr

SPSA

One particularly useful optimization algorithm: **Simulntaneous Perturbative Stochastic Approximation**

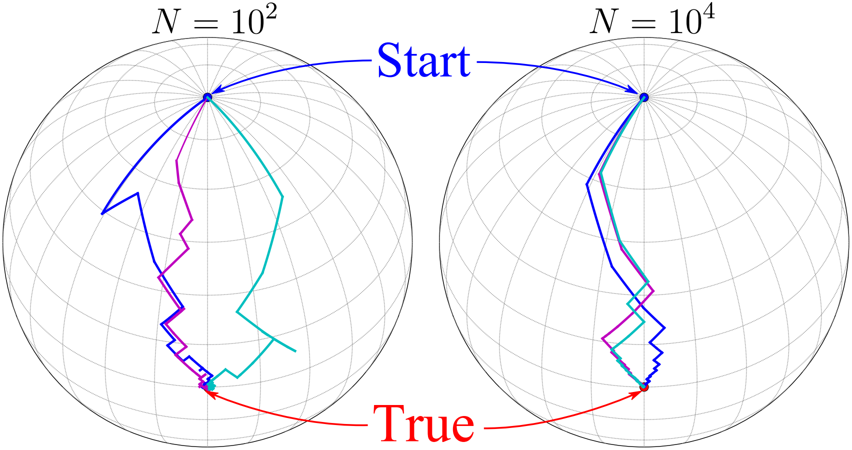
- Pick initial test state $|\phi_0\rangle$ at random.
- For each iteration k:
  - Estimate $f(\phi_k)$ from measurements.
  - Pick perturbation δ at random.
  - Estimate $f(\phi_k \pm \alpha_k \delta)$.
  - Let $\phi_{k+1} = \phi_k + \beta_k (\hat{f}(\phi_k + \alpha_k \delta) - \hat{f}(\phi_k - \alpha_k \delta)) / 2\alpha_k$.

Parameters α_k, β_k control gain and step sizes; choose to decay exponentially with k.

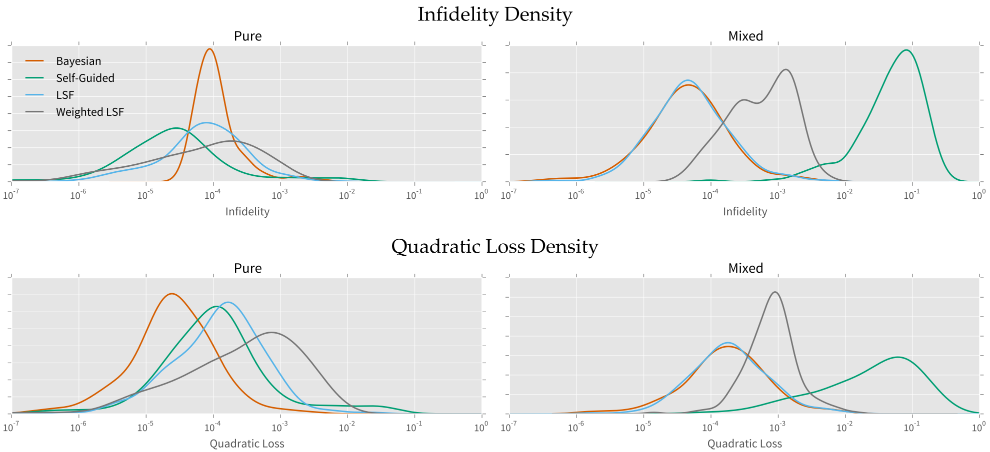
Spall goo.gl/JlQ3dm, Ferrie DOI 10/bchr

Self-Guided Tomography

SPSA gives a particularly useful and simple *heuristic* for choosing test states. (About 5 lines in Python to implement.)

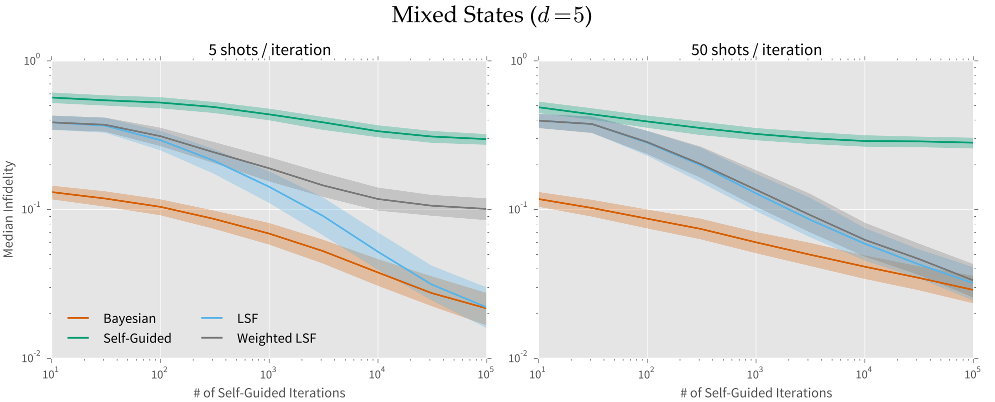


- Perform self-guided tomography, keeping data record.
- Re-analyze with Bayesian inference.

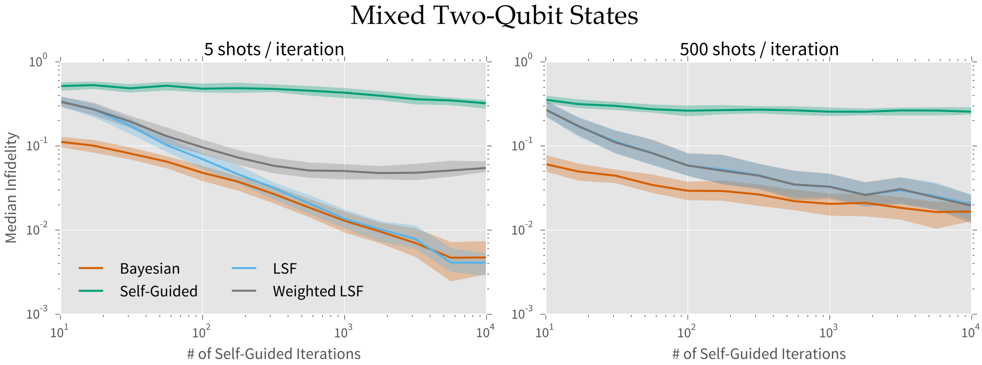


Granade, Ferrie, and Flammia 1605.05039

Important insight: SGT provides useful heuristic *even when its assumptions are wrong*.



NB: Test states for process tomography are product states.



Conclusions

- Bayesian tomography gives principled and easy-to-use approach to state estimation.
- Self-guided tomography gives a practical heuristic for adaptive experiment design.