Quantum Bootstrapping

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Joint work with:

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July 14, 2014

LFQIS 2014

Q100

We want to build a quantum computer.

Q100

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Need to push past what a classical computer can do. How do we get to 100 qubits?

Building Large Systems: Computational Limits

Computational limits affect many aspects of building large quantum systems:

- Characterization of H

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- Verification of control

Building Large Systems: Computational Limits

Computational limits affect many aspects of building large quantum systems:

- Characterization of H
- Characterization of controls
- ~~Design of control sequences~~
- Verification of control

Here, we focus on characterization and verification.

Bootstrapping to Q100

Express challenges in terms of *simulation*, then use quantum simulators.

Use small quantum simulators to characterize and verify large devices, bootstrap up to Q100 scale.

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  - Sequential Monte Carlo: algorithm for Bayesian inference

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- Learning control distortions

Bayesian Approaches to Characterization and Control

Likelihood Function

Model data collection as a probability distribution:

$$\Pr(d|\underline{x};\underline{e})$$

d: data, $\underline{x}$: model, $\underline{e}$: experiment

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Example

Larmor precession at an unknown ω and T_2:

$$H(\omega) = \frac{\omega}{2}\sigma_z, \quad |\psi_{\text{in}}\rangle = |+\rangle, \quad M = \{|+\rangle \langle +|, |-\rangle \langle -|\}$$

$$\Pr(d = 0|\underline{x} = (\omega, T_2); \underline{e} = (t)) = \frac{1}{2}(1 - e^{-t/T_2}) + e^{-t/T_2} \cos^2(\omega t/2)$$

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Given a likelihood, we can reason about

$$\Pr(\underline{x}|d,\underline{e}),$$

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$\implies$ Simulation is a resource for learning.

Estimate $\underline{\hat{x}}$ as the expectation over $\underline{x}$,

$$\underline{\hat{x}} = \mathbb{E}[\underline{x}] = \int \underline{x} \Pr(\underline{x}) \, d\underline{x}.$$

Sequential Monte Carlo

SMC (aka *particle filter*)**:** numerical algorithm for generating samples from a distribution, using a transition kernel.

$$\text{prior} \xrightarrow{\text{Bayes' Rule}} \text{posterior}$$

Posterior samples then approximate $\int / \mathbb{E}$.

SMC Approximation

$$\Pr(\underline{x}) \approx \sum_{i}^{n} w_i \delta(\underline{x} - \underline{x}_i)$$

(Doucet and Johansen 2011; Huszár and Houlsby *10/s86*; *Granade* et al. 2012 *10/s87*)

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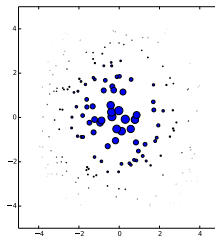
QInfer Open-source implementation for quantum info.

(Doucet and Johansen 2011; Huszár and Houlsby *10/s86*; *Granade* et al. 2012 *10/s87*)

Ambiguity and Impovrishment

Ambiguity in SMC approximation:
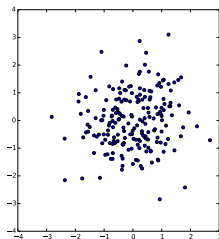
density   weight

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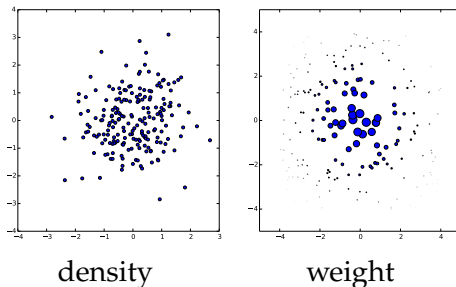
Using weight is less numerically stable, results in smaller *effective* number of particles.

$$n_{\text{ess}} := 1/\sum_i w_i^2$$

Numerical Stability and Resampling

As data D is collected, $\Pr(\underline{x}_i|D) \to 0$ for most initial particles $\{x_i\}$.

- $\Rightarrow n_{\text{ess}} \to 0$ as data is collected.

Resampling: move information from weights to the density of SMC particles.

- Resampling when $n_{\text{ess}}/n \leq 0.5$ preserves stability.
- Monitoring n_{ess} can herald some kinds of failures.

Liu and West Algorithm

Draw new particles $\underline{x}'$ from kernel density estimate:

$$\Pr(\underline{x}') \propto \sum_i w_i \exp\left((\underline{x}' - \underline{\mu}_i)^{\mathrm{T}} \underline{\underline{\Sigma}}(\underline{x}' - \underline{\mu}_i)\right)$$

$$\underline{\mu}_i := a\underline{x}_i + (1-a)\mathbb{E}[\underline{x}] \qquad \underline{\underline{\Sigma}} := h^2 \operatorname{Cov}[\underline{x}] \qquad w_i' := 1/n$$

(West 1993; Isard and Blake 1998 *10/cc76f6*; Liu and West 2001)

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Parameters a and h can be set based on application:

- $a = 1, h = 0$: Bootstrap filter, used in state-space applications like CONDENSATION.
- $a^2 + h^2 = 1$: Ensures $\mathbb{E}[\underline{x}'] = \mathbb{E}[\underline{x}]$ and $\operatorname{Cov}(\underline{x}') = \operatorname{Cov}(\underline{x})$, but assumes unimodality.
- $a = 1, h \geq 0$: Allows for multimodality, emulating state-space with synthesized noise.

(West 1993; Isard and Blake 1998 *10/cc76f6*; Liu and West 2001)

Putting it All Together: The SMC Algorithm

1. Draw $\{\underline{x}_i\} \sim \pi$, set $\{w_i\} = 1/n$.
2. For each datum $d_j \in D$:
   1. $w_i \leftarrow w_i \times \Pr(d_j | \underline{x}_i; \underline{e}_j)$.
   2. Renormalize $\{w_i\}$.
   3. If $n_{\text{ess}}/n \leq 0.5$, resample.
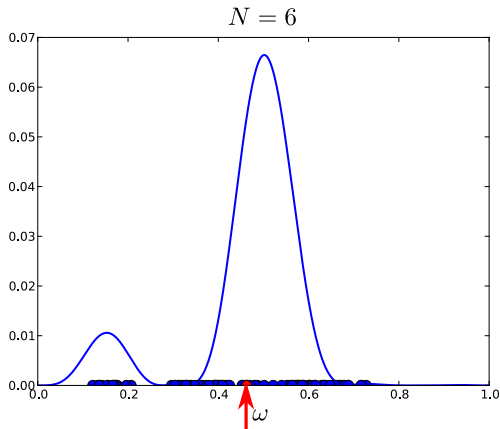3. Report $\hat{\underline{x}} := \mathbb{E}[\underline{x}] \approx \sum_i w_i \underline{x}_i$.

Sequential Monte Carlo
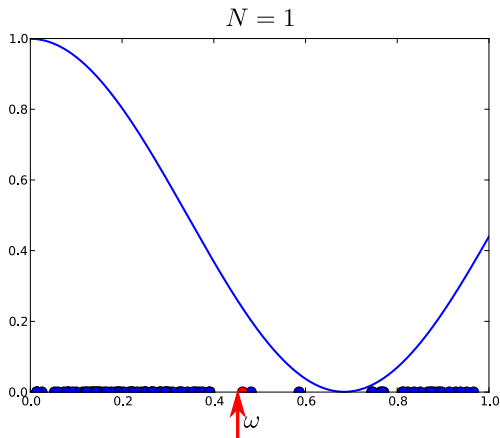
With SMC and resampling, particles move towards the true model as data is collected.

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$N = 6$

Sequential Monte Carlo

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$N = 11$
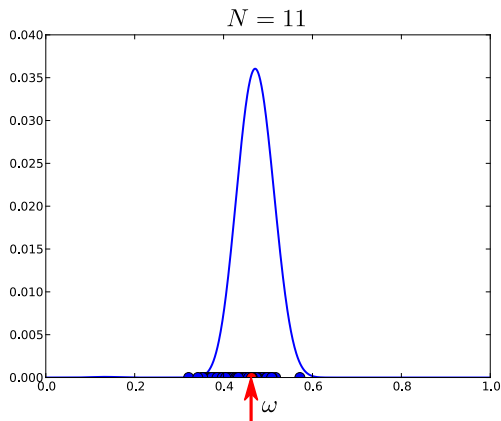
Before bootstrapping, a few examples of SMC w/ classical resources:

Near-Optimality for cos^2

Adaptive experiment design w/ Newton Conjugate-Gradient,
vs. optimum from Bayesian Cramér-Rao Bound:
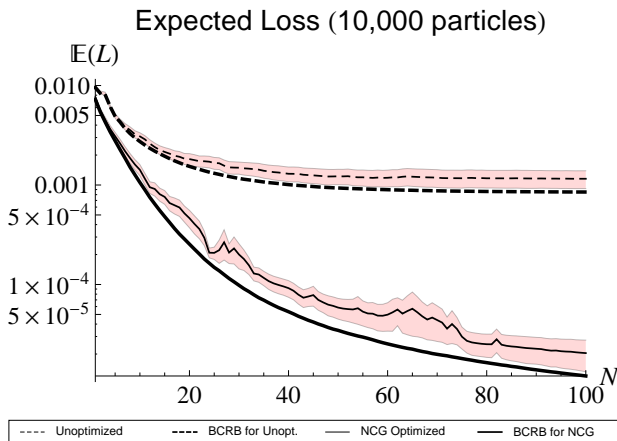
(*Granade* et al. 2012 *10/s87*)

Randomized Benchmarking Example

Applying sequences of random Clifford gates *twirls* errors in a gateset, such that they can be simulated using depolarizing channels.

Imperfect twirling sequence

Equivalent depolarizing channel

Clifford

Decomposition into gates
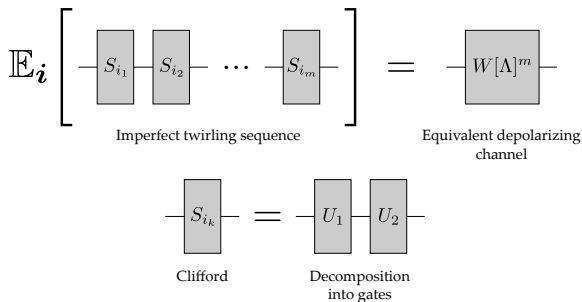
(Knill et al. 2008 *10/cxz9vm*; Magesan et al. 2012 *10/tfz*; Magesan et al. 2012 *10/s8j*)

Randomized Benchmarking Example

Interpret survival probability as likelihood. For interleaved case, the lowest-order model is:

$$\Pr(\text{survival}|A, B, \tilde{p}, p_{\text{ref}}; m, \text{mode}) = \begin{cases} Ap_{\text{ref}}^m + B & \text{reference} \\ A(\tilde{p}p_{\text{ref}})^m + B & \text{interleaved} \end{cases}$$
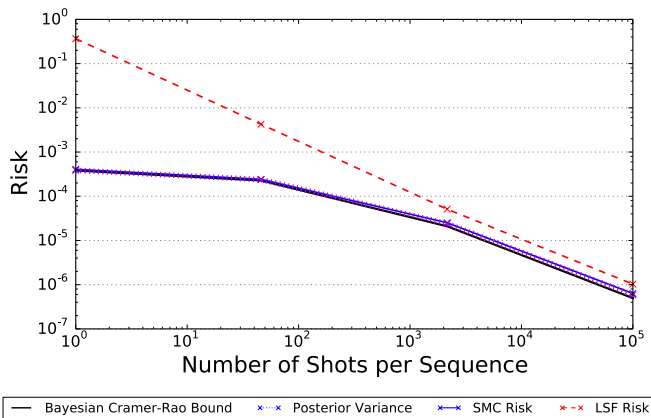
A, B state preparation and measurement

m sequence length

p_{ref} reference depolarizing parameter

$\tilde{p}$ depolarizing parameter for gate of interest

(*Granade*, Ferrie and Cory 2014 *1404.5275*)

Randomized Benchmarking Example

Using SMC, useful conclusions can be reached with
significantly less data than with least-squares fitting.

(*Granade*, Ferrie and Cory 2014 *1404.5275*)

SMC in Nitrogen Vacancy Centers

Would like to learn hyperfine coupling $\underline{\underline{A}}$ between e^- spin $\underline{S}$ and ^{13}C spin $\underline{I}$.

$$H(\underline{x}) = \Delta_{\text{zfs}} S_z^2 + \gamma(\underline{B} + \underline{\delta B}) \cdot \underline{S} + \underline{S} \cdot \underline{\underline{A}} \cdot \underline{I}$$
$$\underline{x} = (\Delta_{\text{zfs}}, \underline{\delta B}, \underline{\underline{A}}, \alpha, \beta, T_{2,e}^{-1}, T_{2,C}^{-1})$$
$$\alpha, \beta : \text{visibility parameters}$$

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$$\alpha, \beta : \text{visibility parameters}$$

- Analytic estimate sensitive to error $\underline{\delta B}$ in static field.
- Use multiple $\underline{B}$ settings to decorrelate $\underline{\delta B}$, $\underline{\underline{A}}$.
- Each experiment informs about multiple parameters

Preliminary Results from Rabi Experiment

As a test, attempt to learn $\underline{\delta B}$, Δ_{zfs} $\delta\omega_{\text{Rabi}}$ and A_N (coupling to nitrogen spin).
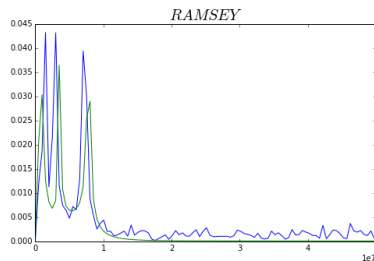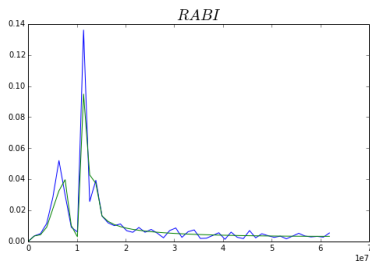
Preliminary Results from Rabi Experiment
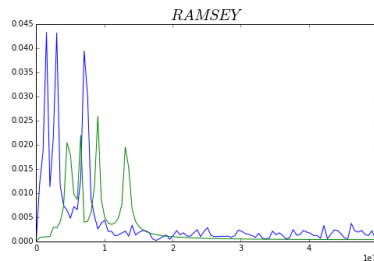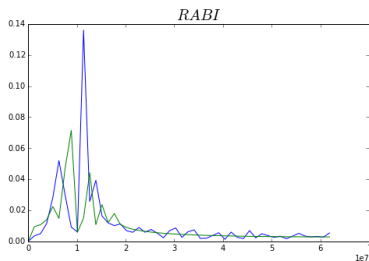
Simulation with prior mean

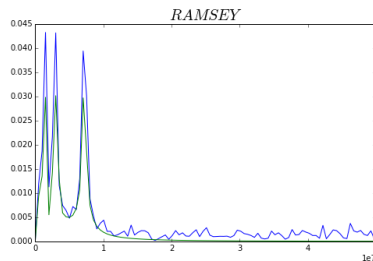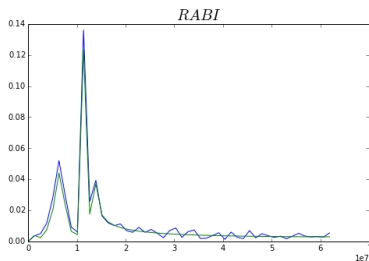Each average: 30k shots per point, 100 Rabi points + 200 Ramsey points

Preliminary Results from Rabi Experiment

RABI

RAMSEY

Simulation with posterior mean, 20 averages

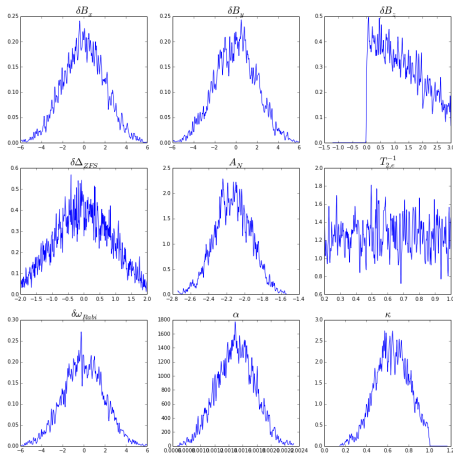Each average: 30k shots per point, 100 Rabi points + 200 Ramsey points

Preliminary Results from Rabi Experiment

Simulation with posterior mean, 486 averages

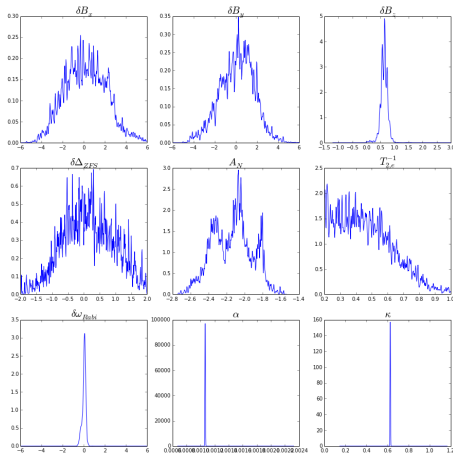Each average: 30k shots per point, 100 Rabi points + 200 Ramsey points

Preliminary Results from Rabi Experiment

Marginals of prior

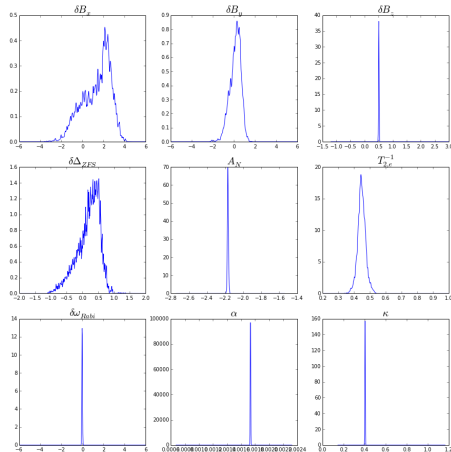Each average: 30k shots per point, 100 Rabi points + 200 Ramsey points

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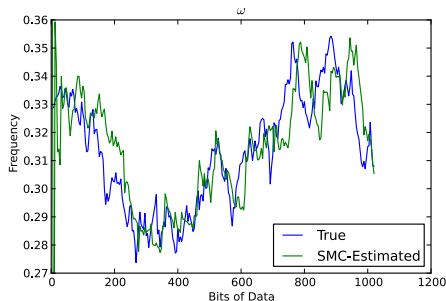

Marginals of posterior, 486 averages

Each average: 30k shots per point, 100 Rabi points + 200 Ramsey points

We can do a few more things with SMC, some of which will be very useful in the semiquantum case.

State-Space SMC

Can move particles at each timestep $\underline{x}(t_k) \sim \Pr(\underline{x}(t_k)|\underline{x}(t_{k-1}))$.

This represents *tracking* of a stochastic process.

Confidence and Credible Regions

Characterizing uncertainty of estimates is critical for many applications:

Definition (Confidence Region)

X_α is an α-confidence region if $\Pr_D(x_0 \in X_\alpha(D)) \geq \alpha$.

(*Granade* et al. 2012 *10/s87*; Ferrie 2014 *10/tb4*)

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Credible regions can be calculated from posterior $\Pr(\underline{x}|D)$ by demanding

$$\int_{X_\alpha} d \Pr(\underline{x}|D) \geq \alpha.$$

(*Granade* et al. 2012 *10/s87*; Ferrie 2014 *10/tb4*)

High Posterior Density

Want credible regions that are *small* (most powerful).

- Posterior covariance ellipses (PCE)— good for approximately normal posteriors
- Convex hull— very general, but verbose description
- Minimum volume enclosing ellipses (MVEE)— good approximation to hull

(*Granade* et al. 2012 *10/s87*; Ferrie 2014 *10/tb4*)

Comparison of HPD Estimators

For multimodal distributions, clustering can be used to exclude regions of small support.
For a noisy coin model (heads probability p, visibility η):
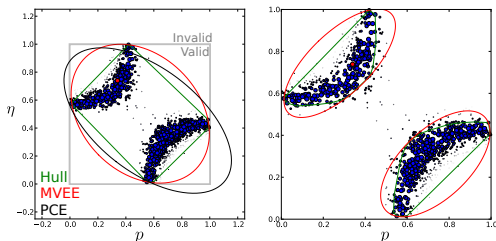
Left, no clustering. Right, DBSCAN.

Plot courtesy of Chris Ferrie. (Ferrie 2014 *10/tb4*)

Bayes Factors and Model Selection

Drunk Under the Streetlights

In SMC update $w_i \mapsto w_i \times \Pr(d|\underline{x}; \underline{e})/\mathcal{N}$,

$$\mathcal{N} = \mathcal{N}(d) \approx \Pr(d|\underline{e}).$$

Is this useful?

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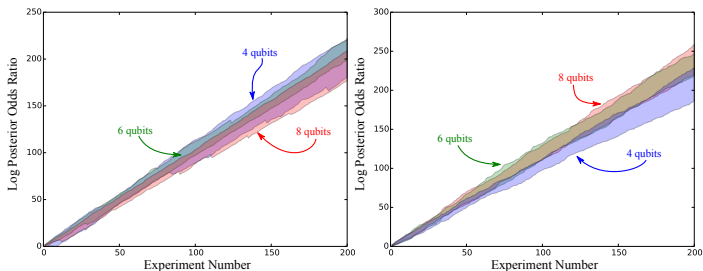$$\mathcal{N} = \mathcal{N}(d) \approx \Pr(d|\underline{e}).$$

Is this useful?

Collecting normalizations $\mathcal{N}_A$ and $\mathcal{N}_B$ for models A, B at each step gives (assuming uniform prior over A, B)

$$\text{Bayes factor} = \frac{\Pr(D|A;\underline{e})}{\Pr(D|B;\underline{e})} \approx \frac{\prod_{d \in D} \mathcal{N}_A(d)}{\prod_{d \in D} \mathcal{N}_B(d)}$$

For full data record, can multiply normalization records to select A versus B.

For example, deciding between linear- (left) and
complete-graph (right) Ising models:

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tdk*)

Towards Bootstrapping

SMC uses *simulation* as a resource for *learning*.

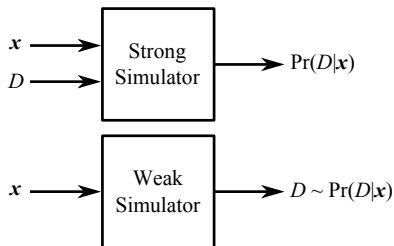Simulation calls: main cost to SMC (n each Bayes update).

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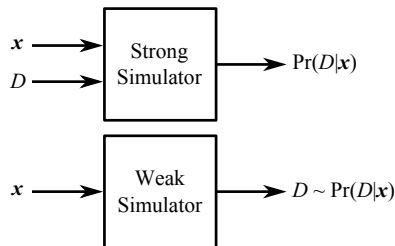Simulation calls: main cost to SMC (*n* each Bayes update).

Big Idea

Use quantum simulation to extend SMC past classical resources.

Weak and Strong Simulation

$x \longrightarrow$ [Strong Simulator] with $D \longrightarrow$ and output $\longrightarrow \Pr(D|x)$

$x \longrightarrow$ [Weak Simulator] and output $\longrightarrow D \sim \Pr(D|x)$

(Ferrie and Granade 2014 *10/tdj*)

Weak and Strong Simulation

Quantum simulation produces data, not likelihoods. Must sample to estimate likelihood.

Adaptive Likelihood Estimation

Solution

Treat estimating the likelihood as a secondary estimation problem:

Learn likelihood of untrusted system from frequencies of trusted system.
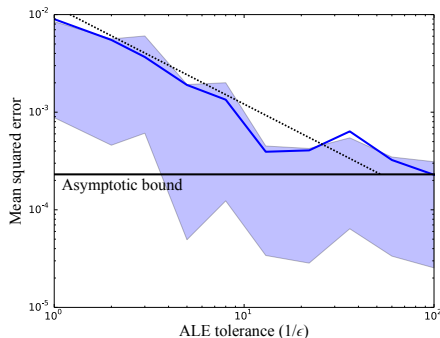
Adaptive Likelihood Estimation

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SMC is robust to likelihood estimation errors.

Performance of SMC+ALE

Ex: Simple 'photodetector' model $\Pr(0|p) = \alpha p + (1-p)\beta$

α, β known bright, dark references

(Ferrie and Granade 2014 *10/tdj*)

ALE Example: Two-Outcome Models

Given:

d result of measurement

D' set of samples from weak simulator

Hedged binomial estimate of likelihood ℓ from frequency k/K:

$$\hat{\ell} = \frac{k + \beta}{K + 2\beta},$$

where $\beta \approx 0.509$, $k := |\{d' \in D' | d' = d\}|$, $K = |\{D'\}|$.

(Ferrie and Blume-Kohout 2012 *10/tf2*, Ferrie and Granade 2014 *10/tdj*)

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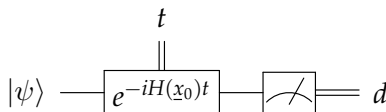where $\beta \approx 0.509$, $k := |\{d' \in D' | d' = d\}|$, $K = |\{D'\}|$.

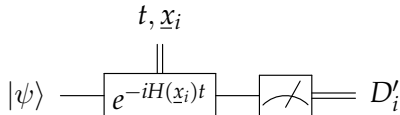Variance well-known, so collect until a fixed *tolerance* is reached.

(Ferrie and Blume-Kohout 2012 *10/tf2*, Ferrie and Granade 2014 *10/tdj*)

Quantum Likelihood Evaluation

Compare *classical* outcomes of unknown and trusted systems.

For each $\underline{x}_i$:

- repeatedly sample from quantum simulation of $e^{-it\underline{x}_i}$, getting D'_i.
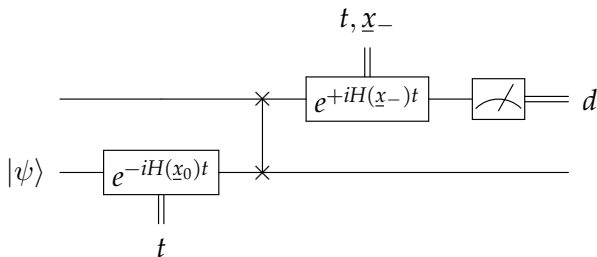- estimate $\hat{\ell}_i$ from D'_i.

SMC update: $\qquad w_i \mapsto w_i \hat{\ell}_i / \sum_i w_i \hat{\ell}_i.$

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tf3*)

QLE can work, but as $t \to \infty$, $\Pr(d|\underline{x}; t) \rightsquigarrow 1/\dim \mathcal{H}$.
Thus, $t \geq t_{\mathrm{eq}}$ is uninformative.

By CRB, error then scales as $O(1/Nt_{\mathrm{eq}}^2)$.
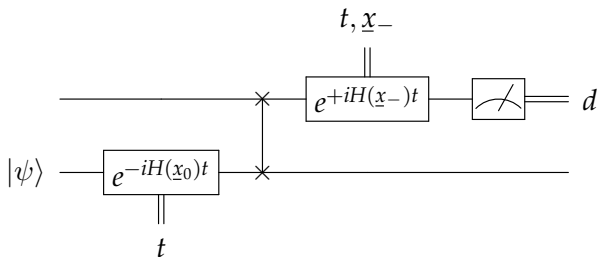
Interactive QLE

Solution: couple unknown system to a quantum simulator, then invert evolution by hypothesis $\underline{x}_-$.

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tf3*)

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Solution: couple unknown system to a quantum simulator, then invert evolution by hypothesis $\underline{x}_-$.

Echo

If $\underline{x}_- \approx \underline{x}_0$, then $\left| \langle \psi | e^{-it(H(\underline{x}_0) - H(\underline{x}_-))} | \psi \rangle \right|^2 \approx 1$.

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tf3*)

Posterior Guess Heuristic

Inversion connects the model and experiment spaces.
Use this duality as a heuristic for experiment design.

- Choose $\underline{x}_-, \underline{x}'_- \sim \Pr(\underline{x})$, the most recent posterior.
- Choose $t = 1/\|\underline{x}_- - \underline{x}'_-\|$.
- Return $\underline{e} = (\underline{x}_-, t)$.

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tf3*)
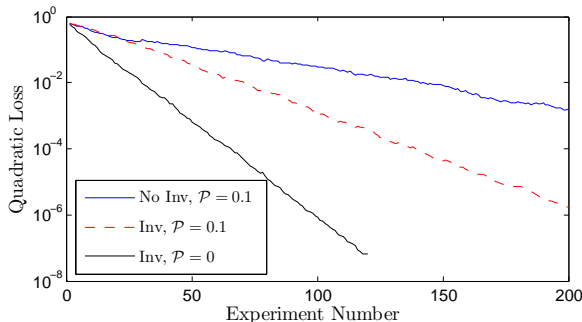
Alternate Interpretation

QHL finds $\hat{\underline{x}}$ such that $H(\hat{\underline{x}})$ most closely approximates "unknown" system H_0.

Gives an α-credible bound on error introduced by replacing $H_0 \to H(\hat{\underline{x}})$.

Ising Model on Spin Chains

Hamiltonian: nearest-neighbor Ising models on a chain of nine qubits.

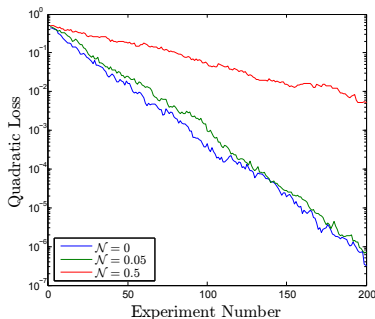Interactivity allows for dramatic improvements over QLE.

$\mathcal{P}$: adaptive likelihood estimation tolerance.

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tf3*)

Ising Model on the Complete Graph

With IQLE, can also learn on complete interaction graphs. We show the performance as a function of the depolarization strength $\mathcal{N}$.
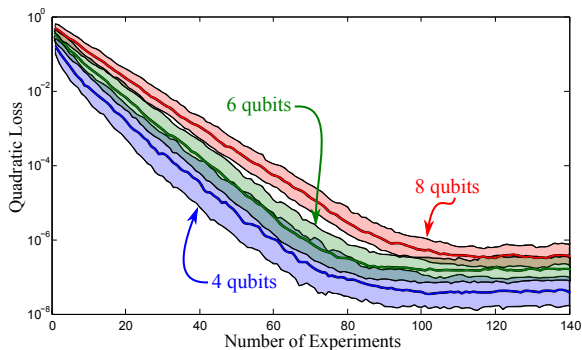

$\mathcal{N}$: depolarizing noise following SWAP gate.

Ising Model with the Wrong Graph

Simulate with spin chains, suppose "true" system is complete, with non-NN couplings $O(10^{-4})$.

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tdk*)

Scaling Parameter

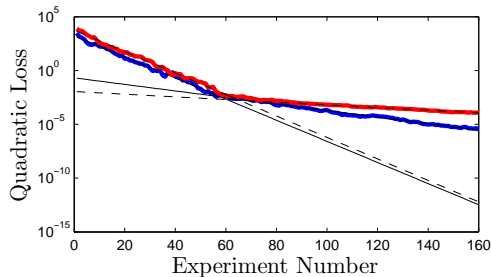dim $\underline{x}$, not dim $\mathcal{H}$, determines scaling of IQLE.

Figure : 4 qubit (red) and 6 qubit (blue) complete graph IQLE

Scaling and Dimensionality

In spin-chain and complete graph, average error decays
exponentially,

$$L(N) \propto e^{-\gamma N}$$

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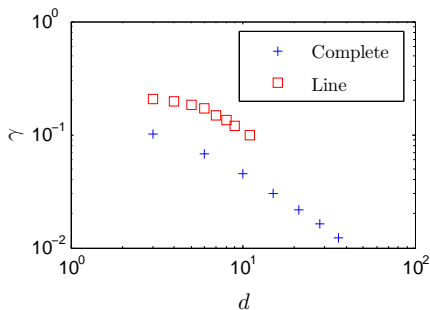Assess scaling by finding $\gamma = \gamma(\dim \underline{x})$:

With quantum simulation, learning *may* scale efficiently.

(Wiebe, *Granade*, Ferrie and Cory 2014 *10/tf3*)
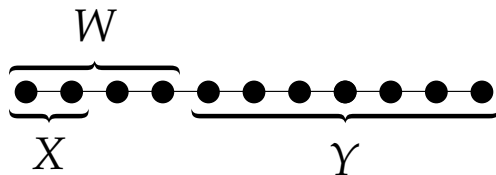
SMC + IQLE:

- Possibly scalable with quantum resources.
- Robust to finite sampling.
- Robust to approximate models.

Still requires simulator be at least as large as system of interest.

Information Locality

To go further, we want to *localize* our experiment, such that we can simulate on a smaller system.

Measure on *X*, simulate on *W*, and ignore all terms with support over *Y*.

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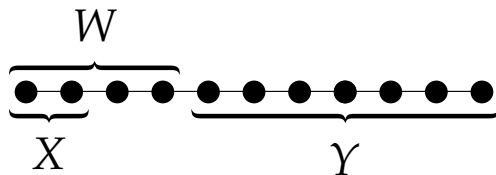
Measure on *X*, simulate on *W*, and ignore all terms with support over *Y*.

Gives *approximate* model that can be used to learn Hamiltonian restricted to *X*.

Local and Global Particle Clouds

To reconstruct the entire system, we need to combine data from different partitions.

Separate out one partition W_k at a time, maintain a *global* cloud of particles.

Local and Global Particle Clouds

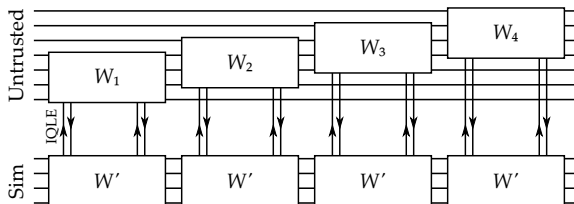Initialize $\{\underline{x}_i\}$ over entire system. Then, for each simulated subregister W_k:

1. Make "local" particle cloud $\{\underline{x}_i|_{W_k}\}$ by slicing $\{\underline{x}_i\}$.
2. Run SMC+IQLE with $\{\underline{x}_i|_{W_k}\}$ as a prior.
3. Ensure that the final "local" cloud has been resampled (has equal weights).
4. Overwrite parameters in "global" cloud $\{\underline{x}_i\}$ corresponding to post-resampling $\{\underline{x}_i|_{W_k}\}$.

In this way, all parameters are updated by an SMC run.

Q50 Example

Goal: characterize a 50-qubit Ising model (complete graph) with unknown ZZ couplings.

All Hamiltonian terms commute, but initial state doesn't. Let A_X be observable, $A_{X'}$ be sim. observable.

$$\|A_X(t) - A_{X'}(t)\| \leq \|A_X(t)\|(e^{2\|H|_Y\|t} - 1)$$
$$\Rightarrow t \leq \ln\left(\frac{\delta}{\|A_X(t)\|} + 1\right)(2\|H|_Y\|)^{-1},$$

where δ is the tolerable likelihood error.

Example Q50 Run
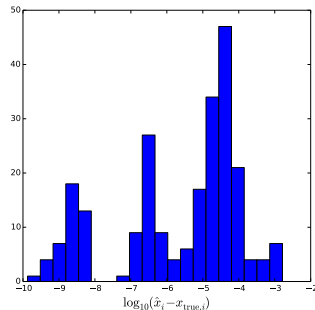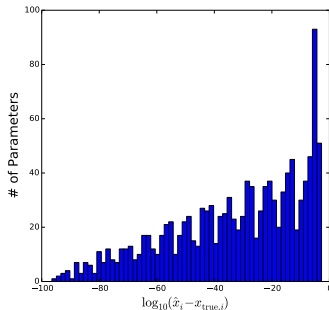
$|X_k| = 4$, $|W_k| = 8$, $n = 20,000$, $N = 500$, exp. decaying interactions.

NB: 1225 parameter model, L_2 error of 0.3%.

Example Q50 Run
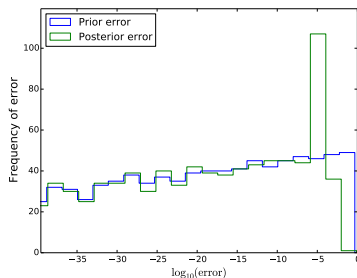
$|X_k| = 4$, $|W_k| = 8$, $n = 20,000$, $N = 500$, exp. decaying interactions.

NB: 1225 parameter model, L_2 error of 0.3%.

Lieb-Robinson Bounds

More generally, for $[H|_W, H_Y] \neq 0$, use *Lieb-Robinson bound*.
If interactions between X and Y decay sufficiently quickly, then
there exists C, μ and v s. t. for any observables $A_X(t)$, B_Y:

$$\|[A_X(t), B_Y]\| \leq C\|A_X(t)\|\|B_Y\|\|X||Y|(e^{v|t|} - 1)e^{-\mu d(X,Y)}$$

This *guarantees* that error due to truncation is bounded if we
choose small t.

(Hastings and Koma 2006 *10/cddqgz*; Nachtergale and Sims 2006 *10/d9xwfg*)

Lieb-Robinson Bounds

Can find bound in terms of Hamiltonian by considering H site-by-site.
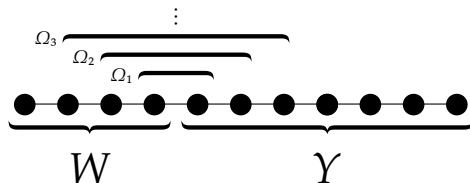
Let H_j be the Hamiltonian term containing distance-j interactions between W and Y, acting on sites Ω_j.

$$\|A(t) - e^{iH|_W t} A e^{-iH|_W t}\| \leq \sum_j C\|A\|\|H_j\|\|X\||\Omega_j|e^{-\mu j}(e^{v|t|} - 1)$$

"Shaking"

Can improve the Lieb-Robinson bound by alternating between simulator and system. Using $r \approx vt$ SWAP gates, error is $O(t)$.
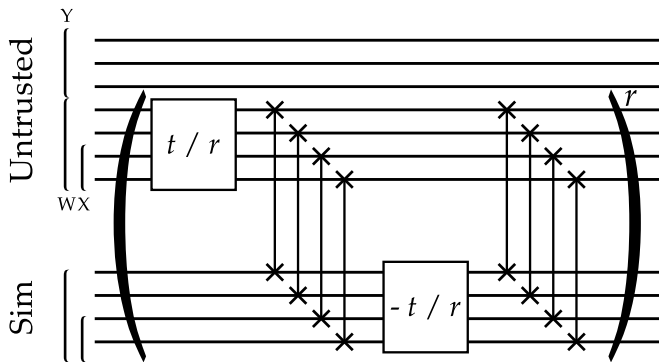
Distortion Operators

Control affected by classical system, *distorts* controls from intended pulse.

$$H(t) = H(t; g[\underline{p}]),$$

where $\underline{p}$ is a pulse, $\underline{q} = g[\underline{p}]$ is the distorted pulse.

(Hincks, Granade, Borneman and Cory *forthcoming*)

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- Cross-talk of control lines
- Phase calibration / reference frame

(Hincks, Granade, Borneman and Cory *forthcoming*)

Learning Controls with IQLE

Learning g is also important to bootstrapping. Suppose:

- $H \neq H(t)$
- $g : \underline{y} \mapsto \underline{x}$ distorts static Hamiltonians
- g parameterized by $\underline{z}$

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Doesn't help with all distortions, but covers a range of important cases.

IQLE Setup

Experiment:
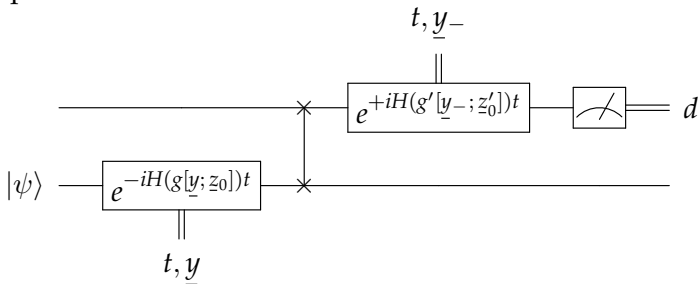
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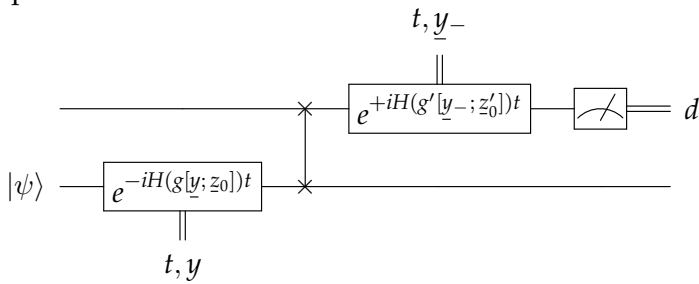
Simulator:
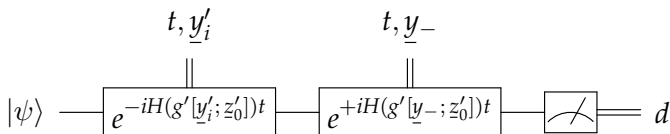
where $g'[\underline{y}'_i; z'_0] = g[\underline{y}; z_i]$.

Control/Distortion Duality

Unknown rescaling example: $g[\underline{y}; \underline{z}] = \underline{y} \odot \underline{z}$.
Straightforward to invert:

$$\underline{y}'_i = \underline{y} \odot \underline{z}_i \oslash \underline{z}'_0 \quad \implies \quad g'[\underline{y}'_i; \underline{z}'_0] = g[\underline{y}; \underline{z}_i]$$

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Shows duality between *hypothesis* and *simulation*.

Experiment		Simulation
hypothesis $\underline{z}_i$		fixed $\underline{z}_0'$
fixed $\underline{y}$	$\longleftrightarrow$	control $\underline{y}_i'$

QHL: special case $\underline{y} = \underline{1}$.
What must we invert by to affect $\mathbb{1}$?

Future: Bootstrapping for Control

Same approach can be used to model cross-talk:

$$\underline{z} = (\text{vec}\,\underline{\underline{G}}, \underline{\epsilon}) \quad g[\underline{y}; \underline{z}] = \underline{\underline{G}}\underline{y} + \underline{\epsilon}$$

Parameter count can be reduced by restricting $\underline{\underline{G}}$ (i.e.: tridiagonal).

Extend to learning time-dependent distortions?

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Extend to learning time-dependent distortions?

Can quantum resources be applied to *design* control?

- Bayesian inference: simulation as a characterization/validation resource.

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- Robust to many practical concerns.
- Can use quantum simulation to offer potential scaling.
- Using robustness of SMC, can truncate simulation $\rightarrow$ bootstrapping.

Further Information

Slides, a journal reference for this work, a full bibliography and a software implementation can be found at *http://www.cgranade.com/research/talks/lfqis-2014/*.

Thank you for your kind attention!

A few definitions help us evaluate estimates $\hat{\underline{x}}$ of $\underline{x}$:

Loss How well have we learned?
$$L_{\underline{\underline{Q}}}(\hat{\underline{x}}, \underline{x}) := (\hat{\underline{x}} - \underline{x})^{\mathrm{T}} \underline{\underline{Q}} (\hat{\underline{x}} - \underline{x})$$

Decision Theory

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Cramér-Rao Bound On average, how well *can* we learn?

Cramér-Rao Bound

Fisher Information

How much information about $\underline{x}$ is obtained by sampling data?

$$\underline{\underline{I}}(\underline{x}) = \mathbb{E}_D[(\nabla_{\underline{x}} \log \Pr(D|\underline{x}))(\nabla_{\underline{x}} \log \Pr(D|\underline{x}))^{\mathrm{T}}]$$

Cramér-Rao Bound

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The Cramér-Rao Bound tells how well any unbiased estimator can do. If $\underline{\underline{Q}} = \mathbb{1}$, then

$$R(\hat{\underline{x}}, \underline{x}) = \mathrm{Tr}(\mathrm{Cov}(\hat{\underline{x}})) \geq \mathrm{Tr}(\underline{\underline{I}}(\underline{x})^{-1}).$$

Bayesian Cramér-Rao Bound

Expectation of Fisher information over prior π: the *Bayesian Cramér-Rao bound*.

$$\underline{\underline{B}} := \mathbb{E}_{\underline{x} \sim \pi}[\underline{\underline{I}}(\underline{x})], \quad r(\pi) \geq \underline{\underline{B}}^{-1}$$

For adaptive experiments, the posterior is used instead of the prior.

The BCRB can be computed iteratively: useful for tracking optimality online.

$$\underline{\underline{B}}_{k+1} = \underline{\underline{B}}_k + \begin{cases} \mathbb{E}_{\underline{x} \sim \pi}[\underline{\underline{I}}(\underline{x}; \underline{e}_{k+1})] & \text{(non-adaptive)} \\ \mathbb{E}_{\underline{x}|d_1,\ldots,d_k}[\underline{\underline{I}}(\underline{x}; \underline{e}_{k+1})] & \text{(adaptive)} \end{cases}$$

Method of Hyperparameters

If "true" model $\underline{x} \sim \Pr(\underline{x}|\underline{y})$, for some *hyperparameters* $\underline{y}$, can est. $\underline{y}$ directly:

$$\Pr(d|\underline{y};\underline{e}) = \int \Pr(d|\underline{x},\underline{y};\underline{e}) \Pr(\underline{x}|\underline{y};\underline{e}) \, \mathrm{d}\underline{x}.$$

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Example

For Larmor precession with $\omega \sim \text{Cauchy}(\omega_0, T_2^{-1})$,

$$\Pr(d|(\omega_0, T_2^{-1}); t) = e^{-tT_2^{-1}} \cos^2(\omega_0 t/2) + (1 - e^{-tT_2^{-1}})/2.$$

Let $\underline{y} = (\omega_0, T_2^{-1})$.

Hyperparameters and Region Estimation

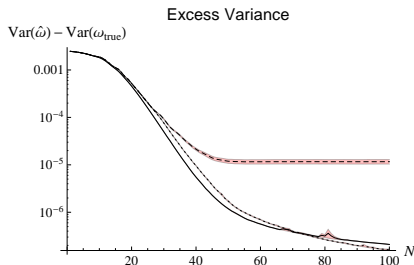In some hyperparameter models, can also express as region estimator on underlying parameters.

Figure : Larmor precession model w/ $\omega \sim N(\mu, \sigma^2)$, three exp. design strategies

Critically, the covariance region for ω is not smaller than the true covariance given by the hyperparameter σ^2. (*Granade* et al. 2012 *10/s87*)