Statistics

Dr. Mohd. Javed

IIIT ALLAHABAD

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Contents

Definition

If a random variable X can only take nonnegative values, then

$$P(X \geq a) \leq \frac{E[X]}{a}, \quad \text{for all } a > 0$$

Definition

Chebyshev Inequality If X is a random variable with mean μ and variance σ^2, then

$$P(|X - \mu| \geq c) \leq \frac{\sigma^2}{c^2}, \quad \text{for all } c > 0$$

The Weak Law of Large Numbers

.Let $X_1, X_2, \ldots, X_n$ be an independent identically distributed (i.i.d) random variables with mean $\mu = E\left[x_i\right]$ and variance σ^2
Sample mean:

$$M_n = \frac{X_1 + \cdots + X_n}{n}$$

$$E\left[M_n\right] = \frac{E\left[x, + \cdots\cdot + X_4\right]}{n} = \frac{n\mu}{n} = \mu$$

$$\mathrm{Var}\left(M_n\right) = \frac{\mathrm{Van}\left(x_1 + \cdots + x_n\right)}{n^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$\mathrm{P}\left(|M_n - \mu| \geq \epsilon\right) \leq \frac{Var[M_n]}{\epsilon^2} = \frac{\sigma^2}{n\epsilon^2} chebyshevineq$$

$$\mathrm{P}\left(|M_n - \mu| \geq \epsilon\right) = \mathrm{P}\left(\left|\frac{X_1 + \cdots + X_n}{n} - \mu\right| \geq \epsilon\right) \to 0, \quad \text{as } n \to \infty$$

Population vs Sample

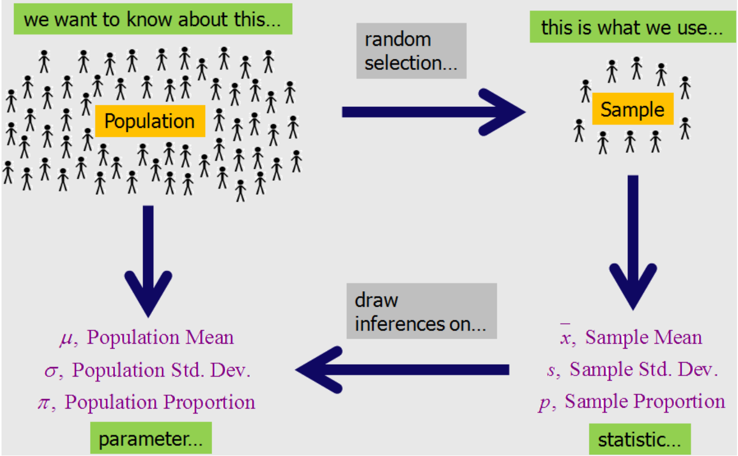


Figure: Population or Sample

The Central Limit Theorem (CLT)

General Idea: Regardless of the population distribution model, as the sample size increases, the sample mean tends to be normally distributed around the population mean, and its standard deviation shrinks as n increases.

- Is a Statistical based Theory
- It will generalize for any type of distribution
- Nature of distribution tends to be a Normal distribution

The Central Limit Theorem (CLT)

$\bullet X_1, \ldots, X_n$ i.i.d., finite mean μ and variance σ^2

$$\bullet S_n = X_1 + \cdots + X_n \text{.......} variance : n\sigma^2$$

$$\bullet \frac{S_n}{\sqrt{n}} = \frac{X_1 + \cdots + X_n}{\sqrt{n}} \text{......} variance : \sigma^2$$

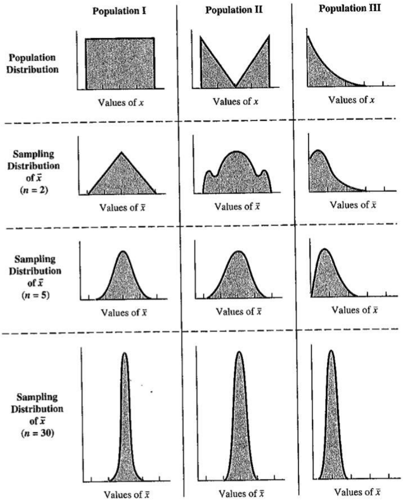
$$Z_n = \frac{S_n - n\mu}{\sqrt{n}\sigma} \quad E\left[Z_n\right] = 0 \,\mathrm{Var}\left(Z_n\right) = 1$$

Let Z be a standard normal r . v. (zero mean, unit variance)

Definition

Limit Theorem: For every z:$\lim_{n\to\infty} \mathrm{P}\left(Z_n \leq z\right) = \mathrm{P}(Z \leq z)$

C.L.T Visualization



Markov Process

Definition

if the future state of the process are independent of the past, and depends only on the present this process is called is Called Markov Process

• Prof. Andrei A. Markov (1856-1922) , published his result in 1906.

• If the time parameter is discrete $t_1, t_2, t_3, \ldots..$, it is called Discrete Time Markov Chain (DTMC).

• If time parameter is continues, (t $\geq$ 0) it is called Continuous Time Markov Chain (CTMC)

• Markov model is a Stochastic model, used to model randomly changing system.

Markov Chain Application

Autonomous vehicle Current state of the vehicle dosen't depends on where the vehicle starting point or In between state

Speech Recoginition Here Next word depens on previous word
I Like .... You

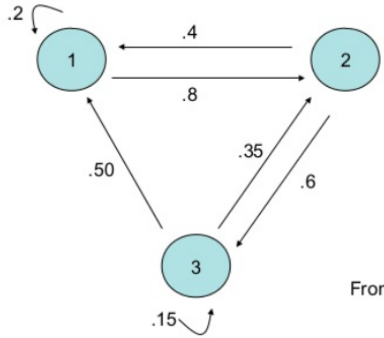
Markov Chain

• Definition: Let $\{X_0, X_1, X_2, \ldots\}$ be a sequence of discrete random variables. Then $\{X_{0,}X_1, X_2, \ldots\}$ is a Markov chain if it satisfes the Markov property:

$$\mathbb{P}\left(X_{t+1} = s \mid X_t = s_t, \ldots, X_0 = s_0\right) = \mathbb{P}\left(X_{t+1} = s \mid X_t = s_t\right)$$

for all $t = 1, 2, 3, \ldots$ and for all states $s_0, s_1, \ldots, s_t, s$
•Memoryless property as the state of the system at future time t_{n+1} is decided by the system state at the current time t_n and does not depend on the state at earlier time instants $t_1, \ldots.., t_{n-1}$

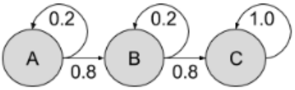
The Transition Matrix



		To		
		1	2	3
	1	.2	.8	0
From	2	.4	0	.6
	3	.5	.35	.15

Figure: Transition Matrix

State estimation Problem



Transition Matrix (**T**)

$$
\begin{array}{c|c}
\text{From... \ To...} & \begin{array}{ccc} A & B & C \end{array} \\
\begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0.2 & 0.8 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1.0 \end{pmatrix}
\end{array}
$$

Base case. $\mathbf{S_0} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

Inductive Step. $\mathbf{S_{t+1} = S_t * T}$

S_0	1.0	0.0	0.0
S_1	0.2	0.8	0.0
S_2	0.04	0.32	0.64
S_3	0.008	0.096	0.896

Random Walk

• Random Walk is a process a Model or rule to generate a path sequence of random motion.

• A random walk is a mathematical object, known as a stochastic or random process, that describes a path that consists of a succession of random steps on some mathematical space such as the integers.

• Many natural phenomena can be modelled as random walk
The path traced by a molecule as it travels in a liquid or a gas
The Path of drunk person
the price of a fluctuating stock and
the search path of a foraging animal
the price of a fluctuating stock and
the financial status of a gambler

Simple random walks on graphs

The sequence of vertices $v_0, v_1, v_2, \ldots, v_k, \ldots$ selected in this way is a simple random walk on 6 . At each step k, we have a random variable X_k taking values on V. Hence, the random sequence

$$X_0, X_1, X_2, \ldots, X_k, \ldots$$

Is a discrete time stochastic process defined on the state space V.

Simple random walks on graphs

- What does "at random" mean?
- If at time k we are at vertex i, choose uniformly an These transition move to.

Let d(i) denote the degree of vertex i.

$$p_{ij} = P\left(X_{k+1} = j \mid X_k = i\right) = \begin{cases} \frac{1}{d(i)}, & \text{if } i.j \in E \\ 0, & \text{otherwise} \end{cases}$$