Ex 9-1.

(a) $KL(q\|p) = -\int q(x)\log p(x)\,dx + \int q(x)\log q(x)\,dx$

$q(x) = \frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left\{-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right\}$ $\quad p(x) = \frac{1}{\sqrt{2\pi\sigma_2^2}}\exp\left\{-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right\}$

$\log q(x) = -\frac{1}{2}\log(2\pi\sigma_1^2) - \frac{(x-\mu_1)^2}{2\sigma_1^2}$

$\log p(x) = -\frac{1}{2}\log(2\pi\sigma_2^2) - \frac{(x-\mu_2)^2}{2\sigma_2^2}$

$KL = \int q(x)\left(\log q(x) - \log p(x)\right)dx$

$= \int \frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma_1^2}\right\}\left(\frac{1}{2}\log(2\pi\sigma_2^2) + \frac{(x-\mu_2)^2}{2\sigma_2^2} - \frac{1}{2}\log(2\pi\sigma_1^2) - \frac{(x-\mu_1)^2}{2\sigma_1^2}\right)$

$= \int q(x)\left(\log\frac{\sigma_2}{\sigma_1} + \frac{(x-\mu_2)^2}{2\sigma_2^2} - \frac{(x-\mu_1)^2}{2\sigma_1^2}\right)dx$

$= \int q(x)\log\frac{\sigma_2}{\sigma_1}\,dx + \int q(x)\left(\frac{(x-\mu_2)^2}{2\sigma_2^2} - \frac{(x-\mu_1)^2}{2\sigma_1^2}\right)dx$

$= \log\frac{\sigma_2}{\sigma_1} + \frac{1}{2\sigma_2^2}\mathbb{E}_q\left[(x-\mu_2)^2\right] - \frac{1}{2\sigma_1^2}\mathbb{E}_q\left[(x-\mu_1)^2\right]$

$= \log\frac{\sigma_2}{\sigma_1} + \frac{1}{2\sigma_2^2}\left[\mathbb{E}_q(x^2) - \mathbb{E}_q(2x\mu) + \mathbb{E}_q(\mu_2^2)\right] - \frac{1}{2\sigma_1^2}\cdot\sigma_1^2$

$= \log\frac{\sigma_2}{\sigma_1} + \frac{1}{2\sigma_2^2}\left[\sigma_1^2 + \mu_1^2 - 2\mu_2\mu_1 + \mu_2^2\right] - \frac{1}{2}$

$= \log\frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1-\mu_2)^2}{2\sigma_2^2} - \frac{1}{2}$

(c) $\mathbb{E}_{q(z)}[\log p(x|z)] = \int [\log p(x|z)] \cdot q(z) \, dz.$

$KL(q(z)||p(z)) = \int q(z) \log \frac{p(z)}{q(z)} dz.$

$\mathbb{E}_{q(z)}[\log p(x|z)] - KL(q(z)||p(z))$

$\downarrow$

$= \int [\log p(x|z)] q(z) dz + \int q(z) \cdot \log \frac{p(z)}{q(z)} dz.$

$= \int q(z) \cdot \log \frac{p(x|z) \cdot p(z)}{q(z)} dz$

$= \int q(z) \log \frac{p(x,z)}{q(z)} dz. = ELBO.$

(d) $z$: hidden variable. $\hat{x}$ is deterministic depent of $z$.

$p(x|z) \rightarrow p(x|\hat{x}).$

$\mathbb{E}_{q(x)} \log p(x|z) \rightarrow \mathbb{E}_{\hat{x}} \log p(x|\hat{x})$

$\uparrow \qquad \uparrow \qquad p(x|\hat{x}) \uparrow$. kind of reconstruction

Ex 9-2

(a) D and G compete each other.

D tries to classify as accurately as possible whether
a certain sample comes from same distribution
as target data ('real')
or whether it has been generated by a generator ('fake').

$D(x) \in [0,1]$ repredents prob that $x$ from target data.

G generates samples looks like real sample. by passing noise $z$.

while training, D better detects 'fake'
G better generats 'real'

In equilibrium, $G \to$ perfect sampleds that from same distribution
$D \to$ cannot classify ½ v.s. ½.

(b) $J^{(D)} = -\frac{1}{2} \mathbb{E}_{x \sim p_{data}(x)} [\log D(x)] - \frac{1}{2} \mathbb{E}_{z \sim p_z(z)} [\log (1 - D(G(z)))]$

prob ↗

cross entropy : $-(y \log p + (1-y) \log (1-p))$

(c) $J^{(G)} = -J^{(D)} = \frac{1}{2} \mathbb{E}_{x \sim p_{data}(x)} [\log D(x)] + \frac{1}{2} \mathbb{E}_{z \sim p_z(z)} [\log 1 - D(G(z$

$\min_G \max_D V(D,G) = J^{(G)} = \frac{1}{2} \mathbb{E}_{x \sim p_{data}(x)} [\log D(x)] + \frac{1}{2} \mathbb{E}_{z \sim p_z(z)} [\cdots]$.

value function

(d) $D(G(z)) \approx 0 \Rightarrow G(z)$ as fake. $\Rightarrow$ D did right. $\Rightarrow$ gredient smal

In the beginning of training