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Rough idea about how to generate probability distribution for surface runoff into pond 1

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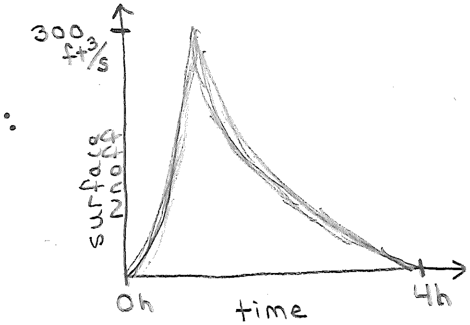
How to estimate probability distribution (rough idea)

- We have estimates of surface runoff into the pond from a design storm. (see fig. 6, Sustech submission)

- A 4-hour portion of the surface runoff looks like:



(1)

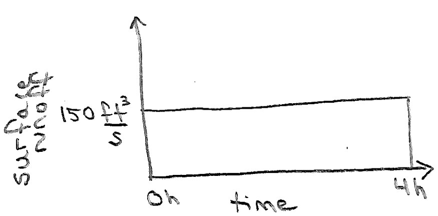
- We want to estimate a probability distribution for surface runoff. For simplicity (for now), we want to have the probability distribution to be the same at each time point. (We can make it more accurate and have the distribution be different at different time points for our journal paper.)

- The surface runoff estimates suggest that over a 4 hour period about

$$\underbrace{300\ \frac{ft^3}{s}}_{[\text{height}]} \cdot \underbrace{4\ \text{hours}}_{[\text{base}]} \cdot \frac{3600\,s}{1\ hr} \cdot \frac{1}{2} = \boxed{2.16 \cdot 10^6\ ft^3}$$

of water in total enters the pond (find the area under the curve, approximate as a triangle).

- So, the average rate of surface runoff is about $\dfrac{2.16 \cdot 10^6\ ft^3}{4h \cdot \frac{3600s}{1hr}} = 150\ \dfrac{ft^3}{s}$ over the 4-hour storm.



(2)

- The graph (2) is a flattened version of (1) with the same area. (amount of water entering over the 4-hour storm).

- The current risk-senstive reachability methods assume a discrete-time horizon (perhaps we can extend this to continuous time in later work).

Let's say the length of our discrete-time interval $\Delta t = 5\text{min} = 300$ seconds, so about

$$150\ \frac{ft^3}{s} \cdot 300\,s = 45000\ ft^3$$ of water should enter the pond every $\Delta t = 5\text{min}$

- Let w_k be a random variable that specifies the amount of water entering the pond during the time interval $[k, k+1)$, which has a duration of $\Delta t = 5\text{min}$.

- Then, we want to estimate the possible values that w_k can take on and their probabilities,

$P_r\{w_k = j\} = p_j$, so that $E[w_k] \approx 45000\ ft^3$, $p_1 + p_2 + \dots + p_N = 1$, $0 \le p_j \le 1$.

The current risk-sensitive reachability methods assume that the values $\{j\}$, and the probabilities $\{p_j\}$ are available, and that there are a finite number of them.

- For example, we could pick

Value	Probability	Value	Probability
$P_r\{w_k = 38000\ ft^3\}$	$= 0.0184$	$P_r\{w_k = 43000\ ft^3\}$	$= 0.0792$
$P_r\{w_k = 39000\ ft^3\}$	$= 0.0273$	$P_r\{w_k = 44000\ ft^3\}$	$= 0.0957$
$P_r\{w_k = 40000\ ft^3\}$	$= 0.0379$	$P_r\{w_k = 45000\ ft^3\}$	$= 0.1133$
$P_r\{w_k = 41000\ ft^3\}$	$= 0.0501$	$P_r\{w_k = 46000\ ft^3\}$	$= 0.1315$
$P_r\{w_k = 42000\ ft^3\}$	$= 0.0639$	$P_r\{w_k = 47000\ ft^3\}$	$= 0.1521$
		$P_r\{w_k = 48000\ ft^3\}$	$= 0.2307$

- I generated these numbers using the attached script (and the CVX solver), so that $\mathbb{E}[W_k] = 45000 \text{ ft}^3$, $\sum_{j=1}^{11} p_j = 1$, $p_j \in [0,1]$.

- If possible, we want the values $\{j\}$ and the probabilities $\{p_j\}$ to better reflect the design storm.

```
% Script to generate probability distribution of surface runoff
into first pond
% Pr{wk = ws(i)} = P(i)
% Fix ws & desired expected value to generate P
```

row vector

```
ws = 38000:1000:48000; %ft^3
```

$ws(i) = i^{th}$ possible value of ω_K

```
nw = length(ws);
```

$P(i)$ = probability that ω_K takes on the value, $ws(i)$

```
cvx_begin

variables P(nw,1)
```

Column vector

```
    minimize ( 1 )

    subject to

        ws*P == 45000;
```

$= \text{expected value of } \omega_K = \sum_{i=1}^{11} ws(i) \cdot P(i)$

11 ← total number of samples

```
        P>=0;

        P<=1;

        sum(P) == 1;

cvx_end
```