Experimentally Determining Drag Coefficients for Vertical Foils with Varying Shapes

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1. Introduction and Background

From centerboards in sailboats to the rudders used to steer cargo ships, there are extensive applications of simple vertical blades in modern technologies. These blades vary in shape and size but share the common goal of maximizing directional guidance while minimizing total drag. Creating efficient blades is crucial for maximizing the speed and efficiency of these moving vehicles. Improved blade designs can enhance the top velocity of America's Cup sailboats and significantly reduce fuel consumption for cargo ships, offering substantial economic and environmental benefits.

When reducing drag on a moving object, designers can modify two main variables: the head-on shape and the bird's-eye shape of the object. For directional guidance, the head-on shape is nearly always a narrow rectangle to maximize steering and minimize unnecessary drag [1]. Based on this commonality, my experiment focused on studying the effect of the bird's-eye shape on the drag of blades.

This experiment aimed to determine whether different shapes experience significantly different drag when pulled through water, and if so, identify the qualities that contribute to an efficient blade design. By controlling the applied force and measuring the resulting velocity of various blades with uniform frontal areas in a tank of water, I obtained the necessary data to determine the drag coefficient for different shapes.

In addition to confirming that shape plays a significant role in determining the drag coefficient for an object, my results suggest that the drag coefficient for a given shape varies with velocity. This finding contradicts some common literature on the subject and emphasizes the importance of experimentally determining these coefficients instead of accepting published values [2].

2. Definitions and Configuration

The term “shape” in this paper refers specifically to the bird's-eye silhouette of the blade being dragged through the water. This simplification aligns with existing literature and reduces verbosity. For this experiment, three uniquely shaped blades were used:

Blade 1, referred to as “Flat Front”;

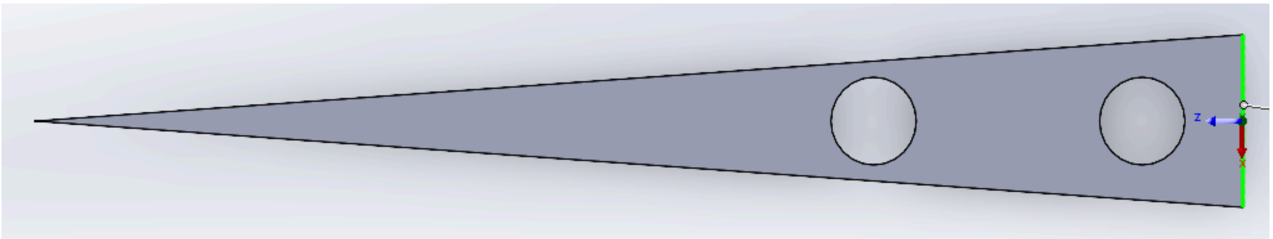
Blade 2, referred to as “Rounded Front”

Blade 3, referred to as “Sharp Front”

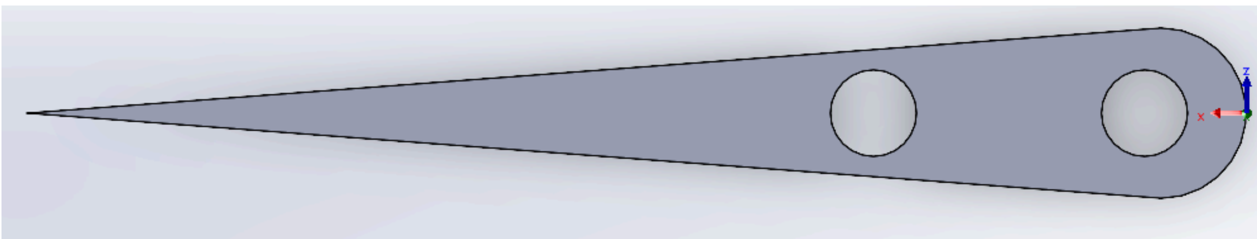
All three blades had identical frontal areas and lengths to maintain consistency throughout the experiment. Blade 1 and 2 were designed on SolidWorks and were produced using a 3D printer. Blade 3 was the same object as Blade 2, but the direction of travel was reversed to make the leading edge the sharp point. The three blades are pictured in Figure 1.

Figure 1: Blade Profiles Used in Experiment

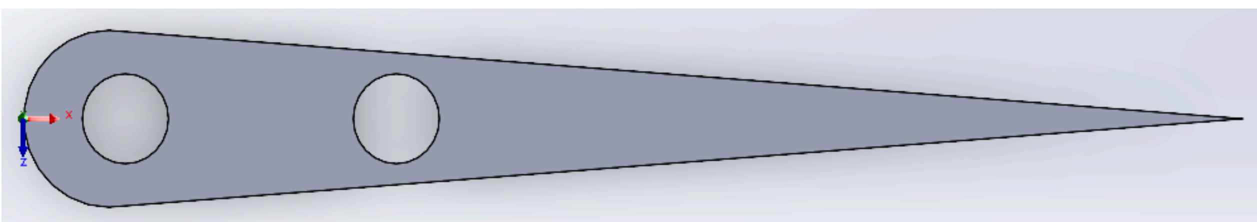
Blade 1 - “Flat Front”



Blade 2- “Rounded Front”



Blade 3- “Sharp Front”



The primary equation used throughout this paper is the Drag Equation:

Equation 1: The Drag Equation

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

In this equation, F_D is the drag force, ρ is the density of the fluid, v is the object’s velocity relative to the fluid, and A is the frontal area, and C_d is the drag coefficient, a unitless constant that depends on factors such as size, shape, material, texture, and fluid flow behavior [3][4].

To maximize consistency throughout the experiment, the blades were designed to have identical frontal areas of 54 cm^2 (18*cm* x 4*cm*) and a length of 18*cm*. The blades were all printed using the same plastic filament, and the surfaces were sanded to the same grain. The only difference between the blades were the shapes.

In all tests, the blades were dragged through a 2*m* x .5*m* x .5*m* glass tank containing tap water. The tank was filled with just enough water to fully submerge the blade. Based on temperature readings in the room, I used the USGS density table to determine $\rho \approx 997\ kg/cm^3$ [5]. The velocity of the blade relative to the water (v) was determined via film analysis of the blade traveling a marked .5*m* interval using a 60 fps camera.

To drag the blade through the water, a track and cart system was designed using the CAD program SolidWorks. The tracks were made of aluminum T-slotted framing and were fastened to the tank using bolts. The cart consisted of two rollers with low friction ball-bearings that rested in the groove of the T slot and were connected by a stiff plexiglass sheet. The blades were then connected to the plexiglass sheet via 0.5-inch adjustable aluminum rods, and the cart was pulled by a thin fishing line attached to its center. A significant portion of this experiment was dedicated to designing this configuration to minimize friction and other potential sources of experimental noise. Machinist Ben King at the Bowdoin machine shop assisted with creating many of the parts, and the design of this experiment was influenced by the guidance of Professor Dale Syphers and lab instructor Ken Dennison of Bowdoin College. The initial SolidWorks assembly is shown in Figure 2 and the completed tank is shown in Figure 3.

Figure 2: SolidWorks Tank Assembly

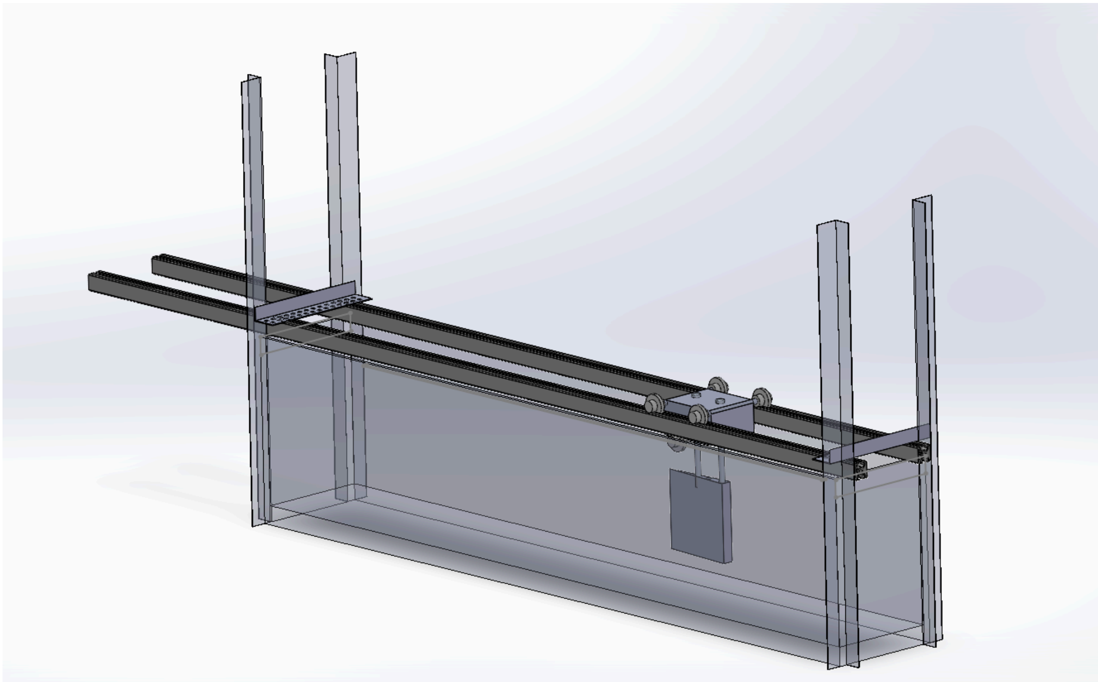
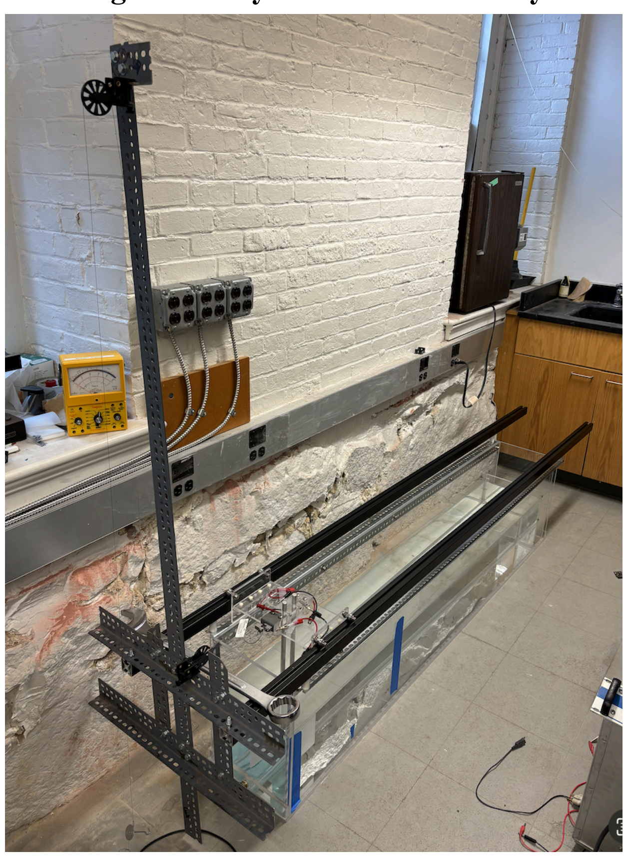


Figure 3: Physical Tank Assembly



Hanging masses released from a vertical arm at the end of the tank moved the cart through the water. This setup ensured a constant applied tension force throughout each trial. The only horizontal forces acting on the blade were the applied tension force and the reactive drag force. At terminal velocity, the drag force equaled the applied tension force, allowing adjustment of the F_D term in the drag equation. By measuring the velocity of each blade for a range of applied forces, the drag coefficient for each shape at various velocities was experimentally determined.

3. Experimental Methods and Data

For each of the 3 blades, I took trials of velocity measurements with 4 different hanging masses (100*g*, 200*g*, 300*g*, 400*g*). I completed 3 trials for each blade-mass pairing to obtain a more accurate average velocity. To account for the inherent friction in the system, I found the hanging mass required to move the cart out of static equilibrium when no blade was attached. This value was found to be 30g, and I subtracted it from the 4 hanging masses to find the "Adjusted Pulling Force". The complete data for each of the 3 blades is shown in Tables 1-3.

Table 1: Complete Data Table for Blade 1 - "Flat Front"						
Blade	**Hanging Mass (kg)**	**Pulling Force on Blade (N)**	**Adjusted Pulling Force (N)**	**Time (sec)**	**Velocity (m/s)**	**Avg Velocity (m/s)**
Flat Front	0.1	0.98	0.69	1.26	0.40	
Flat Front	0.1	0.98	0.69	1.21	0.41	0.41
Flat Front	0.1	0.98	0.69	1.21	0.41	
Flat Front	0.2	1.96	1.67	0.80	0.63	
Flat Front	0.2	1.96	1.67	0.80	0.63	0.62
Flat Front	0.2	1.96	1.67	0.81	0.62	
Flat Front	0.3	2.94	2.65	0.61	0.82	
Flat Front	0.3	2.94	2.65	0.63	0.80	0.80
Flat Front	0.3	2.94	2.65	0.64	0.78	
Flat Front	0.4	3.93	3.63	0.54	0.93	
Flat Front	0.4	3.93	3.63	0.51	0.98	0.93
Flat Front	0.4	3.93	3.63	0.56	0.89	

Table 2: Complete Data Table for Blade 2 - "Rounded Front"						
Blade	**Hanging Mass (kg)**	**Pulling Force on Blade (N)**	**Adjusted Pulling Force (N)**	**Time (sec)**	**Velocity (m/s)**	**Avg Velocity (m/s)**
Rounded	0.1	0.98	0.69	0.93	0.54	
Rounded	0.1	0.98	0.69	0.93	0.54	0.54
Rounded	0.1	0.98	0.69	0.93	0.54	
Rounded	0.2	1.96	1.67	0.55	0.91	
Rounded	0.2	1.96	1.67	0.53	0.95	0.94
Rounded	0.2	1.96	1.67	0.53	0.95	
Rounded	0.3	2.94	2.65	0.43	1.18	
Rounded	0.3	2.94	2.65	0.40	1.25	1.21
Rounded	0.3	2.94	2.65	0.41	1.21	
Rounded	0.4	3.93	3.63	0.39	1.29	
Rounded	0.4	3.93	3.63	0.36	1.38	1.35
Rounded	0.4	3.93	3.63	0.36	1.38	

Table 3: Complete Data Table for Blade 3 - "Sharp Front"						
Blade	**Hanging Mass (kg)**	**Pulling Force on Blade (N)**	**Adjusted Pulling Force (N)**	**Time (sec)**	**Velocity (m/s)**	**Avg Velocity (m/s)**
Sharp Front	0.1	0.98	0.69	1.08	0.47	
Sharp Front	0.1	0.98	0.69	1.10	0.45	0.45
Sharp Front	0.1	0.98	0.69	1.13	0.44	
Sharp Front	0.2	1.96	1.67	0.68	0.74	
Sharp Front	0.2	1.96	1.67	0.70	0.71	0.73
Sharp Front	0.2	1.96	1.67	0.68	0.74	
Sharp Front	0.3	2.94	2.65	0.54	0.93	
Sharp Front	0.3	2.94	2.65	0.48	1.05	0.99
Sharp Front	0.3	2.94	2.65	0.50	1.00	
Sharp Front	0.4	3.93	3.63	0.40	1.25	
Sharp Front	0.4	3.93	3.63	0.43	1.18	1.24
Sharp Front	0.4	3.93	3.63	0.39	1.29	

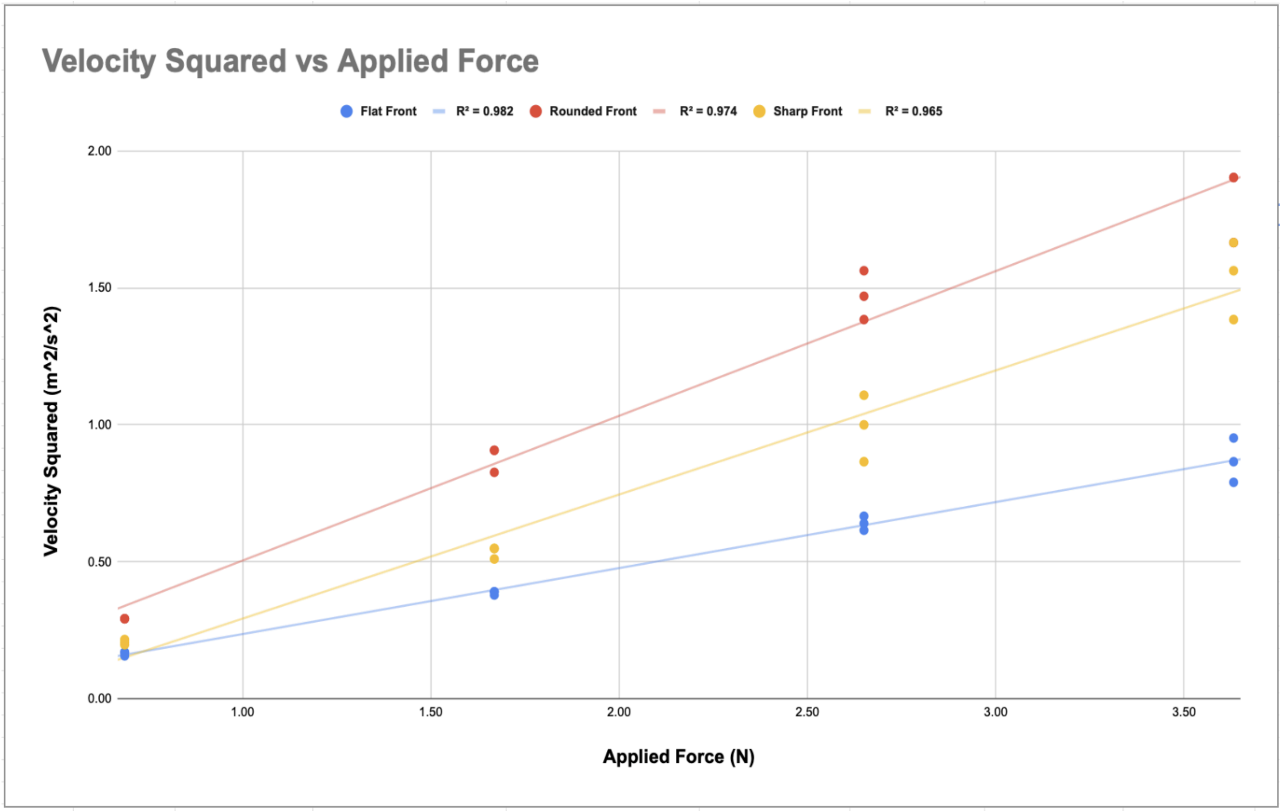
Despite my best attempts, there was some moderate variability for the velocity measurements. The largest standard deviation for the velocity measurements was .061*m/s* for the Sharp Front blade with the 400g mass. Overall, however, the results were relatively consistent with an average standard deviation of .027*m/s*. These values were found using the standard deviation formula for a sample [6]. I plugged the data from Tables 1-3 into Equation 1 to calculate the drag coefficients for the various shapes and velocities. The results of the calculations are shown in Table 4.

Table 4: Drag Coefficient From Average Velocity			
Blade Shape	**Adjusted Pulling Force (N)**	**Average Velocity (m/s)**	**Drag Coefficient**
Flat Front	0.69	0.41	1.52
Flat Front	1.67	0.62	1.61
Flat Front	2.65	0.8	1.54
Flat Front	3.63	0.93	1.56
Rounded Front	0.69	0.54	0.88
Rounded Front	1.67	0.94	0.70
Rounded Front	2.65	1.21	0.67
Rounded Front	3.63	1.35	0.74
Sharp Front	0.69	0.45	1.27
Sharp Front	1.67	0.73	1.16
Sharp Front	2.65	0.99	1.00
Sharp Front	3.63	1.24	0.88

4. Results

According to the Drag Equation (Equation 1) the drag force is exponentially proportional to velocity. Since the measurement of the blade was taken at terminal velocity, I make the assumption that the Adjusted Pulling Force is equal to the drag force. Based on this assumption, there should be a linear relationship between the Adjusted Pulling Force and the velocity of the object squared. Graphing the data from Tables 1-3, we see that this relationship is present in the data.

Figure 4: Velocity Squared vs Applied Force .

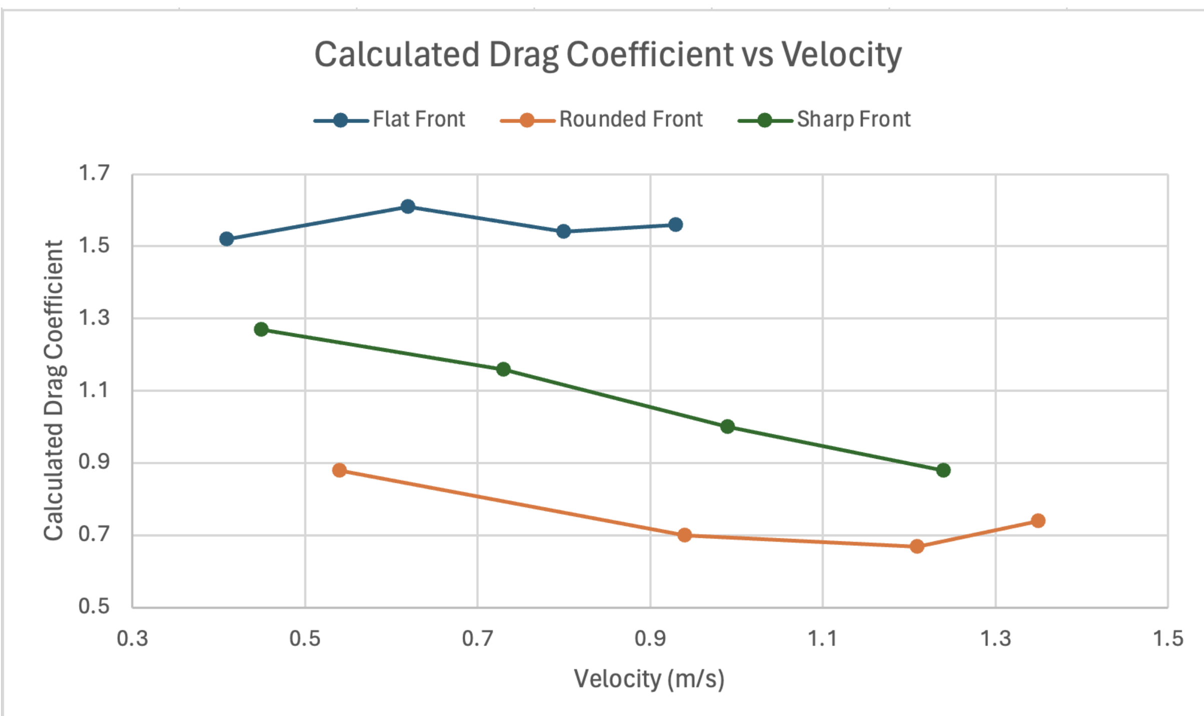


The high r^2 values indicate that the relationship implied in the drag equation holds for all of the blades over a small velocity range. Furthermore, since all other terms in the drag equation (ρ, A) remained the same throughout the experiment, the graph suggests that the drag coefficients for the blades were relatively constant. The differing slopes indicate that the drag coefficients varied between blades. The non-zero y-intercepts were not expected, but their relative proximity allows us to conclude that the deviation is due to experimental noise and imperfect measurement.

Upon further examination, it is made clear that shape had a sizable effect on the drag coefficient of the blades. Taking the average of the 4 trials of each blade in Table 4, we find that that the average drag coefficients were 1.56, 0.75, and 1.08 for the Flat, Rounded, and Sharp blades respectively. These values suggest that shape plays a sizable role in determining the drag coefficient for a given object. With over double the drag coefficient, it is clear that blades with a flat leading surface are far more inefficient compared to identical blades with a rounded front. These results also suggest that blades with rounded fronts are approximately 1.4 times as efficient as blades with sharp fronts. This experiment confirms the common assumption that rounded surfaces are more aero and hydrodynamic than flat surfaces and provides convincing evidence that rounded surfaces are also more efficient than sharp edges.

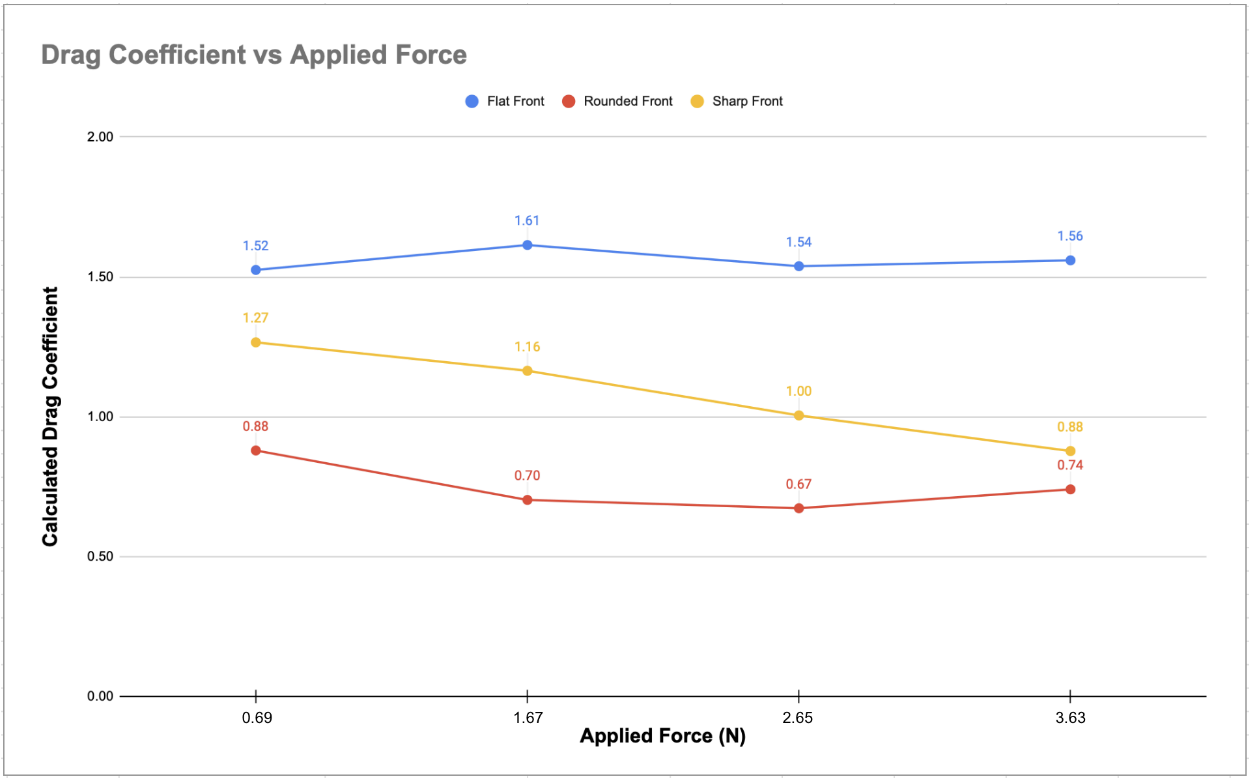
Contrary to what the drag equation implies and various sources suggest, the results in Table 4 indicate that the drag coefficient varies with velocity even when the shape of the object is constant. Looking at Figure 5, we see that the Flat and Rounded blades have relatively constant drag coefficients whereas the drag coefficient for the Sharp blade decreases noticeably as velocity increases.

Figure 5: Calculated Drag Coefficient vs Velocity



Examining the data, we see that the drag coefficient of the Sharp blade decreased by 0.39 between the first and last trial which is a change of over 30%. When the drag coefficient is compared to the adjusted applied force a similar trend is seen (Figure 6).

Figure 6: Calculated Drag Coefficient vs. Applied Force



These results suggest that for certain shapes the drag coefficient is dependent on velocity. This finding not only contradicts various reputable sources, but also indicates the very structure of the drag equation is flawed since it assumes that the drag coefficient for all objects remains constant.

4. Conclusions

This experiment confirmed that shape has a strong effect on the drag coefficient for vertical blades traveling through fluids [7]. As the results reflect, rounded blades exhibit the lowest drag coefficients while flat faced blades have the highest drag coefficients. The blade with the sharp front was more efficient than the flat blade, especially at higher velocities, but still always exhibited more drag than the rounded front. This suggests that, despite what many individuals' intuition may suggest, rounded blades are more efficient than sharp ones. Although it is common practice for boats and vehicles to utilize blades with rounded fronts to improve efficiency, this experiment provides convincing evidence that confirms these decisions.

The results of the experiment also suggest that the drag coefficient is dependent on velocity. The sharp blade exhibited the largest dependency between drag coefficient and velocity, but it is reasonable to conclude that the drag coefficient is not perfectly constant for blades of all shapes. Although my experiment provided little insight into why this dependency occurs, my hypothesis is that as velocity increases the flow dynamics change and the resistance caused by skin friction is outweighed by the resistance caused by form drag [8]. This finding contradicts multiple sources and implies that the drag equation is not a perfect representation for all objects [9].

This experiment suggests that the drag equation can provide an accurate model for objects traveling within a small velocity range, but should not be taken as a perfect equation. For more reliable measurements, the drag for an object should be found through experimentation or computer modeling. Like many other equations in science, the drag equation provides a suitable estimate, but is far from a perfect model. Equations are important for conceptualizing how variables interact, but ultimately experimental confirmation is essential.

5. Future Steps

Overall the results of this experiment were successful and I was pleased with the clarity of the data. However, there are various steps I would take to improve the process in the future. First, I would modify the cart and tank configuration to reduce the friction of the system so the applied force could be more accurately determined. It would also be interesting to slightly modify the experiment and drag the blades at a constant speed using a voltage controlled motor and measure the drag force experienced using a strain gauge. This was my initial plan, but I had to pivot to the hanging mass configuration due to time and material constraints. If I were to repeat the experiment with the hanging masses, I would use more precise techniques for measuring the velocity of the blade such as light gates or a camera with a higher fps. Lastly, a larger tank would be used in future iterations of the experiment because confining the blade to a small area of travel leads to excess pressure and unrealistic fluid dynamics that do not reflect common environmental conditions.

In addition to these improvements, there are some further experiments that I would like to conduct to examine the effects shape has on drag. Using more blades with different shapes would be a simple modification that would help determine which profiles are the most efficient. I am interested in examining how the degree of eccentricity affects drag, and I would like to determine the optimal amount of "roundness". Testing the blades in a direction of travel that is not perfectly head-on would also be interesting since boats and ships often experience lateral forces such as waves and current. I would like to conduct a future experiment that tests the blades over a wider range of velocities to see if the drag coefficients for the Rounded and Flat blades eventually decline like the Sharp faced blade did. Lastly, I would be interested in conducting a similar experiment but using air as the fluid being traveled through in order to determine if these results can be extended to the aviation and automotive industries.

6. Sources

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