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LETTER TO THE EDITOR

Scaling of the active zone in the Eden process on percolation networks and the ballistic deposition model

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Abstract. The interface of the Eden clusters on percolation networks and the ballistic deposition model is studied by Monte Carlo simulations, using a simple definition for the surface thickness. The width of the active zone in the ballistic deposition model is found to diverge differently from the mean height, indicating breakdown of the single scaling length assumption in this model. The exponents ν and ν' describing, respectively, the divergence of the radius and the active zone of the Eden clusters on percolation networks appear to be the same within the statistical errors. The central value of ν', however, is slightly, but systematically, less than ν. The surface thickness of ballistic deposits is shown to exhibit finite-size scaling.

Clusters formed by kinetic growth processes have attracted great interest recently because they appear in a wide variety of phenomena in physics, chemistry, biology and engineering (see e.g. Family and Landau (1984) and references therein). The studies of such processes have mainly been concerned with the geometry of the clusters and with the properties of the cluster size distribution.

The main result concerning these non-equilibrium phenomena was that both the individual clusters and the ensembles of clusters display scaling behaviour similar to that which appears in the experiments and theories on equilibrium phase transitions. It was found that the correlation within large kinetically grown objects such as diffusion limited aggregates (Witten and Sander 1981), clusters generated in kinetic gelation models (Family 1983) or cluster-cluster aggregates (Meakin 1983a, Kolb *et al* 1983) decayed algebraically corresponding to scaling and fractal geometry. The size dependence of the cluster size distribution in the diffusion limited deposition (Rácz and Vicsek 1983) and the cluster-cluster aggregation (Vicsek and Family 1984) models has also been shown to scale in analogy with the behaviour of the same quantity in percolation near the threshold or in Ising models at the critical point.

Very recently numerical evidence was obtained by Plischke and Rácz (1984, Rácz and Plischke 1984) suggesting that the above analogy is violated in the diffusion limited aggregation (DLA) and the Eden model (Eden 1961). In an isotropic equilibrium system there is a single characteristic length diverging at the critical point, and scaling is a consequence of this fact. According to the results of Plischke and Rácz, however, in the DLA and the Eden models as the number of particles in a cluster tends to infinity, the radius of the cluster and the width of the active zone diverge with different exponents.

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The purpose of this letter is to study the problem of surface thickness in qualitatively different growth models and in this way to test how general is the breakdown of the single length scaling assumption. For this reason we have studied the structure of the active zone in the following growth models: the Eden process on the incipient infinite percolation network and the ballistic deposition model. These two processes are different in nature from those studied by Plischke and Rácz. The first model is a space filling growth process on a fractal, while ballistic deposition is an aggregation model which is expected to result in compact (non-fractal) clusters. We define the width of the active zone by the standard deviation of the distances of the active perimeter sites from the origin. This definition is much simpler to implement and gives better statistics for the models we study than the distribution function approach used by Plischke and Rácz. In the Eden clusters at the percolation threshold the width of the active zone and the cluster radius diverge with the same power of the cluster size within the statistical errors. The data, however, give a slightly smaller exponent for the divergence of the width of the active zone. In the ballistic deposition model the surface thickness diverges differently from the mean height, indicating the existence of a second length scale in this model. Finally, we show that in the ballistic deposition model a finite-size scaling behaviour appears in a way which is similar to the same phenomenon in equilibrium systems.

The Eden process on percolation networks is an example of a growth model on a fractal and is expected to have growth properties different from other models, such as DLA. We define this model in the following way: consider a percolation process on a lattice and pick the infinite cluster at p_c, where p_c is the percolation threshold. Then, select a site at random on this cluster and from this site grow an Eden cluster. An Eden cluster is grown by successively choosing at random a perimeter site and occupying it until a large cluster is formed. A perimeter site is any unoccupied site that is the nearest neighbour of an occupied site in the cluster. In order to study the width of the active zone the number and positions of active perimeter sites must first be determined. An active perimeter site is defined to be a perimeter site which has the potential of becoming occupied in the later stages of the growth process. All perimeter sites are active perimeter sites in the Eden process in Euclidean spaces. In contrast, in the Eden process on a percolation cluster only perimeter sites belonging to the percolation cluster are active perimeter sites. Eden perimeter sites falling within the 'holes' in a percolation cluster cannot be occupied and are not active perimeter sites.

In the Eden process we define the surface thickness or the width of the active zone in the following way. Let r_i be the radius of the ith active site from the centre of mass of a cluster of size N. The average radius of active sites, $r(N)$, is given by $r(N) = \Sigma\, r_i/n_p(N)$, where $n_p(N)$ is the number of active perimeters in the N-site cluster. We define the width of the active zone of an N-site cluster by $\sigma(N) = [\Sigma\,(r_i - r(N))^2]^{1/2}$, which is the square root of the variance of radii of active perimeter sites.

In order to study the Eden process on the incipient infinite cluster we first generate the infinite percolation cluster at p_c using the 'growing percolation' method of Alexandrowicz (1980). This process is started with a single occupied site. A site adjacent to this site is randomly selected. This site is considered occupied and is added to the cluster if a random number x attributed to the site is less than a previously fixed value p. The new perimeter sites are found and the process continues in the same way. If the chosen perimeter site is not occupied, then it remains unoccupied, i.e. is 'inactive', forever. The clusters generated in this way, even at $p = p_c$, are not all infinite clusters. In order to obtain the necessary data on the infinite cluster we grow the percolation clusters up to a reasonably large size, say N_{max}, and collect data only up to a much smaller size, say N. Our data obtained for $N_{max} = 15\,000$ and $N = 10\,000$ indicate that this is a reasonable approach.

The Alexandrowicz method is more convenient for the present study than the method of filling a lattice at p_c. The reason is that the Eden process is a random growth process and once a site is randomly chosen and made part of the percolation cluster, it can also be considered as part of the Eden cluster. We therefore expect the results for the structure of the active zone in the Eden clusters on the percolation network to be the same as the structure of the active zone of the 'growing' percolation clusters in the Alexandrowicz model.

The results for the mean radius of active perimeter sites, $r(N)$, the width of the active zone, $\sigma(N)$, and the number of active perimeter sites, $n_p(N)$, of an N-site cluster, are shown in figure 1. These results were obtained from 4000 simulations of percolation clusters of up to $N_{max} = 15\,000$ sites generated at $p = p_c = 0.593$ (Stauffer 1984). We collected data only up to $N = 10\,000$ sites. From figure 1 we see that the data for $r(N)$, $\sigma(N)$ and $n_p(N)$ are consistent with the following scaling relations:

$$r(N) \sim N^{\nu}, \qquad \sigma(N) \sim N^{\nu'}, \qquad n_p(N) \sim N^{\alpha} \tag{1}$$

with

$$\nu = 0.53 \pm 0.015, \qquad \nu' = 0.50 \pm 0.015, \qquad \alpha = 0.40 \pm 0.02.$$

The value of ν agrees with the expected result $\nu = 1/D = 48/91 = 0.527\ldots$, where D is the fractal dimension of percolation clusters. The central value of ν' appears to be smaller than ν, suggesting that the width $\sigma(N)$ diverges more slowly than the cluster radius as $N \to \infty$. However, the actual error for ν' is probably larger than the quoted statistical error, because of the finite-size effects and systematic errors. Thus, considering the errors in ν and ν', the numerical evidence implies $\nu \simeq \nu'$ and $\sigma(N)$ diverges

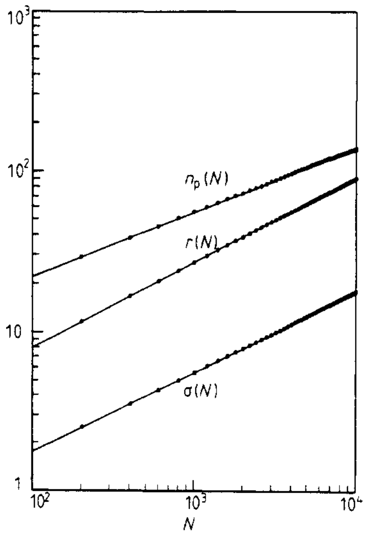

Figure 1. Plots of the mean radius of active perimeter sites, $r(N)$, the width of the active zone, $\sigma(N)$, and the number of active perimeter sites, $n_p(N)$ against the cluster size N. The full lines drawn through the data points have slopes $\nu = 0.53$, $\nu' = 0.50$, and $\alpha = 0.40$.

as the radius. Plischke and Rácz found that ν' is much smaller than ν for the Eden clusters on a square lattice. The reason ν is approximately equal to ν' in the present case is that we have studied the Eden model on a fractal. The exponent α is new and its value agrees with the estimate obtained in a related growth model (Bunde *et al* 1984).

In the ballistic aggregation models (Vold 1963, Sutherland 1966) the particles move along straight trajectories and become part of the growing cluster when they arrive at the surface of the aggregate. The ballistic deposition model investigated in this paper is defined in the following way. Consider a line of L particles, represented by a horizontal line of L sites on a square lattice. Randomly select a site of a horizontal line above the line of particles and place a particle there. Now allow this particle to fall along a straight line vertically downward. The particle sticks to the line, and is made part of the deposit, once it reaches the nearest-neighbour site of a particle on the line. A large deposit is grown by releasing more particles and joining them to the deposit once they have reached the nearest-neighbour site of a particle in the column it was dropped in, or in one of the neighbouring columns. Numerical simulations (Meakin 1983b, Bensimon *et al* 1984a) and a mean-field study (Bensimon *et al* 1984b) of ballistic models indicate that these clusters have a constant density and therefore they are not fractals.

The simulations were carried out with periodic boundary conditions for various values of L and M, where M denotes the number of deposited particles per seed site. The surface thickness in this case is obtained from $\sigma_L(M) = [\Sigma\,(h_i - h(M))^2]^{1/2}$, where h_i is the height of the deposit at site i, and $h(M)$ is the mean height. Figure 2 shows the dependence of $h(M)$ and $\sigma_L(M)$ on M for $L = 2000$. The slopes of the straight

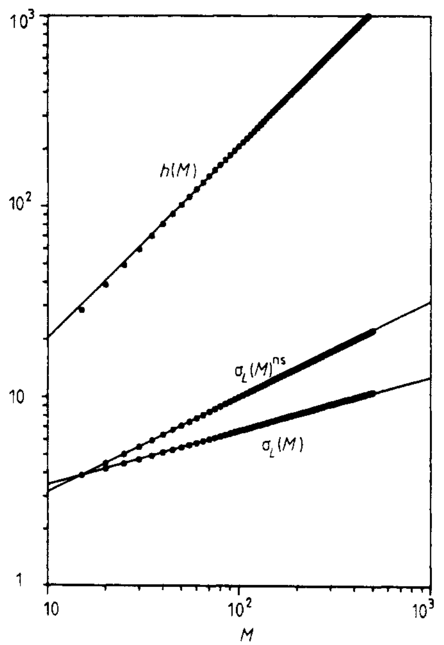

Figure 2. Plots of the mean height, $h(M)$, and the width of the active zone, $\sigma_L(M)$, against the number, M, of deposited particles per seed site for ballistic deposition. The width of the active zone, $\sigma_L(M)^{ns}$, in the non-sticking case is also shown for comparison. The full lines drawn through the data points have slopes $\nu = 1.0$, $\nu'_{ns} = \frac{1}{2}$ and $\nu' = 0.30$.

lines on this log-log plot indicate that both quantities increase with M as a power law. The mean height $h(M)$ scales with M as $h(M) \sim M^{\nu}$ where $\nu = 1.0 \pm 0.01$ in accordance with the earlier works concluding that ballistic deposits are not fractals.

The width of the active zone, however, scales with a different non-trivial exponent as a function of M,

$$\sigma_L(M) \sim M^{\nu'} \tag{2}$$

where

$$\nu' = 0.30 \pm 0.02.$$

For comparison we also show the surface thickness of a non-sticking deposit which is generated in the same way as ballistic deposits except that the falling particles do not stick to the particles in the neighbouring columns. The height of the columns in this problem follows a Poisson distribution and correspondingly, the slope of the active zone data in the non-sticking case has the trivial value $\nu' = \frac{1}{2}$ (see figure 2).

Expression (2) is a good approximation for $\sigma_L(M)$ if M is not too large. According to our simulation results, as M increases, $\sigma_L(M)$ first grows according to (2), but at the later stages of the growth process the width of the active zone saturates. It becomes an L-dependent quantity scaling with the length of the deposit as $\sigma_L(\infty) \sim L^{\beta}$, where $\beta = 0.42 \pm 0.03$.

This result and (2) can be combined into a finite-size scaling expression of the form

$$\sigma_L(M) \sim L^{\beta} f(M/L^{\gamma}) \tag{3}$$

where $f(x)$ is a scaling function defined by

$$f(x) \sim \begin{cases} x^{\nu'} & \text{for } x \ll 1, \\ \text{constant} & \text{for } x \gg 1, \end{cases}$$

and $\gamma = \beta/\nu'$. To test the scaling assumption (3) we have plotted the quantity $\sigma_L(M)/L^{\beta}$ against M/L^{γ} for several values of M and L. Figure 3 shows that our data tend to fall onto a single curve supporting the validity of the scaling form (3). This type of finite-size scaling is similar to that observed in equilibrium models. The fluctuations in the surface height do not grow beyond a size dependent limit just as the susceptibility of a finite width ferromagnet.

The problem of finding the number of active perimeter sites in the Eden model on the percolation network is analogous to the problem of growth sites for the ant in the labyrinth problem (Aharony and Stauffer 1984, Stanley *et al* 1984). The main difference between the two problems is that the ant in the labyrinth, as described by a random walker, can only move to nearest-neighbour occupied sites on a percolation network, whereas in the Eden model the growth can happen at any of the available growth sites.

In conclusion, we have studied the width of the interface in the Eden clusters on percolation networks and the ballistic deposition model. Using a simple definition we have determined the width of the active zone in these models. We find that in the ballistic deposition model the surface thickness diverges differently from the mean height. This indicates that the power law divergence of the width of the active zone is not limited to fractals. In the Eden clusters on percolation networks, we find that in contrast to the Eden clusters on a square lattice, the exponent ν', describing the divergence of the width of the active zone, is the same as the exponent ν, which describes the divergence of the cluster radius, within the statistical errors. However, the central value of ν' is slightly, but systematically, less than ν.

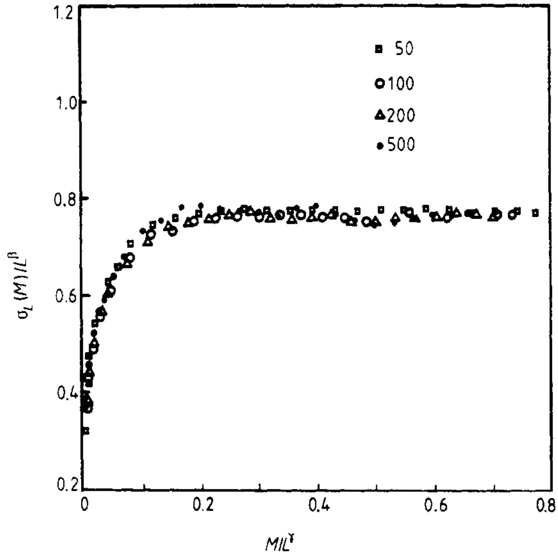

Figure 3. Scaling plot for the ballistic deposition model showing that the data for $\sigma_L(M)/L^\beta$ plotted against M/L^γ for various L and M fall on a single curve supporting the scaling form (3). The data are for various values of M and $L = 50$, 100, 200 and 500, where L is the width of the deposit.

In addition we have presented a finite-size scaling analysis of the ballistic deposition model. The simulation results support our scaling assumption.

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