Complexity Science and Ecology

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MaxEnt: The Formulas

Information entropy of a distribution X: $H(X) = -\sum_{i=1}^{n} p_i \log(p_i)$

Constraints: $\sum_{i=1}^{n} p_i = 1 \qquad \sum_{i=1}^{n} f_k(i) p_i = < f_k >$

Solution of constrained optimisation of H(X):
$$p_i = \frac{e^{-\sum_{k=1}^{K} \lambda_k f_k(i)}}{Z}$$
$$Z = \sum_{i=1}^{N} e^{-\sum_{k=1}^{K} \lambda_k f_k(i)}$$

Number of microstates = number of permutations of the macrostate: (c = number of events/cells, c_i = number of events/cells in which i entities/individuals reside)
$$\Omega = \frac{c!}{c_0! c_1! ... c_n!}$$

Exercises

Using Maximum Entropy

1. Consider again the example of the three-sided dice.
    a. Additional to the normalisation constraint, you know that on average, 2.5 is thrown. What is the MaxEnt distribution? (Hint: get rid of λ_0 and substitute $x := e^{\lambda 1}$ at some appropriate point.)
    b. What is the MaxEnt prediction if the mean is 2?
    c. How can you derive the distribution if also the variance is given (no need for explicit calculations here)?
    d. It can be shown that the MaxEnt solution of p(n) imposing the constraint of the mean and the variance is a normal distribution. We know that a fair dice has a uniform distribution. How can the statements be reconciled?

2. The species-level spatial abundance distribution SSAD describes the probability, that a given area A contains n individuals of a certain species.
    a. If you envision n0 individuals living on a spatial grid of area A0, which is subdivided in cells of area A, and you want to infer the SSAD Π(n|A,n0,A0), what are your state variables and your constraint functions?
    b. What shape does hence the probability distribution have?

Using statistical mechanics/counting

1. Now we want to derive the SSAD with a statistical mechanics/counting approach. First we have to define micro- and macrostates.
    a. Envision a landscape divided into two cells of equal size and two individuals X and Y distributed over the space. Assuming random placement, which are the different microstates the system can reside in? A microstate shall refer to the knowledge of the particular location of each individual across all possible cells.
    b. The macrostate is defined as an unordered count of the number of individuals in each of the cells without regarding the identities of those. Which different macrostates do we see for two individuals distributed over two cells?
    c. Given random placement, what are the likelihoods of the macrostates of two individuals distributed over four cells?

2. The approach in exercise seems perfectly reasonable, except for the fact that the resulting distribution simply doesn't describe ecological reality. This has to do with the definition of a microstate.
   a. Consider the idea of two indistinguishable individuals X and X in two cells. What are mircostates?
   b. What are the probabilities of the macrostates of two indistinguishable individuals distributed over four cells?
   c. Exercise 1 built on the idea of random placement of distinguishable individuals. Another placement rule is called HEAP, Hypothesis of Equal Allocation Probabilities, and allocates individuals over cells according to the following distribution:

      $$P(c_i) = \frac{n_i + 1}{\sum^i n_i + c}$$

      c_i is the i'th cell, n_i the number of individuals in this cell.
      What allocation principle does this distribution describe?
      Convince yourself that applying this rule in the four cells two individuals case leads to the correct distribution.

Degree Distribution

One way of extending the Maximum Entropy Theory of Ecology to food webs is to study the degree distribution of interaction links. What are your state variables? What probability distribution are you possibly looking for? What are your most basic constraints? What shape of probability distribution do you predict? Which other constraints could you envision?

Bonus Question: Species Commonality

A distribution of common interest in ecology is the community level commonality, which describes the co-occurrence of individuals of a certain species in spots with a known distance in between. The species-level commonality $C(m, B, d|n, A, n_0, A_0)$ is defined as the probability that a species with abundance *n0* in *A0* and *n* individuals in *A* (which is contained in *A0*) occurs with *m* individuals in a randomly picked space of area *B* in distance *d* from *A*. How could you use the species-level spatial abundance distribution $\Pi(n|A,n0,A0)$ predicted by METE to derive species commonality?

Further Reading

Tangled Nature Model

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The Maximum Entropy Theory of Ecology

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- Shannon, C. E. (1948). A Mathematical Theory of Communication. *The Bell System Techincal Journal*, *27*(1948), 379–423.

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A Holistic View on Ecology

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