

# High Performance Computing Programming Exercises

27<sup>th</sup> November - 1<sup>st</sup> December 2017  
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On each question, it will be indicated [in brackets] how many marks are available. The marks add up to 100, but together they only count for 60% of your final score for the assignment. The remaining 40% of your final score will be discretionary and based on the overall quality of your text answers and the extent to which they demonstrate your understanding of the topics, the style and quality of the computer code that you handed in and your answers to any of the challenge questions. 'Challenge questions' are therefore not essential but attempting some of them will always improve your result. It is advisable that you only work on 'challenge questions' after you have answered the main questions and checked your answers thoroughly. The reason for this mark scheme is to bring the distribution of marks more in line with that which you would expect from an essay question.

You should hand in three files:

- A typed document giving your answers to the questions marked ★ these will sometimes be written only, other times they require a graph with a caption.
- A single file containing all the commented R code that you ran on your own computer to complete the worksheet.
- A zip file containing all your results files from the cluster along with the shell script and R code that was run on the cluster to produce them.

Many of the functions in your R code file will be marked automatically – so be careful to:

- Name the files by your username e.g. `jrosinde.R`
- Name all your functions exactly as in the instructions.
- Put `"rm(list=ls())"` and `"graphics.off()"` at the top of your file.
- Do not use packages as they should not be needed for this.
- If I run the source command on your file it should run without error and load all the functions into memory so that they can be tested.
- It should not actually run the functions or do anything else.

Deadline 14<sup>th</sup> December 2017

## Neutral Theory Simulations

These questions build on one another step by step so by the end you will have produced your own individual based simulation code in R.

You will store the state of your simulated system as a vector of individuals called '**community**'. Each entry in the vector is a number that tells you the species of the individual in that position.

1.) You will need to know the species richness of your system so write a function '**species\_richness(community)**' to measure the species richness in the input vector '**community**'. For example, **species\_richness(c(1,4,4,5,1,6,1))** should return 4. (Hint: use the 'unique' command) [2 marks]

2.) Write a function **'initialise\_max(size)'** to generate an initial state for your simulation community with the maximum possible number of species for the community of size given by the input number variable 'size'. For example **initialise\_max(7)** should return a vector { 1 2 3 4 5 6 7 } (Hint: use the 'seq' command) [1 mark]

3.) In this type of simulation, it's important to consider the effect of the initial condition so write another function **'initialise\_min(size)'** to generate an alternative initial state for your simulation of a certain size with the minimum possible number of species (that's mono-dominance of one species with a total number of individuals given by the input number 'size'). For example **initialise\_min(4)** should return a vector { 1 1 1 1 }. [1 mark]

Now test what you've done....

species\_richness(initialise\_min(x)) should return 1 (no matter what your value of x was)

species\_richness(initialise\_max(x)) should return x (no matter what your value of x was)

4.) Write a function **'choose\_two(x)'**. This function should first choose a random number according to a uniform distribution between 1 and x inclusive of the endpoints. It should also choose a second random number also between 1 and x but not equal to the first number. The numbers should be returned as a vector of length 2. So **'choose\_two(4)'** should return one of the following vectors with equal probability:

{1 2}, {1 3}, {1 4}, {2 1}, {2 3}, {2 4}, {3 1}, {3 2}, {3 4}, {4 1}, {4 2}, {4 3}

(Hint: use the 'sample' command) [2 marks]

5.) Write a function **'neutral\_step(community)'** to perform a single step of a simple neutral model simulation, without speciation, on a community vector. You will need to pick an individual to die and another to reproduce to fill the gap left by the death - they should not be the same individual (though they could be of the same species). For example **neutral\_step(c(10,5,13))** should return one of the following six community states with equal probability:

{ 5 5 13 } when the first individual dies and is replaced by the second's offspring

{ 13 5 13 } when the first individual dies and is replaced by the third's offspring

{ 10 10 13 } when the second individual dies and is replaced by the first's offspring

{ 10 13 13 } when the second individual dies and is replaced by the third's offspring

{ 10 5 10 } when the third individual dies and is replaced by the first's offspring

{ 10 5 5 } when the third individual dies and is replaced by the second's offspring

(Hint: call your function choose\_two, **but don't think of the numbers it returns and being the species identity of the individuals to die and reproduce**, instead think of them as the **indexes** of your **community** vector where the individuals that will die and reproduce are stored.)

[2 marks]

6.) Write a function **'neutral\_generation(community)'** to simulate several neutral\_steps on a community so that a generation has passed. If the community consists of x individuals, then x/2 individual neutral steps will correspond to a complete generation for the taxa being simulated. A generation is the amount of time expected between birth and reproduction (not the time between birth and death, which is longer if generations overlap). For example, if there are 10 individuals in the system then 5 neutral steps correspond to 5 births and 5 deaths, one generation. Your function should return a vector giving the state of the community after a generation has passed. (Hint: use neutral\_step when writing this function, and use a loop)[2 marks]

7.) Write a function **'neutral\_time\_series(initial,duration)'** that will do a neutral theory simulation and return a time series of species richness in the system. The function should

have two inputs: **initial** (the initial condition community vector, which also determines the simulation size) and **duration** (the number generations that you want to run the simulation for). The function should return a vector giving the species richness at each generation of the simulation run starting with the initial condition species richness. For example **neutral\_time\_series(initial = initialise\_max(7), duration = 20)** should return a vector containing firstly a time series vector of length 21 with the first value being 7. (Hint: use your own function neutral\_generation from above) [2 marks].

8.) ★ Plot a time series graph of your neutral model simulation from an initial condition of maximal diversity in a system size of 100 individuals. Run the simulation for 200 generations. Make sure that the axes are properly labelled and that the write up contains a suitable title and caption for the graph. Include the code you wrote for this question in a function called '**question\_8**' which should require no inputs to run. What state will the system always converge to if you wait long enough? Why is this? (Hint: use your own function neutral\_time\_series from above, as well as the plot command)[3 marks]

9.) Write a new function '**neutral\_step\_speciation(community,v)**' which will perform a step of a neutral model with speciation. In each time step, speciation will replace a dead individual with a new species (with probability  $v$ ) otherwise the dead individual is replaced with the offspring of another individual as before in '**neutral\_step**'. You should leave speciation rate,  $v$ , as a parameter in your function. For example, '**neutral\_step\_speciation(c(10,5,13),v = 0.2)**' should behave like '**neutral\_step(c(10,5,13))**' with probability 0.8, and with probability 0.2 it should instead be equally likely to return any of the following three vectors { 1 5 13 }, { 10 1 13 }, { 10 5 1 } with the number 1 representing the new species. (Hint: use the 'runif' command, also be careful to make sure that any new species really have a unique number assigned to them that has not been used before - it needn't be 1 in the above example but it cannot be 5, 10 or 13. You'll find it easiest to copy and paste your neutral\_step function and then edit it) [3 marks]

10.) Make a new function '**neutral\_generation\_speciation(community,v)**' which uses a neutral simulation with speciation, but otherwise performs in the same way as '**neutral\_generation**' so it advances one generation according to the rules of the model. The new function should have two inputs: the initial community vector and the speciation rate. It should return the state of the community at the end of the generation long set of simulation steps. (Hint: You'll be using neutral\_step\_speciation. It'll be easiest to copy and paste the neutral\_generation function and then edit it to make this function) [1 mark].

11.) Make a new function '**neutral\_time\_series\_speciation(community,v,duration)**' which uses a neutral simulation with speciation, but otherwise performs in the same way as '**neutral\_time\_series**'. The function should have three input parameters: the same two as '**neutral\_time\_series**', and an additional input  $v$  for the speciation rate. The return should be in the same format as before, a time series vector (Hint: You'll be using neutral\_generation\_speciation. It'll be easiest to copy and paste the neutral\_time\_series function and then edit it to make this function) [1 mark].

12.) ★ Perform a neutral theory simulation with speciation and plot species richness against time as you above. Use a speciation rate of  $v = 0.1$ , a community size of  $J = 100$  and run your simulation for 200 generations. Plot two time series on the same axes in different colours showing how the simulation progresses from two different initial states given by **initialise\_max** and **initialise\_min**. Include the code you wrote for this question in a function called '**question\_12**' which should require no inputs to run. Explain what you

found from this plot about the effect of initial conditions. Why does the neutral model simulation give you those particular results? [4 marks]

13.) You are going to study the species abundance distribution of these neutral simulations. First you need to write a function '**species\_abundance**' to tell you what the abundances of all the species are in the system from an input of your community vector. For example **species\_abundance(c(1,5,3,6,5,6,1,1))** should return 3 2 2 1 (in that order - decreasing). This is because there are 3 of species '1', 2 of species '6', 2 of species '5' and 1 of species '3'. (Hint: use table and sort) [3 marks]

14.) Write a function '**octaves**' to bin the abundances of species (e.g. the output of the **species\_abundance** function) into what would be called 'octave classes'. The first value of the returned vector should tell you how many species have an abundance of only 1, the second value of the returned vector should tell you how many species have an abundance of either 2 or 3 and in general the  $n^{\text{th}}$  value of the returned vector should tell you how many species have an abundance greater than or equal to  $2^{n-1}$  whilst strictly less than  $2^n$ . For example, **octaves(c(100,64,63,5,4,3,2,2,1,1,1,1))** is asking us to sort 12 species into bins, the first species has an abundance of 100, the second 64 and the 4 rarest species are all represented by one individual only. **octaves(c(100,64,63,5,4,3,2,2,1,1,1,1))** should return 4 3 2 0 0 1 2 in that order. (Hint: use the log, floor and tabulate functions) [3 marks]

The simulations are stochastic you will therefore need to average the result from a number of independent readings to get an idea of the overall behaviour of the system. You will find that the octave vectors that are not always the same length, so R will not allow you to simply add them, or worse will sum them in a way that you do not intend so will give the wrong answer.

15.) Write a function '**sum\_vect(x, y)**' which accepts two vectors as inputs, x and y, and returns their sum, after filling whichever of the vectors that is shorter with zeros to bring it up to the correct length. For example **sum\_vect(c(1,3),c(1,0,5,2))** should return (2,3,5,2). (Hint: use length and if) [2 marks]

16.) ★ Run a neutral model simulation using the same parameters as in question 12 for a 'burn in' period of 200 generations. Next record the species abundance octave vector. Then repeatedly continue the simulation from where you left off for a further 2000 generations, and record the species abundance octave vector every 20 generations. Produce a bar chart plot of the average species abundance distribution (as octaves). Include the code you wrote for this question in a function called '**question\_16**' which should require no inputs to run. Does the initial condition of the system matter? Why is this? (Hint: it's OK to use a for loop here, it will also be helpful to use the **sum\_vect**, **octaves** and **species\_abundance** and **neutral\_generation\_speciation** functions that you already wrote. You will also find the **%%** function useful). [4 marks]

Challenge Question A: ★ Plot the mean species richness as a function of time (measured in simulation steps) across a large number of repeat simulations using the same parameters as in question 14. Add a 97.2% confidence interval on the species richness at each point in time. Repeat this for both initial conditions (high initial diversity and low initial diversity). Estimate the number of time steps needed for the system to reach dynamic equilibrium. Include the code you wrote for this question in a function called '**challenge\_A**'.

Challenge Question B: ★ Plot a graph showing many averaged time series for a whole range of different initial species richnesses. In each initial community state, each individual should be equally likely to take any species identity. Include the code you wrote for this question in a function called '**challenge\_B**'. (Hint: it's OK both here and elsewhere to make additional functions of your own to help make your code neater)

## Simulations using HPC

You are going to be running a much larger simulation of the same type that you conducted for your answer to question 16 and with more repeat readings. To do this requires use of high performance computing (HPC) and some adaptation of your R code. Make a new R file for running on the cluster and as you code press 'source' every time you run it because that's what will happen on the cluster (still keep your original file for the rest of the worksheet). You will need to copy and paste over some functions from your work so far: `neutral_generation_speciation` (which in turn uses `neutral_step_speciation` and `choose_two`), `species_richness`, `species_abundance`, `octaves` and `initialise_min`. The other functions you have written are not needed but remember how you got them to work because the same kind of code will be needed here.

17.) Create a function '**cluster\_run**' which accepts seven input parameters: `speciation_rate`, `size`, `wall_time`, `interval_rich`, `interval_oct`, `burn_in_generations` and `output_file_name`. [6 marks] This function should:

- Start with a community with size given by the input 'size' and minimal diversity.
- Apply neutral generations with a speciation rate given by 'speciation\_rate' for a predefined amount of time 'wall\_time' measured in minutes. (Hint: if you're not sure where to start get a timer working on its own first and use the `proc.time` command for this).
- Store the species richness at intervals of `interval_rich`, but only during the burn in period, which is measured in generations. After the number of generations exceeds the burn in time, stop recording the species richness. So, suppose `interval_rich` is 2, this means save the species richness every other generation (Hint use a vector to store the species richnesses and use `%%` to help detect when to do this, it's probably easier to have one main simulation loop and use if statements inside it to determine whether the simulation is burning in or not).
- For the entire simulation, until the simulation runs out of time, you should record the species abundances as octaves every `interval_oct` generations. (Hint: use `list`)
- You should save your simulation results in a file with name given by the input 'output\_file\_name' including the following data: the time series recorded during the burn\_in\_time, the list of species abundance octaves, the state of the community at the end of the simulation, the total amount of time actually consumed on the simulation and all six of the input parameters for the function (obviously you don't need to store a reminder of what the file name is inside the file itself).
- Test your code locally before proceeding further using the same parameters from question 16 and a short time limit of 5-10 minutes. For example...  
`cluster_run(speciation_rate = 0.1, size=100, wall_time=10, interval_rich=1, interval_oct=10, burn_in_generations=200 and output_file_name="my_test_file_1.rda")` should run for 10 minutes and return nothing but save a file called "my\_test\_file\_1.rda" which you can then open in R and look at the data from to check you've got everything you need as described above. (Hint: use the `save` command, and don't call your outputs by the same names as your functions - otherwise this will save the functions and not the outputs.)

18.) Now you're ready to write lines of code in your R file around the functions you have written so that when you run the file using the source command you will get the simulation you want. You will not be writing another function, but rather writing R code on the end of the file to be run. Here's what the code needs to do (in this order)...

- a) Clear workspace and turn off graphics
- b) Read in the job number from the cluster. To do this your code should include a new variable *'iter'* and should start with the line: ***iter <- as.numeric(Sys.getenv("PBS\_ARRAY\_INDEX"))***. However, for testing on your own machine this will not work so write the line and then comment it out and instead set iter yourself to 1,2,3,... 100 for testing locally. The last thing you need to do is uncomment the line and remove your setting of iter so that now the cluster chooses what iter is equal to. Your code will be run 100 times in parallel on the cluster, it will be run with iter = 1,2,3, ..., 100 so you should use the variable iter in your code to make sure you don't just repeat the identical simulation 100 times.
- c) You need to control the random number seeds so that each parallel simulation takes place with a different seed. If you run two simulations with the same seed, you will get the same answer regardless of the fact that it's a stochastic simulation. So your function should set the random number seed as iter
- d) Everyone will use the same values for community size J in their simulations (500,1000,2500,5000) but each person will have different speciation rates - handed out separately. You will have to select the correct value for J in each parallel simulation based on the value of iter (e.g. J = 500 when iter = 1,5,9,13,... And J = 1000 when iter = 2,6,10,14,...) - think of this as a todo list with 100 items on that will in the end be done in parallel.
- e) Create a filename to store your results - the end of the file name should be a number given by iter (the number of the random seed). This way simulation files will not overwrite one another on the cluster where there will be no nice warning message asking if you want to replace the file or save under a different name! Also this gives you have a sanity check that no pair of simulations were conducted with the same random seed. (Hint: use the paste command)
- f) Call the cluster\_run function, which will actually do the simulation and save the results. Use a time limit of 12 hours for all your jobs (you will put 11.5 hours into your code and tell the cluster 12 hours just in case). Use interval\_rich = 1, interval\_oct = size/10 (roughly) and burn\_in\_time = 8\*size

Test your code locally for a shorter length of time before running it on the cluster. Then test on the cluster with iter of 1,2,3 only before doing the rest (use -J 1-3 as in the lecture notes) [6 marks for correct test code and correct code for running on the cluster]

19.) Write shell script for running your code on the cluster. Use sftp and ssh to set your jobs running on the cluster as instructed during the lecture (also see lecture notes). Run a small job first just to test, then run the full set of jobs to the cluster [10 marks for all your output files shell script code and R code in a zip]

20.) ★ While your job is running on the cluster you can write R code to read in and process the output files. Your code should provide four bar graphs in a multi-panel graph (one for each simulation size) each showing a mean species abundance octave result from all your simulation runs of that size. Only use data of the abundance octaves after the burn in time is up. (Hint: use the load function on your .rda files and use sum\_vect). Please hand in all your plots and the raw data used to produce the plots (i.e. four vectors of numbers) along with the exact speciation rate that you used. [10 marks for your graphs and results]

Challenge Question C: ★ Plot a graph of mean species richness against simulation generation and use it to inform you more precisely how long should have been allowed as a burn in period for different values of  $J$ . Include the code you wrote for this question in a function called '**challenge\_C**'.

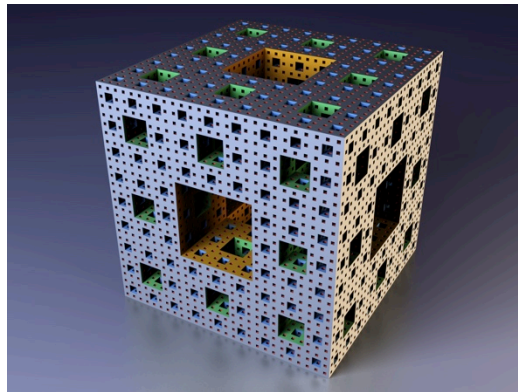
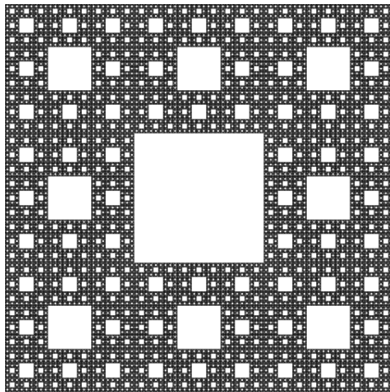
Challenge Question D: ★ Conduct further simulations of the same system using coalescence (see the pseudo code below). Check that your results from the cluster agree with those from coalescence and compare the speed of the two approaches. How many CPU hours were used on the coalescence simulation and how many on the cluster to do an equivalent set of simulations? Why were the coalescence simulations so much faster? Include the code you wrote for this question in a function called '**challenge\_D**'. To get a coalescence simulation in R of the neutral model from question 16 as a function of community size  $J$  and speciation rate  $v$ .

- a. Initialise a vector **lineages** of length  $J$  with 1 as every entry.
- b. Initialise an empty vector **abundances**.
- c. Initialise a number  $N = J$ .
- d. Calculate  $\theta$ , where  $\theta = v \frac{J-1}{1-v}$ .
- e. Choose an index  $j$  of the vector **lineages** at random according to a uniform distribution.
- f. Pick a random decimal number **randnum** between 0 and 1.
- g. If **randnum**  $< \frac{\theta}{\theta + N - 1}$  append **lineages[j]** to the vector **abundances**.
- h. If **randnum**  $\geq \frac{\theta}{\theta + N - 1}$  choose another index  $i$  of the vector **lineages** at random, but not allowing  $i = j$ . Then set **lineages[i] = lineages[i] + lineages[j]**.
- i. remove **lineages[j]** from **lineages** so that the **lineages** vector is now one shorter.
- j. Decrease  $N$  by one so that  $N$  still gives the length of the **lineages** vector.
- k. If  $N > 1$  repeat the code again from e through to here.
- l. Add the only element left in **lineages** to the end of **abundances**.
- m. END: a vector of simulated species abundances is stored in **abundances**.

## Fractals in nature

18.) ★ What are the fractal dimensions of these objects? Show and briefly explain your workings. [4 marks]

*Hint: the object on the right looks the same from all six faces and is hollow in the very center; it should have a dimension somewhere between 2 and 3.*



19.) ★ The chaos game

- Store the following three points that correspond to coordinate on a graph:  $a=(0,0)$ ,  $b=(3,4)$  and  $c=(4,1)$ .
- Initialize the point vector  $X$  to indicate the point  $(0,0)$ .
- Plot a **very small** point on the graph at  $X$ . (hint: use `cex`)
- Choose one of the three points ( $a$ ,  $b$  or  $c$ ) at random and move  $X$  half way towards whichever of the three points you chose.
- Write a loop to repeat the code of c. and d. 100 times – what do you see? Now try increasing the number of repeats to 1000 or more. The function that does this should be called '**chaos\_game**' [8 marks]

Challenge question E: ★ Try starting the chaos game from a completely different initial position  $X$  what happens now and why? Try plotting the first  $n$  steps in a different colour for various values of  $n$  to help you answer this. Try starting with the points of an equilateral triangle as  $a$ ,  $b$  and  $c$  to produce a classic Sierpinski Gasket. If you're feeling super enthusiastic you could have more than 3 points and a distance of movement different from a half towards the next point.

20.) Create a function '**turtle**' in R to draw a line of a given length from a given point (defined as a vector) and in a given direction. So, '**turtle**' will have three inputs: start position, direction and length. As well as drawing the line, turtle should return the endpoint of the line it just drew as a vector. **Turtle should not open the plot it should just draw the line on an already open plot**, this is because in a moment you are going to use successive calls of the function to draw things and you want all the lines to be on the same axes. **So, you will need to open the plot with a command outside of turtle when you want to test it and the same applies to all the below functions too.** (Hint: you need to use `sin` and `cos`). [2 marks]



21.) Now create another function **'elbow'** that calls **'turtle'** twice to draw a pair of lines that join together. **'elbow'** should accept as an input: the starting point, direction and length of the first line. The second line should start at the end point of the first line, have a direction that is 45 degrees ( $\pi/4$  radians) to the right of that of the first line and a length that is 0.95 times the length of the first line. [2 marks]

22.) ★ Now copy and paste your 'elbow' function and rename it **'spiral'**. Spiral will be an iterative function that draws a spiral. Instead of calling **'turtle'** twice to draw the first and second lines, spiral should call **'turtle'** to draw the first line and then call itself **'spiral'** instead of **'turtle'** to draw the second line. What happens and now and why? (Hint: if you get an error message that might be what's expected! Try and think about why you're getting it - think like a computer – run through the code you just wrote in your own head and see where it gets you) [2 marks]

23.) ★ Edit the 'spiral' function calling it **'spiral\_2'**. The edit should make it so that 'spiral\_2' will only act if it's called with a line length that's above a certain size (e, a variable that you can experiment with). Now your code will draw a spiral shape on the graph without crashing or giving any error messages. [3 marks for a working spiral plotting function]

24.) ★ Now, copy and paste the **'spiral\_2'** function and rename the copy **'tree'**. Instead of having **'tree'** call itself only once (as **'spiral\_2'** did), you should have it call itself twice: with directions that are 45 degrees to the right and 45 degrees to the left. Also, make the length of each subsequent call 0.65 times the length of the previous call (instead of 0.95 as it was for drawing the spiral). Don't forget that **'tree'** should still call **'turtle'** once as well as **'tree'** twice. You should get an attractive tree shape as your output plot. [4 marks for a working tree plotting function]

25.) Now copy and paste **'tree'** and rename it to **'fern'**. Change your variables so that whilst one of the two branches goes 45 degrees to the left (as it did in f.) the other goes straight up (instead of to the right). Length multiples should now be 0.38 for the branch going to the left and 0.87 for the branch going straight up (instead of 0.65 for both as it was before). [3 marks]

26.) ★ Now copy and paste the function 'fern' and rename it to **'fern\_2'**. This should have an input parameter 'dir' which will decide whether the side branch of the fern goes to the left or right (it's easiest to do this with a variable that takes the value of either -1 or +1). When calling **'fern\_2'** iteratively from within itself allow the direction of the side branch to alternate by passing on the 'dir' variable that has been multiplied by -1 to reverse the direction. You should now get an attractive fern picture. [4 marks for a working fern plotting function]

Challenge question F: ★ What do you notice about the image produced and the time the program takes to run as you vary the value of e (the line size threshold)? experiment with the variables and colours to produce other types of fern and tree. Try using multiple colours – bonus points for being imaginative. Include the code you wrote for this question in a function called **'challenge\_F'**, you can create a **'challenge\_F2'** and a **'challenge\_F3'** if you need to.

Challenge question G: See how small you can make your code answer to question 26 without breaking it. To beat the record you would need to do it in less than 154 characters on one line of code (it would fit in a single text message). If you attempt

this challenge, please make your shortened code a separate function named **'challenge\_G'** from your main answer to question 26 because you'll have to remove all your comments to shorten the code. In the past there has been a fair amount of spirited debate about who had done this the best! To be clear, the rules are: your code should work fine even after the workspace has been cleared and should require no libraries, also, no marks should appear on your output axis apart from the lines that make up the fern. Finally, the fern should be of reasonable quality and at least very close in appearance to the fern produced in question 26 – if your code is shorter but doesn't produce the correct result then it doesn't count! This is what it should look like when displayed within in a normal window in R on a normal display, though there could be axes around it. (note: the definition of Challenge\_G and open and close {} do not count towards the character count, you should define another function inside Challenge\_G that calls itself)

