PRACTICAL WORK 2

Some of the questions below relate to the lecture on Wednesday, and will be addressed during the Wednesday lecture.

Investigate the phase portrait of the Lotka-Volterra competition model
(1) Use XPP to investigate the dynamical behaviour of the Lotka-Volterra competition model. The file `lvcomp1.ode` contains the Lotka-Volterra competition model, and a set of parameters. It is set up to show you the phase plane (N1-N2). First, have a look at the behaviour against time (easiest done using "*Xi vs t*"), you will have to change the axes, make sure you understand the connection between the plots of the solutions against time, and those in the phase plane (using "*Viewaxes*" plot N2 vs N1). Do you understand how the time plots relate to the phase plane plots? After you have done this, close the window, *quit* XPP and start afresh. (If *File Quit Yes* doesn't work, close the Run window)

For the second part of the exercise, make the screen as large as you can. For this set of parameters, draw the isoclines by clicking on *Nullcline*, the isoclines appear as dashed lines in your diagram. Next, draw the vector field (the direction of the flow, given by arrows). Do this first by clicking on *Dir. Field/flow*, *(D)irect Field* and entering the number arrows in each direction (10 is a reasonable choice.) This draws the arrows, and their size indicates the rate of change in the variables. Next, *Erase* the field, redraw the *Nullclines* and now plot the *(S)caled Dir. Fld*. Do you understand how these fields relate to the phase plane plots?

You can draw a selection of solutions by selection *Dir. Field/flow* and then *Flow* and entering the appropriate number (the actual solutions that will be drawn is the square of this number, so don't put too high a number in). Once you have seen this *Erase* the diagram, and under *Initialconds*, select *Mice*. If you click in the diagram you will see a solution starting at the point you just clicked at. Click at various points in space. What happens? Can you identify the direction of the eigenvectors around the equilibrium?

(2) repeat the above with file `lvcomp2.ode`. This file contains the same model but with a different choice of parameters.

(3) in `lvcomp2.ode` change the parameter b to 0.8333.What happens to the isoclines?

Do a 1 parameter bifurcation diagram of the Lotka Volterra competition model
Use the files `lvcomp1.ode` and `lvcomp2.ode` to draw a bifurcation diagram to plot N2 vs k1. Plot the same diagram for N2 vs k2.
If you look at the bifurcation diagram, you see that there are a lot of different equilibria, some stable, some unstable. Not all of these lines correspond to biologically feasible equilibria. To get some feeling for what the lines mean plot them as functions of N1 and as functions of N2 (you do this is the AUTO window, go to *Axes*, choose *hI-lo* and change the Y-axis). For some of the equilibria you see that either N1=0, or N2=0 or that one variable takes negative values.

Investigate the phase portrait for the model in the Mumby et al paper
The model of Mumby et al. (2007) describes the dynamics of corals and algae. It is implemented in the file `mumby.ode`. Investigate the dynamics in the phase plane. Can you make a bifurcation diagram in which you plot the value of variable *C* at equilibrium against *g*?

Try to make sense of what these lines mean. Do all of them correspond to biologically feasible equilibria? If there are some that still do not make sense: in this model we assume that the amount of algal turf is 1-M-C, and therefore if M+C>1, the amount of algal turf is negative. One way to find out what the value of M and C simultaneously is to *Grab* the point, go back to the XPP window and inspect the initial conditions by clicking on the *ICs* box in the top left corner.

An analysis of the codim 2 cusp bifurcation.

In Scheffer et al. (2001) a minimal model for ecosystem with hysteresis is given. The variable *x* is a property (for instance a nutrient) of the ecosystem, it changes over time as:

$$\frac{dx}{dt} = a - bx + rf(x)$$

the parameter *a* represents an environmental factor that promotes *x*, *b* is the rate at which *x* decays in the system. The property increases due to a positive feedback, and the rate of increases depends on parameter *r*, and covers again as a function of *x*. The rate of increase can increase sharply at a threshold *h.* The function *f* is given by the Hill function:

$$f(x) = \frac{x^p}{x^p + h^p}.$$

Use XPP on the file `scheffer.ode` to find the equilibrium point. Now take the equilibrium point to AUTO. Draw the bifurcation diagram in the parameter *r*. Interpret what you see and use XPP to investigate what the dynamics for different parts of the curve are. Once you have done this, go back to AUTO, Grab the curve and go through until you are at a point where the black and red colours meet (if you will use the tab key you will get there quickly). At this point the line under the plot will say LP under the header *Ty.* LP stands for Limit point, it indicates a fold bifurcation point. Grab this point by hitting the return. Now go to *Axes* and select *Two Par.* Change *Ymax* from 20 to 10 and click Ok. Your screen will now show the *h* vs *r* space. Now hit the *Run* button again. A red line appears. Go to *Numerics*, change the sign on *Ds*, go back to the AUTO window and hit *Run* to calculate the other part of the curve. What does this curve mean? Try to make some sense of it, by investigating the dynamics.
What you have plotted is a codimension 2 bifurcation (a bifurcation diagram in 2 parameters.) This particular bifurcation is called a cusp.

Make a bifurcation diagram for the logistic map.
Use the file `logbif.ode`. After the main window has opened, select *Graphics stuff*, *(E)dit Curve* then input 0. Then change the *Line type* to 0 (so you plot points rather than lines). Next go to *Initialconds* and select *(R)ange.*Range over*: k Change the nr of *Steps* to about 200, Change the begin and end point so k starts at 0 (unless you want it to be larger) and stops at 3. *Reset Storage*: N. Now click OK. The diagram will appear.

Note, this is not really a bifurcation diagram in that we track solutions, but a plot of the curve for different parameters. Not that it matters, but the wording could be confusing.

You can zoom in by changing the range over which you let *k* run, and by using *Viewaxes* you can zoom in on part of the diagram, if you want to.

Investigate the dependence on initial conditions, and how it affects the predictability of the logistic map. To do this, use the file `logpred.ode`. It contains two copies of the logistic map, which only differ in the initial conditions (set at 0.4 and 0.401). Think of one of these models as the real world, and the second copy of your model of the world. I want you to find out what happens if you know exactly what happens in the real world (in the sense that you know the parameter of the real world) apart from the exact current state of the world in which you have made a small mistake, which reflect the small difference in initial conditions. We are going to test the model of the real world by plotting the predicted value against the real value.

Select *Graphics stuff*, *(E)dit Curve* then input 0. Then change the *Line type* to -3 (so you plot points rather than lines. If the points are too big, use a different negative number. If you make it 0 the points are very small). Now go to *Initialconds* and select *(G)o.* look carefully at your output, there are points there. Where are they? Now manually increase the parameter *k* bit by bit (go up slowly, and there is no point taking *k* over 3. The interesting bit is around 2.57). What happens? Why this pattern? Is the world predictable? To what extent? Where does chaos start and predictability end?
