Practical 4 (12 Feb 2015)

Question 1

Question 4 from yesterday's Practical 3.

Question 2

Yesterday's part iii, Question 3. Find the 95% confidence interval for p if we observed $y = 35$ heads out of $n = 50$ independent tosses, where p is the $Prob(head)$. Use R and the appropriate likelihood function to help you.

Question 3

Non-standard regression.

Back to our rabbit data. In traditional linear regression, the variance of the residuals is assumed to be constant (actually, the residuals follow iid normal with mean zero and a common variance). We can see from the scatterplot that the variance seems to be increasing with the number of days. In terms of biology, it makes sense that there is a greater variability in the change in body length if we do not see a rabbit for a longer time. Instead of fitting the usual regression line, we would like to fit the following model:

$$\mathrm{y_i} = b * x_i + \epsilon_i, \text{ with}$$

$$\epsilon_\mathrm{i} \sim N(0, x_i^2 \sigma^2)$$

So now, the variance of residuals is not the same for all data points; the variance increases linearly with x, the number of days before rabbit i is recaptured.

Can you modify the existing log-likelihood model (don't overwrite your original code, of course) to handle with this non-standard model? Maximise the log-likelihood and see if it is better than your original one.

I bet it will be extremely difficult to fit such a model with built-in packages. However if you know MLE it is straight forward.

Question 4

One of your friends comes to you with a log-likelihood contour plot. He/she would like your advice as you're a CMEE.

i. Can you find (roughly) the 95% confidence interval for k1 and k2 respectively by looking at the log-likelihood function? Also, can you circle the joint 95% confidence region for k1 and k2?

Can you say something about the correlation of the two estimators (parameters)? (check with the online bivariate normal simulator)
http://socr.ucla.edu/htmls/HTML5/BivariateNormal/