

Maximum Likelihood Estimation

CMEE MSc

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Agenda

A.M. Lecture

P.M. Practical (with your laptop and R)

Learning outcome

- Define random variables, probability functions and associated concepts
- Understand the idea of maximum likelihood estimation
- Perform statistical modelling/ testing/ estimation under maximum likelihood framework (with R programming)
- Develop your likelihood model instead of using “canned” methods
- You’re not afraid of speaking publicly about maximum likelihood!

- Appreciate statistics, start to believe that it is more than just a subject

Introductory lecture

- What is “probability and statistics”?
- Why do we need statistics?

Example 0: German Tank Problem

- During WWII, the Allies wanted to determine how many tanks German actually produced
- Two methods: conventional intelligence vs statistical estimation
- Statistical estimation made use of the serial numbers on captured or destroyed tanks
- Say, we captured four tanks: #2, #6, #7, #14. What can we say about the total number of tanks produced?
- $\hat{N} = (\text{largest serial number captured}) * \left(1 + \frac{1}{\text{number of tanks captured}}\right) - 1$
- “The largest serial number plus the average gap between observations”

German tank problem

- If we assume the captured tank numbers come from a discrete uniform distribution, that \hat{N} is actually the maximum likelihood estimator (MLE) of the total number of tanks.
- If we look at the real data...

Month	Statistical estimate	Intelligence estimate	German records
Jun 1940	169	1000	122
Jun 1941	244	1550	271
Aug 1942	327	1550	342

- Statistical models/estimations could be useful (Of course!)

Probability vs Statistics

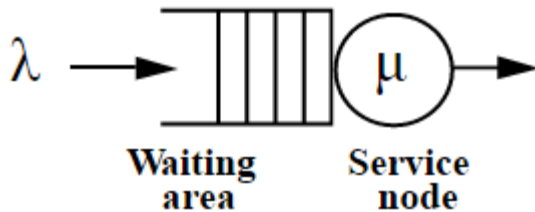
- A Probability question:
 - If the coin is fair, what's the probability of tossing two heads in a row?

Given the random mechanism,
what's the chance of the event?

- A Statistics question:
 - I observed two heads in a row, is the coin fair?

Given the observation, we try to
make inference on the random
mechanism

- Queuing system in fast food shop



@wikicommons

- Beer and nappies (and dad?) association...

BBC NEWS
 You are in: **UK**
 Monday, 13 April, 1998, 07:09 GMT 08:09 UK

Do diapers drive dads to drink?

Men who buy nappies are likely to put some cans of beer in their shopping trolley to shore up their macho image, according to a supermarket chain.

Internet links:

- Tesco
- The Campaign for Real Ale
- Families Need Fathers
- UK Homebrew
- Pubworld

Top UK stories now:

- Postcode lottery in GP services
- Leanne killer jailed for life
- Tories attack Brixton drugs scheme
- Straw defends arms sales change
- IVF mix-up heads for court
- Police shoot man on the M6
- New challenge excites Venables
- Judge urges life sentence shake-up

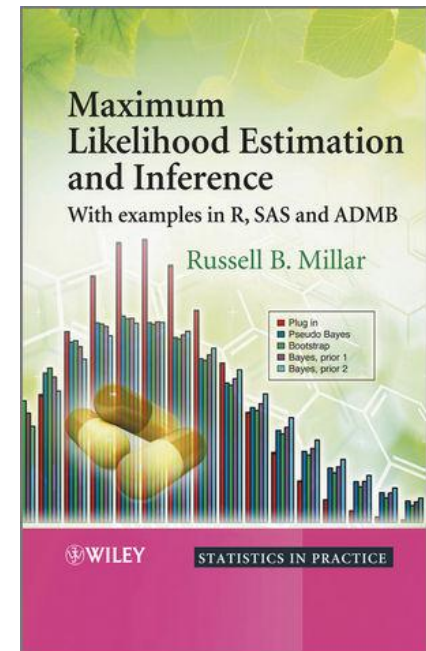
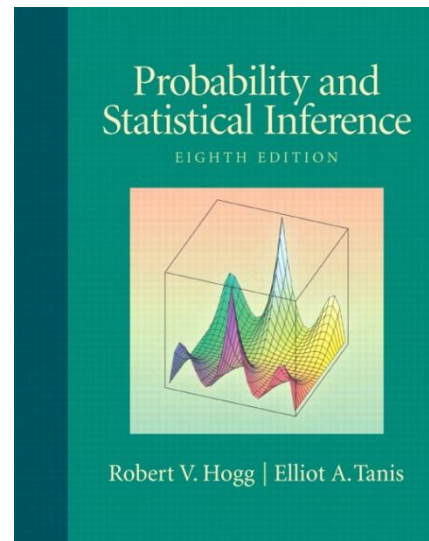
Links to more UK stories are at the foot of the page.

Statistical inference

- Point estimation (estimate a parameter)
- Interval estimation (95% confidence interval)
- Hypothesis testing (H_0 vs H_1)

Suggested readings

- Hogg and Tanis, *Probability and Statistical Inference*.
- Millar, *Maximum Likelihood Estimation and Inference*.
- Crawley, *The R Book*.



More readings

- Silver. *The Signal and the Noise: Why So Many Predictions Fail-but Some Don't.*
- Anderson and Sally. *The Number Game: Why Everything You Know About Football Is Wrong.*

“In war-time, truth is so precious that she should always be attended by a bodyguard of lies. “ – Winston Churchill