

Further Reading

Single species models, stability, the metapopulation concept

The basics are covered in Gotelli, A primer of Ecology (chapter 1, 2 and 4). More on metapopulations can be found in Hanski's books:

Hanski, I. (1999) Metapopulation Ecology. Hanski, I. and Gilpin, M. (1997) Metapopulation Biology, Academic Press (574.5248MET)

A good account of the importance of metapopulation theory applied to the spotted owl in: Gutierrez, R.J. and Harrison, S. Applying metapopulation theory to spotted owl management: a history and a critique. Pages 167-185 in D.R. McCullough (ed) Metapopulations and wildlife conservation.

A good introduction to pair approximations is van Baalen, M. (2000). Pair approximations for different spatial geometries. Pages 359-387 in: U. Dieckmann, R. Law, & J.A.J. Metz, Eds. The Geometry of Ecological Interactions: Simplifying Spatial Complexity. Cambridge University Press, Cambridge. (reprint at <http://ecologie.snv.jussieu.fr/minus/papers/vanBaalen2000.pdf>)

Interspecific competition, phase plane, eigenvalues & eigenvectors

Gotelli, A primer of Ecology chapter 5.

Otto and Day, A biologist's guide to mathematical modelling, chapters 7 and 8 gives a good account of eigenvalues and eigenvectors. The Routh-Hurwitz criteria are in box. 8.2.

Gause's book is still very readable: Gause G.F. (1934) The struggle for existence, ch. 5 <http://www.ggausa.com/Contgau.htm>

The example of the duckweed and pond weed is from: Scheffer M , et al. (2003): Floating plant dominance as a stable state. PNAS 100: 4040-4045

The apparent competition examples are from:

Bonsall, M.B and Hassell, M.P. (1997) Apparent competition structures ecological assemblages. *Nature* 388, 371-373.

Morris, R.B, Lewis, O.T and Godfray, H.C.J. (2004) Experimental evidence for apparent competition in a tropical rainforest. *Nature* 428, 310-313

Alternative stable states, phase shifts and catastrophic transitions

Alternative stable states and phase shifts are discussed in the following papers.

[Dudgeon SR.; Aronson RB.; Bruno JF and Precht WF \(2011\). Phase shifts and stable states on coral reefs. MARINE ECOLOGY-PROGRESS SERIES 413: 201-216.](#) (Please note: the link on the title brings you to a page from which you can download the paper)

Mumby PJ.; Hastings A; Edwards HJ. (2007) Thresholds and the resilience of Caribbean coral reefs. *Nature* 450: 98-101.

Scheffer M; Carpenter S; Foley JA; Folke, C and Walker, B. (2001) Catastrophic shifts in ecosystems. *Nature*, 413:591-596

The Icelandic midge example is from

Ives AR.; Einarsson A; Jansen VAA.; Gardarsson, A. (2008) High-amplitude fluctuations and alternative dynamical states of midges in Lake Myvatn. *Nature* 452: 84-87

Chaos and unpredictability

This is a classic topic in mathematical biology, and it is covered by most textbooks. Gotelli cover it with in section on the discrete logistic model and Otto and Day both cover it in Chapter 4.

The classic review by May is worth reading R.M. May (1976) Simple mathematical models with very complicated behaviour. *Nature* 261, 459 – 467.

The examples of biological chaos are from

Beninca, E. et al. (2008) Chaos in long-term experiment with a plankton community. *Nature* 822-827

Costantino, RF, Cushing, J.M, Dennis, B & Desharnais, RA (1995) Experimentally induced transitions in the dynamics behaviour of insect populations. *Nature* 375, 227-230

The paper by Pearl is:

Pearl, R. (1927) The growth of populations. *Q. Rev. Biol.* 2: 531-548

Predator-prey models, limit cycles, Hopf bifurcation

Gotelli, chapter 6, cover predator-prey models. The basics of the Hopf bifurcation are described in Otto and Day, section 11.4.

The example of the experimental demonstration is from
Fussman GF, Ellner SP, Shertzer KW & Hairston NG (2000) Crossing the Hopf
Bifurcation in a Live Predator-Prey System, *Science*, 290. 1358-1360

Some of the material on spatial predator-prey models is from
V.A.A. Jansen and A.M de Roos (2000) The role of space in reducing predator-prey
cycles. In: The Geometry of Ecological Interactions: Simplifying Spatial Complexity.
(U. Dieckmann, R. Law and J.A.J. Metz, eds). Cambridge University Press (2000)
(downloadable from <http://personal.rhul.ac.uk/ujba/115/chapter11.pdf>)

The classic paper on the spatial Nicholson-Bailey model is
Hassell MP, Comins HN, May RM, (1991) Spatial Structure and chaos in insect
population dynamics. *Nature*, 353, 255-258

The experiments with the mites on floating islands are described in
Ellner SP, McCauley E, Kendall BE, Briggs CJ, Hosseini PR, Wood SN, Janssen A,
Sabelis MW, Turchin P, Nisbet RM, Murdoch WW. 2001. Habitat structure and
population persistence in an experimental community. *Nature* 412: 538-543.

Epidemiological models.

A brief overview of theory can be found in Otto and Day, section 3.5

Two overviews of the use of models in epidemiology are in:

Scherer, A. and McLean, A. (2002) Mathematical models of vaccination. *British Medical
Bulletin* 62, 187-199

D. Louz et al. (2010) Emergence of viral diseases: mathematical modeling as
a tool for infection control, policy and decision making. *Critical Reviews in
Microbiology*, 2010; 36(3): 195–211

The study on phase locking is from

Earn, DJ, Rohani, P, Bolker, BM, Grenfell, BT(2000) A Simple Model for Complex
Dynamical Transitions in Epidemics *Science* 287: 667-670

The parts on the reemergence of measles from a vaccinated population are from
Jansen, V.A.A. et al (2003) Measles outbreaks in a population with a declining vaccine
uptake. *Science*, 301,804.

General references for bifurcation theory

A standard reference is

Guckenheimer J. and Holmes, P. (1983) Nonlinear Oscillations, Dynamical Systems, and
Bifurcations of Vector Fields. Springer-Verlag, 1983, Volume 42

A very comprehensive description, with very helpful illustrations is
Kuznetsov, Yu.A. (1995) Elements of Applied Bifurcation Theory. Springer-Verlag,
New York, 1995

The XPP programme is described in detail (with many examples)

Bard Ermentrout (2002) Simulating, Analyzing, and Animating
Dynamical Systems: A Guide to XPPAUT for Researchers and
Students. SIAM