Practical 2 (10 Feb 2015, Tuesday)

Question 1

i. If X and Y are independent, then the joint density of X and Y is the ______ of their individual densities.

ii. We need three items to perform MLE. What are they?

iii. What is the verbal definition of maximum likelihood estimation? How would you explain MLE to your fellow classmates?

iv. [Circle the correct answer] The likelihood function is a function of parameters/data.

Question 2

i. Let $X_1, X_2, \dots, X_n$ be i.i.d. $Exponential(\lambda)$. What is the likelihood function $L(\lambda)$?

ii. What is the log-likelihood function $l(\lambda)$ then?

iii. Find $\lambda = \hat{\lambda}$ such that the log-likelihood function is maximised.

Question 3

i. Let $X_1, X_2, \dots, X_n$ be i.i.d. $Poisson(\lambda)$. Find the MLE for λ.

ii. If I observed 5, 3, 2, and 6 events within a period of time, what is the best guess for λ?

Question 4

The exercise left to you in class. Let $X_1, X_2, \dots, X_n$ be i.i.d. $N(\mu, \sigma^2)$. Find the MLE for the two parameters. (Hint: Try to break down this big problem into pieces. First, write down the likelihood function, then take the log of the likelihood function. Find the derivatives, set them to zero, solve for unknowns…)

Question 5

A linear regression exercise. It is a spin-off exercise from the original marked-recapture experiment. Some rabbits are tagged, and then 29 of them are recaptured after several days. We measured the difference in their body lengths and how long (in days) they had been in the field before being recaptured. We would like to whether it is possible to predict the growth in body length from the time they spent in the wild. Please refer to today's power point slides. Ask me for the dataset if you don't have. There is a short note about the use of `optim()` on p.32 which may be helpful.