discrete (negative) exponential
= geometric distribution

1. Maximum Entropy

$$H(x) = -\sum_{i=1}^{n} p_i \ln p_i$$

constraints: $\sum_{i=1}^{n} p_i = 1$ $\quad f_0 = 1$

$\sum_{i=1}^{n} f_k(i)\, p_i = \langle f_k \rangle$

solution: $p_i = \dfrac{e^{-\sum_{k=0}^{K} \lambda_k f_k(i)}}{Z}$ $\quad Z = \sum_{i=1}^{n} e^{-\sum_{k=0}^{K} \lambda_k f_k(i)}$

λ_k: $\nabla H(x) = -\lambda_k \nabla f_k(x)$

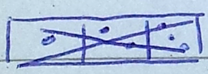
Example: three-sided dice

$n = 3$ $\quad K = 0$

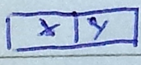
constraint: $\sum_{i=1}^{3} p_i = 1$

solution: $p_i = \dfrac{e^{-\lambda_0}}{Z} = \dfrac{e^{-\lambda_0}}{\sum_{i=1}^{3} p_i} = \dfrac{e^{-\lambda_0}}{3e^{-\lambda_0}} = \dfrac{1}{3}$

2. Counting



distributing microstates over macrostates



2 individuals over two cells - distinguishable

→ microstates
macrostates
probability

⇒ indistinguishable: microstates
macrostates
probability

larger systems

microstates in a macrostate :

$$\Omega = \frac{c!}{c_0!\,c_1! \ldots c_{n_0}!} \begin{array}{l} \leftarrow \text{arranging all cells} \\ \leftarrow \text{arranging cells with } i \text{ individuals} \end{array}$$

macrostate with most microstates: $\max \Omega \stackrel{?}{=} \max \ln \Omega$

$$\ln \Omega = \ln c! - \sum_{i=0}^{n} \ln c_i!$$

Stirling: $\ln c! \approx c\ln c - c$

$$\Rightarrow \ln \Omega = c\ln c - c - \sum_{i=0}^{n} (c_i \ln c_i - c_i)$$

$$= \underbrace{c\ln c}_{\text{constant}} - \sum_{i=0}^{n} c_i \ln c_i$$

$\Rightarrow$ maximize $- \sum_{i=0}^{n} c_i \ln c_i$

$\Rightarrow$ recall solution $c_n = a e^{-\lambda n}$

infinite support : geometric distribution

$$P(n) = p(1-p)^n$$

$$\Rightarrow P(n) = (1-e^{-\lambda})(e^{-\lambda})^n$$

mean : $\frac{1-p}{p} \rightarrow \frac{e^{-\lambda}}{1-e^{-\lambda}} \stackrel{!}{=} \frac{n_0}{c}$

$$\Rightarrow e^{-\lambda} = \frac{n_0/c}{1+n_0/c}$$