Further Reading

Single species models, stability, the metapopulation concept

The basics are covered in Gotelli, A primer of Ecology (chapter 1, 2 and 4). More on metapopulations can be found in Hanski's books:
Hanski, I. (1999) Metapopulation Ecology. Hanski, I. and Gilpin, M. (1997) Metapopulation Biology, Academic Press (574.5248MET)

A good account of the importance of metapopulation theory applied to the spotted owl in:
Gutierrez, R.J. and Harrison, S. Applying metapopulation theory to spotted owl management: a history and a critique. Pages 167-185 in D.R. McCullough (ed) Metapopulations and wildlife conservation.

A good introduction to pair approximations is
van Baalen, M. (2000). Pair approximations for different spatial geometries. Pages 359-387 in: U. Dieckmann, R. Law, & J.A.J. Metz, Eds. The Geometry of Ecological Interactions: Simplifying Spatial Complexity. Cambridge University Press, Cambridge. (reprint at http://ecologie.snv.jussieu.fr/minus/papers/vanBaalen2000.pdf)

Interspecific competition, phase plane, eigenvalues & eigenvectors

Gotelli, A primer of Ecology chapter 5.

Otto and Day, A biologist's guide to mathematical modelling, chapters 7 and 8 gives a good account of eigenvalues and eigenvectors. The Routh-Hurwitz criteria are in box. 8.2.

Gause's book is still very readable: Gause G.F. (1934) The struggle for existence, ch. 5
http://www.ggause.com/Contgau.htm

The example of the duckweed and pond weed is from: Scheffer M , et al. (2003): Floating plant dominance as a stable state. PNAS 100**:** 4040-4045

The apparent competition examples are from:

Bonsall, M.B and Hassell, M.P. (1997) Apparent competition structures ecological assemblages. *Nature* 388, 371-373.

Morris, R.B, Lewis, O.T and Godfray, H.C.J. (2004) Experimental evidence for apparent competition in a tropical rainforest. *Nature* 428, 310-313

Alternative stable states, phase shifts and catastrophic transitions are discussed in the following papers:

[Dudgeon SR.; Aronson RB.; Bruno JF and Precht WF (2011). Phase shifts and stable states on coral reefs.MARINE ECOLOGY-PROGRESS SERIES 413: 201-216.](#) (Please note: the link on the title brings you to a page from which you can download the paper)

Mumby PJ.; Hastings A; Edwards HJ. (2007) Thresholds and the resilience of Caribbean coral reefs. Nature450: 98-101.

Scheffer M; Carpenter S; Foley JA; Folke, C and Walker, B. (2001) Catastrophic shifts in ecosystems. Nature, 413:591-596

Effects of climate change on temperate marine ecosystems are discussed in

Verges A. et al. (2014) The tropicalization of temperate marine ecosystems: climate-mediated changes in herbivory and community phase shifts. Proc. R. Soc. 281: 20140846; DOI: 10.1098/rspb.2014.0846

The Icelandic midge example is from

Ives AR.; Einarsson A; Jansen VAA.; Gardarsson, A. (2008) High-amplitude fluctuations and alternative dynamical states of midges in Lake Myvatn. Nature 452: 84-87

The bee example from

Bryden, J. Gill, R. , Mitton, RAA., Raine, N.E., Jansen, VAA. (2013) Chronic sublethal stress causes bee colony failure Ecology Letters**. doi:10.1111/ele.12188

Chaos and unpredictability

This is a classic topic in mathematical biology, and it is covered by most textbooks. Gotelli cover it with in section on the discrete logistic model and Otto and Day both cover it in Chapter 4**.

The classic review by May is worth reading R.M. May (1976) Simple mathematical models with very complicated behaviour. Nature 261, 459 – 467.

The examples of biological chaos are from
Beninca, E. et al. (2008) Chaos in long-term experiment with a plankton community. *Nature* 822-827

Costantino, RF, Cushing, J.M, Dennis, B & Desharnais, RA (1995) Experimentally induced transitions in the dynamics behaviour of insect populations. *Nature* 375, 227-230

The paper by Pearl is:
Pearl, R. (1927) The growth of populations. Q. Rev. Biol. 2: 531-548

Predator-prey models, limit cycles, Hopf bifurcation

Gotelli, chapter 6, cover predator-prey models. The basics of the Hopf bifurcation are described in Otto and Day, section 11.4.

The example of the experimental demonstration of the Hopf bifurcation is from Fussman GF, Ellner SP, Shertzer KW & Hairston NG (2000) Crossing the Hopf Bifurcation in a Live Predator-Prey System, *Science, 290. 1358-1360*

Some of the material on spatial predator-prey models is from V.A.A. Jansen and A.M de Roos (2000) The role of space in reducing predator-prey cycles. In: The Geometry of Ecological Interactions: Simplifying Spatial Complexity. (U. Dieckmann, R. Law and J.A.J. Metz, eds). Cambridge University Press (2000) (downloadable from http://personal.rhul.ac.uk/ujba/115/chapter11.pdf)

The classic paper on the spatial Nicholson-Bailey model is Hassell MP, Comins HN, MayRM, (1991) Spatial Structure and chaos in insect population dynamics. Nature, 353, 255-258

The experiments with the mites on floating islands are described in Ellner SP, McCauley E, Kendall BE, Briggs CJ, Hosseini PR, Wood SN, Janssen A, Sabelis MW, Turchin P, Nisbet RM, Murdoch WW. 2001. Habitat structure and population persistence in an experimental community. Nature 412: 538-543.

Epidemiological models.

A brief overview of theory can be found in Otto and Day, section 3.5

Two overviews of the use of models in epidemiology are in: Scherer, A. and McLean, A. (2002) Mathematical models of vaccination. *British Medical Bulletin* 62, 187-199

D. Louz et al. (2010) Emergence of viral diseases: mathematical modeling as a tool for infection control, policy and decision making. *Critical Reviews in Microbiology*, 2010; 36(3): 195–211

The study on phase locking is from:

Earn, DJ, Rohani, P, Bolker, BM, Grenfell, BT(2000)A Simple Model for Complex Dynamical Transitions in Epidemics *Science* 287: 667-670

There is a further paper on phase locking in epidemiological models:

Krylova, O and Earn DJD (2013) Effects of the infectious period distribution on predicted transitions in childhood disease dynamics. J R Soc Interface. Jul 6, 2013; 10(84): 20130098. doi: 10.1098/rsif.2013.0098.

The supplementary material provides a detailed description about how their results can be generated by XPPAUT:
http://rsif.royalsocietypublishing.org/content/royinterface/suppl/2013/05/09/rsif.2013.0098.DC1/rsif20130098supp2.pdf

The parts on the reemergence of measles from a vaccinated population are from Jansen, V.A.A. et al (2003) Measles outbreaks in a population with a declining vaccine uptake. *Science*, 301,804.

There is a wealth of current material on Ebola. An early model is by the CDC:
http://www.cdc.gov/mmwr/preview/mmwrhtml/su6303a1.htm

Some further papers I have come across

The effectiveness of control measures is discussed in:

Kucharski, A. J., Camacho, A., Flasche, S., Glover, R. E., Edmunds, W. J., & Funk, S. (2015). Measuring the impact of Ebola control measures in Sierra Leone. *Proceedings of the National Academy of Sciences*, 112, 14366–14371, doi: 10.1073/pnas.1508814112

The same team did a post mortem on the 1976 outbreak

Camacho, A., Kucharski, A.J., Funk, S., Breman, J., Piot, P. and Edmunds, W.J., 2014. Potential for large outbreaks of Ebola virus disease. *Epidemics*, *9*, pp.70-78.

The following is interesting for the anthropological angle:

Hewlett, B.S. and Amola, R.P., 2003. Cultural contexts of Ebola in northern Uganda. *Emerging infectious diseases*, *9*(10), p.1242.

General references for bifurcation theory
A standard reference is
Guckenheimer J. and Holmes, P. (1983) Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer-Verlag, 1983, Volume 42

A very comprehensive description, with very helpful illustrations is
Kuznetsov, Yu.A. (1995) Elements of Applied Bifurcation Theory. Springer-Verlag, New York, 1995

The XPPAUT programme is described in detail (with many examples)
Bard Ermentrout (2002) Simulating, Analyzing, and Animating Dynamical Systems: A Guide to XPPAUT for Researchers and Students. SIAM