Supplement to the Article "Simple Iterative Methods for Linear Optimization over Convex Sets"

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1 Introduction

The article "Simple Iterative Methods for Linear Optimization over Convex Sets" proposes iterative algorithms to solve linear optimization problems

$$\text{OPT} = \max\{\langle c, x\rangle \ : \ x \in K\}$$

whose feasible regions K are only available through separation oracles. In each iteration t of these algorithms, A_t refers to a set of inequalities $\langle a, x\rangle \leq \beta$ that has been separated and γ_t is the value of the best incumbent solution until iteration t. In the experimental part of this article, the performance of one of these methods is evaluated for packing problems. To measure performance, the number of iterations t until the value

$$\text{LP}(A_t) = \max\{\langle c, x\rangle \ : \ \langle a, x\rangle \leq \beta \text{ for all } (a, \beta) \in A_t\}$$

is less than $1.01 \cdot \text{OPT}$ is used, i.e., A_t overestimates OPT by less than one percent. As reference method, a simple cutting plane procedure is used for which performance is measured analogously.

We have tested our method on instances of the maximum matching and stable set problem. While the instances for stable set have been selected from the Color02 symposium[1], the instances of the matching problem have been generated randomly: For each $r \in \{30, 33, \dots, 75\}$ we build an instance by sampling r triples of nodes $\{u, v, w\}$ from a graph with 500 nodes and adding the edges of the induced triangles to the graph.

This supplement provides a detailed overview on the numerical results. For each tested instance and setting, there is a plot visualizing the values γ_t, $\text{LP}(A_t)$, and the dual value of the cutting plane procedure (referred to as "gamma_t", "dual from A_t", and "dual from LP", respectively). In the experiments, tests on instances of the maximum matching and stable set problem have been conducted. Moreover, the impact of different enhancements of the new procedure has been tested: applying a fully corrective step with frequency $k \in \{0, 10, 1\}$, using different initial constraints, and initializing γ_1 with OPT. For details, we refer the reader to the main article. To encode the different settings, the title of each figure contains the

- name of the tested instance;
- frequency of the fully corrective step;

[1] available at `http://mat.gsia.cmu.edu/COLOR02`

- initial constraints (“ub” refers to upper bound constraints, “basic” to basic constraints as explained in the article);
- whether the algorithm is initialized with a standard value (“init-std”) or OPT (“init-opt”).

The following sections contain a summary and the visualizations of the results. Section 2 contains aggregated information on our experiments. Section 3 shows the plots for tests with different frequencies; Section 4 lists the plots for the experiments with different initial constraints. Note that the lines for γ_t and $\mathrm{LP}(A_t)$ need not match at termination, because only the dual bound is used as termination criterion. Moreover, an empty plot shows that no cut was needed to solve this instance.

2 Aggregated Information

Table 1 summarizes our experiments for different frequencies of the fully corrective step both for the standard and optimal initialization. Table 2 shows aggregated information on the iteration count for different selections of initial constraints. Note that we do not report on the iteration count of the cutting plane procedure for maximum matching in Table 2, because no instance could be solved by this method. In all remaining tests, every method was able to solve all instances within the iteration limit.

Table 1: Comparison of iteration counts for different frequencies of fully corrective step.

initialization:	standard		optimal
maximum matching:			
frequency 0	169.25		86.75
frequency 10	51.88		32.25
frequency 1	45.50		30.12
LP		92.62	
maximum stable set:			
frequency 0	62.75		55.88
frequency 10	39.75		36.69
frequency 1	36.31		35.19
LP		106.06	

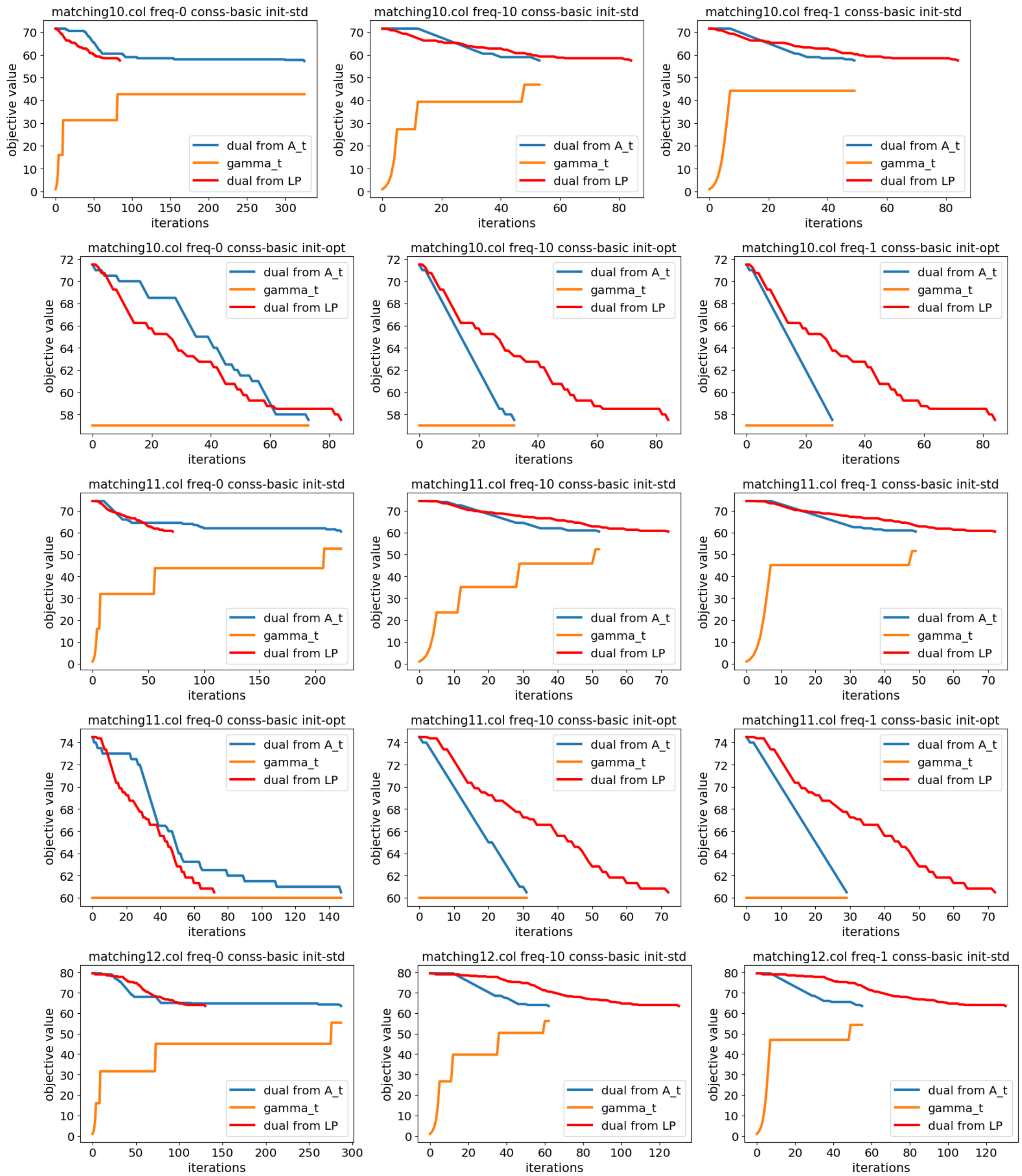
Table 2: Comparison of iteration counts for different initial constraints in fully corrective algorithm.

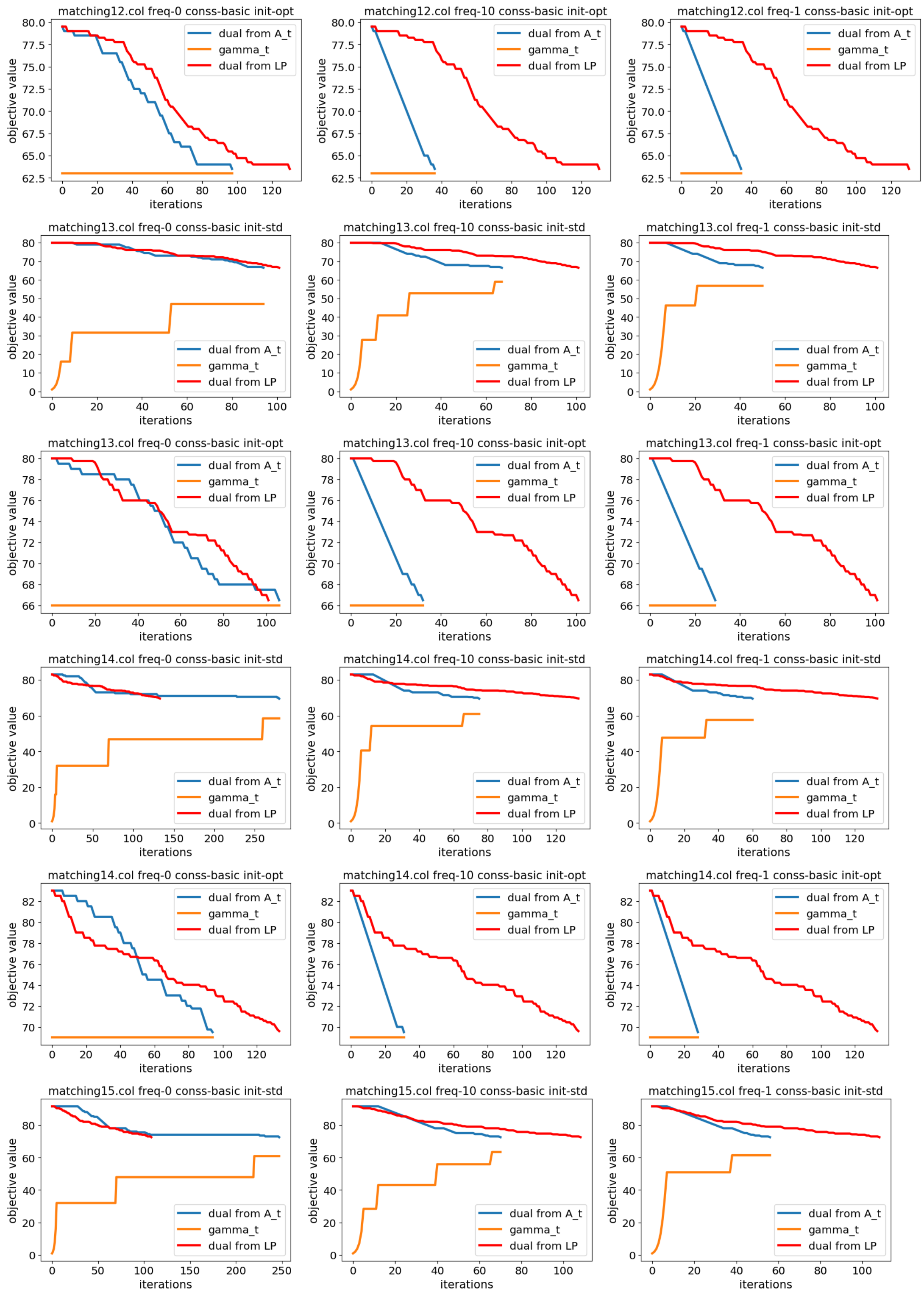
initial constr.	standard	optimal	LP
maximum matching:			
upper bound	73.00	69.75	—
basic	45.50	30.12	92.62
maximum stable set:			
upper bound	60.75	61.62	225.00
basic	36.31	35.19	106.06

3 Experiments for Different Frequencies

This section provides the plots for the experiments with different frequencies. For each instance, the effect of frequency $k \in \{0, 10, 1\}$ has been tested both for init-std and init-opt.

3.1 Experiments for Matching

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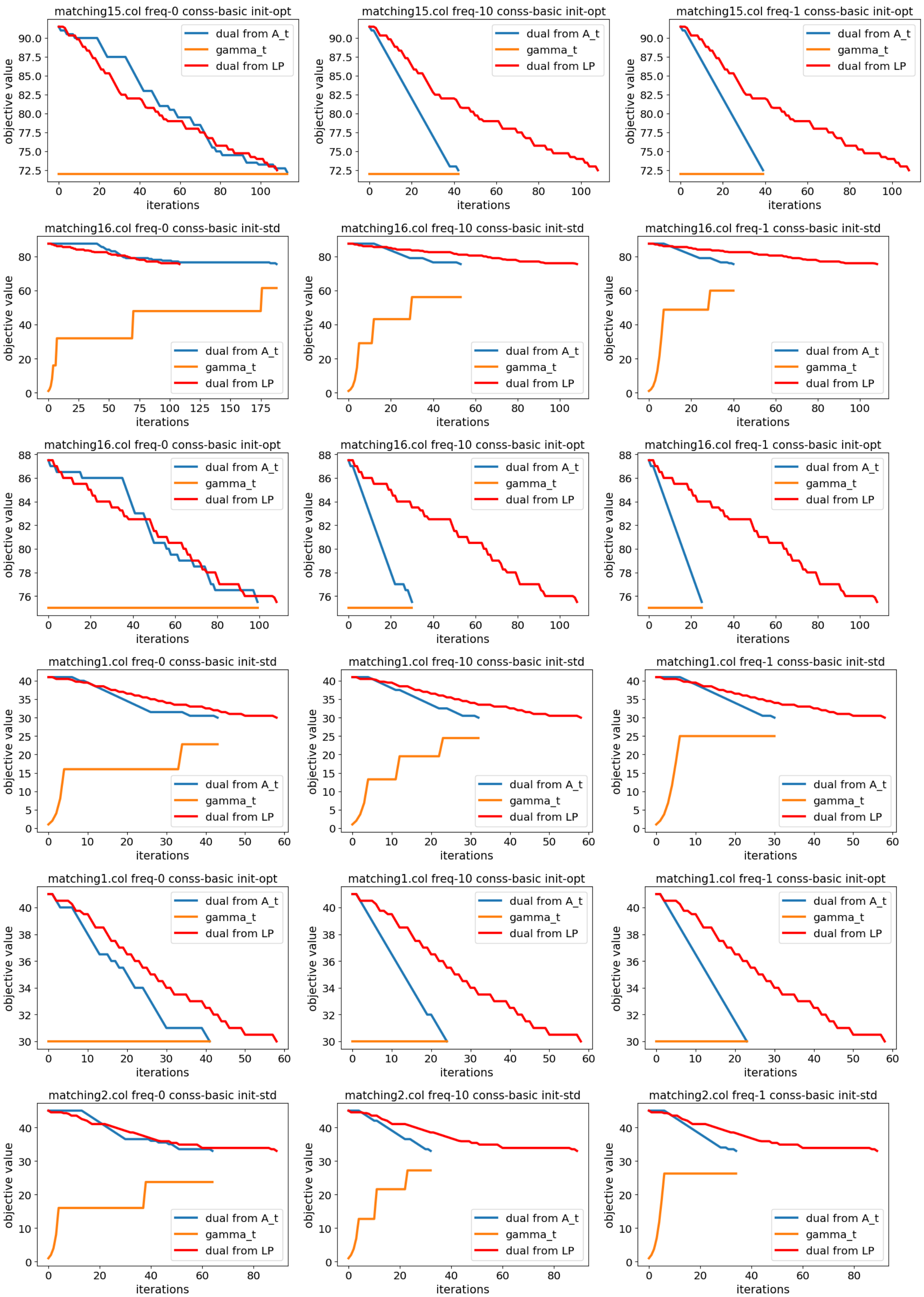
dual from A_t
gamma_t
dual from LP
objective value
iterations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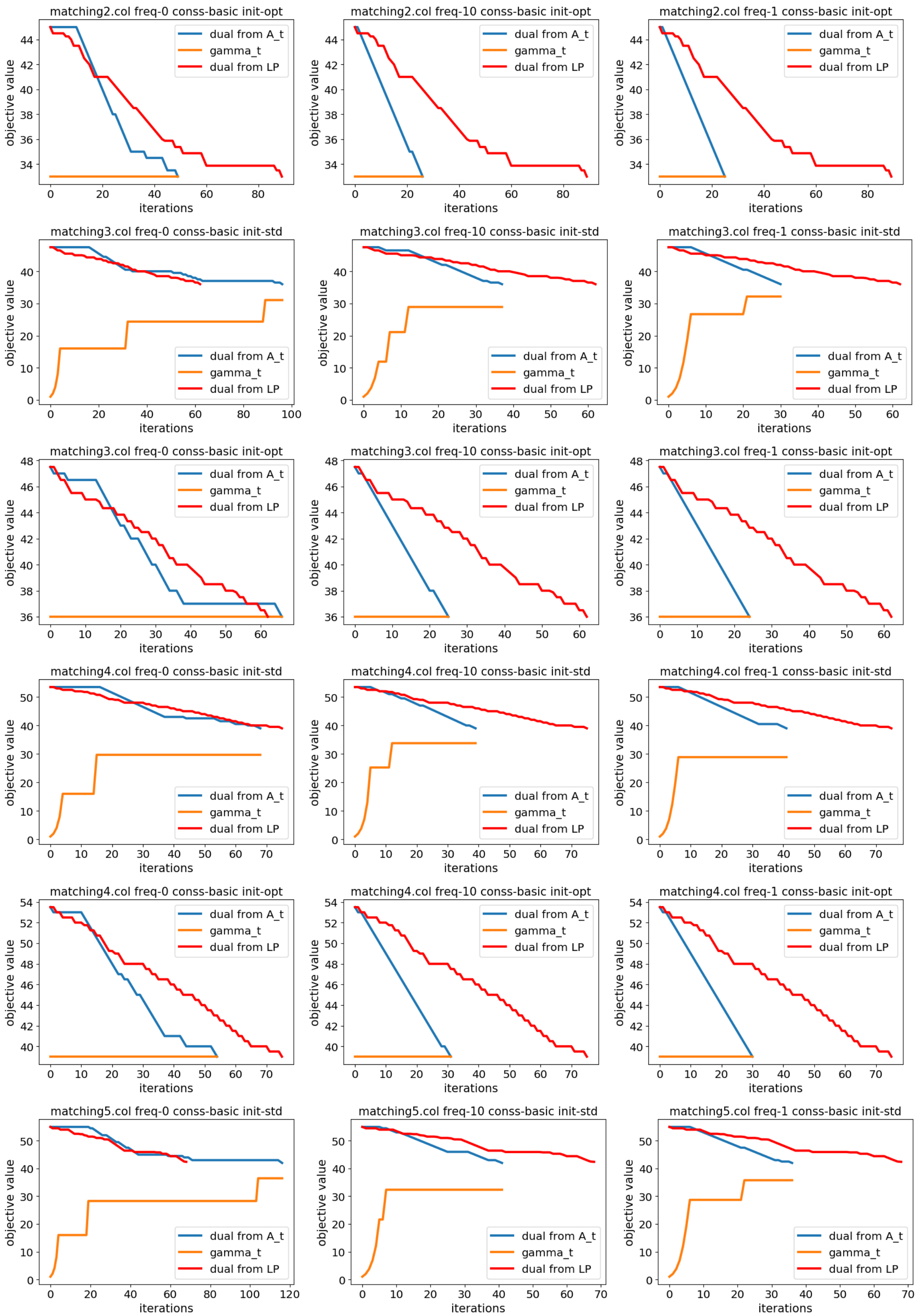
dual from LP
objective value
iterations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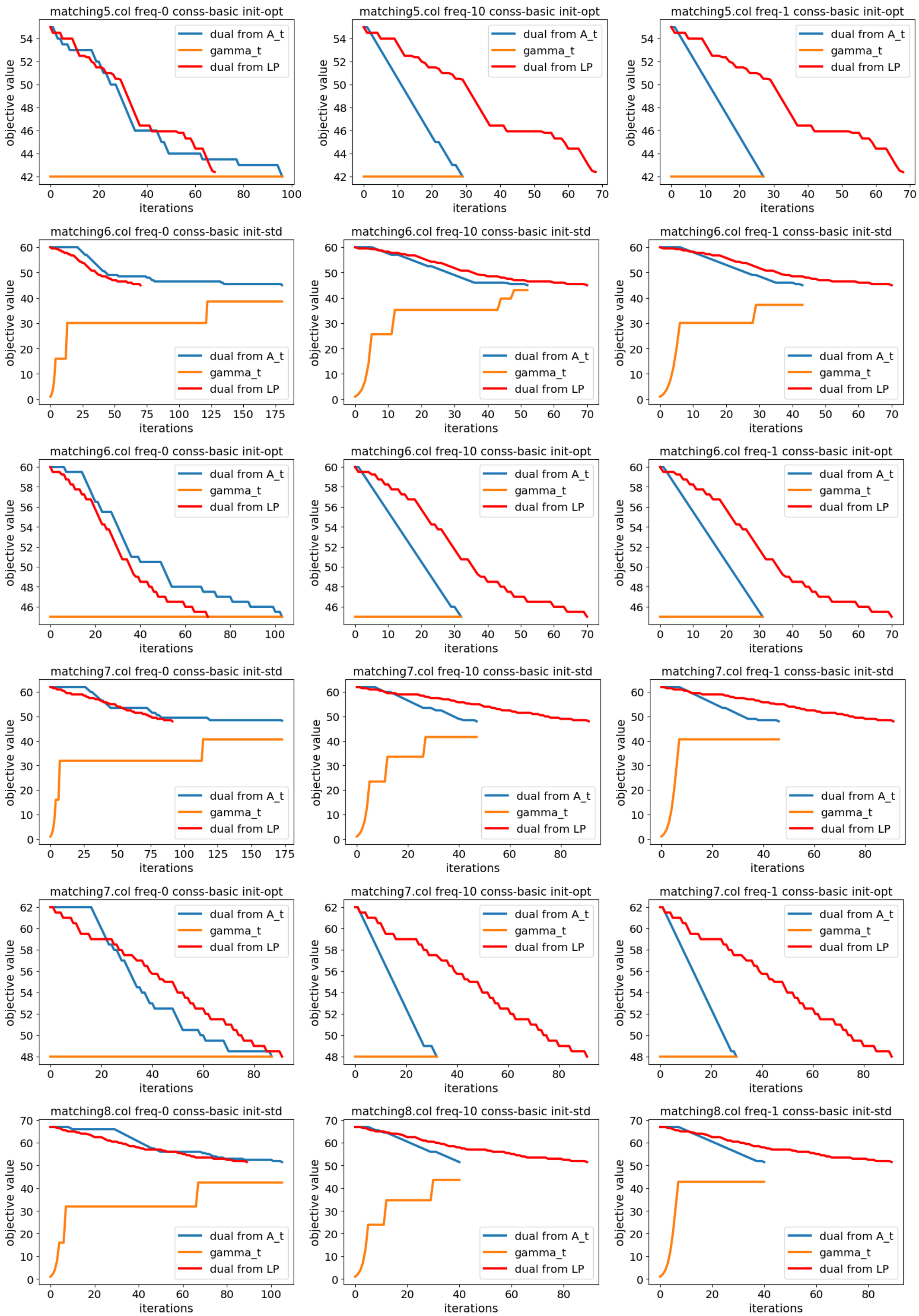
gamma_t
dual from LP
objective value
iterations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dual from LP
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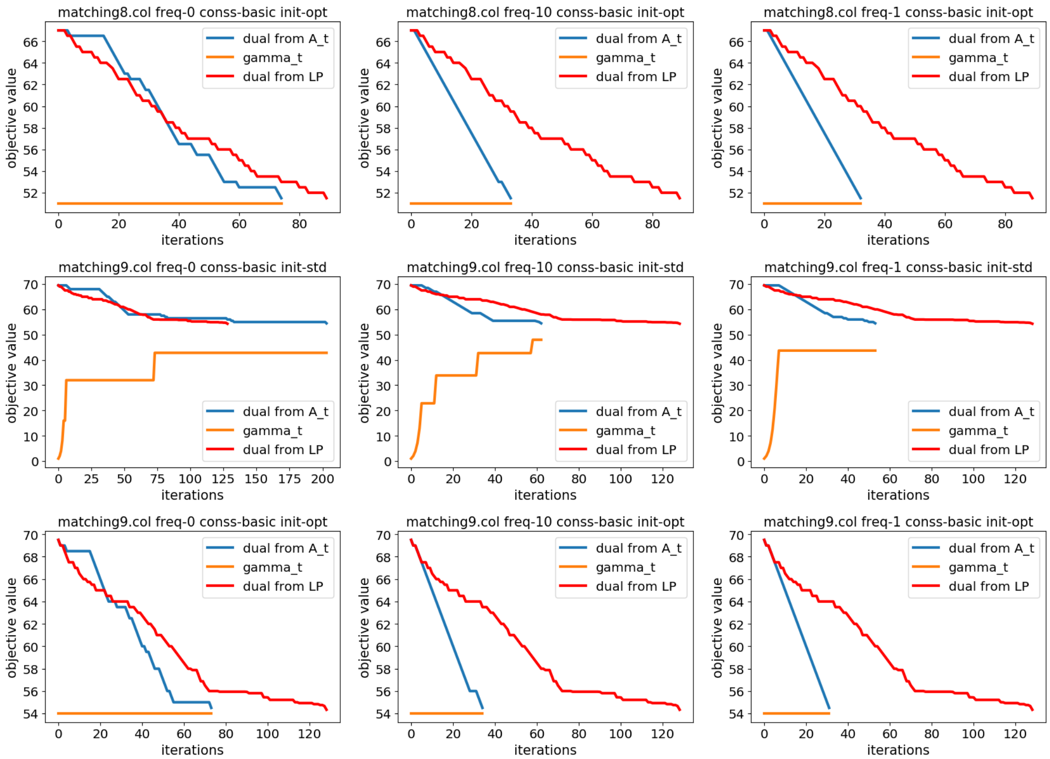
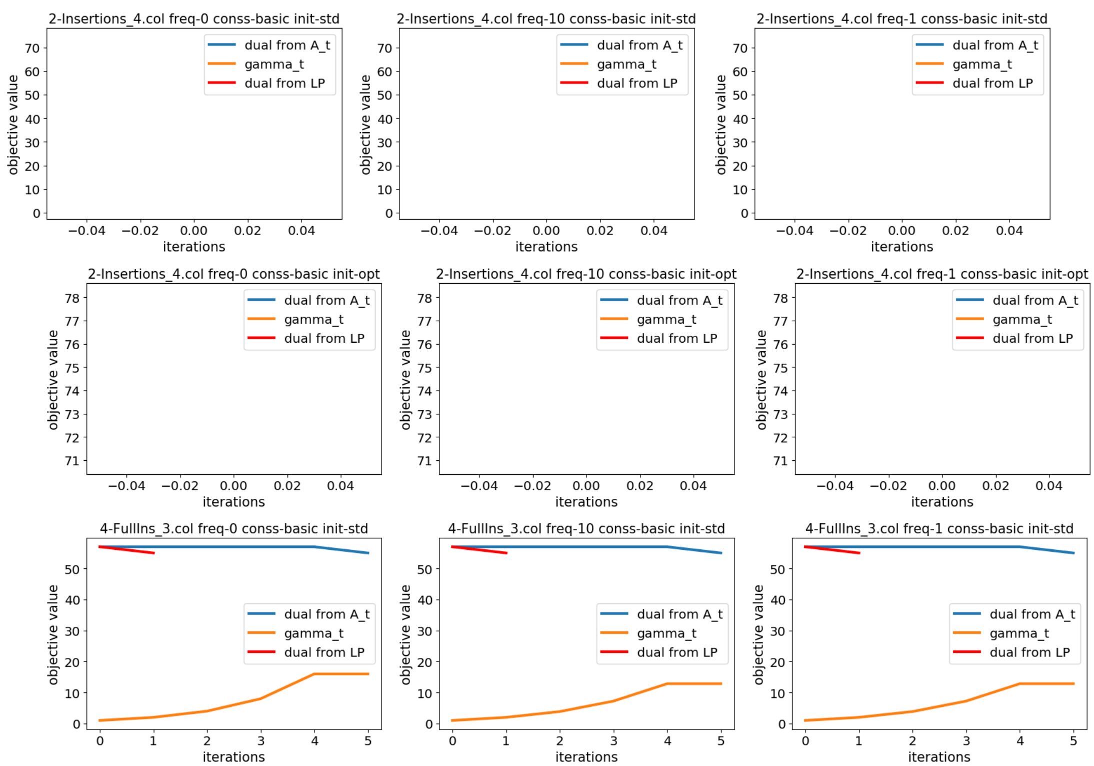
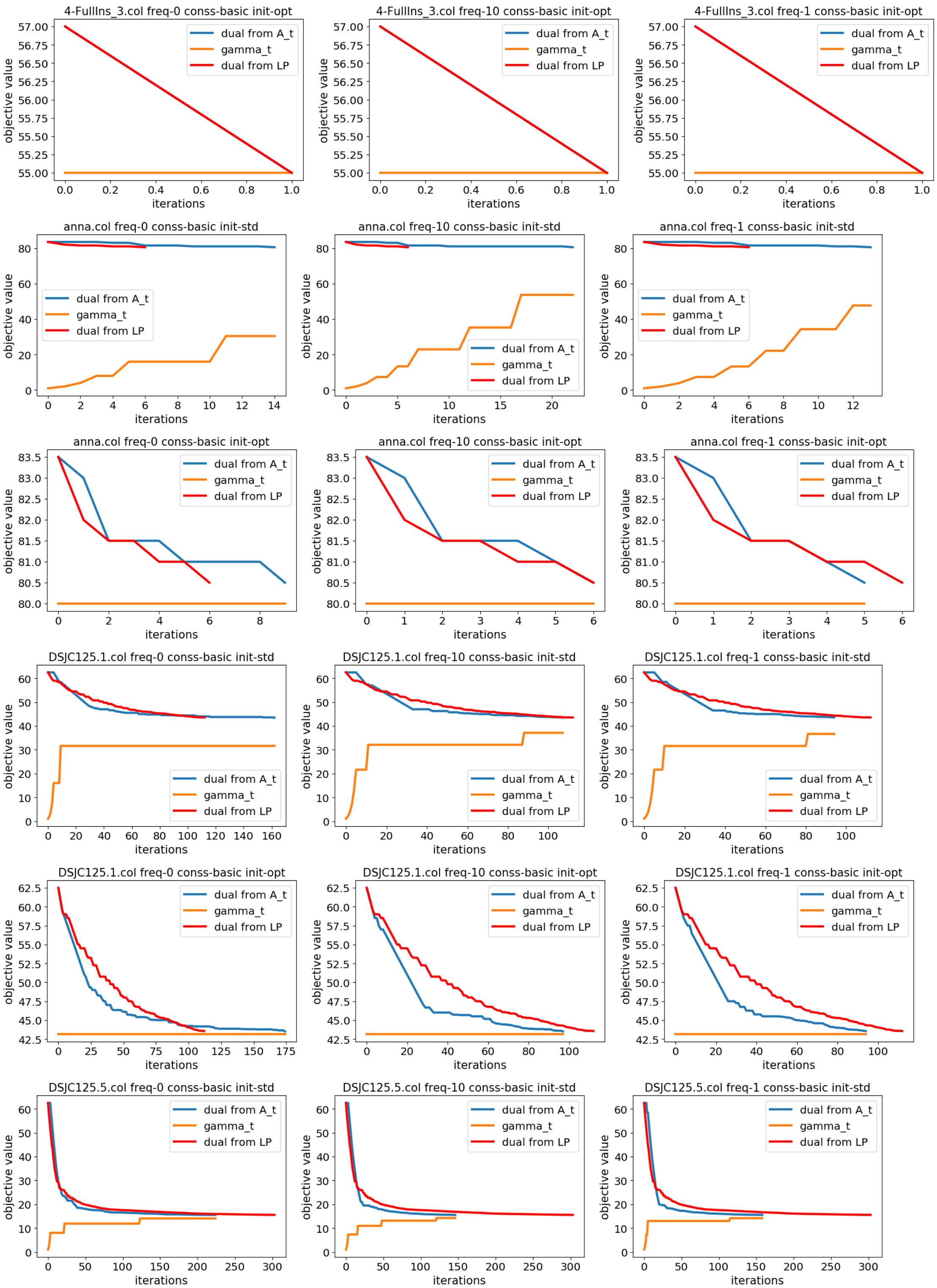
iterations



3.2 Experiments for Stable Set

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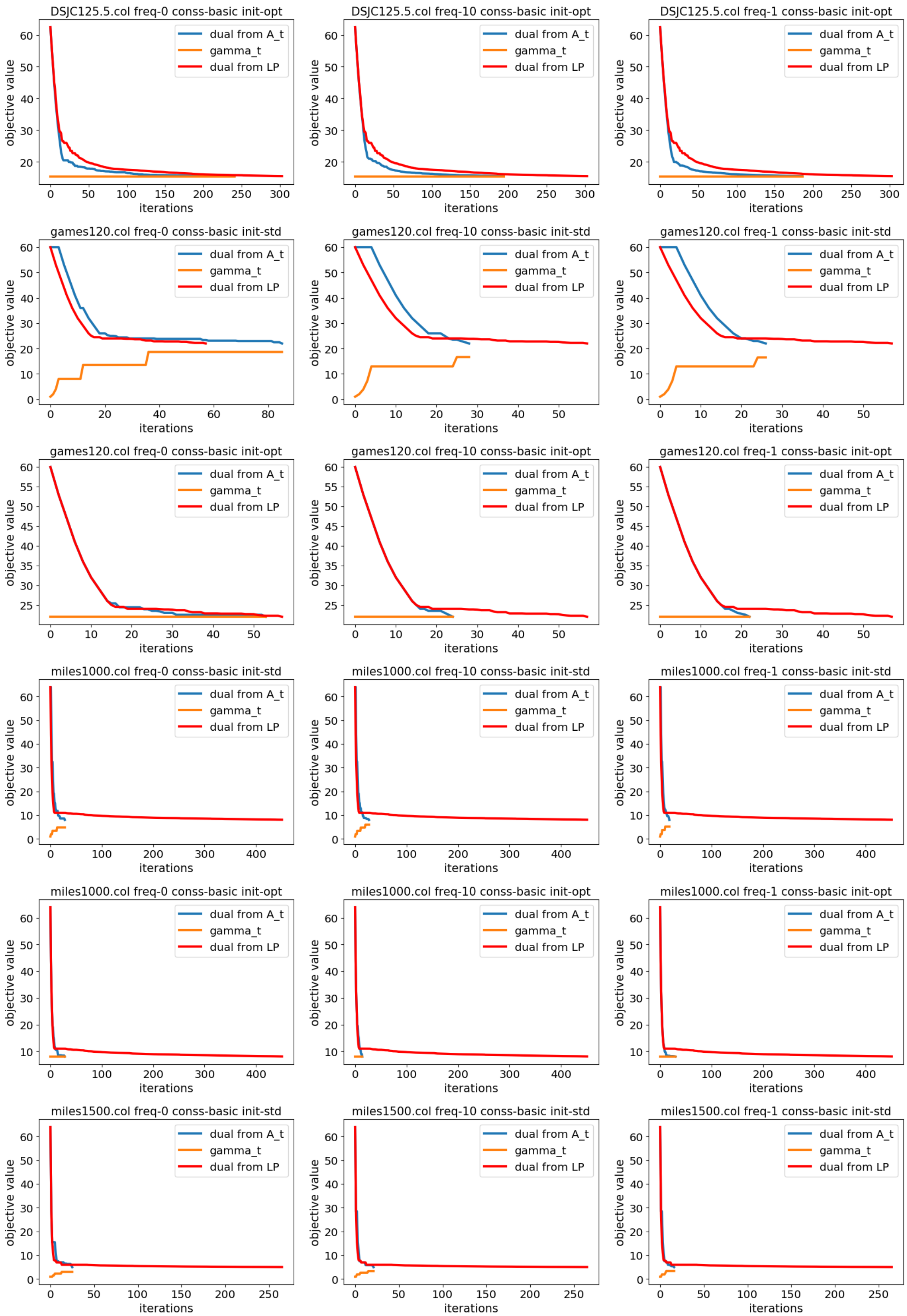
gamma_t
dual from LP
objective value
iterations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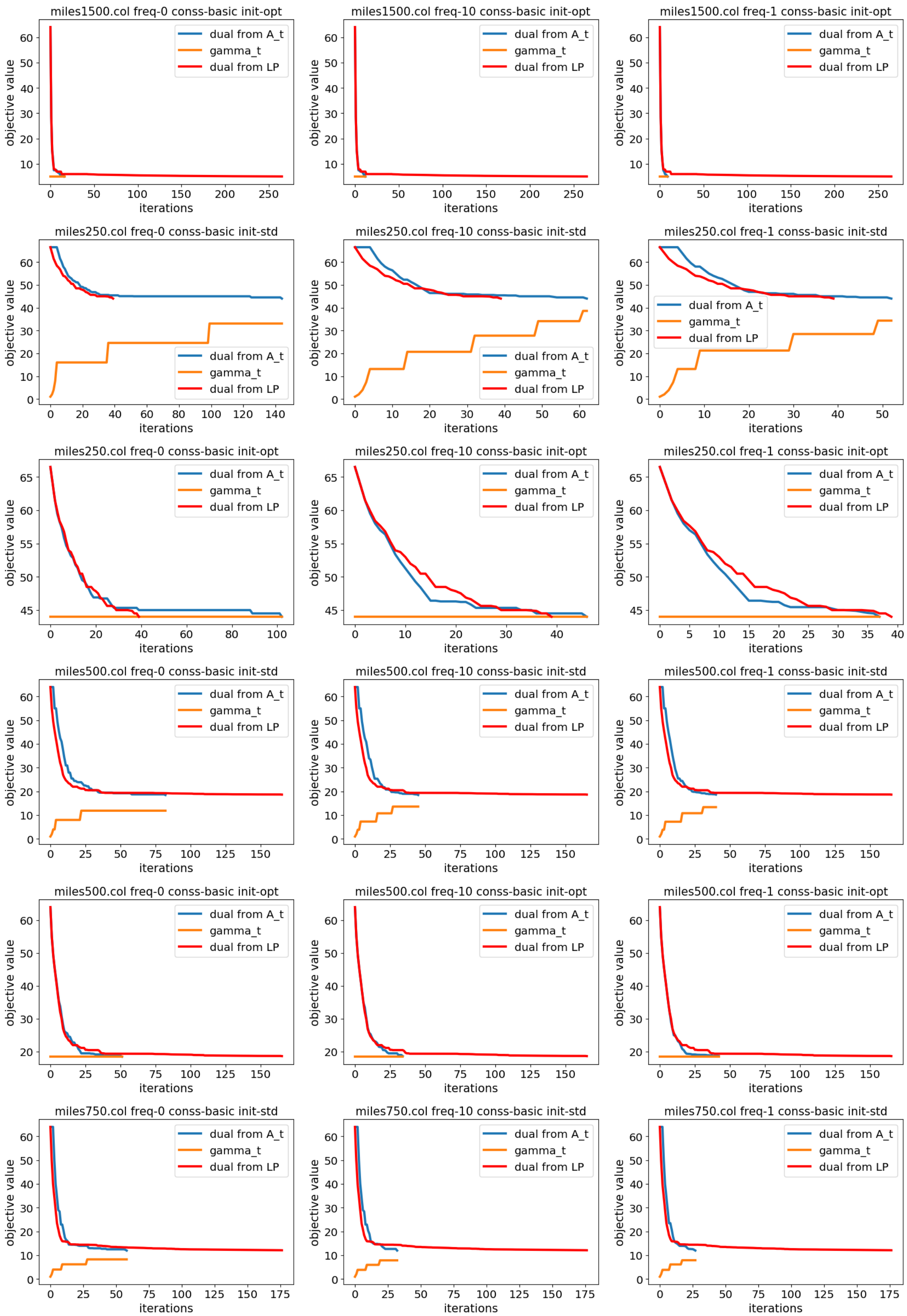
objective value
iterations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dual from LP
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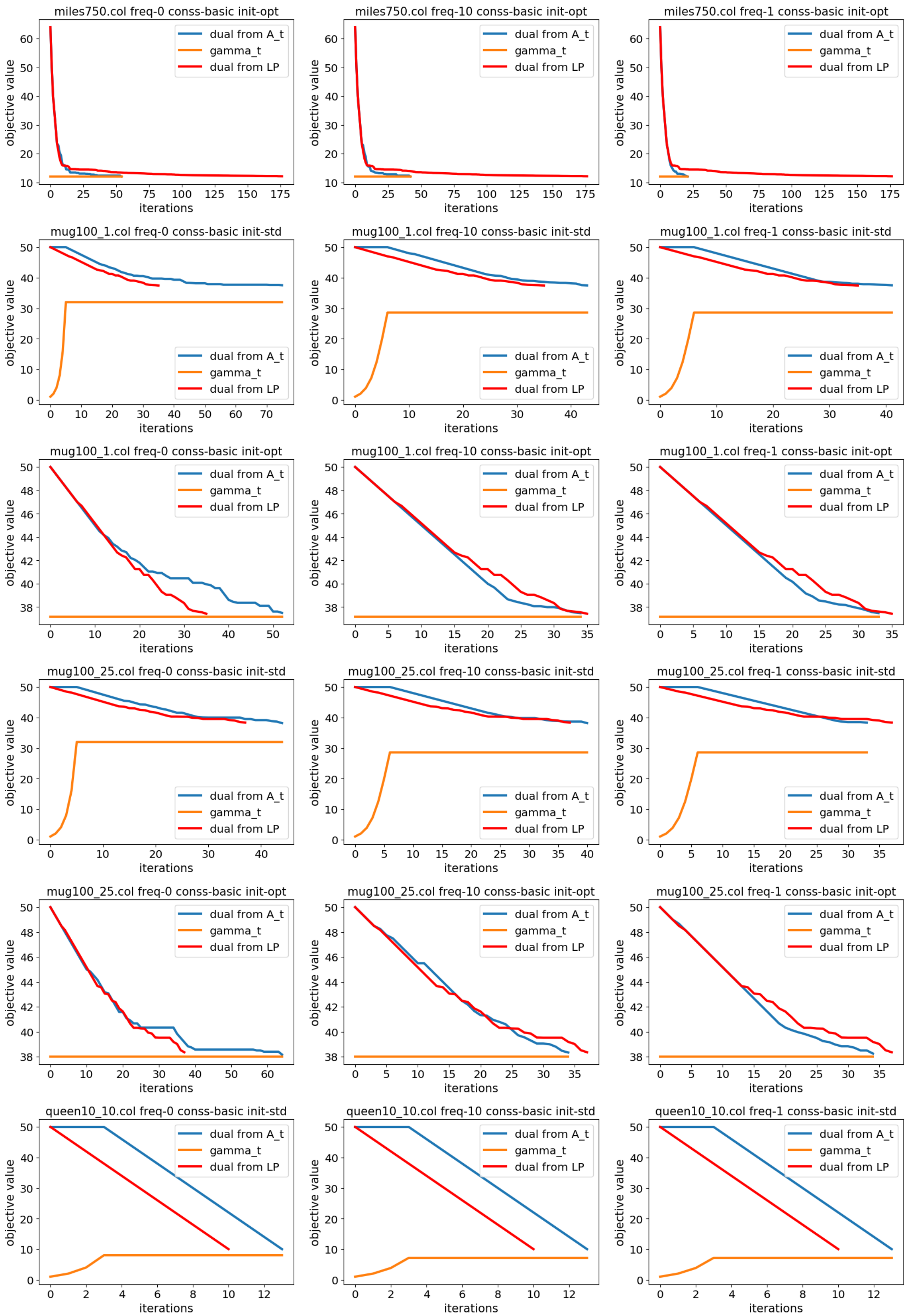
iterations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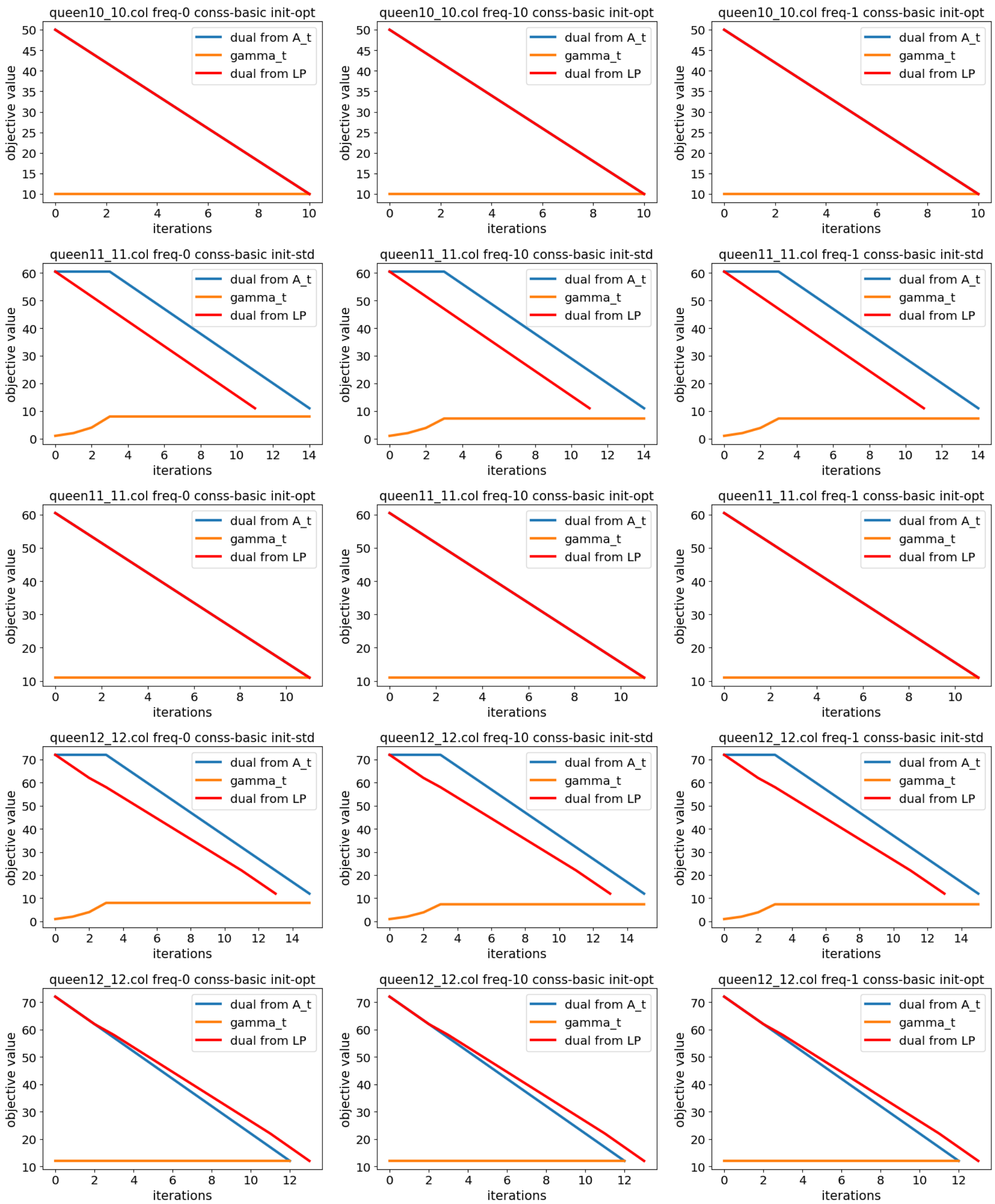
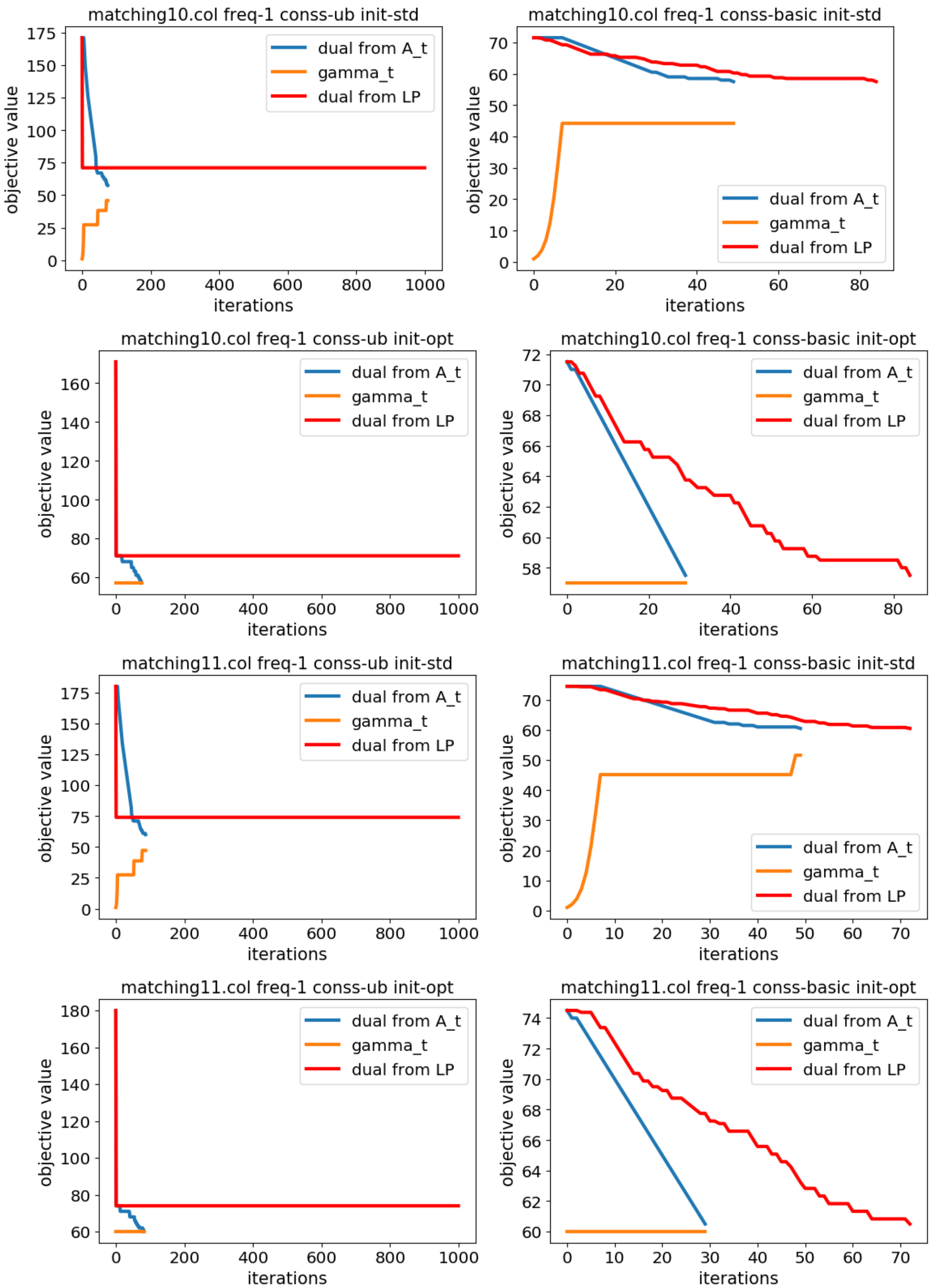
gamma_t
dual from LP
objective value
iterations

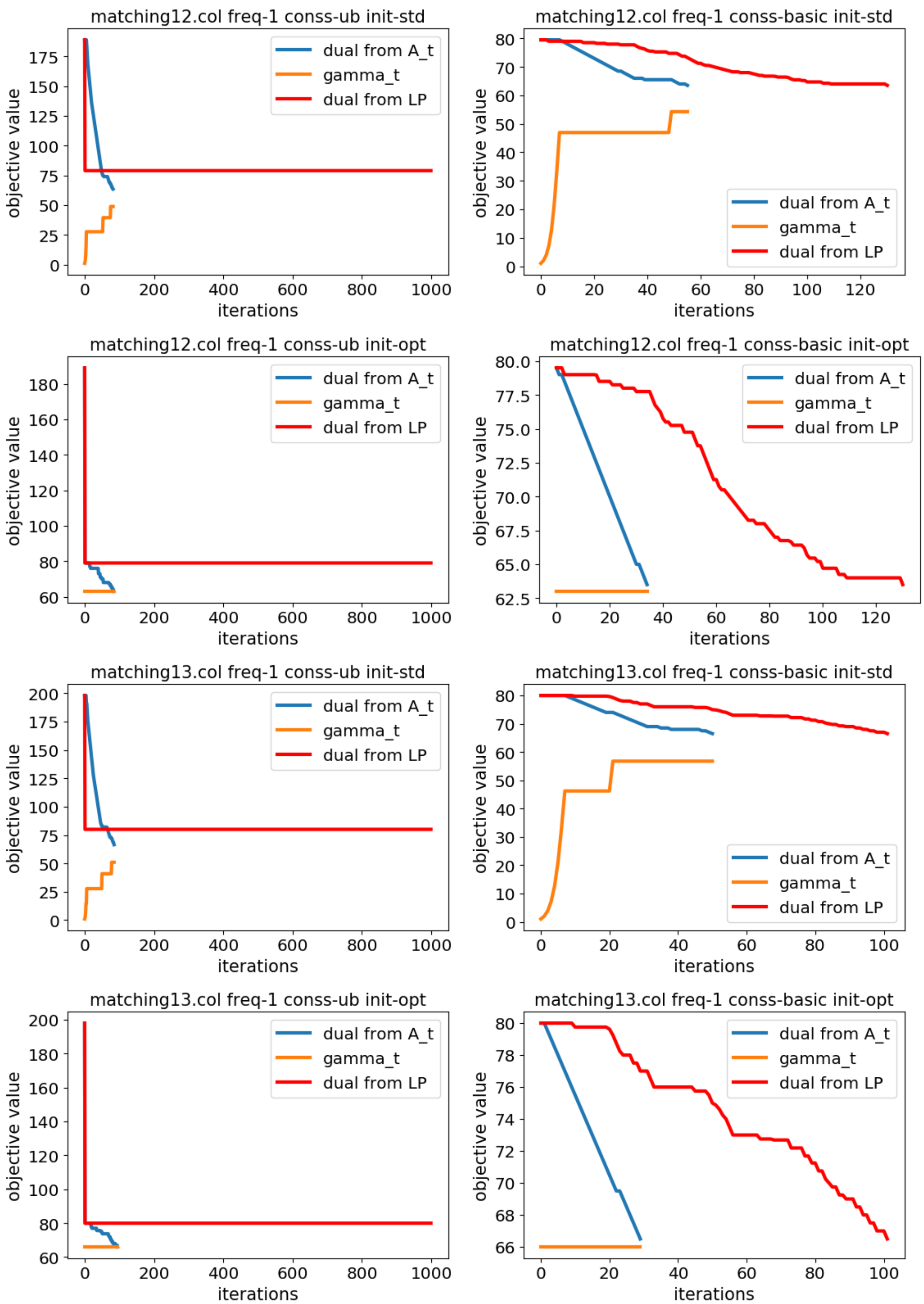


4 Experiments for Different Initial Constraints

This section provides the plots for the experiments with different initial constraints. For each instance, the effect of just upper bound constraints and additional basic constraints has been tested both for init-std and init-opt.

4.1 Experiments for Matching

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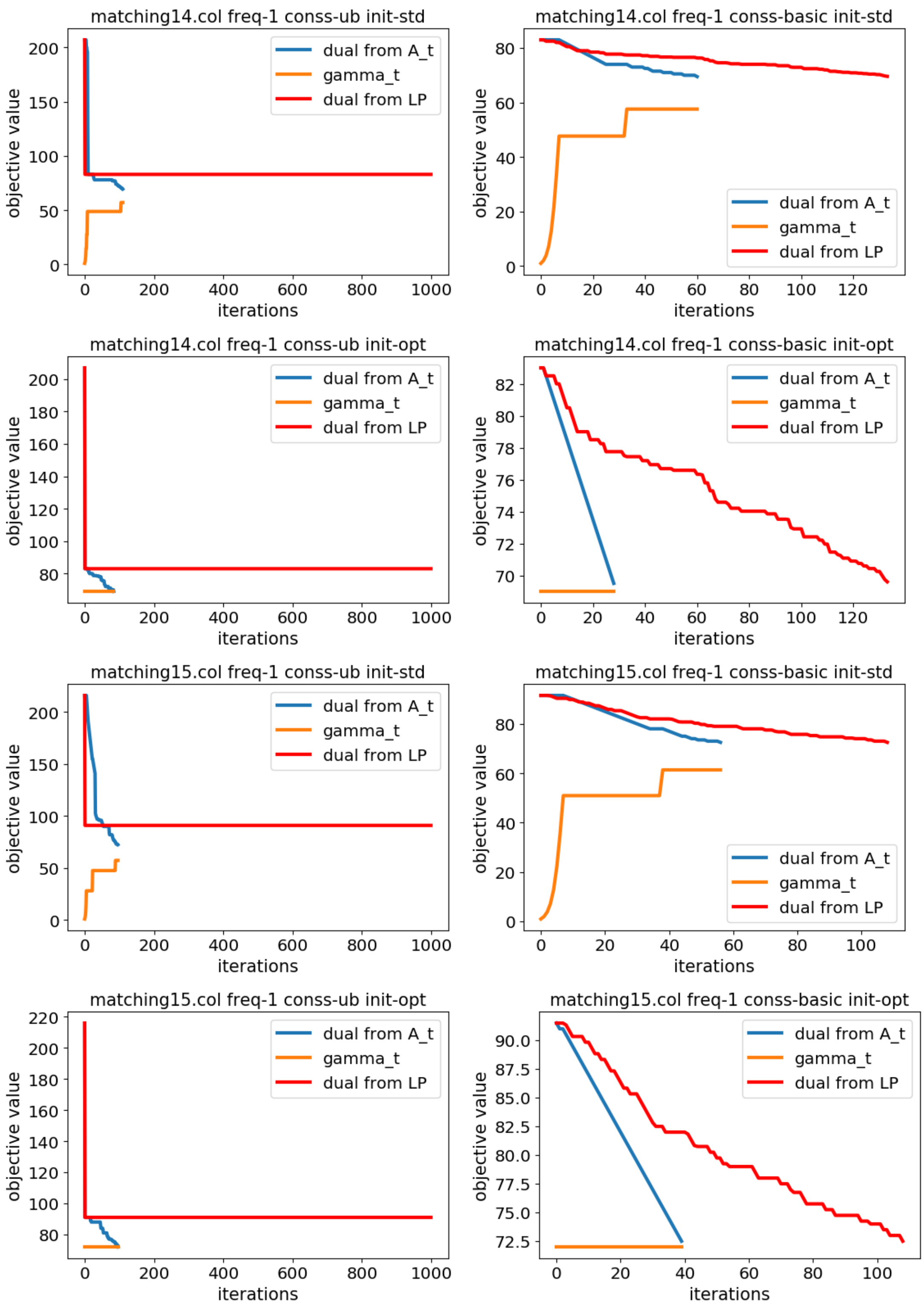
dual from LP
objective value
iterations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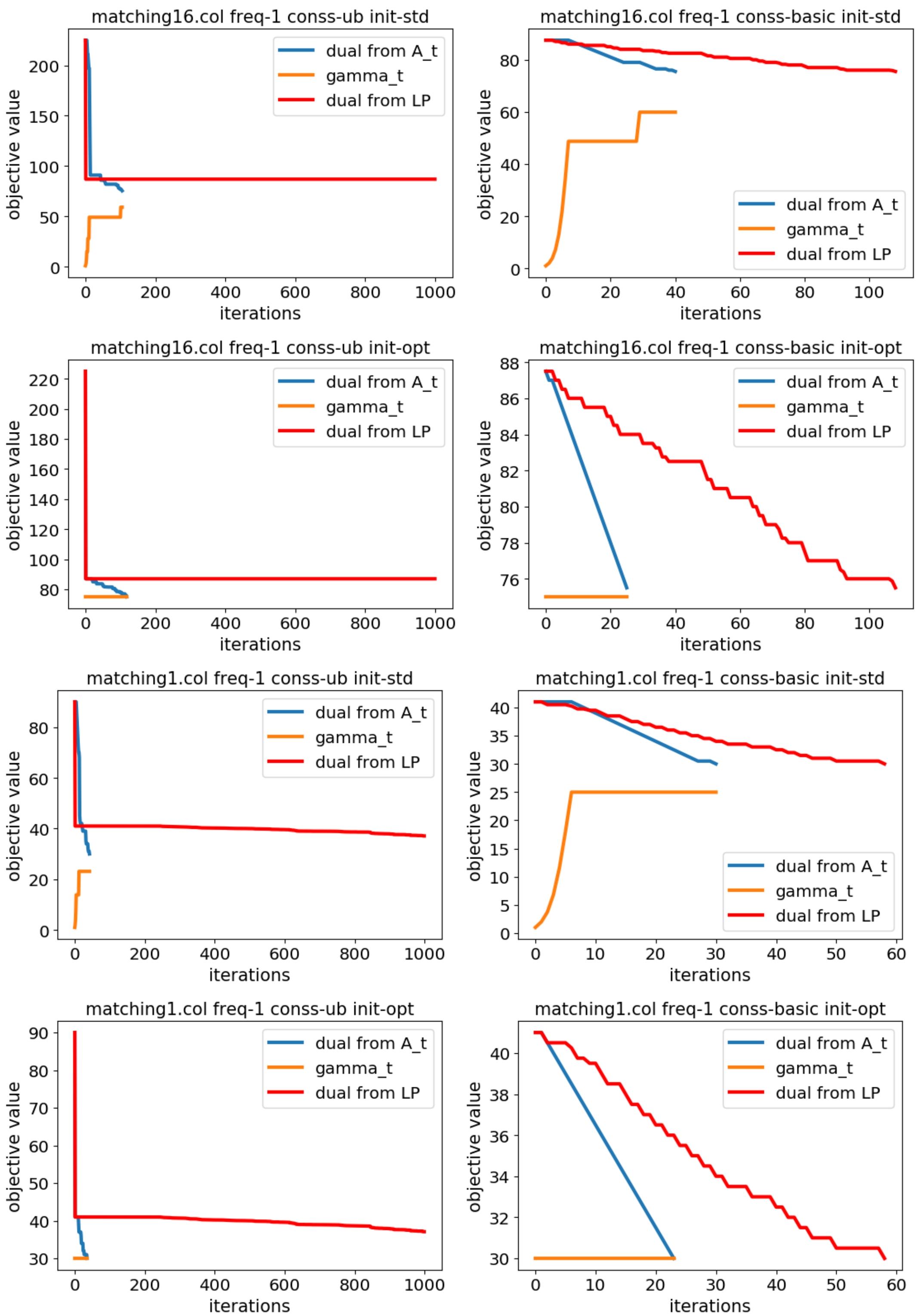
dual from LP
objective value
iterations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dual from LP
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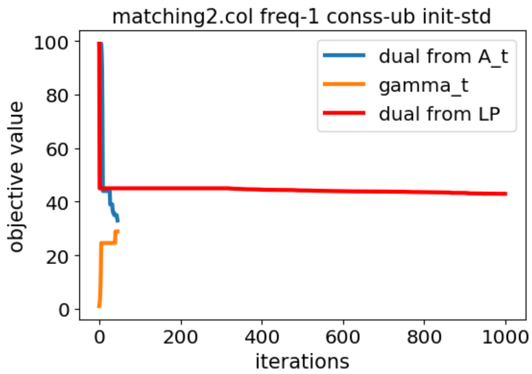
iterations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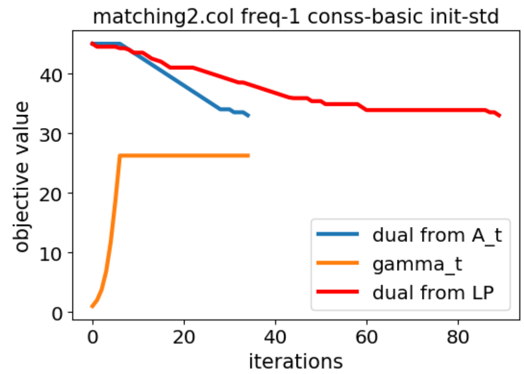
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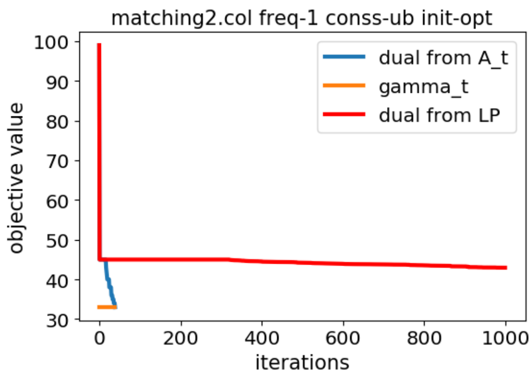
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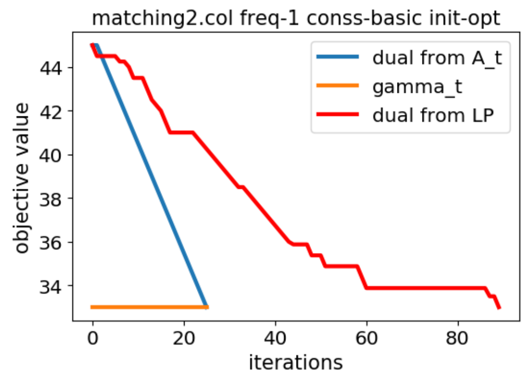
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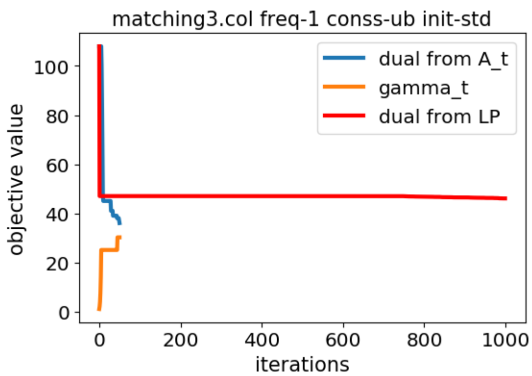
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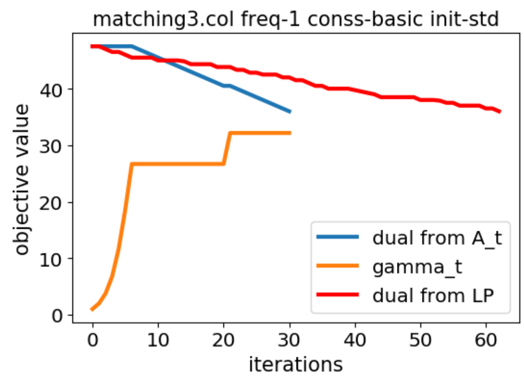
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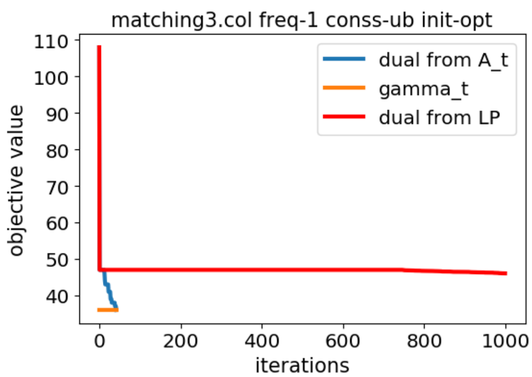
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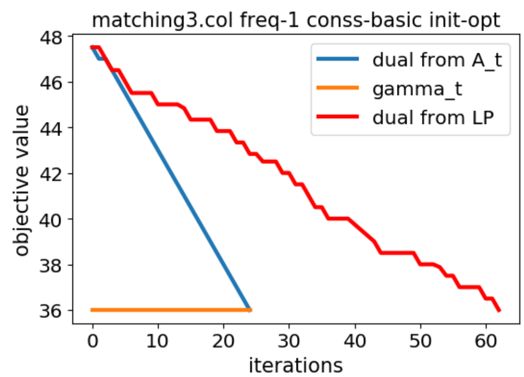
objective value
iterations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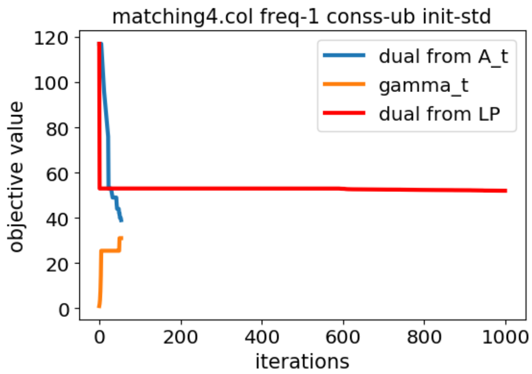
iterations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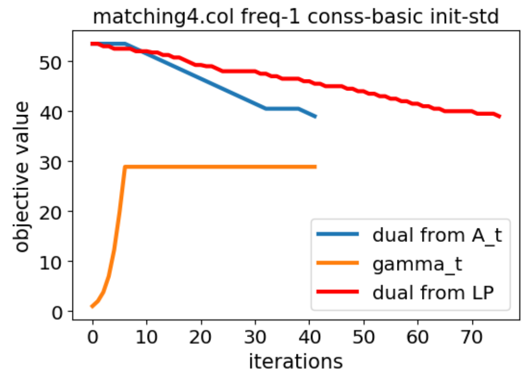
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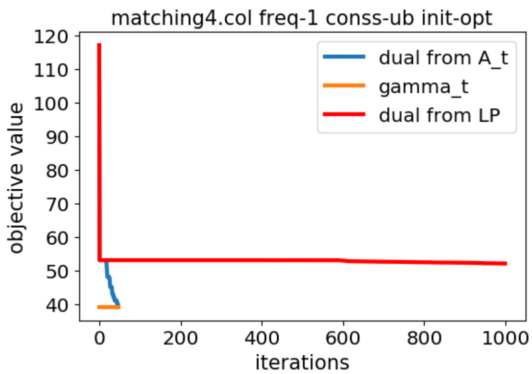
objective value
iterations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gamma_t
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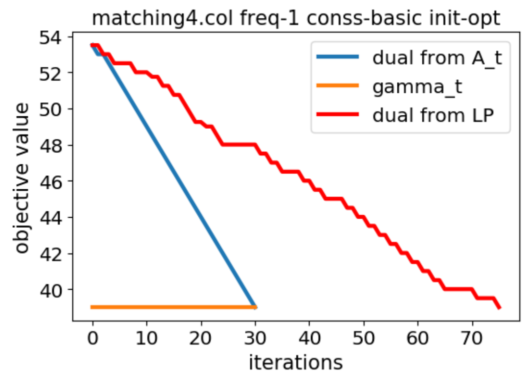
iterations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gamma_t
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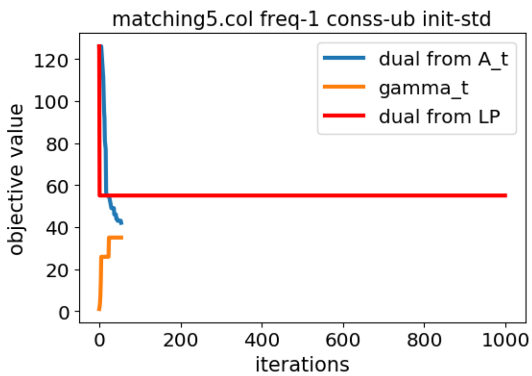
objective value
iterations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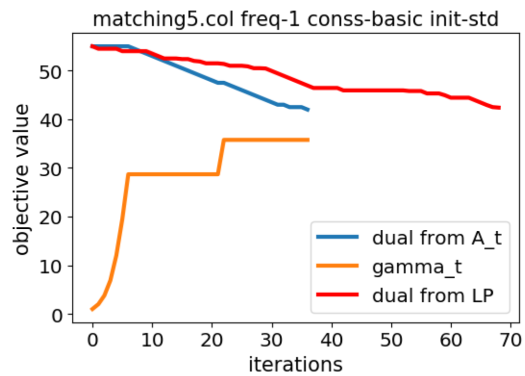
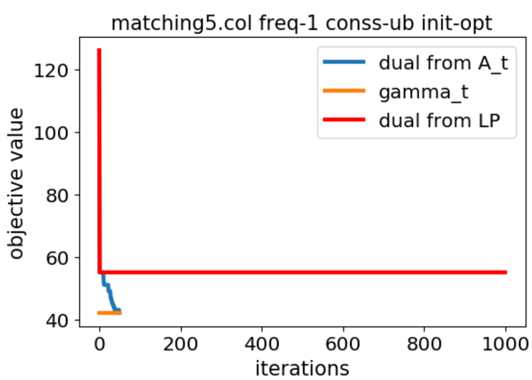
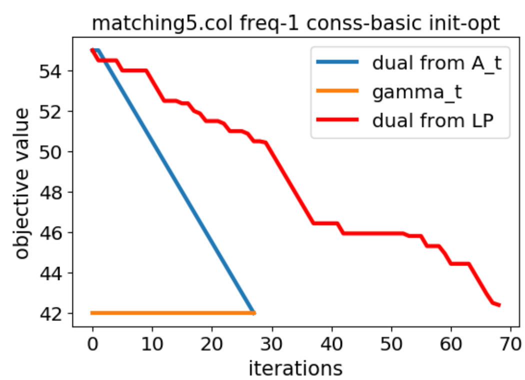
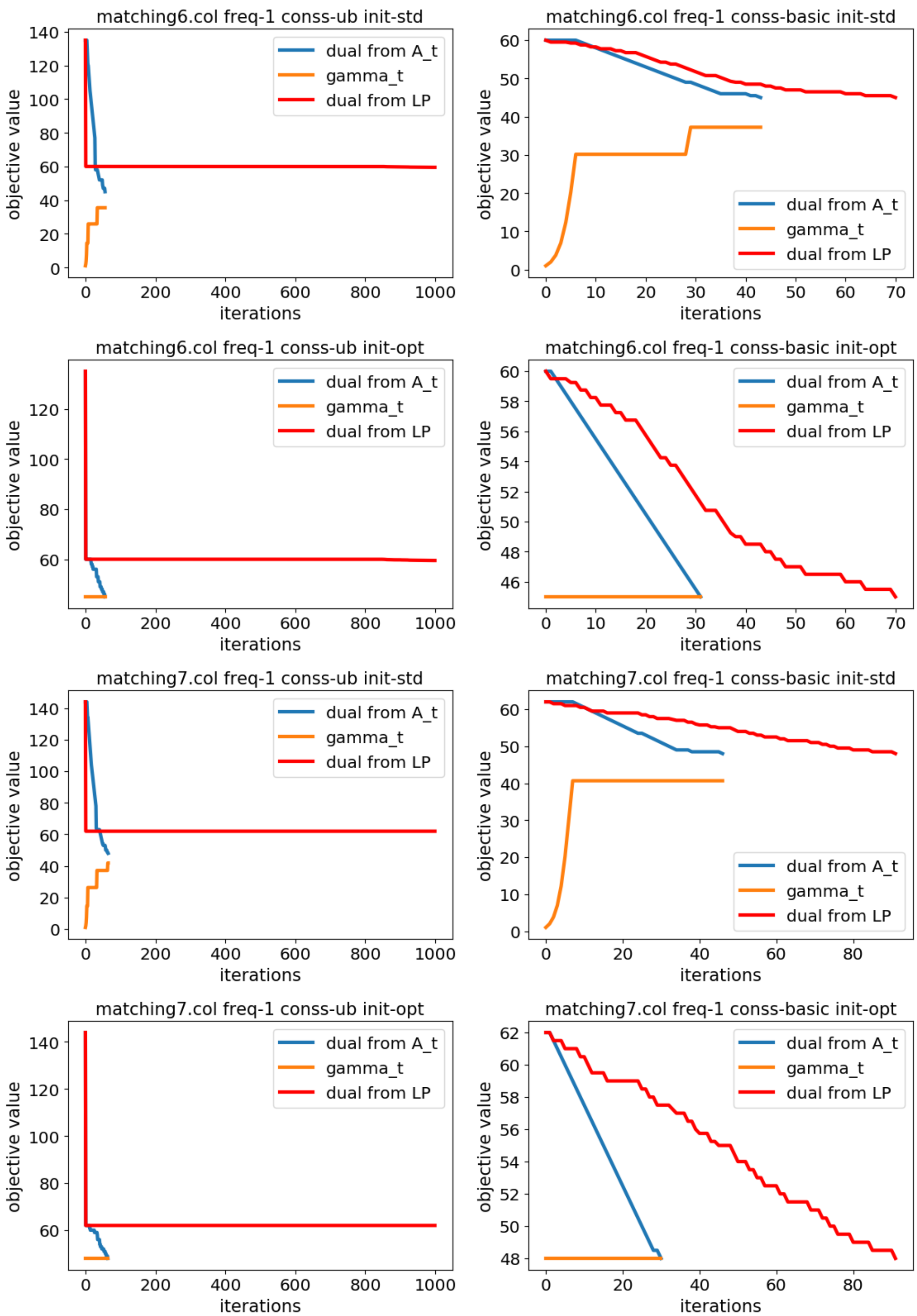
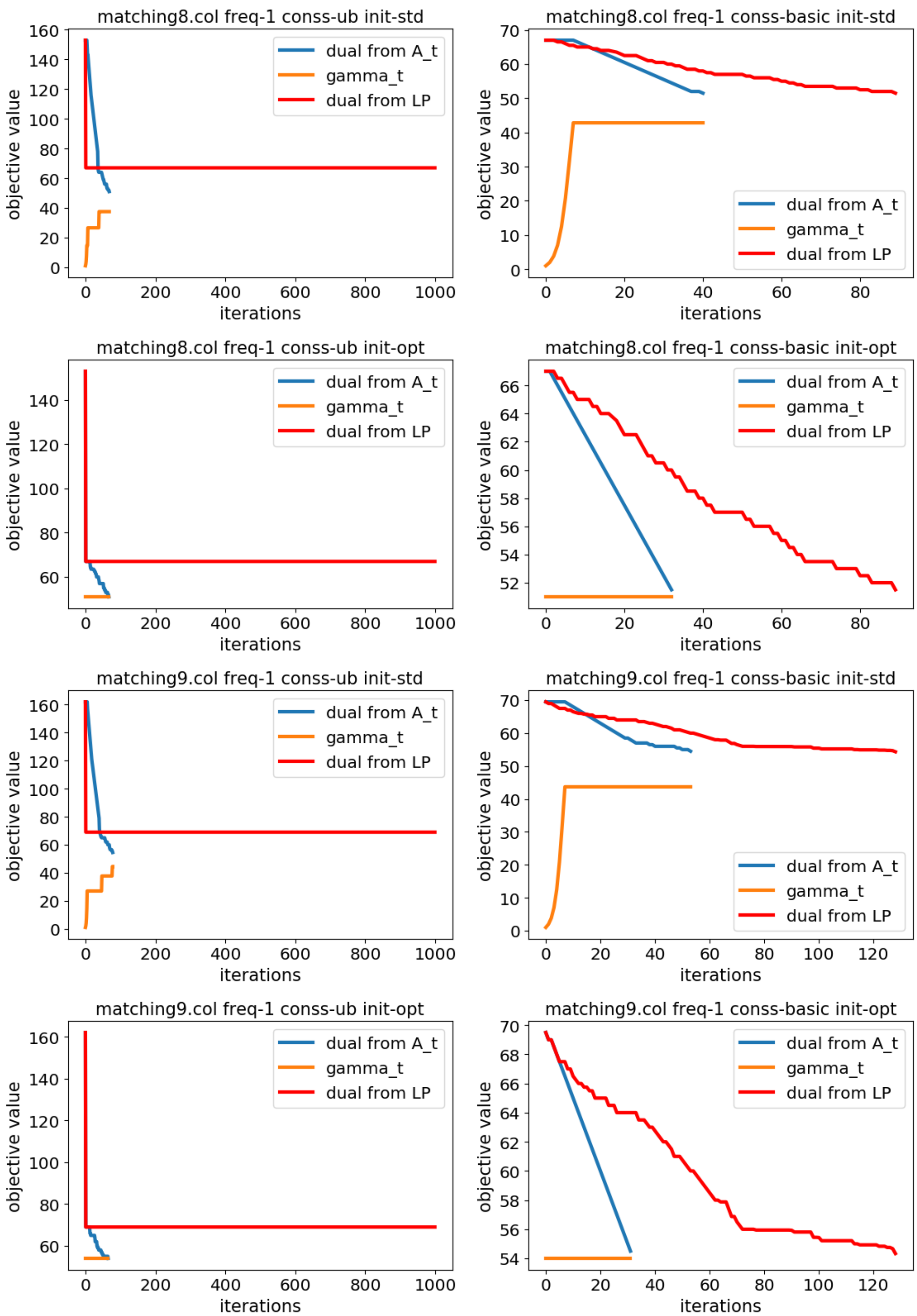
gamma_t
dual from LP
objective value
iterations


matching5.col freq-1 conss-basic init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations

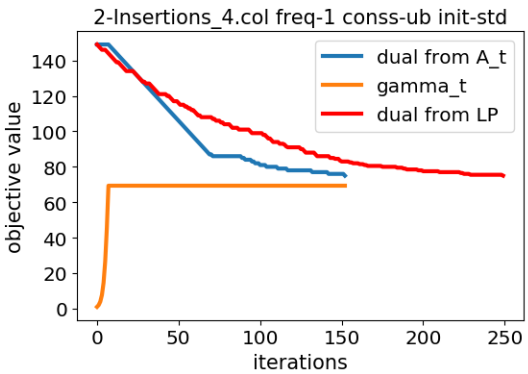

matching5.col freq-1 conss-ub init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations

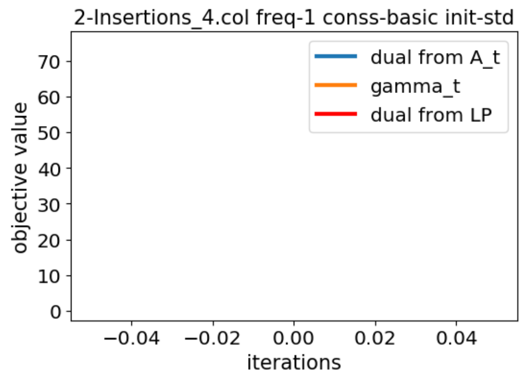

matching5.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations

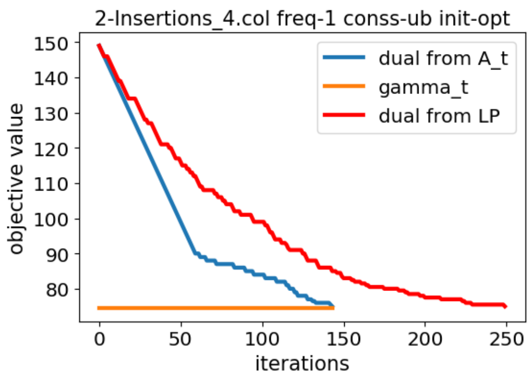

matching6.col freq-1 conss-ub init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching6.col freq-1 conss-basic init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching6.col freq-1 conss-ub init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching6.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching7.col freq-1 conss-ub init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching7.col freq-1 conss-basic init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching7.col freq-1 conss-ub init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
matching7.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations

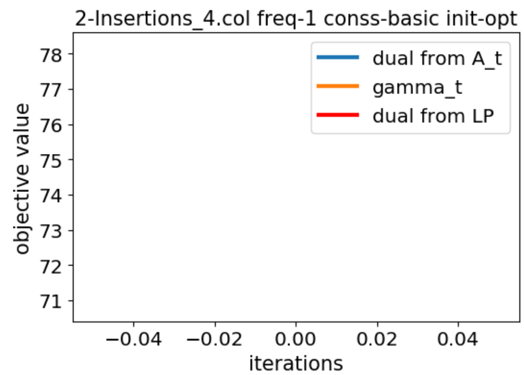

matching8.col freq-1 conss-ub init-std
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching8.col freq-1 conss-basic init-std
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching8.col freq-1 conss-ub init-opt
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching8.col freq-1 conss-basic init-opt
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching9.col freq-1 conss-ub init-std
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching9.col freq-1 conss-basic init-std
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching9.col freq-1 conss-ub init-opt
objective value
iterations
dual from A_t
gamma_t
dual from LP
matching9.col freq-1 conss-basic init-opt
objective value
iterations
dual from A_t
gamma_t
dual from LP

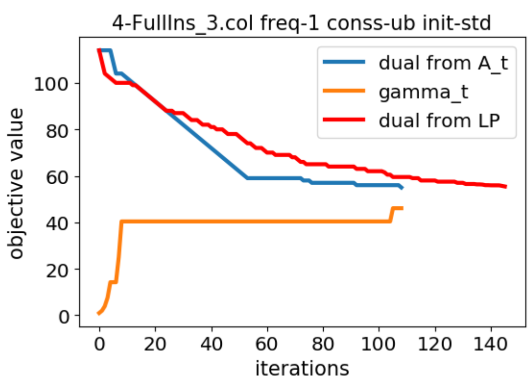
4.2 Experiments for Stable Set

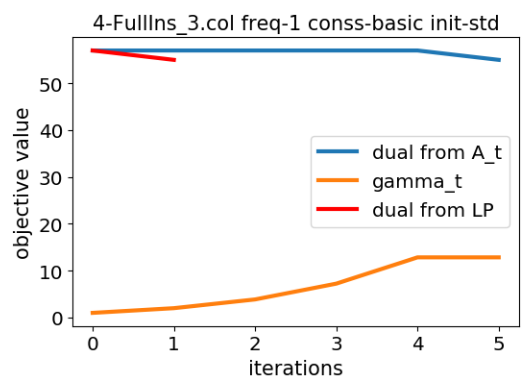


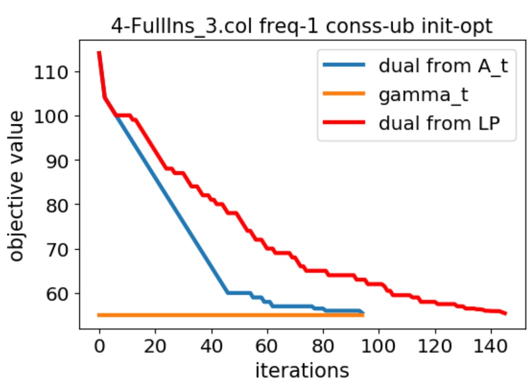


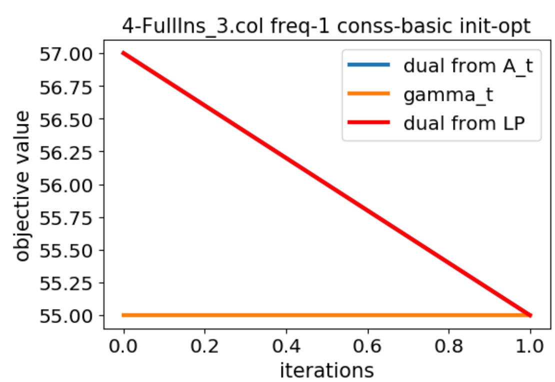


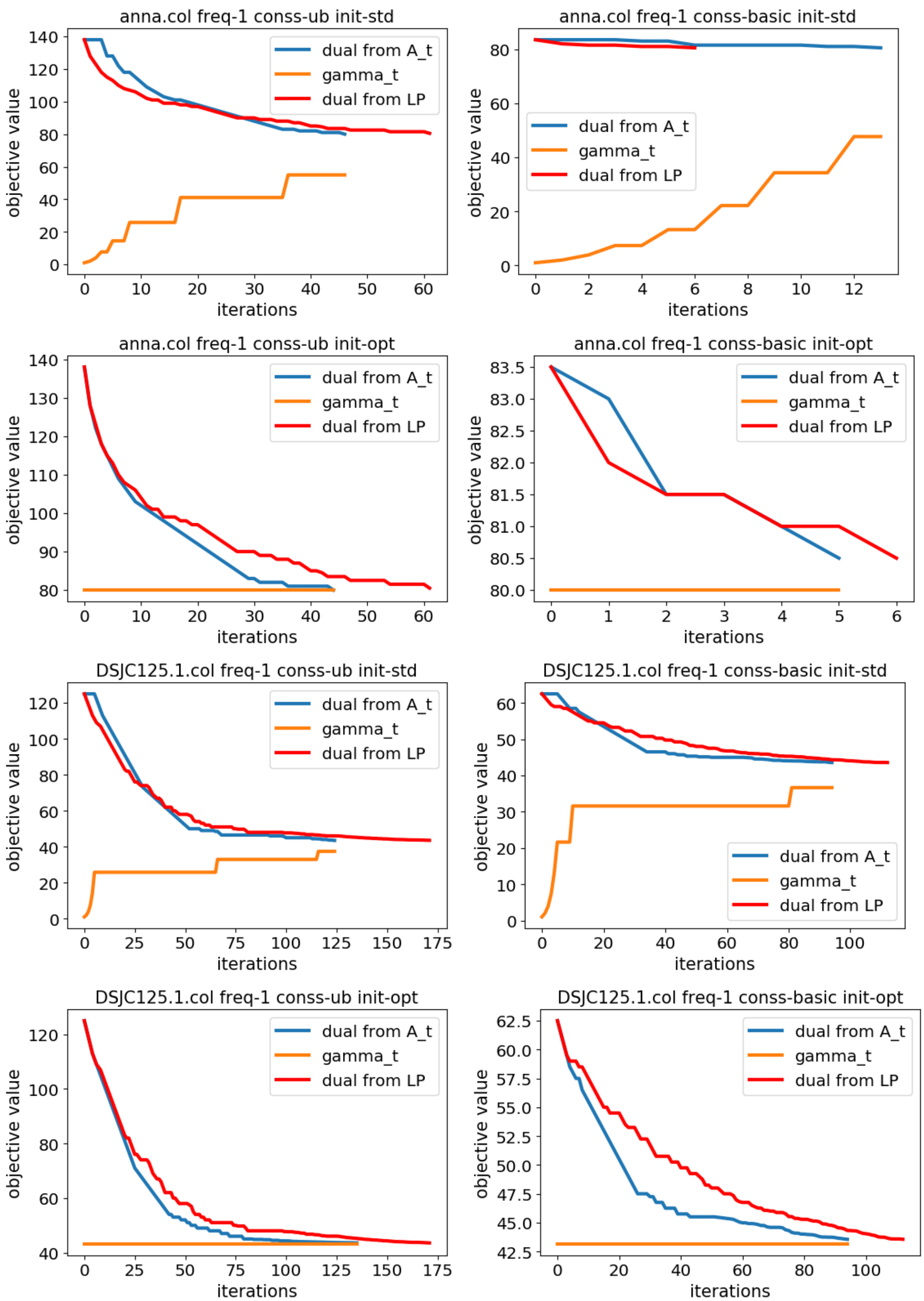


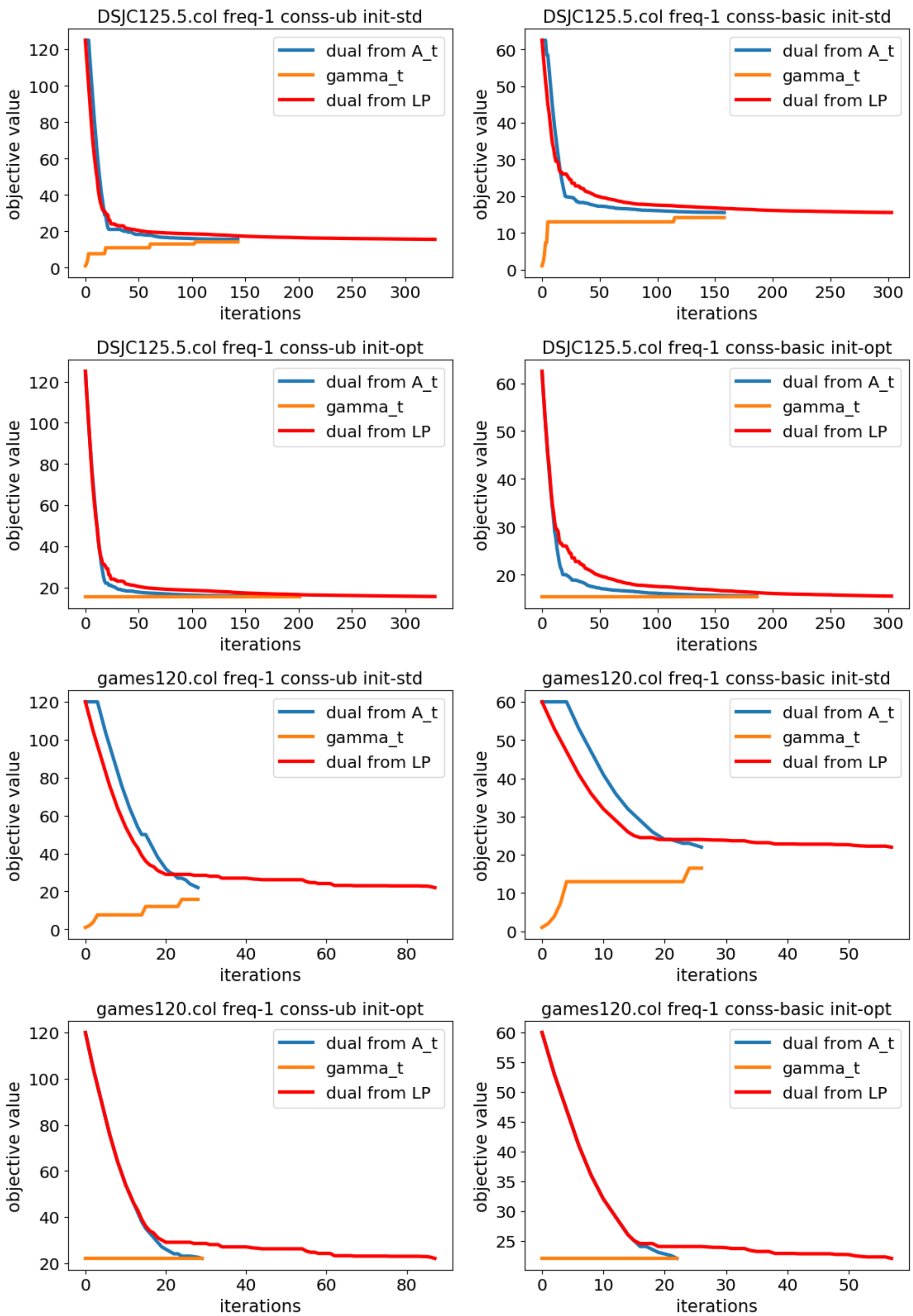


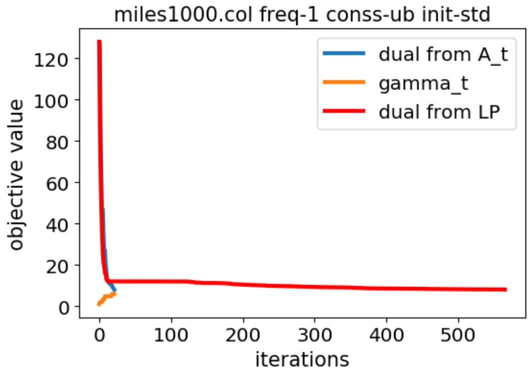


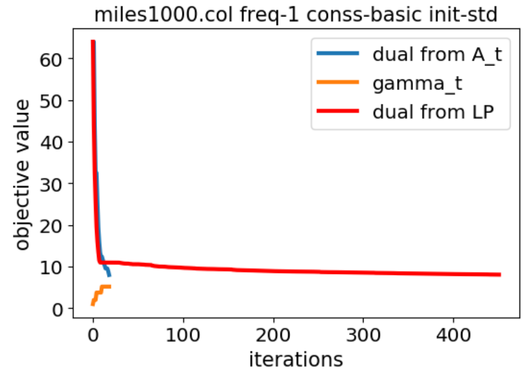


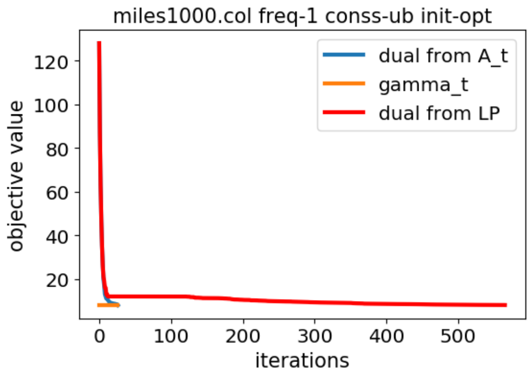


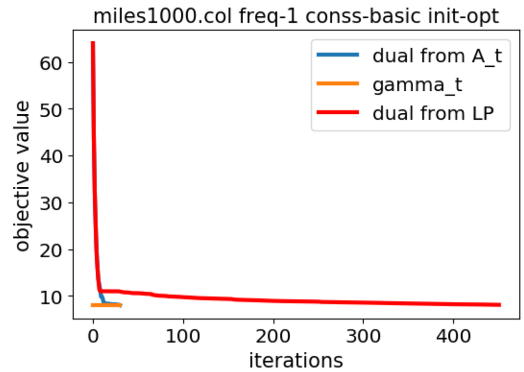

anna.col freq-1 conss-ub init-std
anna.col freq-1 conss-basic init-std
anna.col freq-1 conss-ub init-opt
anna.col freq-1 conss-basic init-opt
DSJC125.1.col freq-1 conss-ub init-std
DSJC125.1.col freq-1 conss-basic init-std
DSJC125.1.col freq-1 conss-ub init-opt
DSJC125.1.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations

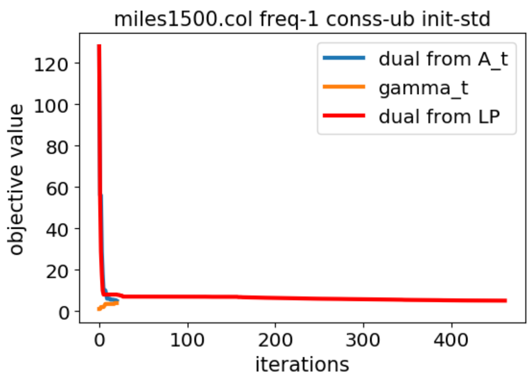

DSJC125.5.col freq-1 conss-ub init-std
DSJC125.5.col freq-1 conss-basic init-std
DSJC125.5.col freq-1 conss-ub init-opt
DSJC125.5.col freq-1 conss-basic init-opt
games120.col freq-1 conss-ub init-std
games120.col freq-1 conss-basic init-std
games120.col freq-1 conss-ub init-opt
games120.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations

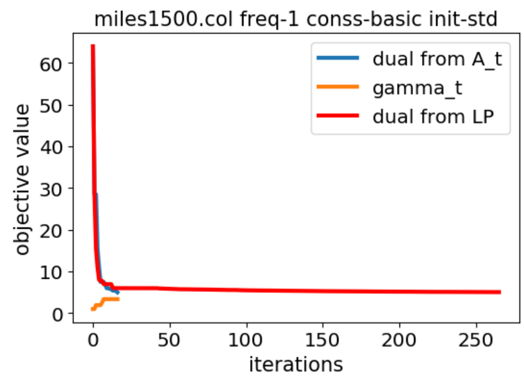

miles1000.col freq-1 conss-ub init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
0 20 40 60 80 100 120
0 100 200 300 400 500

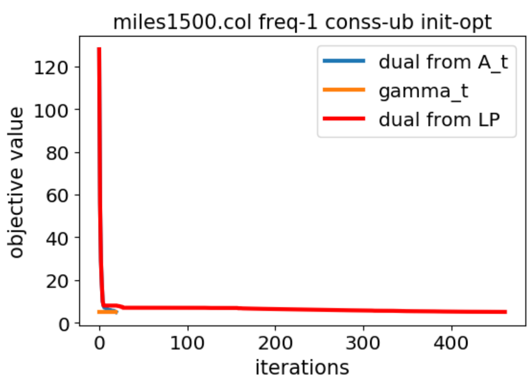

miles1000.col freq-1 conss-basic init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
0 10 20 30 40 50 60
0 100 200 300 400

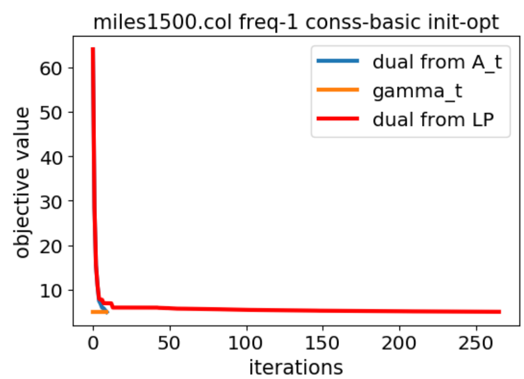

miles1000.col freq-1 conss-ub init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
20 40 60 80 100 120
0 100 200 300 400 500

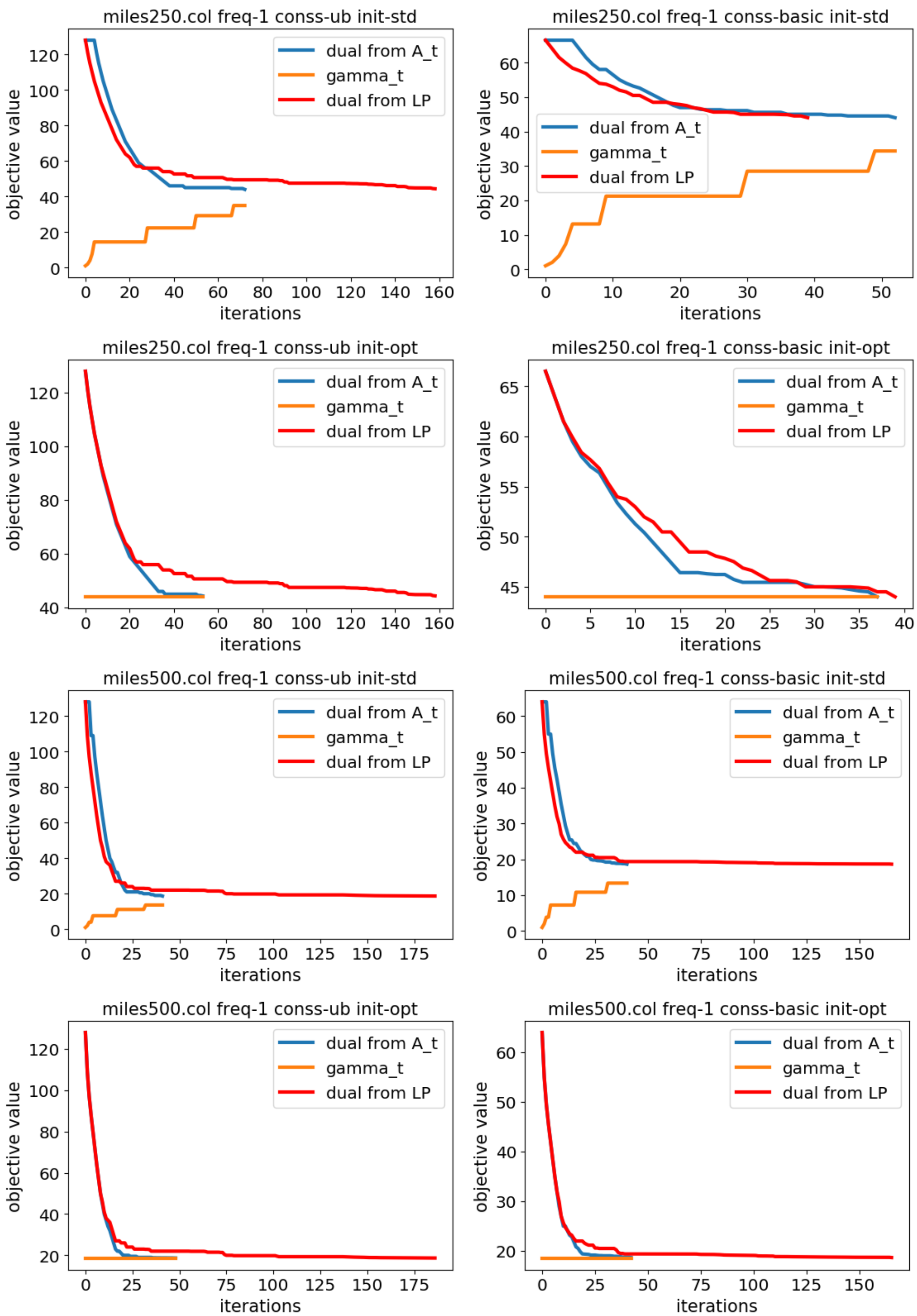

miles1000.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
10 20 30 40 50 60
0 100 200 300 400


miles1500.col freq-1 conss-ub init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
0 20 40 60 80 100 120
0 100 200 300 400

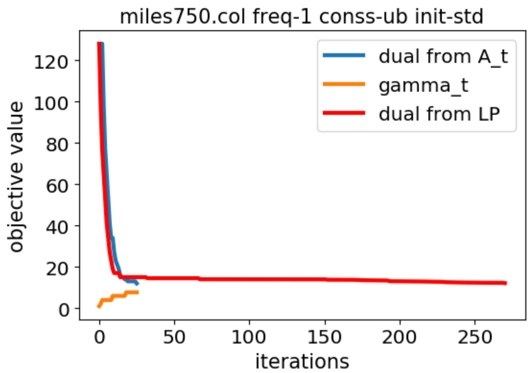

miles1500.col freq-1 conss-basic init-std
dual from A_t
gamma_t
dual from LP
objective value
iterations
0 10 20 30 40 50 60
0 50 100 150 200 250

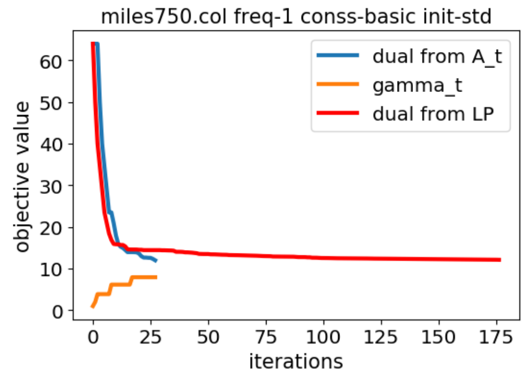

miles1500.col freq-1 conss-ub init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
0 20 40 60 80 100 120
0 100 200 300 400

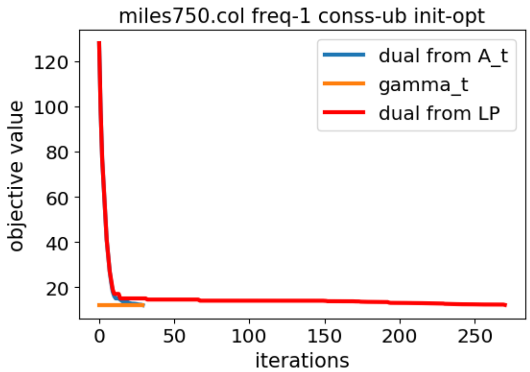

miles1500.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations
10 20 30 40 50 60
0 50 100 150 200 250

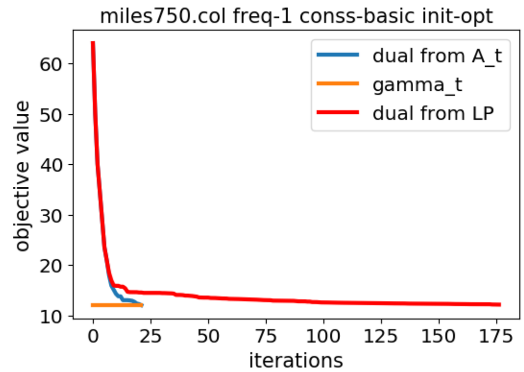


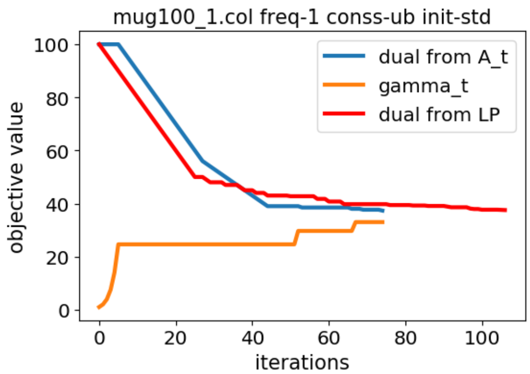
miles250.col freq-1 conss-ub init-std
miles250.col freq-1 conss-basic init-std
miles250.col freq-1 conss-ub init-opt
miles250.col freq-1 conss-basic init-opt
miles500.col freq-1 conss-ub init-std
miles500.col freq-1 conss-basic init-std
miles500.col freq-1 conss-ub init-opt
miles500.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations

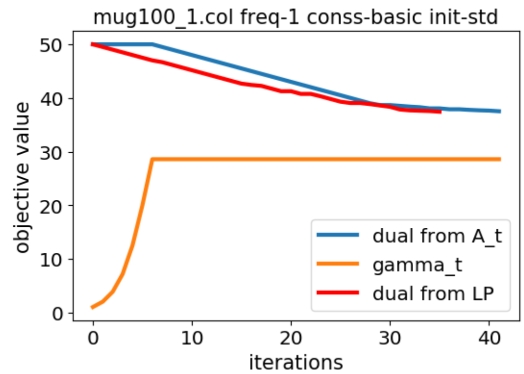

miles750.col freq-1 conss-ub init-std
objective value
iterations
0 50 100 150 200 250
0 20 40 60 80 100 120
dual from A_t
gamma_t
dual from LP

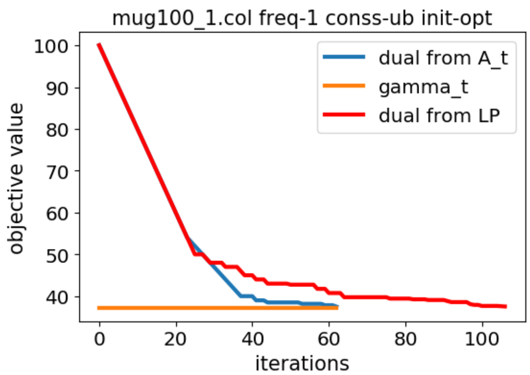

miles750.col freq-1 conss-basic init-std
objective value
iterations
0 25 50 75 100 125 150 175
0 10 20 30 40 50 60
dual from A_t
gamma_t
dual from LP

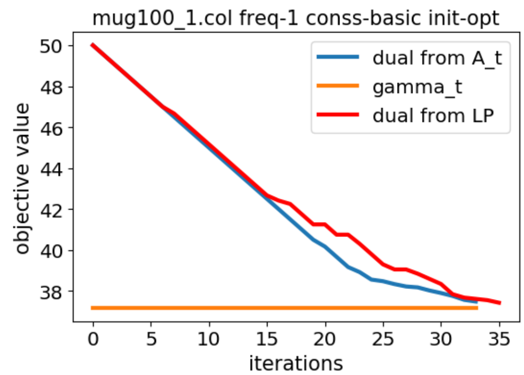

miles750.col freq-1 conss-ub init-opt
objective value
iterations
0 50 100 150 200 250
20 40 60 80 100 120
dual from A_t
gamma_t
dual from LP

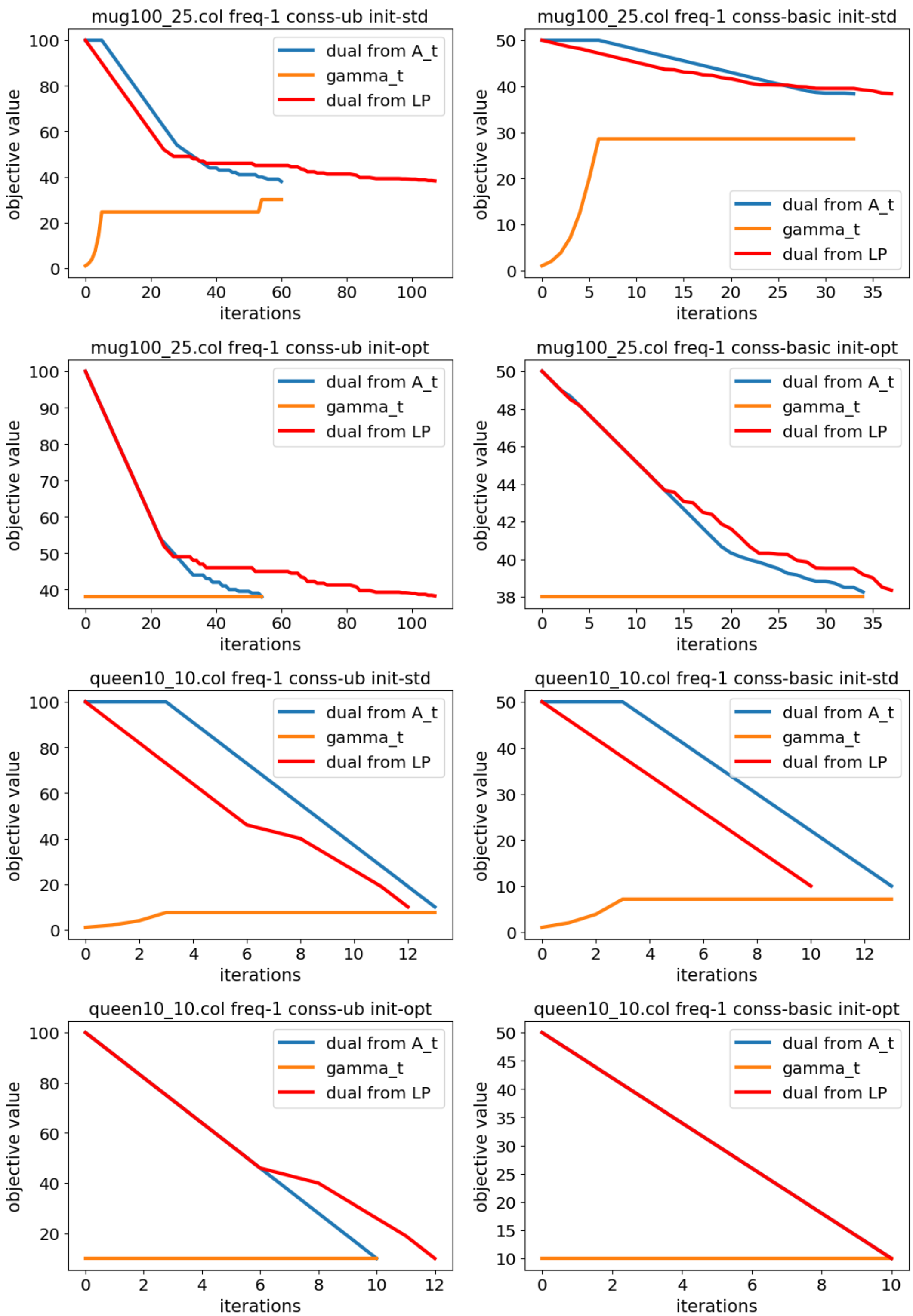

miles750.col freq-1 conss-basic init-opt
objective value
iterations
0 25 50 75 100 125 150 175
10 20 30 40 50 60
dual from A_t
gamma_t
dual from LP


mug100_1.col freq-1 conss-ub init-std
objective value
iterations
0 20 40 60 80 100
0 20 40 60 80 100
dual from A_t
gamma_t
dual from LP


mug100_1.col freq-1 conss-basic init-std
objective value
iterations
0 10 20 30 40
0 10 20 30 40 50
dual from A_t
gamma_t
dual from LP


mug100_1.col freq-1 conss-ub init-opt
objective value
iterations
0 20 40 60 80 100
40 50 60 70 80 90 100
dual from A_t
gamma_t
dual from LP


mug100_1.col freq-1 conss-basic init-opt
objective value
iterations
0 5 10 15 20 25 30 35
38 40 42 44 46 48 50
dual from A_t
gamma_t
dual from LP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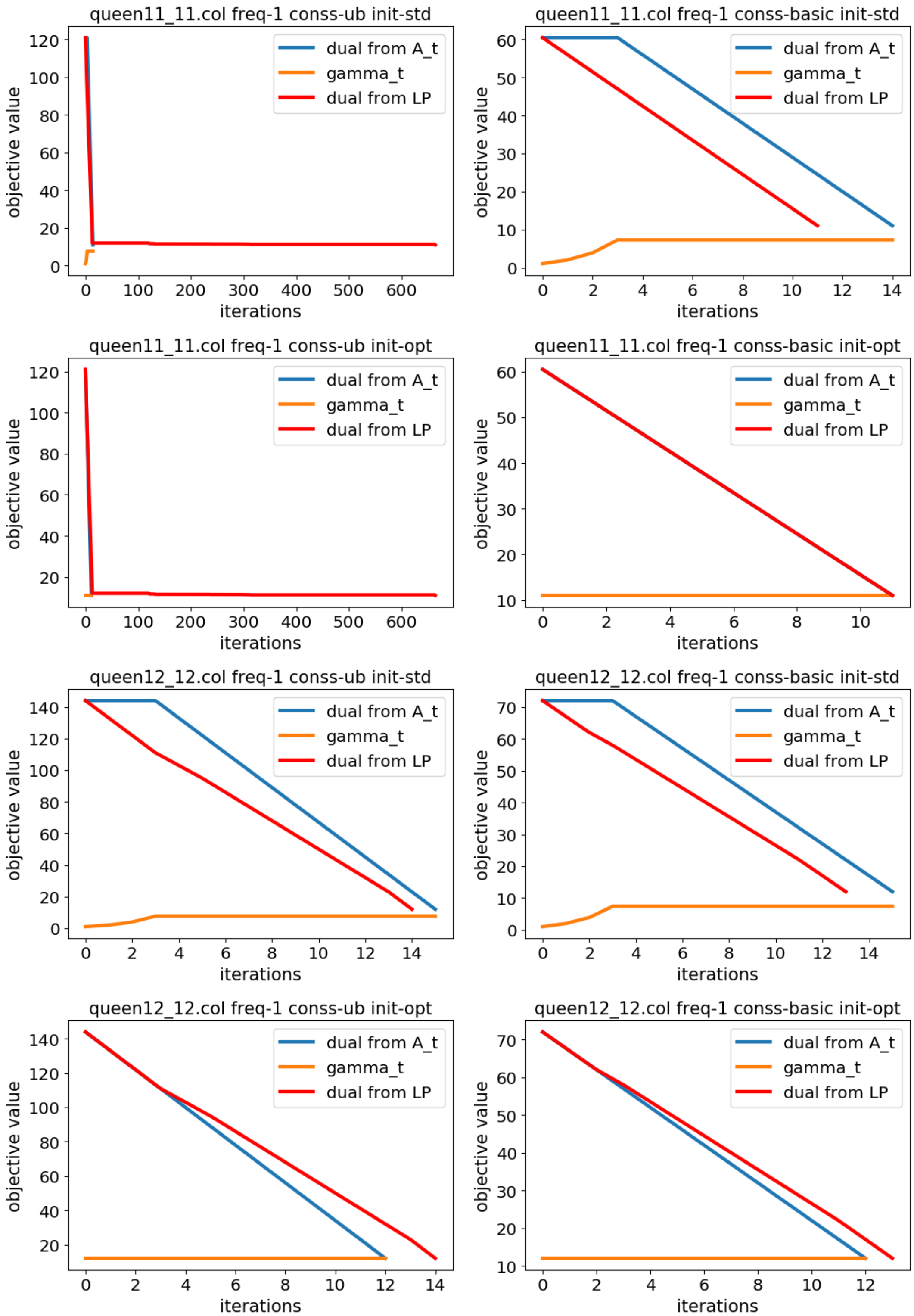
mug100_25.col freq-1 conss-basic init-std
mug100_25.col freq-1 conss-ub init-opt
mug100_25.col freq-1 conss-basic init-opt
queen10_10.col freq-1 conss-ub init-std
queen10_10.col freq-1 conss-basic init-std
queen10_10.col freq-1 conss-ub init-opt
queen10_10.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations


queen11_11.col freq-1 conss-ub init-std
queen11_11.col freq-1 conss-basic init-std
queen11_11.col freq-1 conss-ub init-opt
queen11_11.col freq-1 conss-basic init-opt
queen12_12.col freq-1 conss-ub init-std
queen12_12.col freq-1 conss-basic init-std
queen12_12.col freq-1 conss-ub init-opt
queen12_12.col freq-1 conss-basic init-opt
dual from A_t
gamma_t
dual from LP
objective value
iterations