Naive Bayes

Boston University CS 506 - Lance Galletti

Conditional Probability

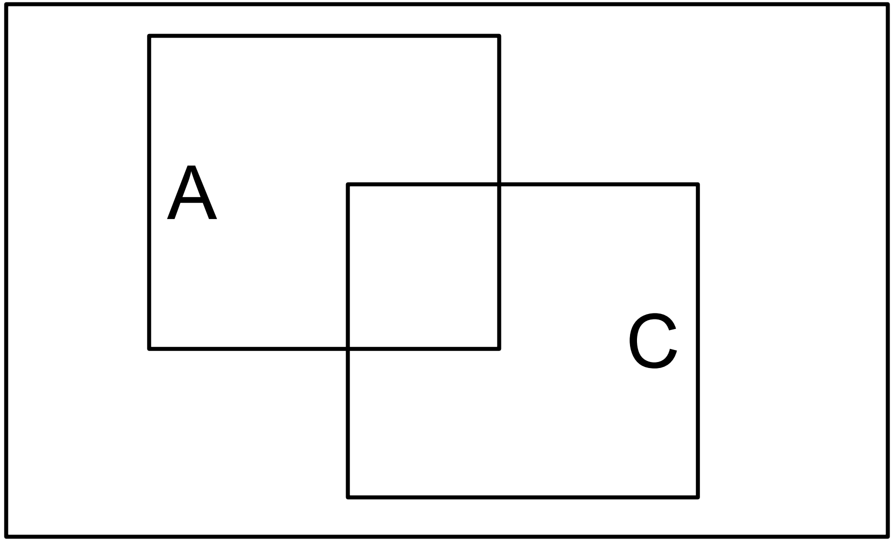
Recall

$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$

Conditional Probability

Recall

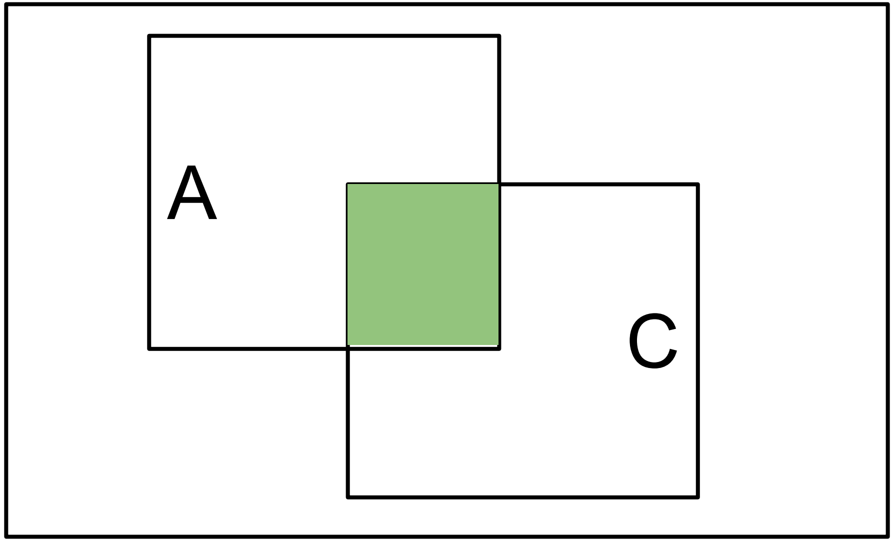
$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$



Conditional Probability

Recall

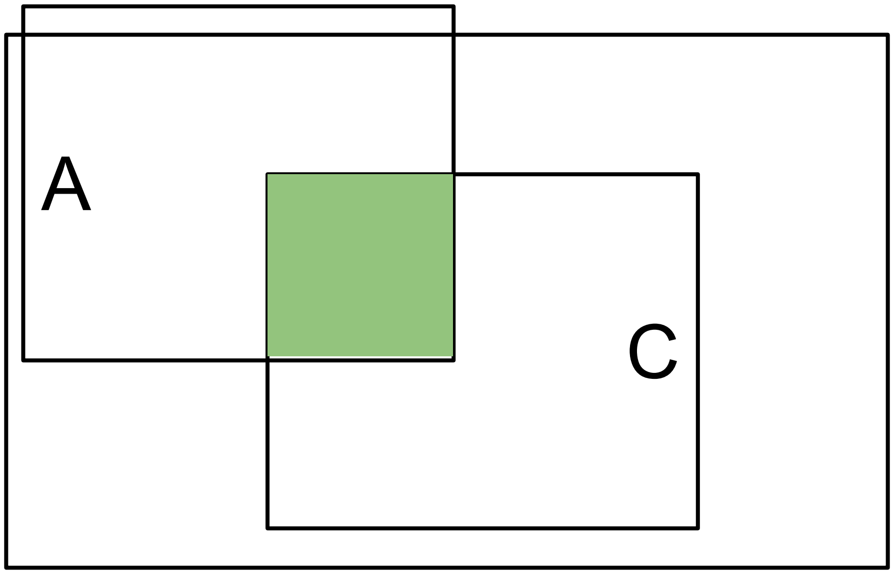
$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$



Conditional Probability

Recall

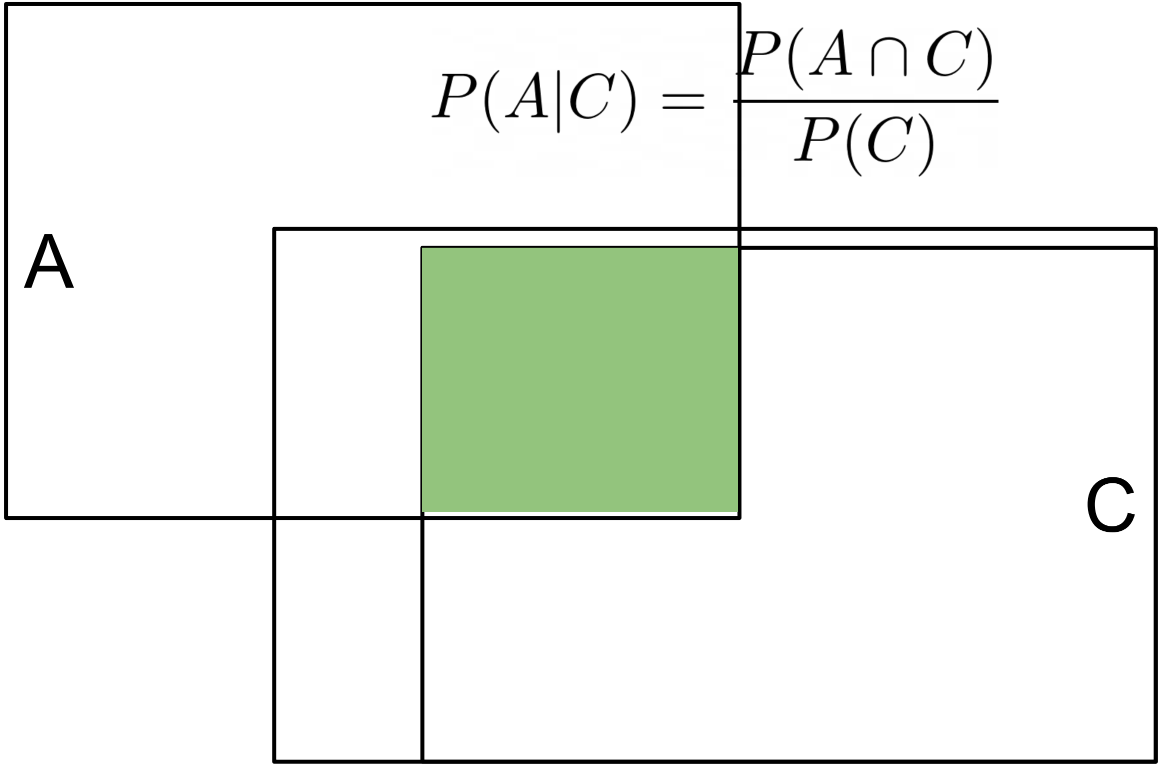
$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$



Conditional Probability

Recall

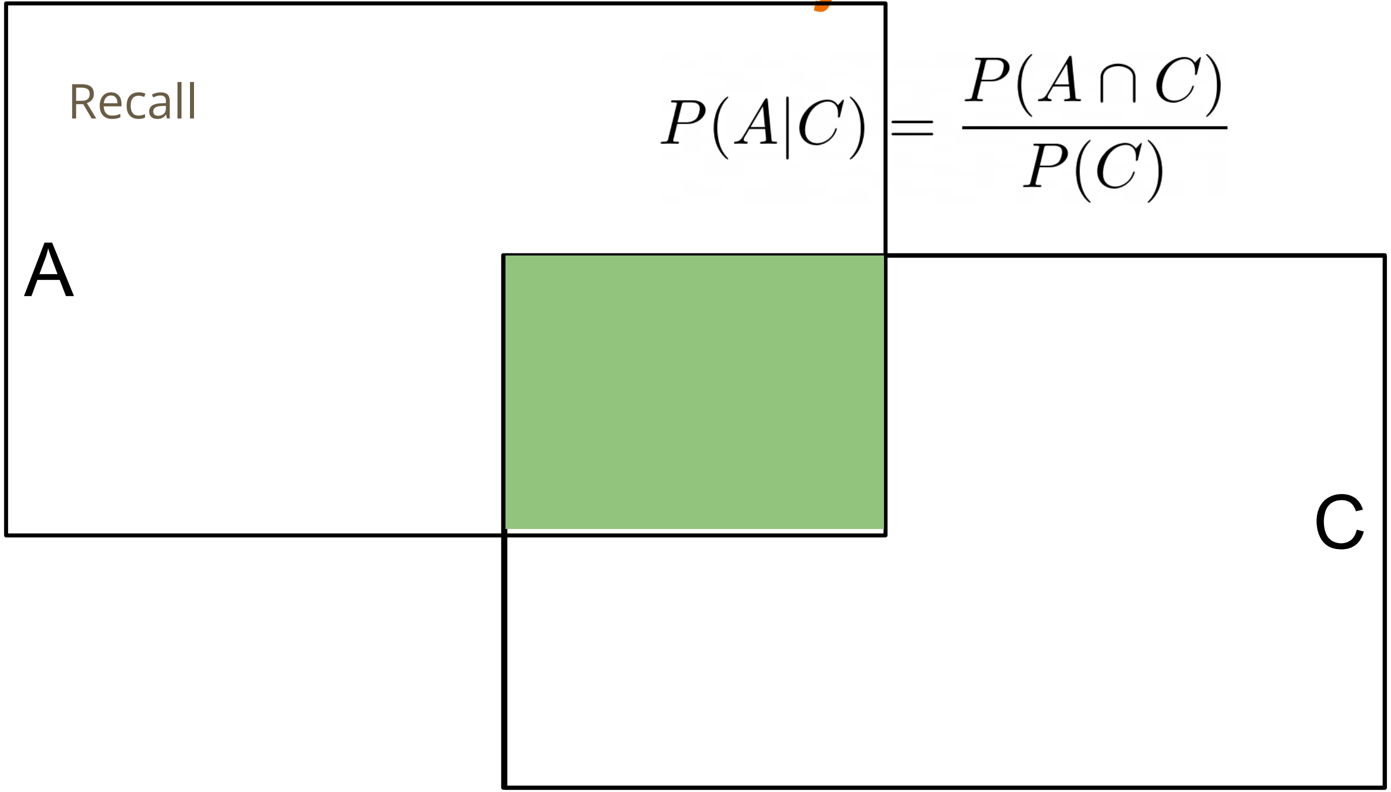
$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$



Conditional Probability

Recall

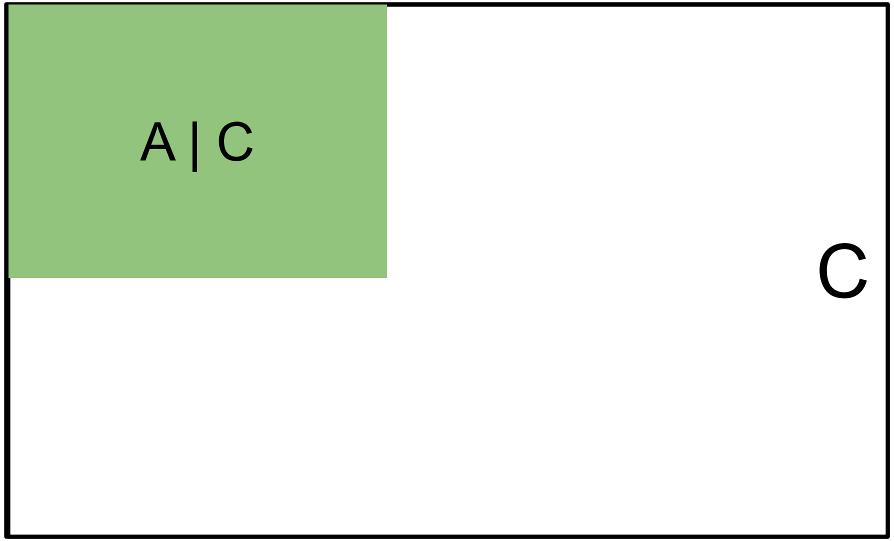
$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$



Conditional Probability

Recall

$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$



Bayes Theorem

$$P(A|C) = \frac{P(C|A)P(A)}{P(C)}$$

Bayes Theorem

$$P(A|C) = \frac{P(C|A)P(A)}{P(C)}$$

$$\frac{P(A \cap C)}{P(A)}$$

Bayes Theorem

$$P(A|C) = \frac{P(C|A)P(A)}{P(C)}$$

$$\frac{P(A \cap C)}{\text{P(A)}}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$

Example

Given:

- Meningitis causes a stiff neck 50% of the time
- Prior probability of any patient having meningitis is 1/50,000
- Prior probability of any patient having a stiff neck is 1/20

If a patient has a stiff neck, what is the probability that they have meningitis?

Example

Given:

- Meningitis causes a stiff neck 50% of the time
- Prior probability of any patient having meningitis is 1/50,000
- Prior probability of any patient having a stiff neck is 1/20

If a patient has a stiff neck, what is the probability that they have meningitis?

$$P(M|S) = \frac{P(S|M)P(M)}{P(S)} = \frac{.5 \cdot 1/50,000}{1/20} = .0002$$

Bayesian Classifiers

Predict the class C that maximizes P(C | some attributes)

Bayesian Classifiers

Predict the class C that maximizes P(C | some attributes)

Example: **binary class {yes, no}**

To classify unseen record **(marital status = “married”, income = 100k)**

Bayesian Classifiers

Predict the class C that maximizes P(C | some attributes)

Example: binary class {yes, no}

To classify unseen record **(marital status = "married", income = 100k)**

1. Compute **P(yes | marital status = "married" and income = 100k)**
2. Compute **P(no | marital status = "married" and income = 100k)**
3. Compare and predict the class that has the highest prob given the attribute values

Bayesian Classifiers

How do we estimate P(C | some attributes) from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n)$$

Bayesian Classifiers

How do we estimate P(C | some attributes) from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{P(A_1 \cap A_2 \cap \cdots \cap A_n|C)P(C)}{P(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

Bayesian Classifiers

How do we estimate P(C | some attributes) from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{P(A_1 \cap A_2 \cap \cdots \cap A_n|C)P(C)}{P(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

Does not depend on C

Bayesian Classifiers

How do we estimate P(C | some attributes) from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{P(A_1 \cap A_2 \cap \cdots \cap A_n|C)P(C)}{P(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

Maximizing $P(C|A_1A_2\ldots A_n)$ is equivalent to maximizing

Bayesian Classifiers

How do we estimate P(C | some attributes) from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{P(A_1 \cap A_2 \cap \cdots \cap A_n|C)P(C)}{P(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

Maximizing $P(C|A_1A_2...A_n)$ is equivalent to maximizing the numerator on the right

Bayesian Classifiers

So how to we estimate $P(A_1 A_2 \ldots A_n | C)P(C)$ from the data?

Bayesian Classifiers

So how to we estimate $P(A_1A_2...A_n|C)P(C)$ from the data?

P(C) is easy we can just count how many instances of each class we have

But $P(A_1A_2...A_n|C)$ is tricky because it requires knowing the **joint distribution** of the attributes...

Bayesian Classifiers

Can we make some assumptions about the attributes in order to simplify the problem?

Bayesian Classifiers

Assume that $A_1 A_2 \ldots A_n$ are independent!

Then

$P(A_1 A_2 \ldots A_n | C) = P(A_1 | C)\ P(A_2 | C)\ \ldots\ P(A_n | C)$

Can we estimate $P(A_j | C)$ from the data?

Bayesian Classifiers

Assume that $A_1A_2 \ldots A_n$ are independent!

Then

$P(A_1A_2 \ldots A_n | C) = P(A_1 | C) \, P(A_2 | C) \ldots P(A_n | C)$

Can we estimate $P(A_j | C)$ from the data?

Yes! Just count the occurrence of A_j for that class.

Example

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

$P(C = Yes) = 3/10$

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Single” | C = Yes)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Single” | C = Yes)

Refund	Marital Status	Income	Class
No	Divorced	90k	Yes
No	Single	85k	Yes
No	Single	90k	Yes

P(Marital Status = “Single” | C = Yes)
= 2/3

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Married” | C = No)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Married” | C = No)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Married	60k	No
Yes	Divorced	220k	No
No	Married	75k	No

P(Marital Status = “Married” | C = No)
= 4/7

Worksheet a)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

$$P(\text{Income} = 120\text{k} \mid C = \text{No})$$

Continuous Attributes

- Binning / 2-way or multi-way split
  - Create new attribute for each bin
  - Issue is that these attributes are no longer independent
- Pdf estimation
  - Assume attribute follows a particular distribution (example: normal)
  - Use data to estimate the parameters of the distribution

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Assume normal distribution

P(Income = 120k | C = No)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Assume normal distribution

P(Income = 120k | C = No)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Married	60k	No
Yes	Divorced	220k	No
No	Married	75k	No

Assume normal distribution

P(Income = 120k | C = No)

Sample mean = 110

Sample variance = 2975

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Married	60k	No
Yes	Divorced	220k	No
No	Married	75k	No

Assume normal distribution

P(Income = 120k | C = No)

Sample mean = 110

Sample variance = 2975

$$P(Income = 120|No) = \frac{1}{\sqrt{2\pi}(54.54)} e^{-\frac{(120-110)^2}{2(2975)}} = .0072$$

Putting it all together

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Test Record:

X = (Refund = No, Married, Income = 120k)

- P(X | No) =

- P(X | Yes) =

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Test Record:
X = (Refund = No, Married, Income = 120k)

- P(X | No) = P(Refund = No | No) P(Married | No) P(Income=120k | No) = 4/7 * 4/7 * .0072 = .0024

- P(X | Yes) =

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Test Record:
X = (Refund = No, Married, Income = 120k)

- P(X | No) = P(Refund = No | No) P(Married | No) P(Income=120k | No) = 4/7 * 4/7 * .0072 = .0024

- P(X | Yes) = P(Refund = No | Yes) P(Married | Yes) P(Income=120k |Yes) = 1 * 0 * 1.2 * 10^{-9} = 0

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Test Record:
X = (Refund = No, Married, Income = 120k)

- P(X | No) = P(Refund = No | No) P(Married | No) P(Income=120k | No) = 4/7 * 4/7 * .0072 = .0024

- P(X | Yes) = P(Refund = No | Yes) P(Married | Yes) P(Income=120k |Yes) = 1 * 0 * 1.2 * 10^{-9} = 0

Since P(X|No)P(No) > P(X|Yes)P(Yes)
=> predict No

Limitation

If one of the conditional probabilities is zero, the entire expression becomes zero...

Original estimate of $P(A_i \mid C) = N_{ic} / N_c$

Laplace estimate : $P(A_i \mid C) = (N_{ic} + 1) / (N_c + \text{constant})$

m-estimate : $P(A_i \mid C) = (N_{ic} + mp) / (N_c + m)$

p = prior probability
m = parameter

Worksheet b)