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Bachelor Thesis

A Combined Column-Generation and Lagrangian-Relaxation Approach for a Decentralized Scheduling Strategy of Heating Systems

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Kurzfassung

Zur dezentralen Koordinierung von Heizsystemen wurde in dieser Arbeit eine Planungsstrategie entwickelt und implementiert mit dem Ziel einer Integration von lokal erzeugtem Strom aus regenerativen Energiequellen . Durch die Verwendung von Wärmespeichern als integrierte Puffer können Verbrauch und Erzeugung von Wärmeenergie entkoppelt werden, um ungleiche Lastverteilungen auszugleichen. Die Planung der Heizsysteme kann als gemischt ganzzahliges Optimierungsproblem (MILP) formuliert werden, das z.B. mit einem Column-Generation Verfahren gelöst werden kann. Dieses Verfahren beruht auf der Dantzig-Wolfe Dekomposition des Problems in ein Masterproblem und verschiedene Teilprobleme und wird iterativ gelöst. Anstatt das Masterproblem in jeder einzelnen Iteration zu lösen, wird in dieser Arbeit in Verfahren vorgeschlagen, das die Column-Generation Methode mit einem Lagrange Relaxationsansatz verbindet. Dieser Ansatz nutzt eine Subgradienten-Methode, um die Preissignale von der Dualseite des Problems aus zu optimieren. Die Vorteile des Dekompositionsansatzes, wie z.B. Flexibilität, Skalierbarkeit und ein verringerter Datenaustausch, werden dabei erhalten.

Die Ergebnisse für ein Szenario aus 100 Gebäuden zeigen eine deutliche Verbesserung der Konvergenz des Column-Generation Verfahrens. Die berechneten Lösungen des Optimierungsproblems befinden nah am Optimum und können innerhalb einer annehmbaren Zeitspanne gerechnet werden.

Darüber hinaus untersucht diese Arbeit verschiedene Lösungen, um am Ende des Verfahrens zulässige ganzzahlige Lösungen zu generieren. Durch uneinheitliche Preissignale aus dem Column-Generation Verfahren entstehen hierbei ökonomische Probleme, die die vorgeschlagene Methode zu beseitigen versucht. Obwohl diese Methode in den Untersuchungen keine Lösungen ausreichender Qualität sicherstellen kann, zeigen die Ergebnisse dennoch einen vielversprechenden Ansatz mit großem Verbesserungspotential.

Abstract

In this thesis, a decentralized day-ahead scheduling strategy for the coordination of heating supply systems has been implemented. The aim is thereby to enhance the integration of renewable energy sources and balance electricity demand and supply. This scheduling problem can be formulated as a mixed integer linear program (MILP) and solved using a column generation algorithm based on the Dantzig-Wolfe decomposition technique. Instead of solving the Dantzig-Wolfe masterproblem in each iteration, we propose a concept that combines a column generation algorithm with a Lagrangian relaxation approach and uses the subgradient method to optimize the shadow prices from the dual side. Thereby, the proposed algorithm preserves the advantages of a conventional column generation approach, i.e. its flexibility, scalability and limited data exchange.

The results for a cluster of 100 buildings show that this method substantially improves the convergence of the column generation algorithm and delivers fairly optimal solutions for the scheduling problem in a reasonable amount of time.

Moreover, different strategies for obtaining integer solutions at the end of the algorithm have been investigated to resolve problems arising from non- unique shadow prices. While the proposed integering step in this thesis does not deliver solutions of acceptable quality, the basic approach seems promising with room for improvement.

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Glossary

Symbol and Unit

Symbol	Description	Unit
c	electricity cost	€/kWh
c_w	specific heat capacity of water	J/(kg·K)
COP	coefficient of performance	
m	mass	kg
P	electrical power	W
$\dot{Q}$	heat flow	W
T	temperature	K
t	time	s
x	binary working status of primary heating device	
y	binary working status of secondary heating device	

Greek Symbols

Symbol	Description	Unit
$\lambda_{i,p}$	weighting variables	
π_t	shadow price of resource constraint t	€/W
σ_i	shadow price of convexity constraint i	€
ω_{chp}	overall efficiency of a CHP unit	
η_{chp}	electrical efficiency	
σ_{chp}	CHP coefficient	
η_{boiler}	boiler efficiency	
η_{heater}	heater efficiency	

Indices and Abbreviations

Symbol	**Description**
PV	photovoltaic systems
RES	renewable energy sources
MILP	mixed integer linear program
LP	linear program
CHP / chp	combined heat and power unit
HP / hp	heat pump unit
CG	Convetional column generation algorithm
LR	Combined column generation and Lagrangian relaxation algorithm
LR	Lagrangian relaxation
LD	Lagrangian dual problem
DLD	Dual of Lagrangian dual problem
DW	Dantzig-Wolfe masterproblem
LDW	linear relaxation of Dantzig-Wolfe masterproblem
DLDW	Dual of linear Dantzig-Wolfe masterproblem
RDW	restricted Dantzig-Wolfe masterproblem
LRDW	linear relaxation of restricted Dantzig-Wolfe masterproblem
nom	nominal capacity of an electro-thermal device
d	demand
el	electricity
amb	ambient

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1 Introduction

In recent years the effort to integrate an increasing share of Renewable Energy Sources (RESs) into electricity systems has been steadily grown all over the world. The main goal of this transition is to reduce greenhouse gas emissions and gain indepence from fossil energy resources like coal or natural gas. To encourage this development in Germany, there are several political incentives implemented, e.g. the renewable energy law or the European Union Emissions Trading System. Therefore, our electricity system is undergoing a significant transformation from centrally producing power plants to a distributed generation of electricity. Over the past years, this development came along with a shift of residential buildings from passive energy consumers to active prosumers, mostly through the proliferation of combined heat and power units (µCHP) and photovoltaic systems (PV). This uncontrolled distributed electricity generation and the increased penetration of volatile Renewable Energy Sources are expected to cause challenges for the security of electricity supply and the stability of the electricity grid. One possible way to tackle this growing challenge is to deploy energy concepts on a building and micro grid level that take advantage of flexible loads and generation capacities on the demand side. By means of these energy management strategies the consumption and generation of electricity can be balanced to minimize the exchanging electricity between a micro grid and the connected macro grid.The implementation of such a concept depends on critical aspects such as the flexibility, extendability and scalability of the system as well as the protection of data privacy.

Therefore, a decentralized day-ahead scheduling strategy for the coordination of heating supply systems has been implemented. Hereby, a cluster of buildings has been modeled containing heat pumps (HPs) and µCHPs coupled with a thermal storage unit decoupling heat production and demand. The scheduling problem of the heating devices can be formulated as a Mixed Integer Linear Program (MILP) with a block-angular constraint matrix. The special structure of the constraint matrix allows for decomposing a complex problem into smaller subproblems which can be handled by each participating building individually. A central coordinator is needed to gather the individual subproblem solutions and associate each one with a weighting factor. This basic decomposition approach is called Dantzig-Wolfe decomposition. In order to deal with the large number of variables of this problem, a column generation algorithm is implemented that restricts the Dantzig-Wolfe reformulation to a subset of all variables. New variables coming from the subsystems are then iteratively added to this restricted optimization problem only if they have potential to improve the solution. Potentially useful variables, also called columns or proposals in this context, can be computed by setting a shadow price that basically serves as an incentive for the subsystems to shift electricity consumption in a certain direction. In a final Integering step, one distinct schedule for each building is chosen from the set of proposals minimizing the overall cost. Although this basic approach has some advantages in terms of scalability, flexibility as well as data privacy, it comes with some disadvantages too. When solving large scale combinatorial optimization problems with column generation algorithms, a slow convergence towards the optimum in the final phase of the algorithm is generally observed, the so-called "tailing-off" effect [Barnhart et al., 1998; Vanderbeck and Wolsey, 1996]. Moreover, there is some unequal treatment between the participating buildings when a final schedule is chosen at the end of the column generation procedure, because the proposals have been computed with varying shadow prices.

A promising way to reduce the computational effort of column generation algorithms that has been proposed in the literature is the integration of another decomposition method called Lagrangian relaxation. This technique also exploits the block-angular structure of the original constraint matrix by "relaxing" the complicating resource constraints connecting all the subsystems. The amount by which these constraints are violated is then introduced into the objective function and associated with a penalization price. Instead of finding building schedules that minimize the overall cost of the micro grid, the Lagrangian relaxation approach focuses on finding the optimal penalization price. This optimization problem is called the Lagrangian dual problem and is solved using a subgradient optimization method in this thesis. Fortunately, the shadow price that is computed in the column generation algorithm corresponds to the penalization price of the Lagrangian relaxation formulation. In fact, the Dantzig-Wolfe reformulation and the Lagrangian relaxation formulation are actually dual problems of each other. Moreover, they share almost the same subproblems. Therefore, the subgradient optimization procedure of the Lagrangian dual problem can be used to generate new promising columns for the column generation algorithm. The primary advantage of this procedure is the much better convergence of the subgradient optimization algorithm in comparison with the column generation algorithm. Additionally, the iterative steps in this procedure are computationally less expensive and easy to implement. Although the subgradient method computes proposals for each building while optimizing the shadow prices, it does not weight these proposals to compute final schedules. Therefore, both approaches are applied to combine the speed of the subgradient method with the exact primal schedule solution of the column generation algorithm.

Furthermore, this thesis proposes an additional step in the algorithm to obtain feasible schedules for each building without any unequal treatment. Hereby, a fixed approximated optimal shadow price from the combined algorithm is used to compute a set of final proposals. In this final Intering step, it is tried to find the basic columns of the Dantzig-Wolfe formulation contributing to the optimal linear solution of the problem by iteratively solving the subproblems again. The actual schedules for each building are then chosen only from this final set of subproblem proposals.

2 Theoretical background

The following chapter is meant as a short outline of the mathematical optimization topics relevant to this thesis. For a detailed discussion of these specific topics or mathematical optimization in general, it is recommended to have a look at "Introduction to linear optimization" by Bertsimas and Tsitsiklis [1997], "Nonlinear programming" by Bertsekas [1999] and "Applied mathematical programming" by Bradley, Hax, and Magnanti [1977].

2.1 Mixed integer programming

The scheduling of electro-thermal heating systems i.e. heat pumps and combined heat and power units can be formulated as a MILP (Mixed Integer Linear Program) in minimization form. Such optimization problems characteristically contain decision variables that take only integer values as well as decision variables that can take every feasible value within the given constraints. For solving these problems, commercial optimizers generally use linear-programming based branch-and-bound algorithms. These algorithms find the optimal integer values by solving a series of linear relaxations of the original problem while systematically introducing further constraints that cut out part of the continuous solution space. During this procedure most of the non-promising integer points can be discarded without testing them and the algorithm sets lower as well as upper bounds on the optimal integer solution that can be used for early termination. The set of all linear relaxations can be described in a decision tree called branch-and-bound tree where each node represents a solution of a LP. If a solution does not satisfy the integrality conditions of the original problem, further constraints are added to the problem formulation. This process is called branching. If it does, the node is marked as fathomed. Once all nodes of the branch-and-bound tree have been fathomed, the particular integer solution with the most favorable objective value gives the optimal solution of the original nonlinear integer programming problem. An improved version of this method is the branch-and-cut algorithm which is a combination of the branch-and-bound algorithm and cutting planes. Cutting planes are constraints that are added to the problem with the aim to reduce the size of the solution space.

By restricting some or all variables to take integer values, the solution space of integer problems has to be the same or a subspace of the solution space of the same optimization problem without these restrictions. Hence, it can be concluded that the optimal solution of MILP or IP problems can at best take the same value as its linear counterpart, but is likely to be inferior. The gap between the optimal integer solution and the optimal solution of the linear relaxation is called MIP- or IP-gap

As described above, algorithms for solving MILP problems usually depend on solving LP problems efficiently. The most common algorithm for solving LP problems is still the simplex method, which was proposed by George Dantzig in 1947. It uses the fact that if a general LP problem is bounded and has an optimal solution which is feasible, then there exists a vertex that is optimal. By ”moving” from one vertex to another while ensuring that the objective value always improves, the optimal solution can be found. Several improvements have been introduced e.g. the dual or primal-dual simplex method. The primary alternative to the simplex method is the barrier or interior-point method. In practice both approaches have advantages and disadvantages so that neither of the methods dominates the other. One big advantage of the improved dual simplex methods are the dual solutions as a byproduct that can be used for bounds on the optimal objective value or the decomposition methods used in this thesis.

2.2 Duality

To minimize a function subject to equality constraints, for example calculate the closest Euclidian distance from any point on a constraint function to the zero point, Lagrange multipliers are often used in calculus. The main idea is to "relax" constraints and allow them to be violated, but associate a multiplier, or price, that penalizes the amount by which they are violated. When the price is chosen properly, minimizing the cost function gives the same optimal solution as the problem with enforced constraints would do.
This basic approach can as well be applied to LP optimization problems. By relaxing all the equality constraints and introducing them into the objective function with the associated prices, an optimization problem is attained whose optimal solution is a function of the price vector. It can be shown that every optimal solution to this particular problem always gives a bound on the optimal objective value of the original problem no matter which price vector has been chosen [Bertsimas and Tsitsiklis, 1997]. More precisely, it gives lower bounds for LP problems in minimization form and upper bounds for LP problems in maximization form. In the example given above, the optimal price can easily be calculated through basic algebraic operations. For large LP problems with multiple equality constraints this is a bit more complicated. Thus, a new optimization problem can be formulated that contains the price vector as its decision variable and searches for the tightest possible bound on the original LP problem. This optimization problem is called the dual and can be formulated as a LP problem as well. The original problem is called the primal. Considering only optimization problems in minimization form, every feasible solution of the dual problem gives a lower bound on the optimal primal objective value. This property is called weak duality. The goal of

the dual problem is finding the tightest bound on the primal problem. Moreover the objective value of the optimal dual solution equals the objective value of the optimal primal solution. Accordingly this property is called strong duality. In general, it can be shown that dualizing the dual problem leads again to the primal problem, which illustrates the strong relationship between the two problem formulations. This important observation is exploited in many efficient algorithms for solving optimization problems.

2.2.1 Shadow prices

The theory of duality has numerous applications, e.g. the dual simplex method, and some properties that can be very useful in sensitivity analysis and mathematical economics. Analyzing incremental changes in the primal LP problem leads to an economical interpretation of the dual solution vector that will be helpful for the algorithm implemented in this thesis. It can be shown that a small change d in the right-hand side vector of a primal equality constraint i results in a change equal to $\pi_i \cdot d$ in the optimal primal cost, with π_i being part of the optimal solution to the dual problem [Bertsimas and Tsitsiklis, 1997]. Therefore the optimal dual solutions can economically be interpreted as the marginal cost per unit increase of a requirement associated with a particular constraint. In this thesis, this vector is called shadow price vector.

2.2.2 Convex-hull pricing model

The thoughts leading to the definition of shadow prices are based on assumptions that hold only for LP problems. To attain the optimal shadow prices when solving MILP problems, a linear relaxation of the MILP problem has to be introduced that allows the integer variables to take every value from within the convex hull of the integer solution space. Therefore the optimal dual solution to this problem is called convex hull price. In an economic dispatch problem, the convex hull price minimizes the uplift payments, which can be interpreted as minimizing the residual load of a microgrid in this thesis [Gribik et al., 2007].

2.3 Decomposition methods and algorithms

The straight forward implementation of the MILP optimization problem can be interpreted as a centralized coordination scheme. Although the centralized scheduling approach is easy to implement and delivers good coordination results for a small number of buildings, it has several big disadvantages, e.g. its extendibility or data privacy. Therefore a decentralized scheduling approach has been proposed that takes advantage of the problem structure of the MILP and uses a decomposition technique called Dantzig-Wolfe decomposition and column generation to solve the problem

more efficiently. The algorithm, that implements the scheduling problem of heatings systems, will be discussed in detail in section 2.3.1 and 2.3.2.
To further improve column generation algorithms, (Reference) propose a method that combines column generation and a decomposition technique called Lagrangian relaxation. This new formulated problem is closely related to the theory of duality and requires non-linear optimization techniques for its solution. It will be discussed in section 2.3.3.

2.3.1 Dantzig-Wolfe decomposition

The main idea of Dantzig-Wolfe decomposition is to decompose an optimization problem into a master- and several subproblems by substituting the variables in the original problem formulation with a convex combination of the extreme points of the subproblems. It was originally developed by George Dantzig and Philip Wolfe and initially published in 1960. By introducing convexity constraints to the masterproblem, the subproblem constraints will inherently be satisfied weighting the extreme points. Furthermore, it is possible to separate easy to solve substructures from the complicating resource constraints. Solving the resulting masterproblem can be described as finding the optimal weights for the subproblems extreme points to minimize or maximize the objective function with respect to the complicating resource constraints. The subproblems goal is to compute the extreme points with respect only to the subproblem specific constraints. Of course the original problems constraint matrix needs to exhibit a certain structure called block-angular to be applicable to Dantzig-Wolfe decomposition. Block-angular matrices, such as the scheduling constraint matrix in this thesis, typically consist of one or several comprehensive resource constraints and some additional subproblem-specific constraints. Dantzig-Wolfe decomposition can be useful if the new formulated optimization problem is easier to solve, e.g. due to the reduction of constraints. In fact, the new optimization problem is just a reformulation of the equivalent original problem.
The key advantage of Dantzig-Wolfe decomposition is its applicability to column generation. For very large optimization problems, it is not practicable to add every possible subproblem extreme point to the masterproblem. Instead, it is much more efficient to restrict the masterproblem to a selection of extreme points and add additional extreme points - or columns - only if they have potential to further improve the solution. This method dealing with a large number of variables is called column generation. Since the restricted Dantzig-Wolfe masterproblem contains only a subset of all extreme points, its solution is naturally bounded by the Dantzig-Wolfe solution of the complete set of extreme points.

2.3.2 Column generation algorithm

Although Dantzig-Wolfe decomposition and column generation work very well together, the possibility to restrict optimization problems only to a subset of variables comes in handy for a variety of problems and improves the tractability of many large-scale optimization problems. This is especially true for problems that have a great number of variables taking zero values. In case of Dantzig-Wolfe decomposition there are usually many extreme points that do not contribute to the final solution and therefore don't have to be added to the masterproblem.

To find the columns with potential to improve the solution of the restricted masterproblem, an additional optimization problem is formulated called pricing problem. This pricing problem essentially consists of finding the column, extreme point, or variable, that has the greatest potential for improving the master solution. The key value that has to be taken into account here is called reduced cost. It can be interpreted as the cost per unit increase in a variable x_j minus the cost of the compensating change in the basic variables necessitated by the associated constraints. Let c_j be the cost factor associated with x_j, let A_j be the constraint vector associated with x_j and let π be the optimal solution vector to the restricted dual masterproblem. Then the reduced cost $\bar{c}_j$ of a variable x_j can be defined as

$$\bar{c}_j = c_j - \pi A_j . \tag{2.1}$$

As previously mentioned, the dual solutions of a LP can economically be interpreted as shadow prices and describe the change in the value of the objective function when increasing the right hand side of the constraint by one unit. When using modern solvers like CPLEX or Gurobi for solving LP problems, the dual solutions are determined automatically and can therefore be used without additional computational costs. To understand the concepts applied in this thesis, it is very important to note that shadow prices make sense only for a masterproblem in linear form, whereas the pricing subproblems can be solved as integer problems. This is due to the fact that shadow prices derive from incremental analysis, which doesn't apply for integer problems. To obtain an integer solution in the end anyhow, the algorithm has to be slightly adjusted, which is explained in section 2.5.

Considering LP problems in minimization form like the scheduling of heating systems, potentially useful columns mean negative reduced costs. Hence the pricing problem minimizes the reduced cost by selecting the potentially most useful column, in case of Dantzig-Wolfe decomposition the potentially most useful subproblem extreme point. Columns with positive reduced cost don't have any potential to improve the overall solution and can be disregarded.

When solving LP problems, all considered columns can be divided into basic and non-basic columns. Basic columns are those contributing to a solution and non-basic are those taking zero values. Basic columns characteristically have reduced cost of zero, which is an important property for the algorithm implemented in this thesis.

To solve the LP optimization problem to optimality or approximate optimality, the algorithm solves the pricing problems of every subproblem and adds the new found columns to the masterproblem. Then, with the added columns the masterproblem has to be solved again to compute new shadow prices. The updated shadow prices can now be used to solve the pricing problems again. This procedure is to be repeated until the masterproblem is solved to optimality, which can be determined by non-negative reduced cost for all subproblems, or a termination criterion is reached, e.g. a time-limit or sufficient bounds for the optimal solution (see section 2.3.4). Figure 2.1 illustrates the basic principle of column generation algorithms and the transfer of information between a decomposed masterproblem and two subproblems.

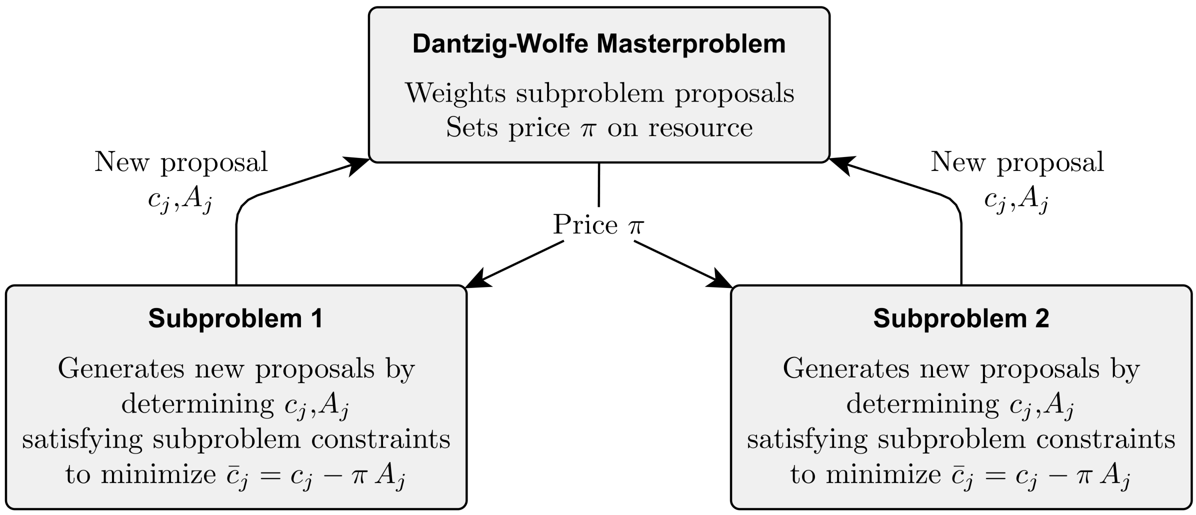

Figure 2.1: Information transfer in column generation [Bradley et al., 1977]

2.3.3 Lagrangian relaxation

In section 2.2 the theory of duality is discussed as an outgrowth of the Lagrange multiplier method. By introducing all equality constraints of a LP problem to its objective function, a new optimization problem is derived whose goal is to find the optimal associated multipliers. A very similar approach can also be used as a decomposition method to exploit the structure of block-angular matrices. Instead of relaxing all the equality constraints, only the complicating constraints linking the subproblems are introduced into the objective function. Obviously, this leads to a problem formulation without any linking constraints. Therefore the new formulated problem can be solved by splitting the problem into its subproblems and solving them separately. The solution of this new problem is simply the sum of all subproblem solutions. This decomposition approach is called Lagrangian relaxation.

Although the described Lagrangian relaxation problem is not exactly the same as the dual problem

discussed in section 2.2, it shares some of the important properties, for example weak and strong duality. Hence, the Lagrangian relaxation problem always gives a lower bound for minimization problems and the Lagrangian dual problem consists of finding the maximum lower bound which is equal to the linear solution of the original problem as a function of the price vector.
The problem of finding the optimal Lagrangian multipliers is typically solved using an iterative procedure. Hereby, in each iterative step the Lagrangian relaxation has to be solved to update the multipliers afterwards. In this thesis, the subgradient method [Fisher, 1979] is used for approximating the optimal multipliers, but there also exist some other, more advanced methods such as the bundle method [Lemaréchal et al., 1995] or the volumne algorithm [Barahona and Anbil, 2000]. Section 2.4 gives a short introduction to the theory of the subgradient method.

2.3.4 Relationship and comparison

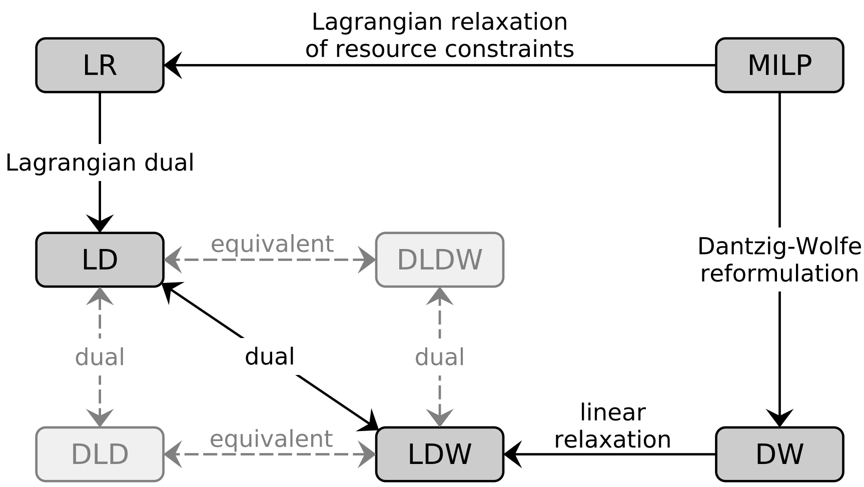

Figure 2.2: Relationship between Dantzig-Wolfe decomposition and Lagrangian relaxation [Nishi et al., 2009]

To speed up the column generation procedure, the combined optimization algorithm first proposed by Barahona and Jensen [1998] and used for the proposed algorithm in this thesis exploits the strong relationship between Dantzig-Wolfe decomposition and Lagrangian relaxation. In particular, it can be shown that the Langrangian dual problem, that derives from the Lagrangian relaxation reformulation, is the dual of the linear relaxation of the Dantzig-Wolfe reformulation and vice versa [Geoffrion, 1974; Fisher, 1979]. Hence, the optimal dual variables for the resource constraints of the Dantzig-Wolfe masterproblem correspond to the optimal multipliers of the Lagrangian relaxation formulation [Magnanti et al., 1976]. Figure 2.2 illustrates this relationship between Dantzig-Wolfe decomposition and Lagrangian relaxation. Moreover, there exists another useful relationship be-

tween the two problems that can be exploited. The pricing subproblems that have to be solved in the column generation procedure are almost the same as the subproblems that derive from splitting up the Lagrangian relaxation except for a constant. This will be shown later in more detail.
When solving large scale optimization problems with the described column generation algorithm, in each iterative step the restricted Dantzig-Wolfe masterproblem has to be solved to get the optimal dual solutions known as shadow prices from the simplex algorithm. As the algorithm proceeds the masterproblem gets bigger with each iteration which leads to a significant slowdown in the algorithm. Vanderbeck and Wolsey [1996] and Barnhart et al. [1998] observed that after a few first iterations in which the objective value improves fast, column generation algorithms tend to converge very slow towards the optimum in the final phase. This is called "tailing-off" effect and leads to an enormous number of iterations to attain an optimal or even near-optimal solution for large scale problems. Furthermore, it can be observed that the lower bounds computed from the pricing subproblem solutions of a single iteration are very bad at the beginning and need lots of iterations to converge towards the optimal value.
Optimizing the Lagrangian relaxation of the original MILP can be done with subgradient methods. Hereby, in each iterative step the Lagrangian relaxation problem has to be solved through its subproblems to compute new shadow prices in an updating step. This approach focuses on the dual problem an doesn't provide any primal solutions. Usually, this procedure is easy to implement and the updating step is computationally very inexpensive. Moreover, the computational effort doesn't grow with the number of iterations, which is another great advantage. In comparison to the dual solutions derived from solving the restricted masterproblem in the column generation procedure, the dual solutions in the subgradient update method are expected to converge much faster towards their optimal values. In fact, this can be seen as one of the main advantages of this procedure, because the pricing subproblems can be solved with shadow prices closer to optimality. Unfortunately, there are some problems with the convergence of the subgradient optimization procedure, which is why the algorithm is usually stopped after a fixed number of iterations.
Inequality 2.2 shows how the solutions of the different problem formulations relate to each other for a MILP problem in minimization form. When using Dantzig-Wolfe decomposition to reformulate the original MILP problem, the two problem formulations are actually equivalent problems and thus share the same optimal solution z_{MILP} and z_{DW}. Relaxing the integer variables of the Dantzig-Wolfe masterproblem most likely leads to an improved solution z_{LDW} of the masterproblem, because of the extended solution space. As discussed earlier, the Lagrangian dual problem is actually the dual of the linear relaxation of the Dantzig-Wolfe masterproblem. By strong duality, it can be concluded that its optimal objective value z_{LD} must be the same as z_{LDW}. Obviously, the optimal objective value for LP problems is attained if the optimal decision variables are found, in case of the Lagrangian dual problem the optimal shadow price vector. Following the weak duality theorem, this optimal Lagrangian dual solution gives an upper bound on every other optimal solution of the

Lagrangian relaxation problem with non-optimal shadow prices.

$$z_{LR}(\pi) \leq z_{LD} = z_{LDW} \leq z_{DW} = z_{MILP} \tag{2.2}$$

The reason for using methods like column generation is, again, to avoid solving the full scale optimization problem, which is in many cases computationally too expensive. Therefore a restricted version of the problem is solved, containing only a subset of all variables and reducing the computational effort. Of course, by restricting the optimization problem just to a selection of variables, the solution space of the MILP problem is being pruned and its optimal solution doesn't necessarily have to be inside this restricted solution space. Therefore the optimal solution of the restricted Dantzig-Wolfe masterproblem z_{LRDW} can be interpreted as an upper bound on the optimal solution of the full masterproblem z_{LDW}, because its solution space is only a subspace of the full masterproblem. With every variable introduced in an iteration, the objective value of the restricted masterproblem is improving or at least remaining constant due to the extension of the solution space until the optimal solution is found:

$$\begin{aligned} z_{LDW} &\leq z_{LRDW} \\ z_{DW} &\leq z_{RDW} \end{aligned} \tag{2.3}$$

2.3.5 Combined algorithm

In the literature, the possibility to combine column generation and Lagrangian relaxation has been discussed for several large scale combinatorial optimization problems. A method that uses the Lagrangian relaxation formulation and the subgradient optimization method to generate new columns inexpensively without solving the Dantzig-Wolfe masterproblem in each iteration was first proposed by Barahona and Jensen [1998] for a plant location problem and by Degraeve and Peeters [2003] for the cutting stock problem. Huisman et al. [2005] describe the procedure depicted in figure 2.3 as a nested double loop. In the outer loop, the Dantzig-Wolfe masterproblem is solved with a simplex algorithm to obtain the optimal dual variables as known from the column generation algorithm. In the inner loop, the subgradient optimization method is applied on the Lagrangian relaxation formulation of the problem to iteratively update the shadow prices. As the subproblems that have to be solved in the Lagrangian relaxation formulation are effectively the same as the pricing subproblems in the column generation procedure, the subproblem solutions of the Lagrangian relaxation can also be used as new columns for the Dantzig-Wolfe masterproblem. Using this technique, new columns can be computed without solving the masterproblem in each iteration. Moreover, the subgradient optimization procedure has some advantages regarding the speed of convergence for the dual solutions. Also, the Lagrangian relaxation provides lower bounds on the optimal value of the linear masterproblem, which can be used as an estimation of the quality of the solution and as termination criterion.

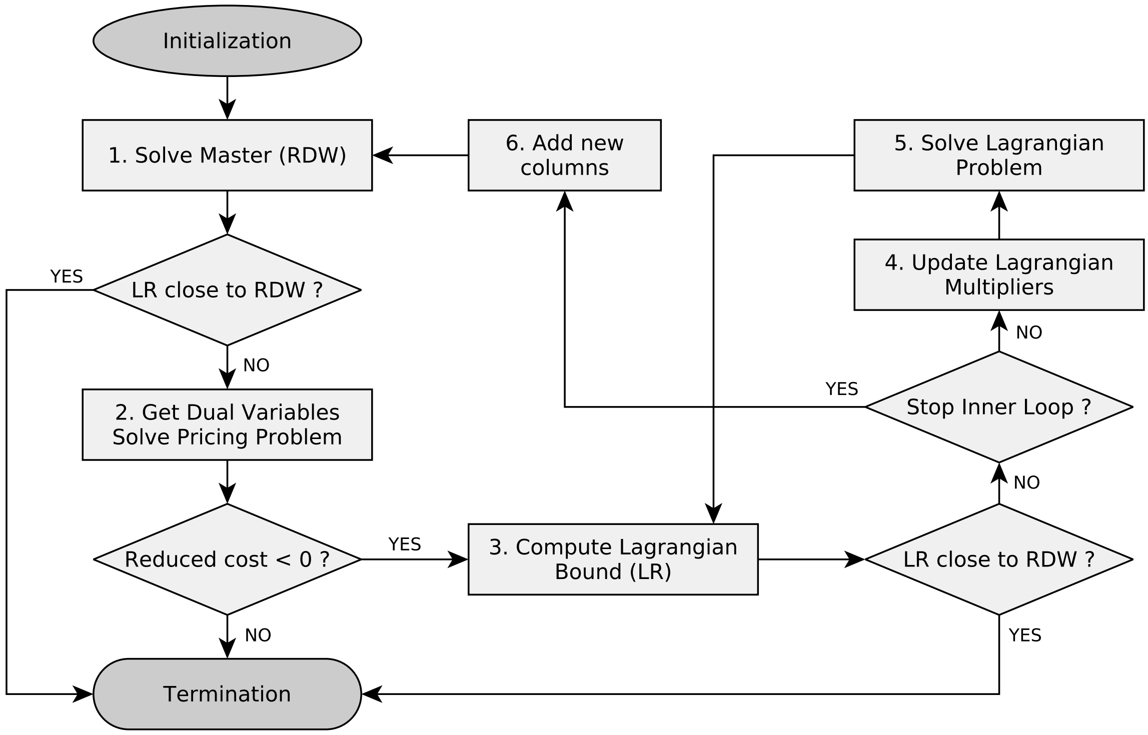

Figure 2.3: Combined column generation and Lagrangian relaxation algorithm [Huisman et al., 2005]

2.4 Subgradient method

The subgradient method is an approach for solving dual problems with nondifferentiable, convex cost functions. Although it looks similar as the gradient projection method, there are some important differences, e.g. the descent properties and stepsize formulation. It was originally developed by Shor in the Soviet Union in the 1970s. The basic reference on subgradient methods is his book [Shor et al., 1985]. For a detailed discussion of nonlinear optimization topics and decomposition methods, it is recommended to have a look at "Nonlinear Programming" by Bertsekas [1999] as well.

Given a primal problem of the form

$$\begin{aligned}
&\text{minimize} && f(x) \\
&\text{subject to} && x \in X \\
& && g_j(x) \le 0, \;\; j = 1, \dots, r\,,
\end{aligned}$$

where $f : \mathbb{R}^n \mapsto \mathbb{R}$, $g_j : \mathbb{R}^n \mapsto \mathbb{R}$ are given functions and X is a subset of $\mathbb{R}^n$, the dual problem can be formulated as

$$\begin{aligned}
&\text{maximize} && q(\pi) = \inf_{x \in X} \Big(f(x) + \pi g(x) \Big) \\
&\text{subject to} && \pi \ge 0\,.
\end{aligned}$$

The subgradient method consists of iteratively generating dual feasible points according to the following updating formula:

$$\pi^{k+1} = \left[\pi^k + s^k g^k \right]^+. \tag{2.4}$$

g^k is the so-called subgradient from the definition of the dual problem, $\left[\,\right]^+$ denotes a projection on the closed convex set of feasible dual solutions and s^k is a positive scalar called stepsize. An important property of the subgradient method is that the iteration step does not necessarily has to improve the dual cost in each iteration for all values of the stepsize. Thus, for some iterations the dual cost may be

$$q\big(\pi^{k+1}\big) < q\big(\pi^k\big), \;\; \forall s > 0. \tag{2.5}$$

What actually is improving in each iteration step, if the chosen stepsize is sufficiently small, is the distance of the current dual solution vector π^k to its optimum π^*:

$$\left\| \pi^{k+1} - \pi^* \right\| < \left\| \pi^k - \pi^* \right\|. \tag{2.6}$$

Bertsekas [1999] shows that inequality 2.6 is true for all stepsizes s^k such that

$$0 < s^k < \frac{2\,\left(q\left(\pi^*\right) - q\left(\pi^k\right)\right)}{\|g^k\|^2}. \tag{2.7}$$

In practice, unfortunately, the optimal dual cost $q(\pi^*)$ are usually unknown. Therefore, in most cases the stepsize has to be approximated, e.g. by the updating formula proposed by Fisher [1979]:

$$s^k = \frac{\alpha^k\,\left(f^k - \hat{q}^k\right)}{\|g^k\|^2}, \tag{2.8}$$

where $0 < \alpha^k < 2$, f^k is the best known upper bound of the optimal cost, $\hat{q}^k$ is the best known value of the dual cost

$$\hat{q}^k = \max_{0<i<k} \left(q(\pi^i) \right) \tag{2.9}$$

and g^k is the subgradient of the dual problem. When implementing the algorithm introduced in section 2.3.5, the solution of the restricted masterproblem gives an upper bound on the optimal cost value and can be used as f^k in the formula above. Therefore, the outer loop of the algorithm can as well be understood as updating the upper bound in the stepsize formula for the subgradient optimization procedure. The update formula given above is the one that is used for the optimization procedure in this thesis, but it should be noted that there are numerous different ways of approximating the stepsize.

2.5 Integering step

As explained earlier, to apply the concept of column generation and shadow prices to MILP problems, the masterproblem has to be solved as a linear relaxation of the original MILP. In particular, the weighting variables on the subproblem proposals can and will most likely take non-integer values. That is, having found a sufficiently optimal solution to the LP problem doesn't mean that an integer solution is found yet.

One possible way to obtain integer solutions is to use the same procedure as in branch-and-bound algorithms, i.e. systematically introduce further constraints in a branching tree and solve the linear relaxation of the masterproblem in each node until a solution satisfying the integrality constraints has been found. Solving the linear relaxation of each node of the branch-and-bound tree by means of column generation is called branch-and-price [Desrosiers and Lübbecke, 2011]. Unfortunately, this technique is not suitable for the scheduling problem of heating systems in this thesis, because solving the linear relaxation of one node often takes too much time already. In addition, the convex-hull pricing model would have to be reconsidered.

Hence, there needs to be a final integering step in which an integer solution to the MILP problem is found using the information of the previously solved LP. This can be done, for example, with rounding-heuristics that use the fractional primal solution to obtain a feasible integer solution. Another, more precise method proposed by Belov and Scheithauer [2006] is to implement a final optimization step of the masterproblem using all the proposals obtained in every iteration and declare the weighting factors as integers. This technique is implemented in the conventional column generation algorithm in this thesis. In the implementation of the combined algorithm, a slightly different method is used to obtain integer solutions, because of the convex-hull pricing model. This will be explained in section REF.

3 Optimization model

In this chapter, the original optimization problem is formulated that serves as a basis for the decomposition approaches applied in this thesis. This optimization model can be seen as a straight foward implementation of a centralized coordination technique that gathers all the information needed from the microgrid including the renewable energy sources. Then, with the input data a MILP problem is formulated to compute individual schedules for each registered CHP or HP unit with the goal to minimize the overall cost.

3.1 Microgrid model

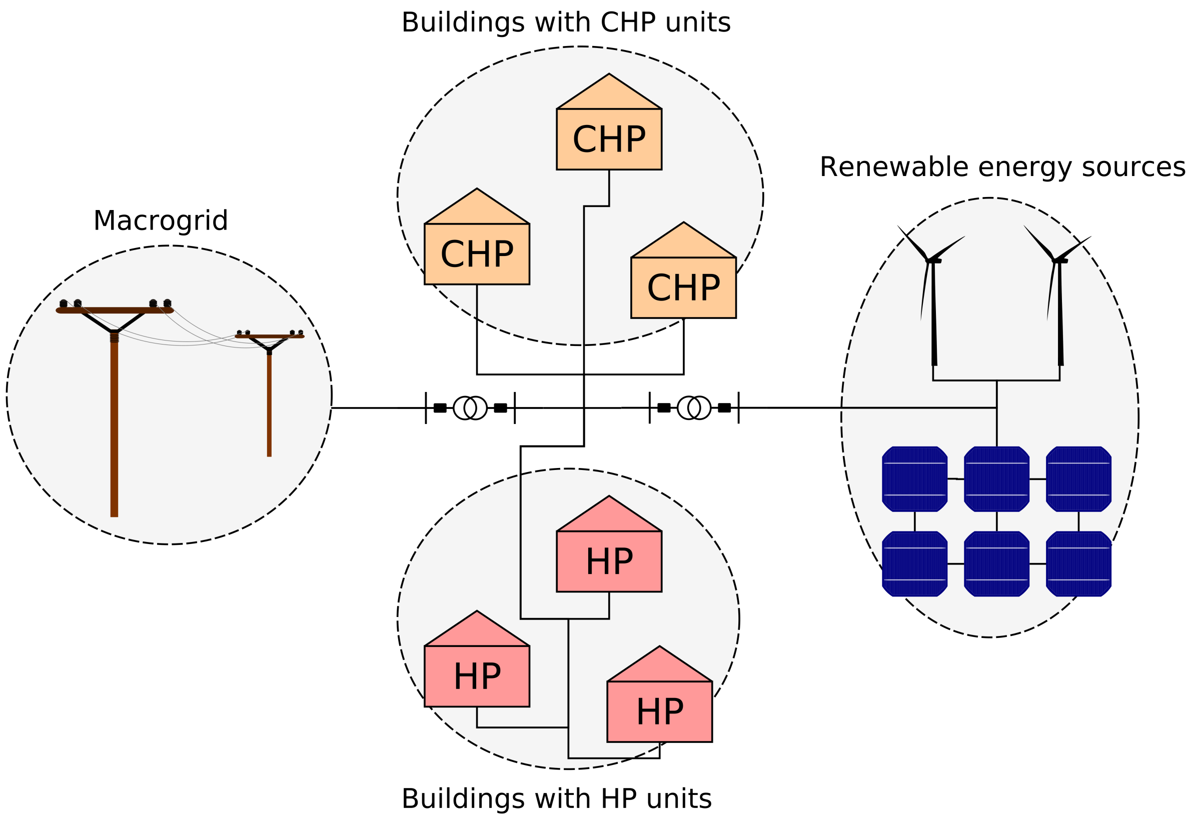

Figure 3.1: Structure of the microgrid

The model of the microgrid examined in this thesis consists of a cluster of 102 buildings and some renewable energy sources. Each of the buildings either has a built-in CHP unit or a HP unit that can be used to satisfy the buildings heat demand and balance the residual load. More precisely, there are 51 buildings with CHP units and 51 with HP units. The heating systems of each building are designed as bivalent heating systems, meaning that each building has a secondary heating device to cover peak loads. To balance the electrical consumption and generation each building is modeled with a buffer storage tank that enables load shifting and a flexible operation of heating supply systems. Figure 3.1 illustrates the simplified model of the microgrid, which is connected to a macrogrid for managing energy shortage or surplus.
In the following sections, the scheduling problem is step by step formulated as a MILP problem in minimization form.

3.2 Objective function

The process of generating feasible schedules for the electro-thermal heating systems is designed as MILP with the objective of minimizing the total costs of the cluster:

$$
\begin{aligned}
z_{MILP} = \min_{P_{hp}, P_{chp}, P_{im}, P_{ex}} \Bigg(& \sum_{t=t_0}^{t_{end}} \left(P_{import}(t) \cdot c_{backup} - P_{export}(t) \cdot c_{grid} \right) \cdot \Delta t \\
& + \sum_{t=t_0}^{t_{end}} \sum_{i=1}^{n_{chp}} \left(\frac{P_{el,i}^{chp}(t) + \dot{Q}_i^{chp}(t)}{\omega_i} + \frac{\dot{Q}_i^{boiler}(t)}{\eta_i^{boiler}} \right) \cdot c_{gas} \Delta t \Bigg).
\end{aligned}
\tag{3.1}
$$

$P_{import}(t)$ and $P_{export}(t)$ represent the imported and exported electrical power from the cluster which are penalized by c_{backup} and remunerated by c_{grid}. They are not allowed to take negative values. $P_{el,i}^{chp}$ is the electricity and Q_i^{chp} is the heat produced by the i'th CHP unit with an over all efficiency of ω_i . Q_i^{boiler} is the heat produced by the auxiliary gas heaters of the i'th CHP unit with an efficiency of η_i^{boiler}.

3.3 Resource constraints

The following electricity balance must be satisfied at all times t

$$
\begin{aligned}
P_{import}(t) + P_{RES}(t) + \sum_{i=1}^{n_{chp}} P_{el,i}^{chp}(t) = & \\
& P_{export}(t) + P_d(t) + \sum_{j=1}^{n_{hp}} \left(P_{el,j}^{hp}(t) + P_{el,j}^{heater}(t) \right) \quad \forall t
\end{aligned}
\tag{3.2}
$$

where $P^{hp}_{el,j}(t)$ is the electricity consumed by a heat pump unit and $P^{heater}_{el,j}(t)$ the electricity consumed by an electrical heater. $P_d(t)$ is the aggregated electrical demand of lights and appliances of participating buildings. $P_{RES}(t)$ represents the electricity available from renewable energy sources. All variables are discrete in time with an interval length of $\Delta t = 15\,\mathrm{min}$.

3.4 CHP subsystem

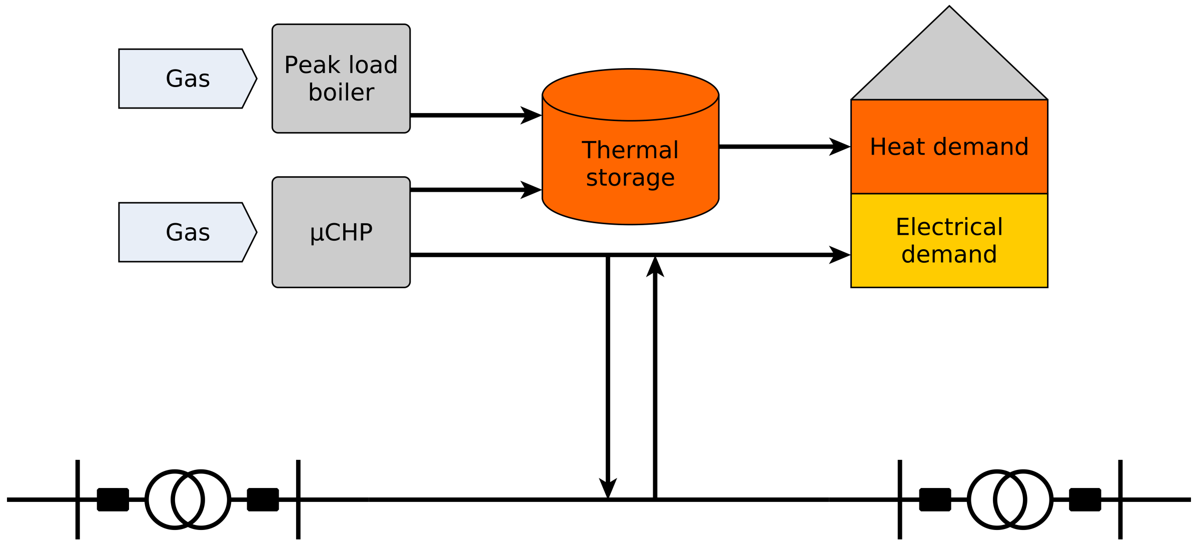

Figure 3.2: Energy system of a house with a CHP unit and a peak load boiler

Figure 3.2 illustrates the simplified model of a building with a bivalent CHP heating unit and a thermal storage tank. Both, CHP unit and boiler produce thermal energy to "charge" the thermal storage tank by burning gas. The buildings heat demand, which has to be satisfied at all times, is supplied by the tank. This kind of heating system setup allows for separating the production and consumption of energy in heat form. Additionally, CHP units produce electrical energy that can be used to satisfy the buildings electrical demand. Any surplus or shortage in electrical energy has to be exported or imported from the microgrid, respectively.
In this thesis, the boiler is designed as an adjustable heating device. Its heat output $Q_i^{boiler}(t)$ can be described by

$$0.3 \cdot y(t) \cdot Q^{boiler}_{nom,i} \leq \dot{Q}^{boiler}_i(t) \leq y(t) \cdot Q^{boiler}_{nom,i} \tag{3.3}$$

where $Q^{boiler}_{nom,i}$ denotes the maximal heat output and $y(t)$ is binary variable describing the working status of the boiler.
Furthermore, the simplified, linearized storage equations have to be satisfied for each discrete time

step:

$$m_{sto,i} \cdot c_w \cdot \frac{T_{t,i} - T_{t-1,i}}{\Delta t} = x_i(t) \cdot \dot{Q}^{chp}_{nom,i} + \dot{Q}^{boiler}_i(t) - \dot{Q}_{dem,i}(t) - \dot{Q}_{loss,i}(T_{t,i}) \quad \forall t \tag{3.4}$$

where $m_{sto,i}$ is the mass of the water in the storage tank and c_w its specific heat capacity, T is the temperature of the water in the tank and $x(t)$ is a binary variable describing the working status of the CHP unit. $Q^{chp}_{nom,i}(t)$ denotes the nominal heat output of the CHP unit, $Q_{dem,i}(t)$ is the heat demand of the whole building and $Q_{loss,i}(T_{t,i})$ is the heat loss of the storage tank modeled as a linear function of its temperature. Its capacity is limited by a maximum temperature T_{max} which is set to 70°C in this thesis. To be able to provide the required heat demand, the storages minimum temperature has be at least equal to the flow temperature T_{flow} of the heating system.
The electrical energy produced by a CHP unit can be expressed through the power coefficient σ_i and the heat output $Q^{chp}_{nom,i}$:

$$P^{chp}_{el,i}(t) = x_i(t) \cdot Q^{chp}_{nom,i} \cdot \sigma_i. \tag{3.5}$$

This relationship can also be used to express the energy consumed by a heat pump in equation 3.1 as a function of the nominal heat output $Q^{chp}_{nom,i}(t)$ with η_i denoting the electrical efficiency of a CHP unit:

$$\frac{P^{chp}_{el,i}(t) + \dot{Q}^{chp}_i(t)}{\omega_i} = \frac{Q^{chp}_{nom,i} \cdot \sigma_i}{\eta_i} \cdot x(t) \tag{3.6}$$

3.5 HP subsystem

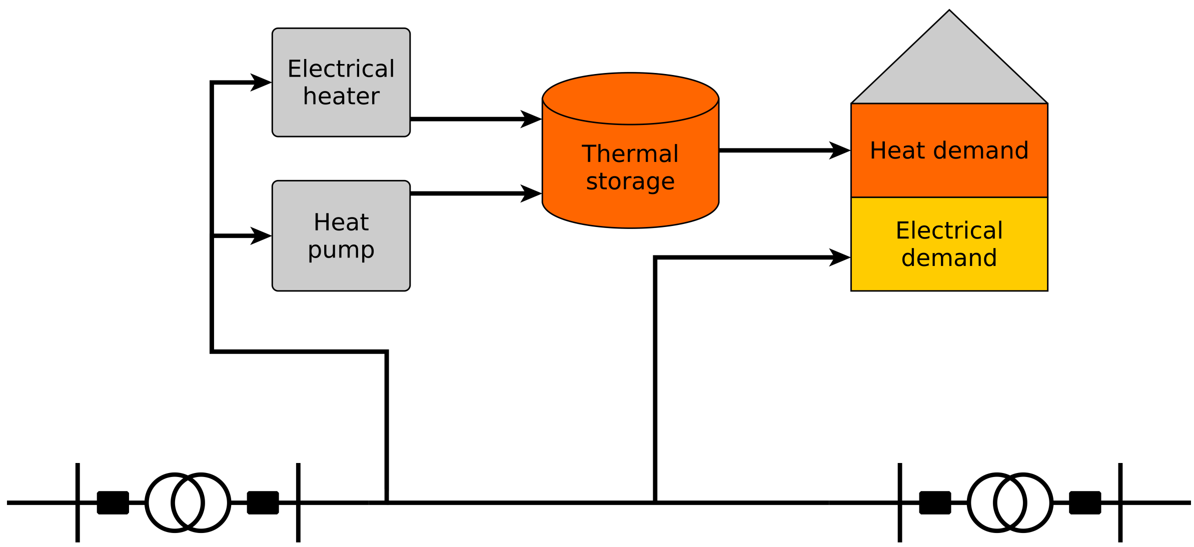

Figure 3.3: Energy system of a house with a heat pump and electrical heater

Similar to the CHP building model, figure 3.3 shows a simplified model of a building with a heat pump system. Instead of a CHP unit and a boiler, a heat pump and an electrical heater are installed charging the thermal storage tank by consuming electrical energy from the microgrid. In contrast to the CHP building model, the electrical energy consumed by HP buildings has to be completely imported from the microgrid. The storage tank equations are very similar to equation 3.4 with Q_j^{heater} denoting the heat output of the secondary, electrical heater and $Q_j^{hp}(t)$ denoting the heat output of the heat pump. The heat output $Q_j^{hp}(t)$ is a function of the ambient temperature and therefore differs over time:

$$m_{sto,j} \cdot c_w \cdot \frac{T_{t,j} - T_{t-1,j}}{\Delta t} = x_j(t) \cdot \dot{Q}_j^{hp}(t) + \dot{Q}_j^{heater}(t) - \dot{Q}_{dem,j}(t) - \dot{Q}_{loss,j}(T_{t,j}) \quad \forall t. \tag{3.7}$$

The temperature limits of the storage tank are set to the same boundaries as the CHP storage tank. When turned on, the electrical energy consumed by a heat pump can be computed beforehand through its coefficient of performance (COP), which is itself a function of the ambient temperature and describes the relation of heat output and electrical consumption. The electrical heater basically directly transforms the electrical energy $P_{el,j}^{heater}(t)$ into the heat output $Q_j^{heater}(t)$ with an efficiency η_j^{heater} set to 1 in this thesis.

3.6 Problem structure

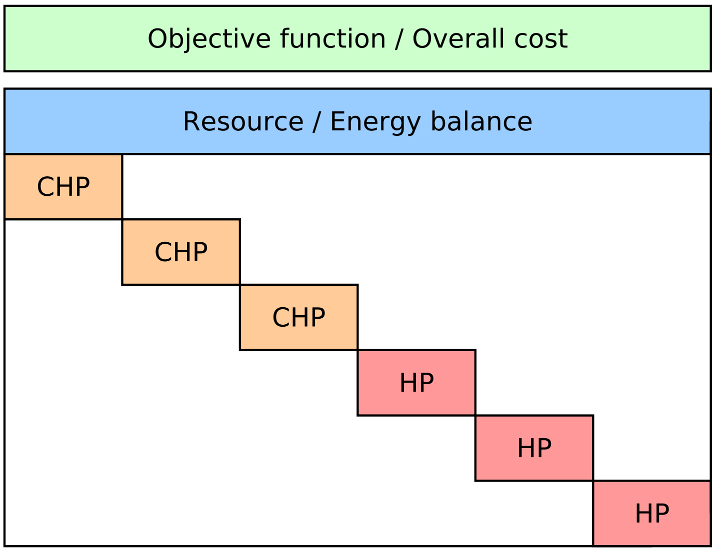

Figure 3.4: Problem structure of the centralized MILP formulation

The optimization problem as formulated above has a special problem structure called block-angular.

Optimization problems with block-angular constraint matrices typically consist of one or several resource constraints that connect the different subproblems and some constraints that only comprise variables from a single subproblem. The resource constraints are sometimes also called complicating constraints, because without these the problem could just be solved by individually optimizing each subproblem and summing up the solutions. In the given optimization problem the energy balance equations can be interpreted as resource constraints and each building with its storage equations as a subproblem. Figure 3.4 illustrates this special kind of problem structure, which can be exploited by the decomposition methods applied in this thesis.

3.7 Why using decomposition methods?

Unfortunately, solving the optimization problem as formulated above causes some problems, both mathematical and technical. First of all the formulated MILP problem is very large in terms of variables and constraints. Solving a scheduling problem for a cluster of 100 buildings over a time period of 2 days to optimality takes too much time for real life applications. Obviously, this gets even worse with a growing number of buildings. Second of all the coordinator in this centralized scheme has to be provided with a certain amount of sensible information which is a drawback concerning the data privacy of the participants.

Therefore, a decentralized coordination scheme is presented, which is based on the decomposition of the original optimization problem into one master problem, that deals with the coupling resource constraint (equation 3.2) and several subproblems that deal with the local constraints of each heating supply system on a building level independently. The subproblems are solved by the local intelligence within the buildings, thus reducing the amount of sensible information that needs to be shared with the coordinator.

4 Decentralized column generation algorithm

To solve the optimization problem introduced in the previous chapter both more efficiently and with a higher emphasis on data privacy, its inherent problem structure is used to separate the building subsystems from the resource constraint. This can be done with the Dantzig-Wolfe decomposition technique, which expresses the subproblem variables through a convex combination of its extreme points. By introducing convexity constraints, this approach ensures that the subproblem constraints are always satisfied without explicitly using them in the masterproblem formulation. The key advantage of this procedure is its applicability to column generation. This is very helpful because of the large number of extreme points of each subproblem. By using a column generation algorithm, the Dantzig-Wolfe masterproblem can be restricted to a subset of all subproblem extreme points. Then, new subproblems are formulated searching for extreme points with the highest potential to improve the current overall solution and add them iteratively to the restricted masterproblem. These subproblems are called pricing problems and compute the extreme points with the best reduced cost.
This chapter introduces a reformulated optimization problem (see chapter 3) by applying the Dantzig-Wolfe decomposition method and shows how feasible schedules are computed with a column generation algorithm.

4.1 Masterproblem

As discussed, the reformulated Dantzig-Wolfe masterproblem gathers all the subsystem proposals and weights them to minimize the overall cost with respect to the resource constraints. In each iteration, every subsystem $i = 1, \ldots, (n_{buildings} = n_{chp} + n_{hp})$ computes a single proposal containing the consumed or produced electricity $P^p_{el,i}(t)$ for every timestep of a subsystem i and the corresponding costs c^p_i. For this thesis, buildings with HP units are modeled as systems without any non-electricity related cost. Therefore, the corresponding cost c^p_i of each HP subsystem can be set to zero. A negative value for $P^p_{el,i}(t)$ indicates electricity consumption and a positive value indicates electricity production. The masterproblem is a linear relaxation of the original MILP, because of the problems arising from column generation applications described in chapter 2. With $P_{import}(t)$ and $P_{export}(t)$

describing the exchange of electricity with a macrogrid, the new formulated objective is

$$z_{LRDW} = \min_{0\le\lambda_i^p\le 1, P_{im}, P_{ex}} \left(\sum_{t=t_0}^{t_{end}} \left(P_{import}(t)\cdot c_{backup} - P_{export}(t)\cdot c_{grid} \right)\cdot \Delta t + \sum_{i=1}^{n_{buildings}} \left(\sum_{p\in P} c_i^p \lambda_i^p \right) \right). \quad (4.1)$$

The penalization of slack variables in the objective function is a stabilizing technique for column generation known as boxstep method [Amor et al., 2009]. This allows for accelerating the convergence of the algorithm as well as avoiding the deterioration of the solution. Moreover, c_{backup} and c_{grid} also represent the price boundaries of the inner market of the microgrid, as will be shown later in detail (see chapter 5). The price range in between these boundaries is used to coordinate the heating units. Therefore, the values of the imported and exported electricity always need to be non-negative:

$$P_{import}(t), P_{export}(t) \ge 0 \quad \forall t \quad (4.2)$$

The resource constraints of the centralized coordination scheme (equation 3.2) then reduce to

$$\sum_{i=1}^{n_{buildings}} \left(\sum_{p\in P} P_{el,i}^p(t)\cdot\lambda_i^p \right) + P_{RES}(t) + P_{import}(t) - P_{export}(t) = 0 \quad \forall t. \quad (4.3)$$

Goal of the Dantzig-Wolfe masterproblem is to weight every proposal from every subsystem to minimize the overall cost. Hence, there exist a continuous variable λ_i^p for each proposal p of a building subsystem i. The set of proposals of a subsystem is defined as P. To make sure that the overall solution of the masterproblem is also feasible for every subsystem, convexity constraints are introduced:

$$\sum_{p\in P} \lambda_i^p = 1 \quad \forall i \quad (4.4)$$

$$0 \le \lambda_i^p \le 1 \quad \forall i, p\in P \quad (4.5)$$

4.2 Pricing subproblems

When the linear restricted masterproblem above is solved using modern software like Gurobi or CPLEX, dual solutions are usually automatically determined by simplex algorithms. The optimal dual solutions of a LP can also be interpreted as shadow prices (or marginal costs) $\pi(t)$ of the resource constraints (equation 4.3) and σ_i of the convexity constraint (equation 4.4). The shadow price of a constraint is the change in the value of the objective function when increasing the right hand side of the constraint by one unit. Therefore, the shadow price $\pi(t)$ is a vector with 96 entries[1] and is interpreted as the current price for exchanging electricity within the microgrid. The values

[1] The schedule is determined for a period of time of 24 hours with a time resolution of $\Delta t = 15min$. This results in 96 discrete time steps.

of $\pi(t)$ are bounded by the low price for exports c_{grid} and the high price or imports that were introduced in the objective function. Basically, these prices are incentives for the subsystems to shift their operation into the most attractive time periods. The shadow prices σ_i are scalar values that are individual for each subsystem i and are interpreted as the marginal costs for each subsystem.
The process of finding new proposals to add to the restricted masterproblem is called pricing and can as well be formulated as an optimization problem. In contrast to the linear masterproblem, these pricing subproblems can be solved as a MILP. The potential to improve the linear solution of the masterproblem can be evaluated by computing the reduced cost of a subproblem proposal (see section 2.3.2). Since the goal of the masterproblem is to minimize the overall cost, the pricing subproblems are searching for proposals with negative reduced cost. Basically, the reduced cost compute the change in the objective value when marginally introducing a new proposal to the solution of the masterproblem. If the optimal solution of a subproblem is non-negative, it has no potential to improve the overall solution. If all subproblems return non-negative solutions, there are no proposals that have potential to improve the overall solution. Therefore, the optimal solution of the restricted masterproblem is the also the optimal solution of the original MILP.

4.2.1 CHP subproblems

Equation 4.6 shows the objective function of a CHP subproblem. The non-electricity related costs of a proposal p are condensed in a single cost factor c_i^p and the vector $P_{el,i}^p(t)$ comprises the overall electricity exchange of a building with the microgrid, where $P_{d,i}(t)$ denotes the demand of appliances and lighting of a building i and $P_{el,i}^{chp}$ the electricity produced by a CHP unit. $\pi(t)$ and σ_i are the shadow prices derived from the solution of the masterproblem of the current iteration.

$$\begin{aligned} z_{PP,i}^{chp}(\pi) = \min_{P_{chp}} \Bigg(& \underbrace{\sum_{t=t_0}^{t_{end}} \left(\frac{P_{el,i}^{chp}(t) + \dot{Q}_i^{chp}(t)}{\omega_i} + \frac{\dot{Q}_i^{boiler}(t)}{\eta_i^{boiler}} \right) \cdot c_{gas} \Delta t}_{=c_i^p} \\ & + \sum_{t=t_0}^{t_{end}} \underbrace{\left(P_{d,i}(t) - P_{el,i}^{chp}(t) \right)}_{=-P_{el,i}^p(t)} \cdot \pi(t) \Bigg) - \sigma_i \end{aligned} \tag{4.6}$$

In order to obtain a feasible subsystem proposal, the constraints of a CHP building discussed in section 3.4 have to be introduced to the pricing subproblem as well to satisfy the local thermal demand and device specific restrictions.

4.2.2 HP subproblems

Buildings with HP units use electricity from the microgrid to power the heating devices and therefore don't cause any non-electricity related costs in this thesis. The price factor c_j^p can be set to zero for each proposal of a HP subsystem j. Thus, the reduced cost in equation 4.7 only comprise of the cost of interacting with the microgrid, while satisfying the subsystem constraints of section 3.5. The total consumed electricity is the sum of the buildings electricity demand $P_{d,j}(t)$, the consumed electricity of the heat pump $P_{el,j}^{hp}(t)$ and consumed electricity of the heater $P_{el,j}^{heater}(t)$.

$$z_{PP,j}^{hp}(\pi) = \min_{P_{hp}} \left(\sum_{t=t_0}^{t_{end}} \underbrace{\left(P_{el,j}^{hp}(t) + P_{el,j}^{heater}(t) + P_{d,j}(t) \right)}_{=-P_{el,j}^{p}(t)} \cdot \pi(t) \right) - \sigma_i \tag{4.7}$$

4.3 Column generation algorithm

Figure 4.1 gives an overview on the column generation algorithm from a coordinator agents perspective. In the beginning, the masterproblem needs to be initialized with a first set of proposals to start the iterative column generation procedure. This can be done by solving the subproblems with an arbitrary shadow price vector to return a set of feasible proposals. In this first step the shadow prices don't have to satisfy the dual feasibility conditions, therefore they can be set to zero to get solutions quickly. These initial proposals will be removed after the first iteration of the algorithm, because as random proposals they are not expected to contribute to the optimal solution in the end. Then, the initial masterproblem is solved using the initial proposals to get a feasible shadow price vector.

With this first shadow price vector the iterative column generation procedure is started. In each iteration, the shadow prices are sent to all buildings to compute new proposals by solving the individual pricing subproblems. Afterwards, the computed proposals are added to the basis of the masterproblem, which is then solved again to attain new shadow prices. To terminate the column generation procedure, a termination criterion is checked in each iteration, e.g. the optimality conditions of the subproblems (each $z_{PP,i} \geq 0$), a time limit or a minimum change in the value of the masterproblems objective function. In this thesis, a time limit is set to $t_{max} = 10\,\text{min}$ and an optimality condition is checked. A minimum change criterion is not implemented in this thesis to be able to evaluate and compare the convergence of the algorithm with the proposed new algorithm.

After the column generation procedure has been terminated, the coordinator has to provide feasible integer solutions for each building. Thereby, the weighting factors λ_i^p are declared as integer variables in the final step and the resulting indices of the chosen subproblem proposals are sent to the corresponding local intelligence of the individual buildings (see also section 2.5).

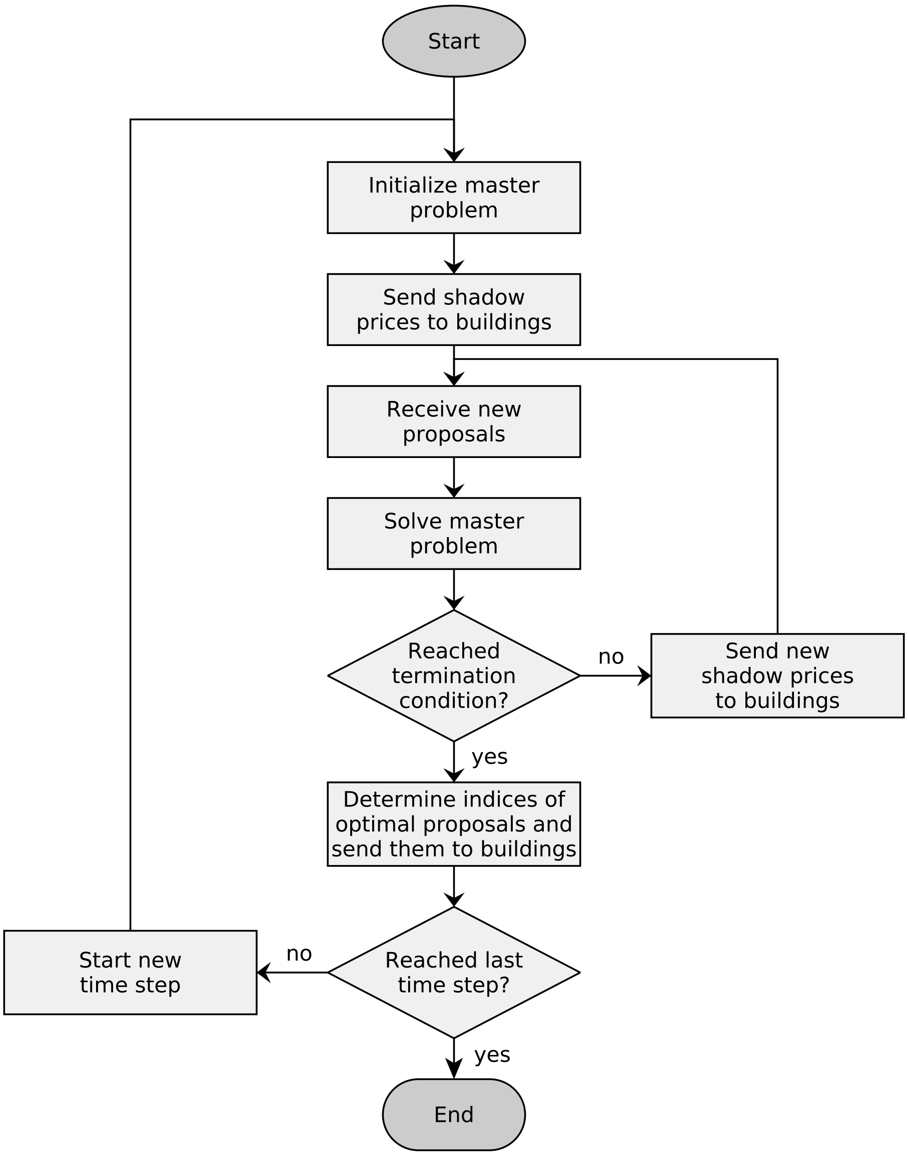

Figure 4.1: Decentralized column generation algorithm from a coordinators perspective

4.4 Discussion

Although the centralized coordination scheme discussed in chapter 3 provides quite good solutions for a small number of buildings, the computational time solving the problem rises exponentially with an increasing number of participants. This is obviously a drawback in terms of scalability and extendability, because solving the scheduling problem for a large number of buildings might be intractable for conventional solvers in a reasonable amount of time. Furthermore, the central coordinator has to be provided with some sensible information regarding the participants energy consumption.

The decentralized column generation algorithm discussed in this chapter exploits the block-angular structure of the constraint matrix (see figure 3.4) to separate the easy-to-solve subsystems from the complicating resource constraints. This coordination scheme reduces the computational effort for solving large-scale optimization problems, because the Dantzig-Wolfe masterproblem can be restricted to a selection of variables and only those potentially improving the overall solution are added in each iteration. Thus, the decentralized column generation approach shows significant advantages regarding stability, scalability and extendability, since no modifications to the masterproblem are required when adjusting the number of participants. Figure 4.2 illustrates how the column generation scheme is using the block-angular problem structure (see figure 3.4) to decompose the optimization problem. Moreover, solving the subproblems by a local intelligence leads to a reduction in the amount of sensible information that needs to be shared.

Despite the clear advantages in comparison with the centralized coordination scheme, the decentralized column generation approach unfortunately comes with some drawbacks regarding the convergence of the algorithm and the shadow price principle. When solving large-scale optimization problems with column generation algorithms, the objective value of the masterproblem usually develops a fairly good approximation fast, but then tends to "tail off" by converging very slowly towards its optimal value. This effect can be observed in many different applications using column generation and is often referred to as tailing-off effect. As will be shown later in this thesis, the convergence towards its optimal values is often even worse for the dual solutions of the masterproblem, also discussed as shadow prices. When terminating the algorithm after 10 minutes of computation time, the shadow prices are in many cases far from optimal and would need a lot more iterations to even reach an approximate optimal solution. This is a problem for two reasons: First of all, when computing integer solutions in the final step, the algorithm chooses one distinct proposal from all iterations for each building. These proposals have been computed with the suboptimal dual solutions of each iteration that often fluctuate in a wide range during the course of the algorithm. The coordinator agent now selects proposals for each building that have possibly been computed with fairly different shadow prices, which is an unequal treatment of the participating buildings. Therefore, the convex-hull pricing model is introduced (see section 2.2.2). The convex-hull price of the inner market of the microgrid is the optimal dual solution of the unrestricted linear Dantzig-Wolfe masterproblem developed in this chapter. Unfortunately, the slow convergence of the shadow prices prevent its usage as unique convex-hull prices for generating subproblem proposals, which is the second big disadvantage of the column generation procedure.

Although the masterproblem is restricted to a selection of variables and can be solved very fast in the first iterations, it is growing with the number of proposals in every iteration. While the computational effort to solve the subproblems stays the same in every iteration, the computation time to solve the masterproblem is growing exponentially, which makes the convergence of the algorithm even slower in the end and should be noted as another disadvantage of the procedure.

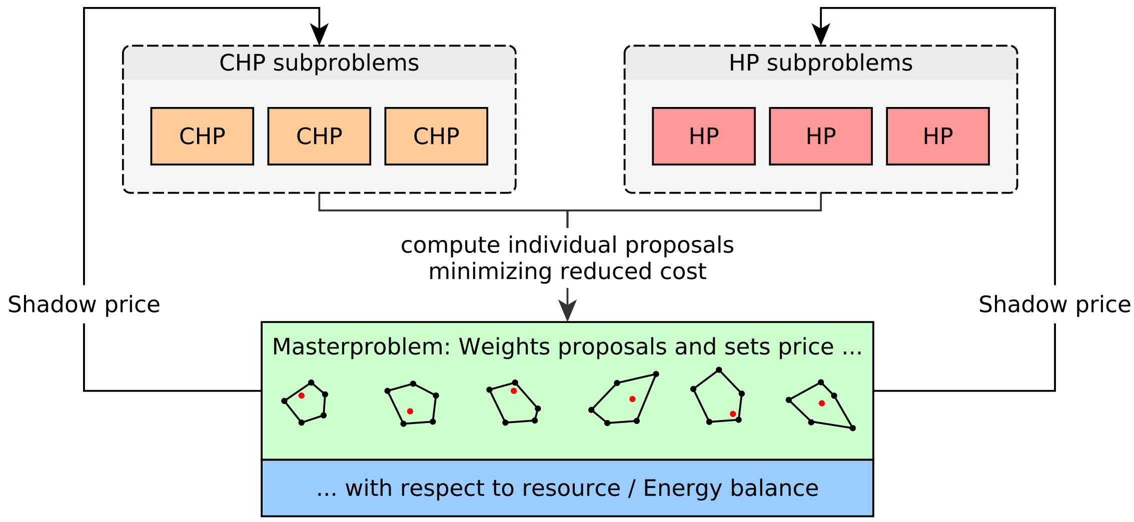

Figure 4.2: Information transfer between the Dantzig-Wolfe masterproblem and several subproblems (compare fig. 2.1 and fig. 3.4)

5 Decentralized combined algorithm

In chapter 2 an algorithm has been discussed that combines two different decomposition approaches to speed up the column generation procedure. In this thesis, the algorithm combining Dantzig-Wolfe decomposition and Lagrangian relaxation in a single column generation procedure is applied to the scheduling problem of heating systems. It is expected to speed up the convergence both of the primal and the dual solutions. To obtain feasible integer solutions for every participant with a unique shadow price vector and avoid the problems discussed in section 4.4, a final integering step is proposed. In this final step, the algorithm uses the approximated dual solution of the column generation procedure as convex-hull price and computes a set of different proposals for each building using the exact same shadow prices.
In this chapter, the Lagrangian relaxation method is applied to the scheduling problem and its subproblems are formulated. After that, the updating procedure using the subgradient optimization method is discussed and in the last part the actual algorithm is explained in detail.

5.1 Lagrangian relaxation

The idea of the Lagrangian relaxation approach is, again, to introduce the complicating resource constraints of optimization problems with block-angular constraint matrices into the objective function and associate Lagrange multipliers that penalize the amount by which the resource constraints are violated. This basic decomposition approach of the presented scheduling problem is illustrated in figure 5.1. When the optimization problem is freed of all comprehensive constraints connecting the subsystems, the structure of the new constraint matrix allows to solve the problem as a sum of several independent subproblems. Each subproblem consists of the building-specific variables and constraints. An additional subproblem comprises all the other variables in the objective function that do not belong to any building subsystem.
The new objective function of the Lagrangian relaxation problem is shown in equation 5.1. It consists of the original objective function of the MILP (see equation 3.1), the resource constraints formulated as deviation (see equation 3.2) and the associated prices $\pi(t)$. The optimal objective value z_{LR} of the new formulated problem is a function of the price vector π, which boundaries will be developed later. By grouping all the variables belonging to a specific building together, the optimization problem can be split into several independent subproblems without any connection to each other. These subproblems will be discussed in the next sections.

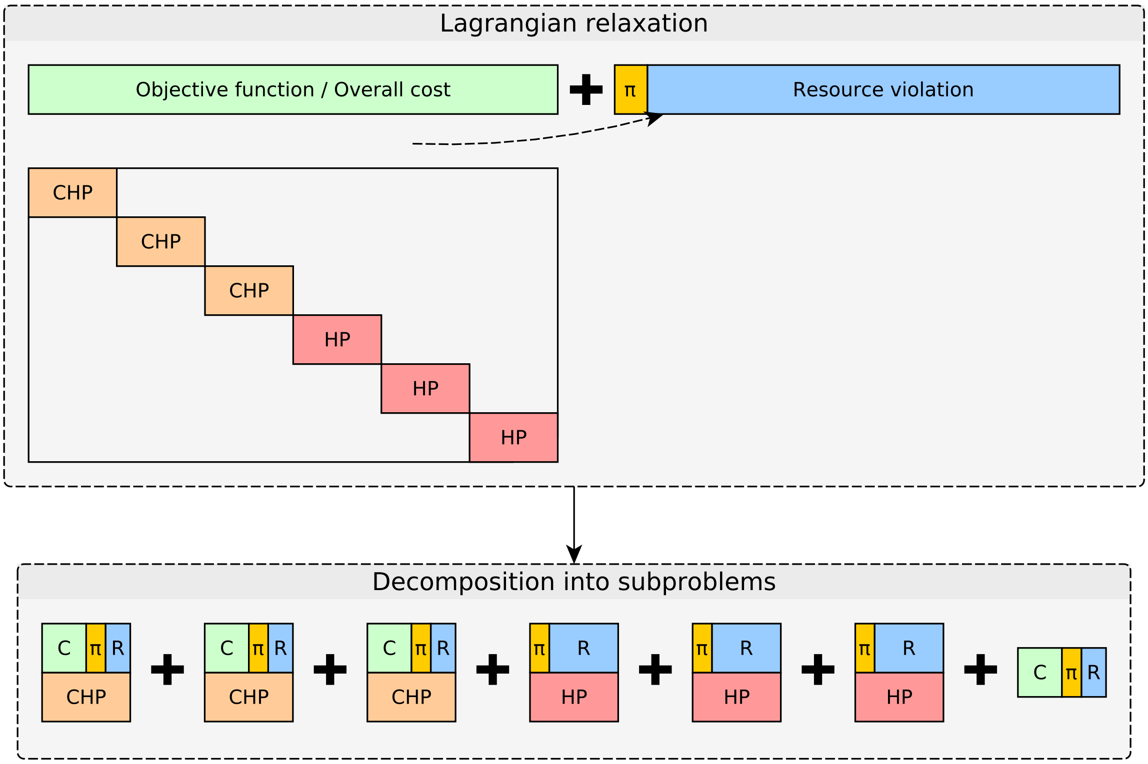

Figure 5.1: Lagrangian relaxation and decomposition of the scheduling problem

$$\begin{aligned} z_{LR}(\pi) = \min_{P_{hp}, P_{chp}, P_{im}, P_{ex}} \Bigg(& \sum_{t=t_0}^{t_{end}} \Big(P_{import}(t) \cdot c_{backup} - P_{export}(t) \cdot c_{grid} \Big) \cdot \Delta t \\ & + \sum_{t=t_0}^{t_{end}} \sum_{i=1}^{n_{chp}} \left(\frac{P_{el,i}^{chp}(t) + \dot{Q}_i^{chp}(t)}{\omega_i} + \frac{\dot{Q}_i^{boiler}(t)}{\eta_i^{boiler}} \right) \cdot c_{gas} \Delta t \\ + \sum_{t=t_0}^{t_{end}} \pi(t) \cdot \Bigg(& \sum_{i=1}^{n_{chp}} P_{d,i}^{chp}(t) + \sum_{j=1}^{n_{hp}} P_{d,j}^{hp}(t) + P_{export}(t) \\ & + \sum_{j=1}^{n_{hp}} \Big(P_{el,j}^{hp}(t) + P_{el,j}^{heater}(t) \Big) - P_{import}(t) - P_{RES}(t) - \sum_{i=1}^{n_{chp}} P_{el,i}^{chp}(t) \Bigg) \Bigg) \end{aligned} \tag{5.1}$$

5.1.1 CHP subproblems

Equation 5.2 shows the subproblems objective function consisting of all CHP-related variables with a part coming from the original objective function and a part from the Lagrangian relaxation of the resource constraints. The constraints of the subproblem are the same as the ones developed in section 3.4. It can easily be seen that this subproblem is mostly the same as the pricing subproblem of the Dantzig-Wolfe/column generation procedure (see section 4.2.1). In fact, the only difference is the missing constant factor σ_i denoting the shadow price of the convexity constraint. Therefore, solving a subproblem of the Lagrangian relaxation problem and solving a pricing subproblem of the column generation procedure is essentially the same. This is a very useful property of the Lagrangian relaxation approach, because its subproblem solutions can as well be used as proposals c_i^p, $P_{el,i}^p(t)$ for the column generation procedure.

$$\begin{aligned} z_{LR,i}^{chp}(\pi) = \min_{P_{chp}} \Bigg(& \underbrace{\sum_{t=t_0}^{t_{end}} \left(\frac{P_{el,i}^{chp}(t) + \dot{Q}_i^{chp}(t)}{\omega_i} + \frac{\dot{Q}_i^{boiler}(t)}{\eta_i^{boiler}} \right) \cdot c_{gas} \Delta t}_{= c_i^p} \\ & + \sum_{t=t_0}^{t_{end}} \underbrace{\left(P_{d,i}(t) - P_{el,i}^{chp}(t) \right)}_{= -P_{el,i}^p(t)} \cdot \pi(t) \Bigg) \end{aligned} \tag{5.2}$$

5.1.2 HP subproblems

The HP-related subproblems of the Lagrangian relaxation formulation exhibit the same similarities with the pricing subproblems (see section 4.2.2) as the CHP-related subproblems. In section 3.5 the constraints associated with HP subsystems have been developed. Equation 5.3 shows the objective function of the subproblem with the missing constant factor and its usage as proposal $P_{el,j}^p(t)$.

$$z_{LR,j}^{hp}(\pi) = \min_{P_{hp}} \left(\sum_{t=t_0}^{t_{end}} \underbrace{\left(P_{el,j}^{hp}(t) + P_{el,j}^{heater}(t) + P_{d,j}(t) \right)}_{= -P_{el,j}^p(t)} \cdot \pi(t) \right) \tag{5.3}$$

5.1.3 Additional subproblem

In addition to the subproblems comprising all the building-specific variables and constraints, there are some variables left in the objective function forming an additional minimizing problem, shown in equation 5.4. In particular, the problem consists of the two slack variables $P_{import}(t)$ and $P_{export}(t)$ and the constant vector $P_{RES}(t)$ denoting the electricity production from renewable energy sources. It is important to remember that $P_{import}(t)$ and $P_{export}(t)$ can't take any negative values.

$$\begin{aligned} z_{LR}^{add}(\pi) = \min_{P_{im}, P_{ex}} \Bigg(& \sum_{t=t_0}^{t_{end}} \left(P_{import}(t) \cdot c_{backup} - P_{export}(t) \cdot c_{grid} \right) \cdot \Delta t \\ & + \sum_{t=t_0}^{t_{end}} \left(P_{export}(t) - P_{import}(t) - P_{RES}(t) \right) \cdot \pi(t) \Bigg) \end{aligned} \tag{5.4}$$

By rearranging the equation above into equation 5.5, it can be seen that there are two possible solutions to this problem. Since $P_{import}(t)$ and $P_{export}(t)$ have to take non-negative values, the solution of the problem depends on the sign of the coefficients. If any coefficient is negative at any timestep, the minimization problem is unbounded and its objective is $-\infty$. If, on the other hand, all coefficients have non-negative values, $P_{import}(t)$ and $P_{export}(t)$ take zero values to minimize the function:

$$z_{LR}^{add}(\pi) = \underbrace{\min_{P_{im},P_{ex}} \left(\sum_{t=t_0}^{t_{end}} P_{import}(t) \cdot \left(c_{backup}\Delta t - \pi(t) \right) + \sum_{t=t_0}^{t_{end}} P_{export}(t) \cdot \left(\pi(t) - c_{grid}\Delta t \right) \right)}_{\begin{cases} 0, & \text{if } c_{grid}\Delta t \le \pi(t) \le c_{backup}\Delta t, \\ -\infty, & \text{otherwise} \end{cases}} - \sum_{t=t_0}^{t_{end}} P_{RES}(t) \cdot \pi(t). \tag{5.5}$$

Since the coefficient values are functions of the Lagrange multipliers or shadow prices, boundaries for them can now be formulated. In fact, these boundaries represent precisely the feasibility conditions of the Lagrangian dual problem (see also section 2.3.3):

$$c_{grid}\Delta t \le \pi(t) \le c_{backup}\Delta t \tag{5.6}$$

With the assumption of shadow prices satisfying condition 5.6, the solution of the additional problem can simply be computed by

$$\Rightarrow \underset{c_g\Delta t \le \pi \le c_b\Delta t}{z_{LR}^{add}(\pi)} = -\sum_{t=t_0}^{t_{end}} P_{RES}(t) \cdot \pi(t). \tag{5.7}$$

5.1.4 Lagrangian relaxation and the Lagrangian dual problem

To obtain the solution of the Lagrangian relaxation problem, the solutions of the independently solved subproblems simply have to be added up:

$$\underset{c_g\Delta t \le \pi \le c_b\Delta t}{z_{LR}(\pi)} = \sum_{i=1}^{n_{chp}} z_{LR,i}^{chp}(\pi) + \sum_{j=1}^{n_{hp}} z_{LR,j}^{hp}(\pi) - \sum_{t=t_0}^{t_{end}} P_{RES}(t) \cdot \pi(t), \tag{5.8}$$

where π is a shadow price vector satisfying the feasibility conditions (equation 5.6). The solution of the Lagrangian relaxation problem is a function of the shadow price vector π and it can be shown that its objective value is a lower bound on the objective value of the original optimization problem . Finding the tightest such bound can be formulated as a new optimization problem and is called the Lagrangian dual problem, which is in fact the dual problem of the linear Dantzig-Wolfe masterproblem (see section 2.3.4):

$$z_{LD} = \max_{c_g \Delta t \leq \pi \leq c_b \Delta t} \left(z_{LR}(\pi) \right) \tag{5.9}$$

Therefore, the optimal objective value of the Lagrangian dual problem z_{LD} is exactly the same as the optimal objective value of the linear Dantzig-Wolfe masterproblem z_{LDW}. Unfortunately, optimizing the Lagrangian dual problem is not tractable by standard linear optimization methods, because its objective function is non-differentiable. However, there are some methods for solving non-linear optimization problems like this and in this thesis a technique called subgradient optimization is used.

5.2 Subgradient optimization

In section 2.4 a short introduction was given to the theory of subgradient optimization. The method essentially uses the convex structure of the objective function to update the dual variables iteratively and approximate the optimal dual solution. The relaxed resource constraint in the objective function of the problem can be interpreted as a sugradient, which is a generalization of a gradient. The subgradient optimization procedure basically multiplies this subgradient g^k of a given iteration k by a certain stepsize s^k and adds it to the current dual variable π^k. For sufficiently small stepsizes, this procedure eventually converges towards the optimal dual solution. When the updated dual variable exceeds the dual boundaries, the value is set to its limits. This basic updating procedure is expressed by the following equation:

$$\pi_t^{k+1} = \begin{cases} c_{grid}\,\Delta t, & \text{if} \quad \pi_t^k + s^k g_t^k \leq c_{grid}\,\Delta t \\ \pi_t^k + s^k g_t^k, & \text{if} \quad c_{grid}\,\Delta t \leq \pi_t^k + s^k g_t^k \leq c_{backup}\,\Delta t \,. \\ c_{backup}\,\Delta t, & \text{if} \quad \pi_t^k + s^k g_t^k \geq c_{backup}\,\Delta t \end{cases} \tag{5.10}$$

Equation 5.11 shows the subgradient g^k of the Lagrangian dual problem. With the assumptions made in section 5.1.3, the subgradient can be expressed without the slack variables and represents the residual load of the microgrid when using all the schedules proposed in a single iteration. It can be computed by a coordinator using the proposals from each building, which is an advantage in terms of the information transfered.

$$g_t^k = \underbrace{\sum_{i=1}^{n_{chp}} P_{d,i}^{chp}(t) - \sum_{i=1}^{n_{chp}} P_{el,i}^{chp,k}(t)}_{= -\sum_{i=1}^{n_{chp}} P_{el,i}^{p}(t)} + \underbrace{\sum_{j=1}^{n_{hp}} P_{d,j}^{hp}(t) + \sum_{j=1}^{n_{hp}} \left(P_{el,j}^{hp,k}(t) + P_{el,j}^{heater,k}(t) \right)}_{= -\sum_{j=1}^{n_{hp}} P_{el,j}^{p}(t)} - P_{RES}(t) \tag{5.11}$$

As discussed in section 2.4, it can be shown that for sufficiently small stepsizes s^k the Euclidian distance between any dual solution π and the optimal dual solution π^* is always decreasing after applying the updating procedure. Unfortunately, in most cases the proper stepsize is unknown and can only be estimated. In the literature, many stepsize formulas can be found for the updating procedure of the dual variables. In this thesis, a stepsize formula is used that has originally been proposed by Fisher [1979]. During the algorithm that is presented in the next section, it uses both the best found primal solution and the best found dual solution of the problem to estimate the stepsize by

$$s^k = \alpha \cdot \frac{z_{RDW} - z_{LR,max}}{\sum_{t=t_0}^{t_{end}} (g_t^k)^2}, \tag{5.12}$$

where z_{RDW} is the current primal solution and $z_{LR,max}$ is the best dual solution. While the objective value of the restricted Dantzig-Wolfe masterproblem is always improving with each iteration, the solution of the dual problem can also deteriorate sometimes when using the subgradient optimization method. Hence, the lower bound used in the stepsize formula is always the best bound found yet:

$$z_{LR,max} = \max_{0<i\leq k} \left\{ z_{LR}(\pi^i) \right\} \tag{5.13}$$

The factor α is usually set to a constant value satisfying the following condition, which is derived from the stepsize formula in section 2.4.

$$0 \leq \alpha \leq 2 \tag{5.14}$$

In the implemented algorithm, after testing the α value was set to the constant value of 1.

5.3 Combined algorithm

5.3.1 General idea

For speeding up the column generation procedure, an algorithm was discussed in section 2.3.5 that combines the Dantzig-Wolfe column generation approach presented in chapter 4 with the subgradient optimization of the Lagrangian dual problem. This is possible because of the strong relationship between the two problems. First of all, the Lagrangian dual problem is equivalent to the dual problem of the linear Dantzig-Wolfe masterproblem and, second of all, their subproblems are nearly the same. Therefore, the shadow prices of the Dantzig-Wolfe masterproblem correspond to the dual variables of the Lagrangian dual problem.

In the previous column generation procedure, the subproblems compute proposals by receiving shadow prices from the solution of the restricted masterproblem. Afterwards, these proposals are added to the basis of the masterproblem, which is then solved again to attain updated shadow prices. Section 4.4 presented some of the disadvantages of this procedure, which is a computa-

tionally expensive masterproblem and slowly converging primal and dual solutions. The combined algorithm approach instead uses the subgradient optimization procedure to update the shadow prices. Thereby, the dual solutions are expected to converge faster and the optimization procedure is computationally less expensive. Unfortunately, the subgradient optimization procedure comes with some convergence problems too. In particular, it is often difficult to estimate the proper stepsize for the updating procedure, especially if the gap to the optimal dual solution gets smaller. Therefore, the subgradient optimization procedure is stopped after a fixed number of iterations to compute a new primal solution that is itself used to update the stepsize again.
Of course, this method doesn't solve the problem of non-unique shadow prices in the column generation procedure. So far, it is only expected to speed up the column generation algorithm and hopefully deliver better results for the primal and dual solutions.
Therefore, in this thesis a method is proposed that separates the optimization of the shadow prices and the generation of feasible schedules for the participating buildings. In the first step, the combined algorithm is used to approximate the optimal shadow prices, which can be interpreted as convex-hull price (see section 2.2.2). In a second step, the approximated convex-hull price is fixed and a set of different proposals is computed for each building. The idea behind this approach is based on an important property of reduced cost. If the column generation algorithm has found the optimal convex-hull solution of the MILP problem, there is, by definition, no new subproblem proposal that could possibly further improve the solution. This means that there is no subproblem returning a negative solution or in other words no proposal with negative reduced cost. At this point, every subproblem proposal that doesn't contribute to the linear solution of the masterproblem has a positive reduced cost value. More importantly, all the subproblem proposals that do actually contribute to the solution of the linear masterproblem, have a reduced cost value of zero. That is, with optimal shadow prices there are multiple optimal solutions for the subproblems, all returning the same objective value. The proposed algorithm tries to find these subproblem solutions, also referred to as basic columns, to obtain a set of feasible proposals for the final integering step. In this final step, the Dantzig-Wolfe masterproblem is solved again using only the proposals computed with the convex-hull price and declaring the weighting variables as binary variables.

5.3.2 Implementation

The algorithm proposed in this thesis, is illustrated in figure 5.2. The procedure can be divided into three different parts that will be called Initialization, Pricing Step and Integering step. In the first part of the algorithm, initial proposals and shadow prices are computed that can be used in the first iteration of the pricing procedure. The second part of the algorithm applies the combined column generation and Lagrangian relaxation method to approximate the optimal dual solution, also known as convex-hull price and in the last part this shadow price vector is used to generate feasible schedules for every participating building.

Initialization

The initialization procedure starts with generating random shadow prices satisfying the dual feasibility conditions, which are then used to compute proposals for every building. These random proposals are added to the basis of the masterproblem, but only for the first three iterations. When solving the masterproblem, these random proposals seem to substantially improve the solution of the masterproblem during the first iterations, which is useful for estimating the proper stepsize for the subgradient optimization procedure. Therefore, in the third step of the algorithm the masterproblem consisting of the random proposals is solved and its solution is used in the stepsize formula. To obtain an initial value for the Lagrangian lower bound used in the stepsize formula, the shadow prices coming from the solution of the initial masterproblem are used to compute proposals for every building again. A lower bound is then computed using the solutions from these subproblems. Of course, the lower bound could also be computed by using the random subproblem solutions computed in the beginning, but its value is usually inferior. It is important to generate good initial solutions of z_{RDW} and z_{LR} to get a good estimation of the proper stepsize for the subgradient method and accelerate the convergence of the procedure.

Pricing step

The Pricing step of the procedure is a simplification of the algorithm presented in section 2.3.5 consisting of a nested double loop. In the inner loop, a fixed number of subgradient optimization steps is applied to generate new proposals and update z_{LR} in the stepsize formula iteratively. In the outer loop, the Dantzig-Wolfe masterproblem is solved using all generated proposals to obtain a primal solution z_{RDW} that can be used again to update the stepsize. The reason why the Dantzig-Wolfe masterproblem has to be solved at all is the convergence of the subgradient method. Finding the proper stepsize seems to be a critical point in the subgradient optimization procedure and improving the value of z_{RDW} tightens the stepsize. Therefore, the outer loop can be understood as a way to adjust the stepsize when the subgradient optimization doesn't improve z_{LR} anymore. The procedure starts with updating the shadow prices obtained by solving the masterproblem during the initialization procedure. Recall that in the column generation procedure the primal solution usually improves very fast in the first few iterations and tends to "tail-off" then (see section 2.3.4). This effect is even intensified with the growing computational effort of the masterproblem. Therefore, the proposed algorithm has been implemented with a variable number of iterations in the inner loop to exploit the fast improvement of the primal solution in the beginning of the column generation procedure. Again, computing good primal solutions helps to estimate a proper stepsize value. In particular, at the beginning the inner loop is stopped after a single iteration for the first three iterations of the outer loop. Then, the number of iterations of the inner loop is increased by one with every outer loop iteration.

The combined column generation algorithm constantly computes upper bounds (z_{RDW}) and lower bounds (z_{LR}) on the optimal objective z_{LDW} of the linear Dantzig-Wolfe masterproblem. Another great advantage of the procedure is that theses bounds can be used to evaluate the quality of the current solution and terminate the procedure if a sufficiently optimal solution has been found. Hence, the algorithm is terminated, if the difference between z_{RDW} and z_{LR} falls below a limit ϵ. In this thesis, ϵ is set to zero. The subproblem solutions are usually approximate solutions that are actually slightly larger than the optimal subproblem solutions. Unfortunately, the computed lower bound inherits this inaccuracy. Moreover, the solution of the Dantzig-Wolfe masterproblem is an approximate solution as well. Therefore, a termination criterion ϵ with a zero value doesn't necessarily mean that an optimal solution has been found. In addition, a time criterion is introduced that terminates the Pricing step after a timespan of 10 minutes.

Integering step

In the last step of the algorithm, the Integering step, the last computed shadow price is fixed and used to compute several different proposals. Optimally, these computed proposals represent the basic columns of the optimal solution of the linear Dantzig-Wolfe masterproblem. Thereby, the subproblems are solved multiple times and each subproblem solution is added to the basis of a new Dantzig-Wolfe masterproblem. To obtain differing solutions, in each iteration a constraint is added to the subproblems that excludes the previous solution from the solution space of the subproblem. If x and y are the binary vectors of the primary and secondary heating devices of a building, respectively, then A represents the set of all timesteps t where x_t is one and C represents the set of all timesteps t where y_t is one. B and D, however, represent the set of all timesteps where x and y are zero. The constraint excluding a particular solution (x, y) from the subproblem solution space can then be expressed by

$$\sum_{t\in A} x_t - \sum_{t\in B} x_t + \sum_{t\in C} y_t - \sum_{t\in D} y_t < \sum_{t\in A} 1 + \sum_{t\in C} 1. \tag{5.15}$$

In this thesis, five different proposals are computed in this procedure. The final masterproblem is then solved with weighting variables declared as binary variables to choose one proposal from the final set for each building.

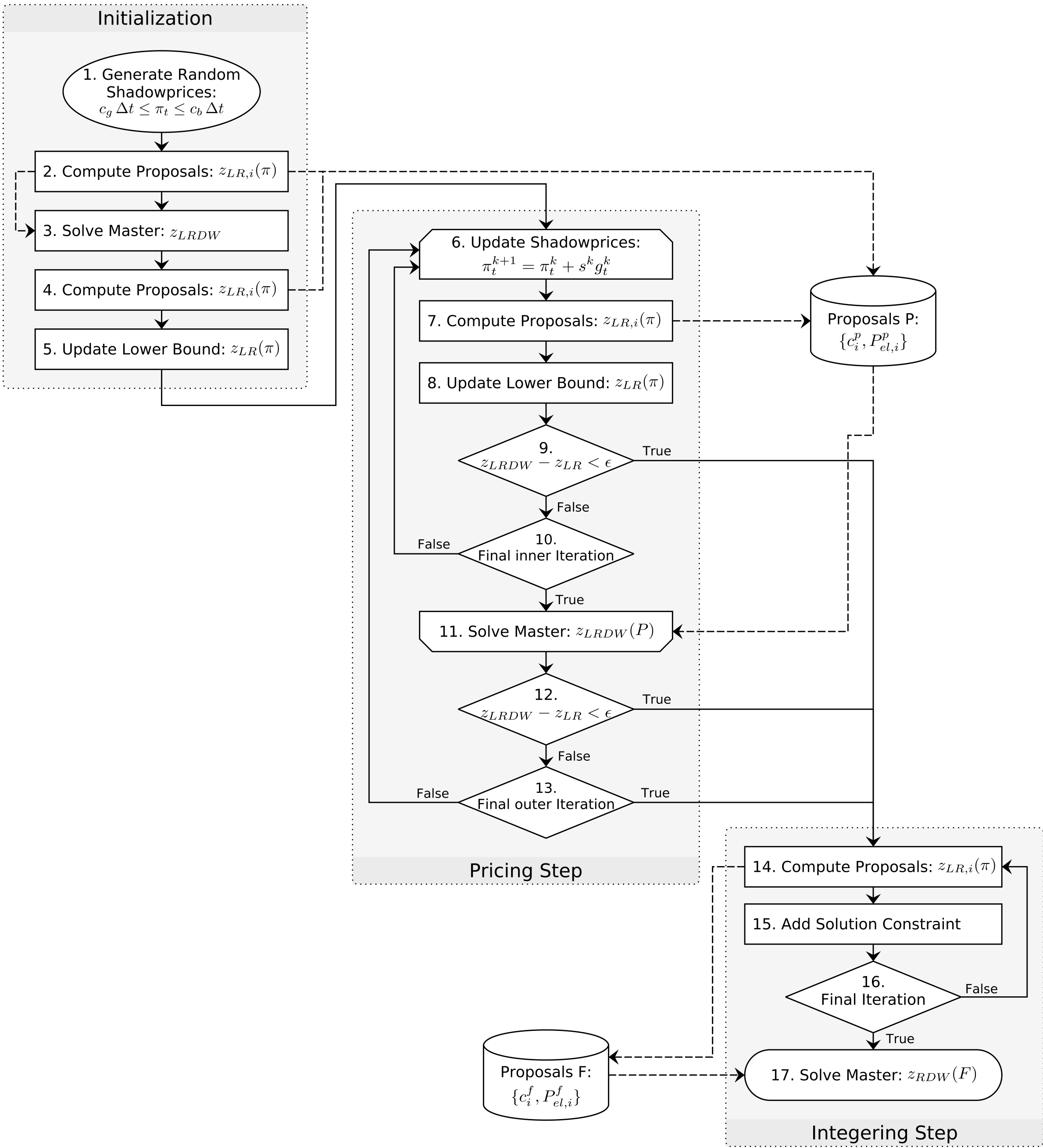

Figure 5.2: Proposed algorithm combining column generation and Lagrangian relaxation to solve the scheduling problem of heating systems.

6 Results and discussion

In this thesis, a conventional column generation algorithm presented in chapter 4 and the proposed combined algorithm discussed in chapter 5 have both been used to generate heating supply system schedules for each day over the course of a whole year. Both schemes are implemented using Python[1] and the commercial optimization software Gurobi[2] under a free academic license and tested on a personal Linux[3] machine with a quad-core CPU[4].

To compute schedules for a whole year, the optimization problem is formulated for a time period of 48 hours starting with the first two days of the simulated year. The thermal storage tanks are initialized with a predefined initial temperature of 50°C for this first scheduling problem. Then, after feasible schedules have been computed by the algorithm the storage tank temperature at the end of the first day is used as initial temperature for the next scheduling problem. Now, a new 48 hour problem can be formulated starting from the second day of the year, and so on. This approach ensures that the algorithm does not fully discharge the storage tank at the end of the first scheduled day, which would be detrimental with regard to the cost of a whole year. When solving optimization problems with the Gurobi software, a MIP gap is set to 1%. The scheduling procedure starts with the first day of January.

This chapter starts with a short discussion of the optimization model setup. Afterwards, both algorithms are compared with a focus on all scheduled days of the year. To investigate the convergence of both algorithms and the quality of the proposed Integering step solutions, the last part of the chapter focuses on the results of a single scheduled day.

To distinguish the solutions of both algorithms from each other, results from the column generation algorithm are labeled with a "CG" prefix and results from the combined algorithm with "LR". Furthermore, the linear masterproblem solution of the combined algorithm Pricing step serves as a benchmark, because its value z_{LRDW} seems to be very close to the optimal value z_{LDW}. This benchmark z_{LRDW} can be interpreted as the solution of a continuously variable heating supply system.

[1] Python version: 2.7.6 (www.python.org)
[2] Gurobi version: 6.0.0 (www.gurobi.com)
[3] Linux Kernel-version: 3.13.0-46-generic
[4] CPU: 4 × Intel ®Core ™i5-4300U CPU @ 1.90GHz

6.1 Setup

Hereby, a cluster of 102 residential buildings is considered consisting of 51 HP heating systems and 51 CHP heating systems. The heating units are designed as bivalent systems according to conventional standards, i.e. the CHP units are designed to run at least 4000 h/a and a bivalence temperature of -2°C is set for the heat pump systems. Both CHPs and HPs are modeled as non modulated units coupled with a thermal storage tank to enable flexible operation. The bivalent heating systems are modeled with an additional gas boiler for every CHP unit and an integrated electrical heater for HP systems to cover peak loads. Some additional information are given in appendix A.

6.2 Benchmark z_{LRDW}

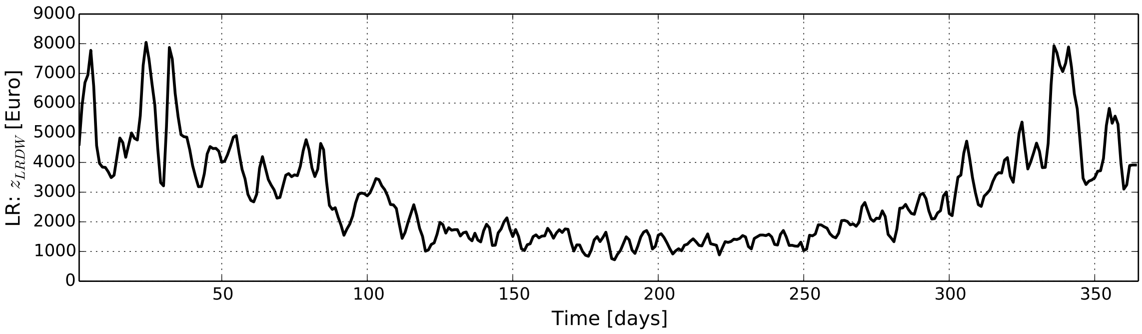

Figure 6.1: Best found lower bound z_{LRDW} from the combined algorithm over a whole year

Figure 6.1 illustrates the linear solution z_{LRDW} of the restricted masterproblem from the combined algorithm Pricing step for all scheduled days. This value is a lower bound on the integer solution of the problem and serves as benchmark. As expected, the total linear costs[5] of the micro grid reach a maximum of approximately 8000 Euro in winter and a minimum of approximately 700 Euro in summer. The high costs in winter can be explained with the higher electricity and heat demand, as well as the reduced electricity production from PVs. In contrast, the low total costs in summer originate from lower electricity and heat demand, as well as a higher production of electricity from PVs. To evaluate the solution quality of the scheduling problems and explore underlying dependencies, z_{LRDW} is used as a characteristic value for each scheduling problem to sort the solutions in the next sections.

[5]computed over a time period of 48 hours.

6.3 Computation time

A comparison of the termination time is illustrated in figure 6.2. The termination time is stopped after the last linear restricted masterproblem has been solved in both algorithms. It can be seen that both algorithms are often terminated due to the time limit of 10 minutes, especially for scheduling problems with low total costs. The conventional column generation algorithm seems to work very well for some scheduling problems with very low or very high costs and can even be solved to optimality before reaching the time limit. However, scheduling problems with very low or high costs are generally the "easier" problems, because the heating devices either have to run almost all the time or almost never. Scheduling problems with medium costs are typically more difficult to solve, because of the many possibilities regulating the heating devices. While the conventional column generation algorithm always reaches the time limit for scheduling problems with medium costs, the combined algorithm seems to work very well and can often be terminated before the time limit.

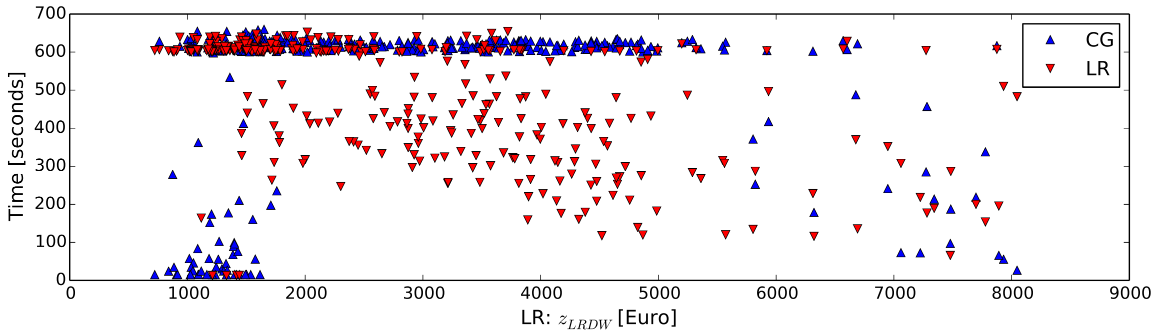

Figure 6.2: Termination time of the column generation procedure. (CG: Conventional column generation algorithm; LR: Combined algorithm (Pricing step))

6.4 Primal and dual solutions

The following section focuses on the best found primal and dual solutions for each scheduled day. The linear masterproblem solution is again used as a benchmark to evaluate the results. To compare both column generation methods and the quality of the integer solutions with each other, final schedules have been computed both by using the final proposals coming from the proposed Integering step and by using the proposals generated in the Pricing step of the combined algorithm.

Figure 6.3 shows how the best found objective values of the combined and the conventional column generation algorithm relate to each other. The upper plot compares the linear solutions with each other, while the lower part illustrates the relation of the integer solutions. Note that both integer solutions have been computed with non-unique shadow prices. The proposed method using fixed

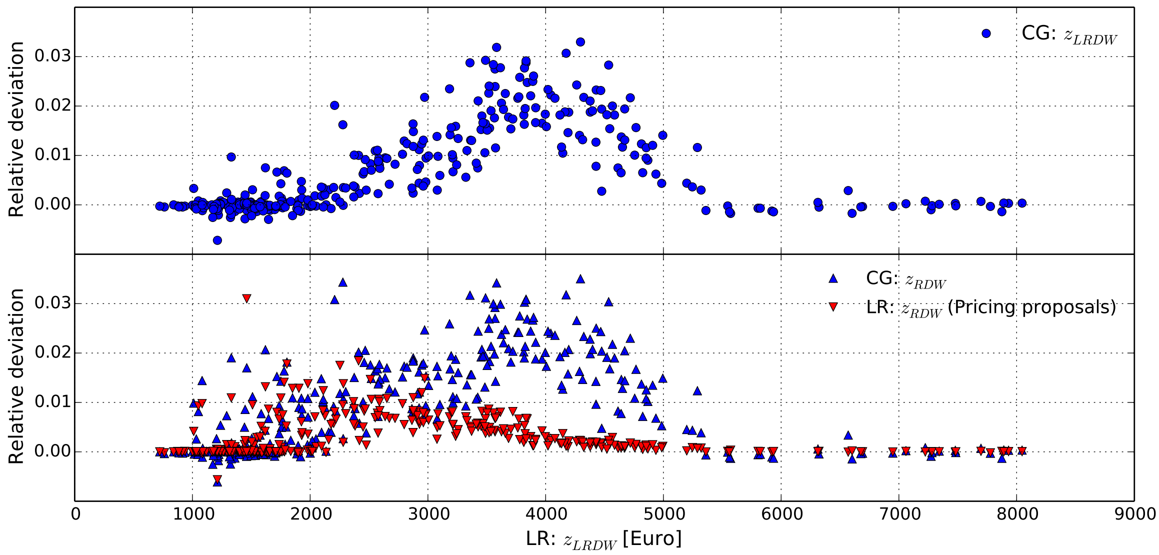

Figure 6.3: Primal solutions (CG: Conventional column generation algorithm; LR: Combined algorithm)

shadow prices is discussed later on. As figure 6.2 already indicated, the combined algorithm seems to work best for scheduling problems with medium costs, while the conventional algorithm has its strength when computing high or low cost scheduling problems. However, for those days the conventional column generation algorithm delivers only slightly better results. The biggest difference between the two algorithms occurs for scheduling problems with medium costs of 4000 Euro and amounts to more than 3%. While the linear and the integer solutions of the conventional algorithm don't seem to differ much at all, scheduling problems with costs below 4000 Euro show a small, but notable gap between the combined algorithm solutions.

As shown in chapter 5, decomposing the Lagrangian relaxation formulation of the scheduling problem into subproblems leads to optimization problems resembling the column generation pricing problems. This relationship was used to compute feasible proposals while optimizing the Lagrangian dual with the subgradient method. However, this relationship can also be exploited the other way round. By solving the pricing problems during the column generation algorithm, the value of the Lagrangian relaxation problem can be computed and used as a lower bound for the conventional column generation algorithm. Although there is no linear dependence between the shadow price vector π and the solution of the Lagrangian relaxation formulation $z_{LR}(\pi)$, its value can be used to roughly evaluate the quality of the shadow prices.

In figure 6.4 the lower bounds computed in both algorithms are shown. Because of the strong duality theorem, the optimal values of z_{LRDW} and z_{LR} are actually the same. Clearly, the lower bounds

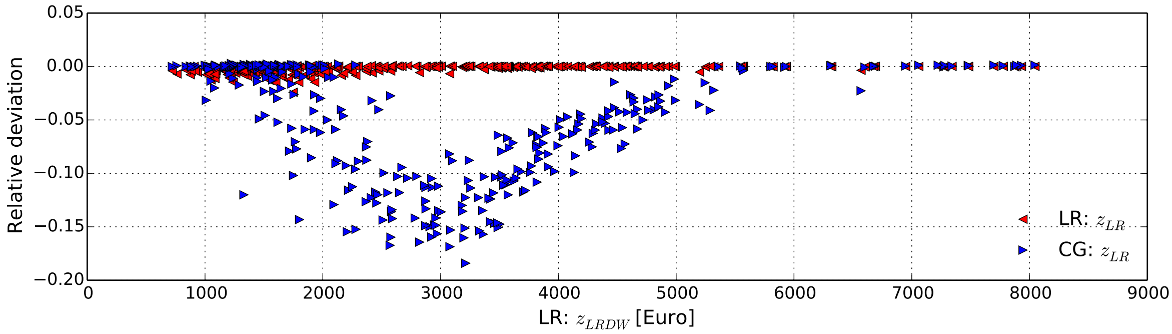

Figure 6.4: Dual solutions (CG: Conventional column generation algorithm; LR: Combined algorithm)

computed by using the column generation pricing problems are are far worse than the ones derived from optimizing the Lagrangian dual problem in the combined algorithm. Again, scheduling problems with costs above 5000 Euro or below 2000 Euro deliver fairly good results for the column generation and the biggest difference occurs for medium cost problems around 3000 Euro. This big difference of the lower bounds illustrates quite nicely where the strength of the combined algorithm is coming from, because the convergence of the dual solutions seems to be much faster. Moreover, it can be seen that for the combined algorithm, the linear objective value and the lower bound are very close together. Only scheduling problems at around 2000 Euro show a slightly bigger gap up tp 2%. Therefore, it can be concluded that the found values of z_{LRDW} and z_{LR} are close to the optimal values z_{LDW} and z_{LD}.

6.5 Results of a single day

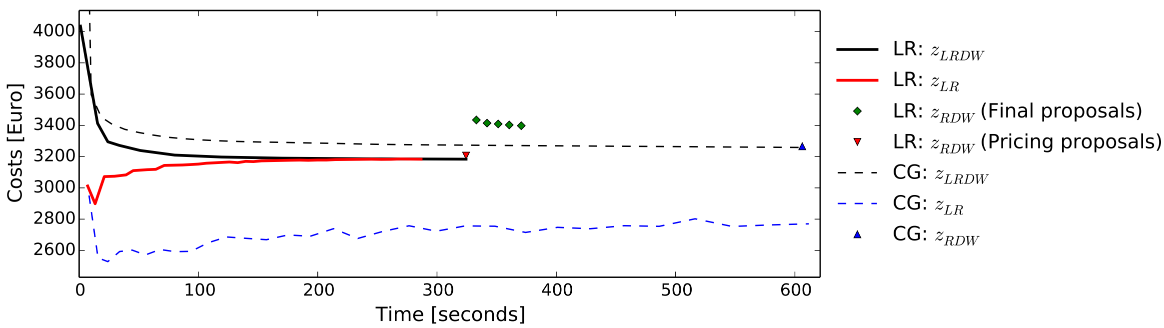

Figure 6.5: Convergence of primal and dual solutions for a day in January (CG: Conventional column generation algorithm; LR: Combined algorithm)

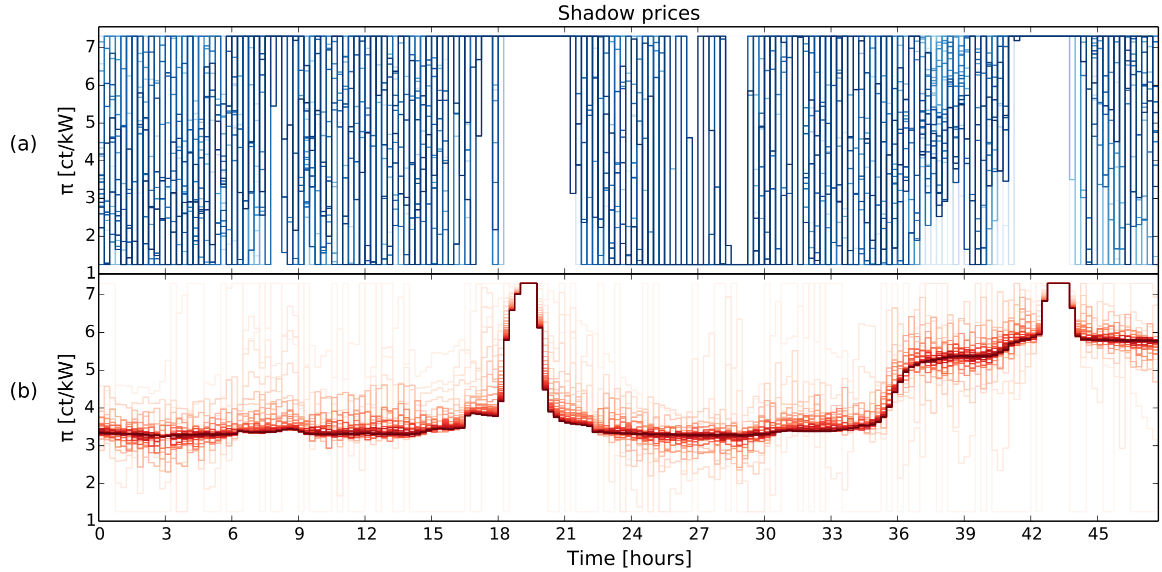

Figure 6.6: Convergence of the shadow prices for a day in January; (a) CG: Conventional column generation algorithm; (b) LR: Combined algorithm

This section focuses on the discussion of a single scheduling problem. As an exemplary scheduling problem, a day in January has been chosen. Figure 6.5 illustrates the convergence of both algorithms. The combined algorithm has found a satisfying solution before the time limit and can therefore be terminated. The conventional algorithm on the other hand converges very slowly towards the optimal value, which is called "tailing-off" effect. Even after optimizing for 10 minutes, there is still a significant gap between the primal and dual solutions, especially because of the poor lower bound. This indicates that the shadow prices of the conventional algorithm are still far from optimal when reaching the time limit.

The convergence of the shadow prices is depicted in figure 6.6. In the upper plot (a), the shadow prices of each iteration in the conventional column generation procedure are shown. The lower plot (b) shows how the shadow prices of the combined algorithm converge over each iteration. It becomes very clear, that the shadow prices computed from solving the linear Dantzig-Wolfe masterproblem fluctuate very much during the column generation procedure and would likely need many more iterations exceeding the time limit to converge towards the optimal values eventually. The shadow prices computed with the subgradient method, in contrast, converge very fast towards specific values and vary only slightly then. Furthermore, figure 6.6 shows an example of how typical optimal shadow prices or convex-hull prices look like. At the end of the first and the second day, there usually is a time period where little electricity is available in the microgrid. During these time periods the electricity generation of CHP units is encouraged by setting the shadow price to a maximum. For the early hours of the day, there typically is an abundance of generated electricity in the microgrid, which can be used to charge the thermal storage tanks by activating the heat pumps. This behavior is encouraged by setting the shadow prices to a minimum.

6.6 Integering step

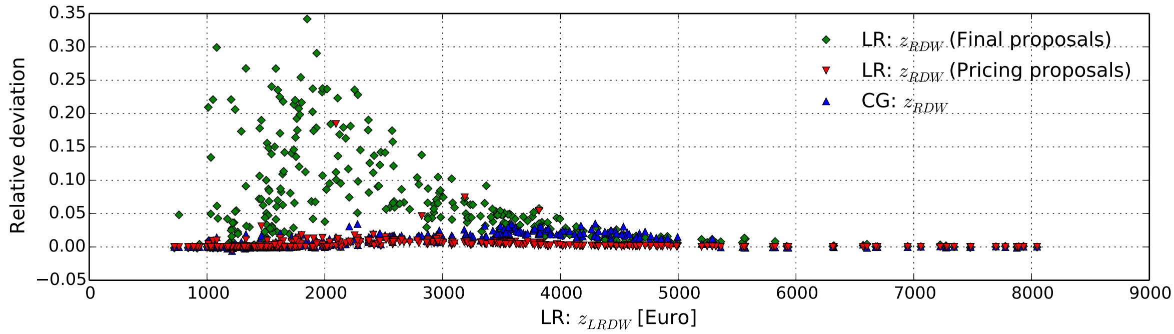

Figure 6.7: Primal integer solutions with unique shadow prices (CG: Conventional column generation algorithm; LR: Combined algorithm)

The quality of the integer solutions computed by the proposed Integering step is presented in figure 6.7. For scheduling problems with costs above 4000 Euro the Integering step with proposals from fixed shadow prices seems to achieve at least a solution that is comparable to the convetional column generation solution. However, the integer solutions for scheduling problems with costs below 4000 Euro are not of accetable quality and differ more than 30% for some problems.

Finally, the electricity exchange of the microgrid with a macrogrid is shown in figure 6.8 for the integer solutions computed by using only the proposals with the fixed shadow price (a) and the integer solutions derived from solving a MILP masterproblem with all pricing proposals. While the latter solution manages to balance the residual load of the microgrid and minimizes the electricity exchange with the macrogrid, the solution from fixed shadow price proposals exhibits sharp fluctuations during the day. The amount of imported and exported electricity, however, seems to balance out over the course of the whole day. An explanation for this behavior could be that the Integering step is failing to compute the basic proposals as it is supposed to. Recall that the subproblem solutions are not solved to strict optimality. Furthermore, the shadow price vector is just an approximation of the convex-hull price, the optimal dual solution. Therefore, the Integering algorithm often returns sufficiently optimal solutions that resemble each other too much. To optimally balance the residual load, it is desired to compute a set of very different proposals, because gives the algorithm more flexibility to regulate the heating systems. This lack of flexibility coming from very similar final proposals could explain the fluctuating electricity exchange and therefore the poor integer solution quality for low cost days. For days with growing costs and heat demand, this lack of flexibility becomes less important, because there are more timesteps where all CHP unit just have to run to satisfy the demand and don't need to be regulated. Thus, the proposals don't need to be different. For the coldest days of the year with maximal costs, the primary heating devices essentially have to run all the time to satisfy the heat demand and don't have to be regulated at all.

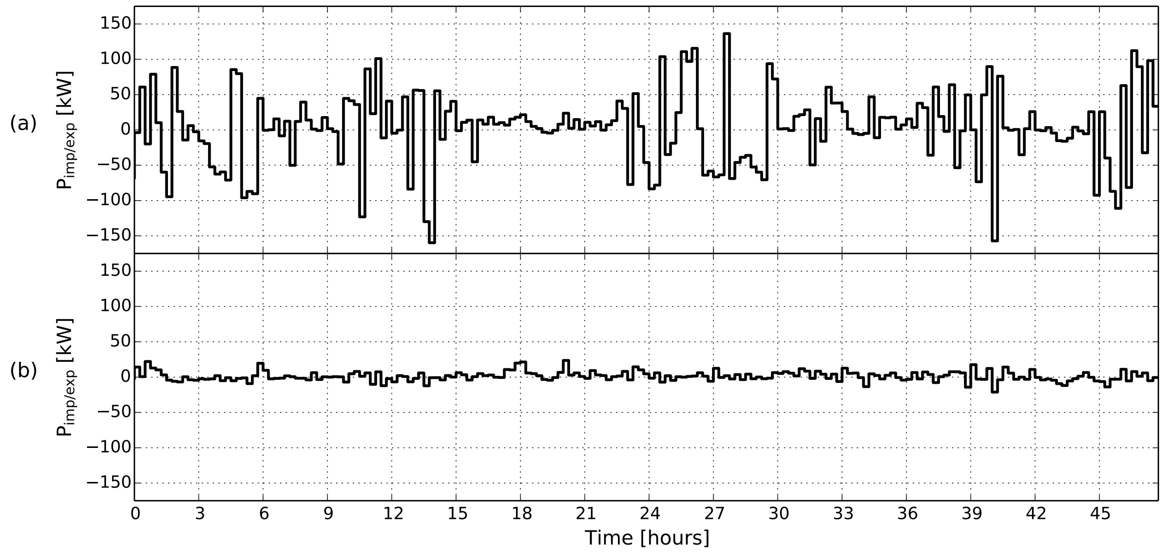

Figure 6.8: Electricity exchange with macrogrid for a day in January; (a) Integering step with final proposals; (b) Integering step with pricing proposals

7 Conclusion and outlook

For this thesis, a decentralized day-ahead scheduling strategy for the coordination of heating supply systems has been implemented combining a column generation algorithm with a Lagrangian relaxation approach. This algorithm uses the subgradient optimization method to optimize the shadow prices from the dual side instead of solving the Dantzig-Wolfe masterproblem in each iteration. Thereby, the proposed algorithm preserves the advantages of a decentralized approach, i.e. its flexibility, scalability and limited data exchange.

As expected, the combined column generation algorithm solves the scheduling problems in an efficient manner and delivers good results close to the optimal solution. In comparison with a conventional column generation algorithm, the method seems superior in almost every aspect. Especially for scheduling problems with medium overall costs, the implemented method computes higher quality solutions in less time. Though the conventional column generation algorithm seems to have its strengths for very high or very low demand scheduling problems, it does not significantly stand out in terms of the solution quality for these problems.

Additionally two different methods for obtaining feasible integer solutions were implemented and compared with each other, because of the problems arising from non-unique shadow prices. While the integer solutions computed from the Pricing step proposals have fairly optimal values close to the linear solution, the proposed Integering step can't compute integer solutions of acceptable quality. However, it is assumed that a more elaborate Integering step that somehow accomplishes to properly approximate the basic proposals of the optimal linear Dantzig-Wolfe masterproblem solution will deliver significantly better results with the same basic approach.

In this thesis, the linear masterproblem in the column generation procedure is used to adjust the stepsize of the subgradient optimization procedure. Albeit the linear solution of the masterproblem is a convenient tool to evaluate the solution quality of the implemented algorithms, it is not a necessity to solve the Lagrangian dual problem and its solution requires some computational effort. Therefore, in addition to enhancing the Integering step, future work could focus on solving the Lagrangian dual problem without any primal solution at all potentially further reducing the computation time.

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Appendix

A Input data

A.1 Renewable Energy sources

The available renewable energy sources (RES) consist of electricity from PV panels and electricity from wind energy. The generated energy from PV panels is modeled as follows

$$P_{solar}(t) = I_0(t) \cdot A_{solar} \cdot \eta_{solar} \tag{A.1}$$

where $I_0(t)$ is the direct solar radiation, A_{solar} is the area of installed PV modules and η_{solar} is the efficiency of the PV modules efficiency of the PV modules. The area of installed PV modules is set to 20 m^2 per building so the available electricity can be scaled with the number of buildings.

The model for the amount of electricity generated from wind turbines is divided into three parts. Wind turbines usually have a cut in wind speed v_{in} where they can start to operate. Until the wind speed $v_w(t)$ reaches the nominal wind speed of the turbine v_n the generated power $P_w(t)$ increases exponentially by the power of three. When v_n is reached the generated power is throttled to a constant value and is shut down when $v_w(t)$ reaches a critical value v_out . The cut in wind speed is set to $v_{in} = 2m/s$, the nominal wind speed is set to $v_n = 12m/s$ and the cut out velocity is set to $v_out = 25m/s$. The model for the electricity produced by wind turbines is comprised by the following equations

$$P_w(t) = \begin{cases} 0 & \text{if} \quad 0 \leq v_w(t) < v_{in} \quad \text{or} \quad v_w(t) > v_{out} \\ \frac{1}{2} \cdot \rho_a \cdot v_w(t)^3 \cdot \frac{\pi}{4} d_R^2 \cdot \eta_w & \text{if} \quad v_{in} \leq v_w(t) < v_n \\ \frac{1}{2} \cdot \rho_a \cdot v_n^3 \cdot \frac{\pi}{4} d_R^2 \cdot \eta_w & \text{if} \quad v_n \leq v_w(t) \leq v_{out} \end{cases} \tag{A.2}$$

ρ_a is the volumetric mass density of air, d_R is the diameter of the rotor and η_w is the efficiency of the considered wind turbine. The density of air is set to $\rho_a = 1.2kg/m^3$, the diameter of the rotor is set to $d_R = 12m$ and the efficiency is set to $\eta_w = 0.42$. The total amount of renewable energies adds up to

$$P_{RES}(t) = P_{solar}(t) + n_w \cdot P_w(t) \tag{A.3}$$

where n_w is the number of installed wind turbines set to 5.

A.2 Electrical and thermal demand

The electrical demand of each building is assumed to follow a standard load profile [1] (SLP). The SLP provides a normalized energy consumption for a year with a temporal resolution of 15 minutes. To determine the electrical power consumption of every single building the value of the normalized SLP has to be multiplied with the total energy consumption of one year. Table A.1 shows the total electricity consumption of a household depending on the number of residents. The heat demand

Table A.1: Yearly electricity consumption of a household [Oberascher, 2012]

Number of residents	Electricity consumption [kWh/a]
1	1798
2	2850
3	3733
4	4480
5	5311
6	5816

is simulated with Modelica/Dymola using a detailed house model. The model takes into account several physical effects i.e. losses due to insulation, occupancy and heat gains from solar radiation or dissipated production heat.

A.3 Dimensioning

A.3.1 Heat pumps

In this thesis buildings equipped with a heat pump system are supplied with high efficiency heat pumps based on the data sheets of a manufacturer [Glen-Dimplex-Deutschland-GmbH, 2009]. These heat pumps are able to provide a flow temperature of up to 55°C which makes them suitable for a heating system. The data sheets in Glen-Dimplex-Deutschland-GmbH [2009] provide detailed parameter descriptions and characteristic curves of the heat pump models such as COP curves. This data is used to compute the heat output and electrical power consumption as a function of the ambient temperature. For each building a heat pump unit is selected whose heat output at the bivalence temperature is closest to the buildings heat demand.

[1] E.ON Westfalen Weser AG. Netznutzung: Synthetische Verfahren. URL: http://www.eon-westfalenweser.com/pages/ewa_de/Netz/Strom/Netznutzung/Synthetisches_Verfahren/index.htm. Accessed on April 21, 2014

A.3.2 Combined heat and power units

A µCHP is designed to operate for a certain amount of full load hours τ_{min}. The heat demand that can't be covered by the CHP unit is covered by an additional boiler. In this thesis all µCHPs are designed to run at least $\tau_{min} = 4000 h/a$ and selected from [Arbeitsgemeinschaft für Sparsamen und Umweltfreundlichen Energieverbrauch, 2011].

B Solutions of different scheduling problems

B.1 Day 145

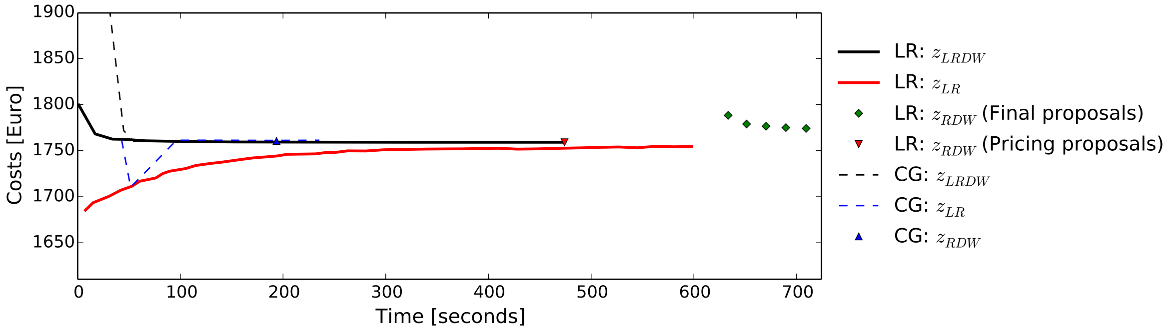

Figure B.1: Convergence of primal and dual solutions for a day in May (CG: Conventional column generation algorithm; LR: Combined algorithm)

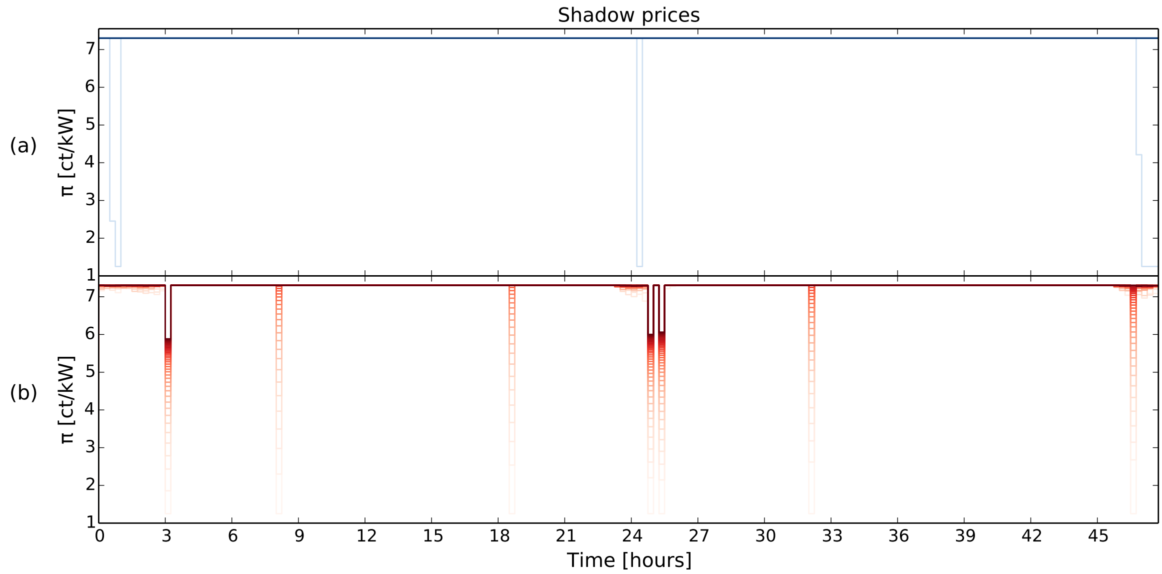

Figure B.2: Convergence of the shadow prices for a day in May; (a) CG: Conventional column generation algorithm; (b) LR: Combined algorithm

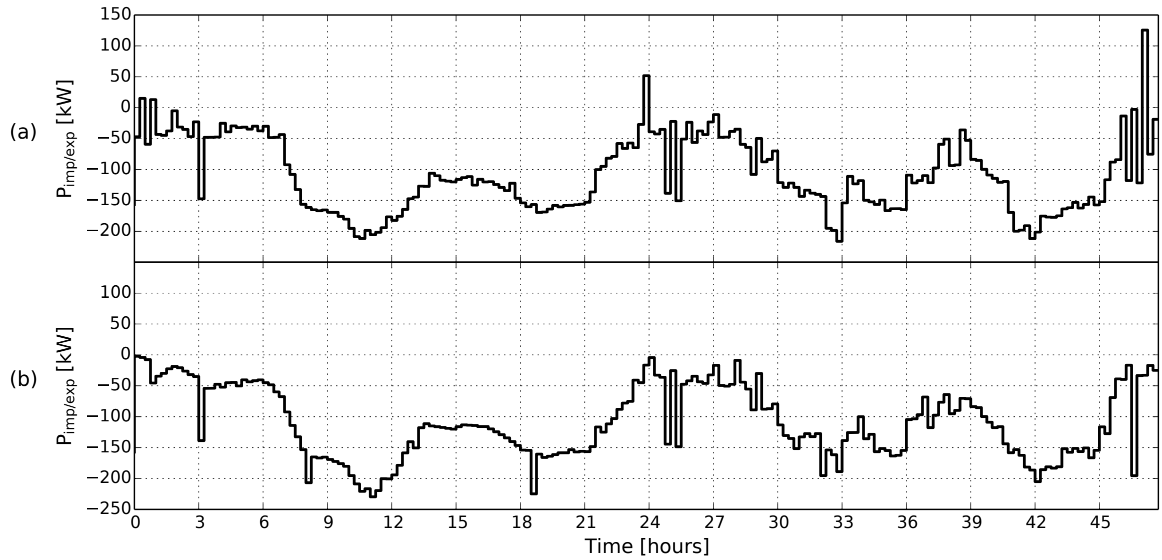

Figure B.3: Electricity exchange with macrogrid for a day in May; (a) Integering step with final proposals; (b) Integering step with pricing proposals

B.2 Day 167

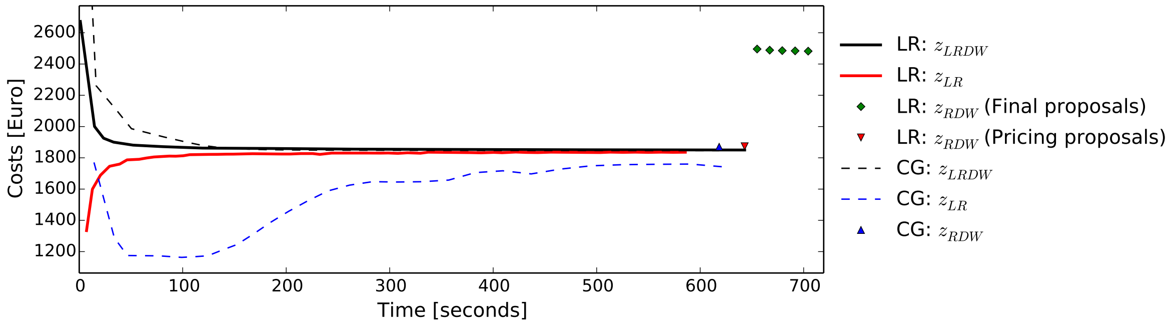

Figure B.4: Convergence of primal and dual solutions for a day in September (CG: Conventional column generation algorithm; LR: Combined algorithm)

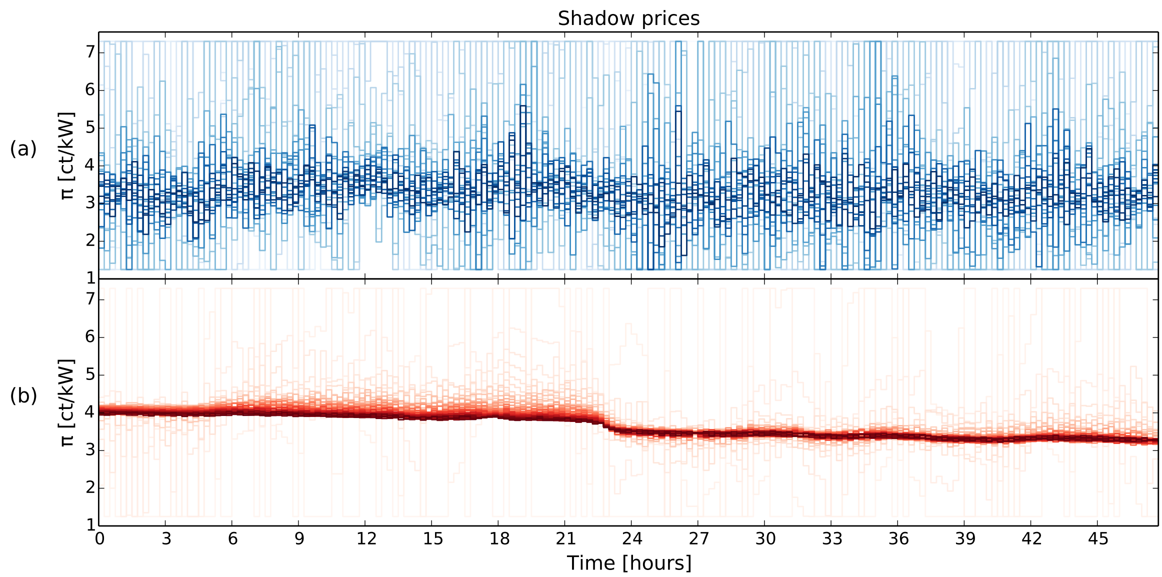

Figure B.5: Convergence of the shadow prices for a day in September; (a) CG: Conventional column generation algorithm; (b) LR: Combined algorithm

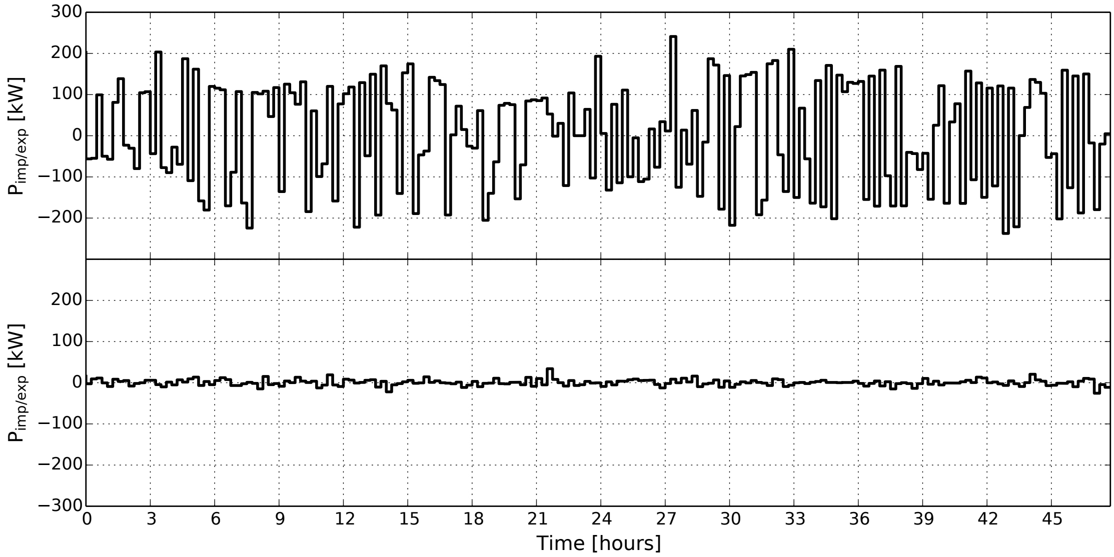

Figure B.6: Electricity exchange with macrogrid for a day in September; (a) Integering step with final proposals; (b) Integering step with pricing proposals

B.3 Day 18

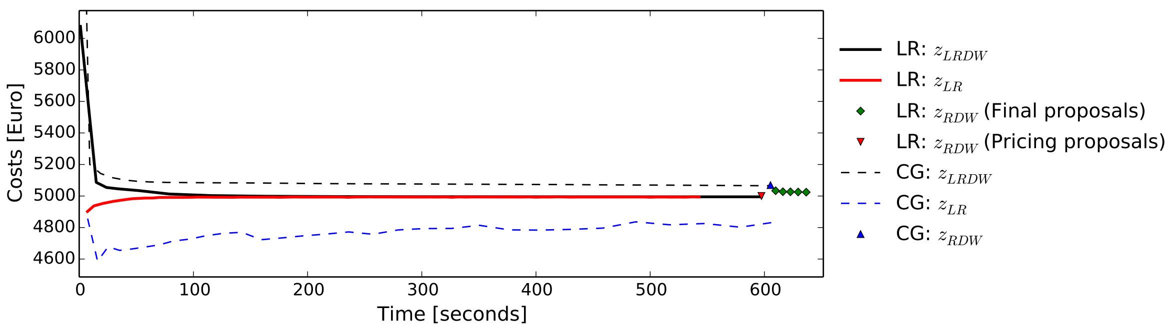

Figure B.7: Convergence of primal and dual solutions for a day in January (CG: Conventional column generation algorithm; LR: Combined algorithm)

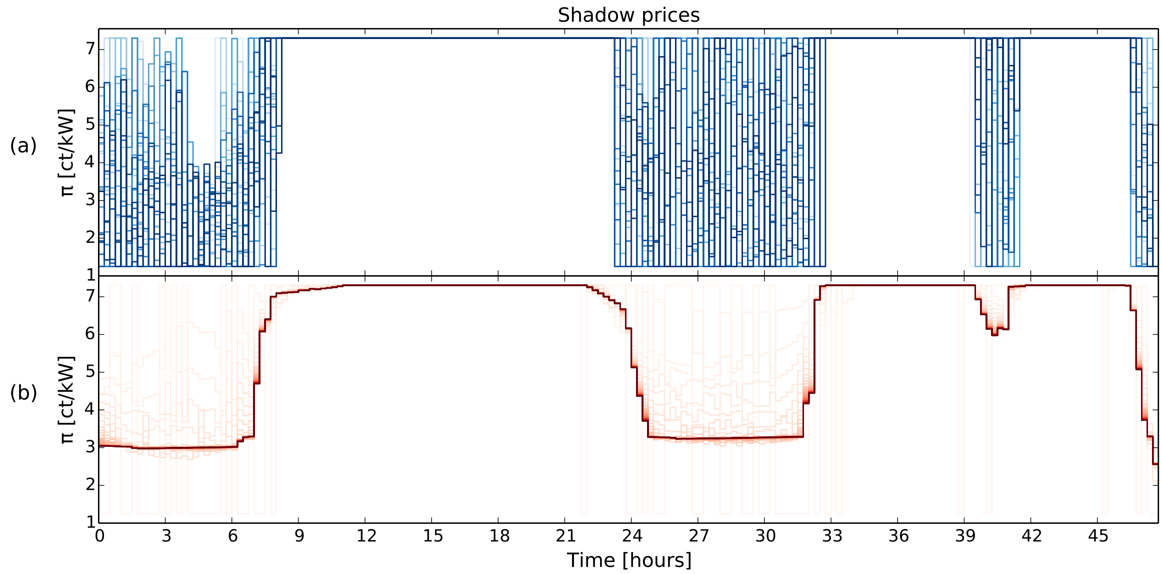

Figure B.8: Convergence of the shadow prices for a day in January; (a) CG: Conventional column generation algorithm; (b) LR: Combined algorithm

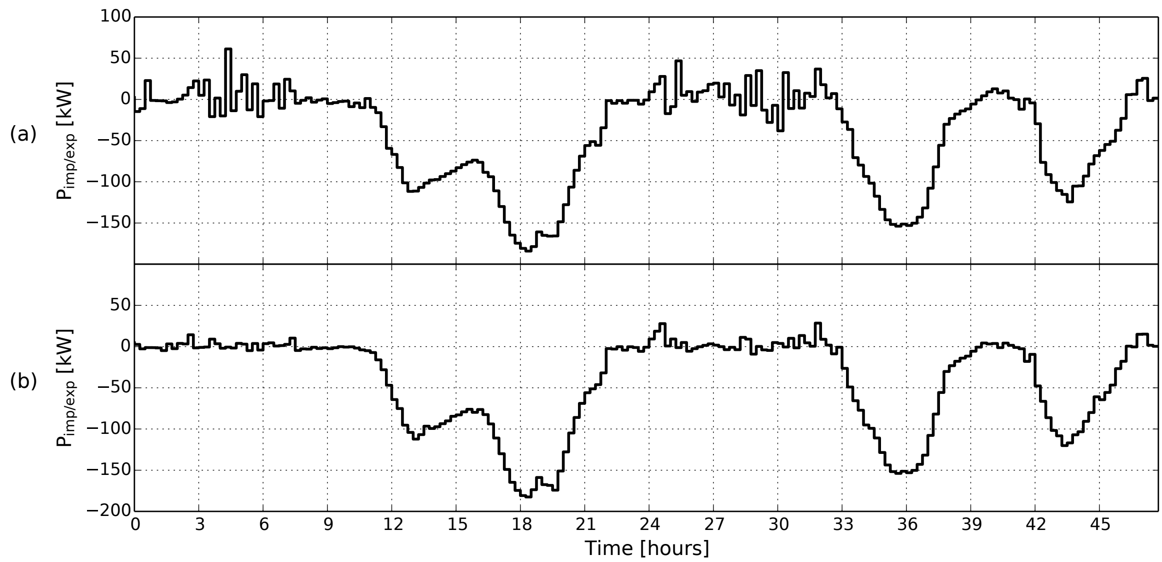

Figure B.9: Electricity exchange with macrogrid for a day in January; (a) Integering step with final proposals; (b) Integering step with pricing proposals

B.4 Day 31

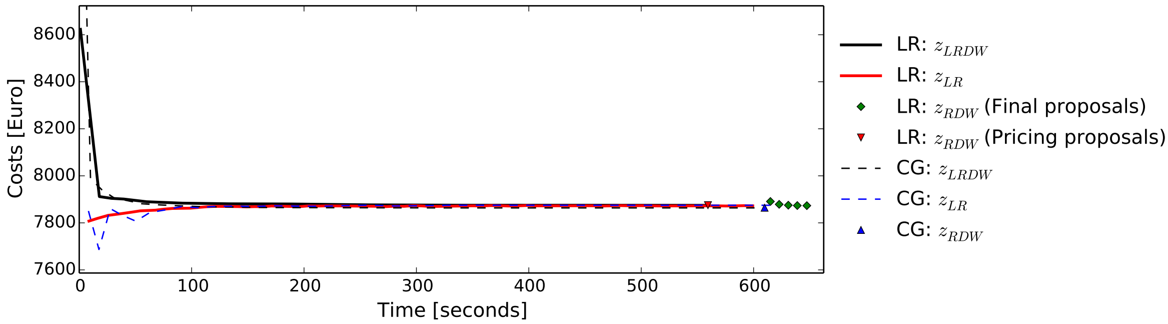

Figure B.10: Convergence of primal and dual solutions for a day in January (CG: Conventional column generation algorithm; LR: Combined algorithm)

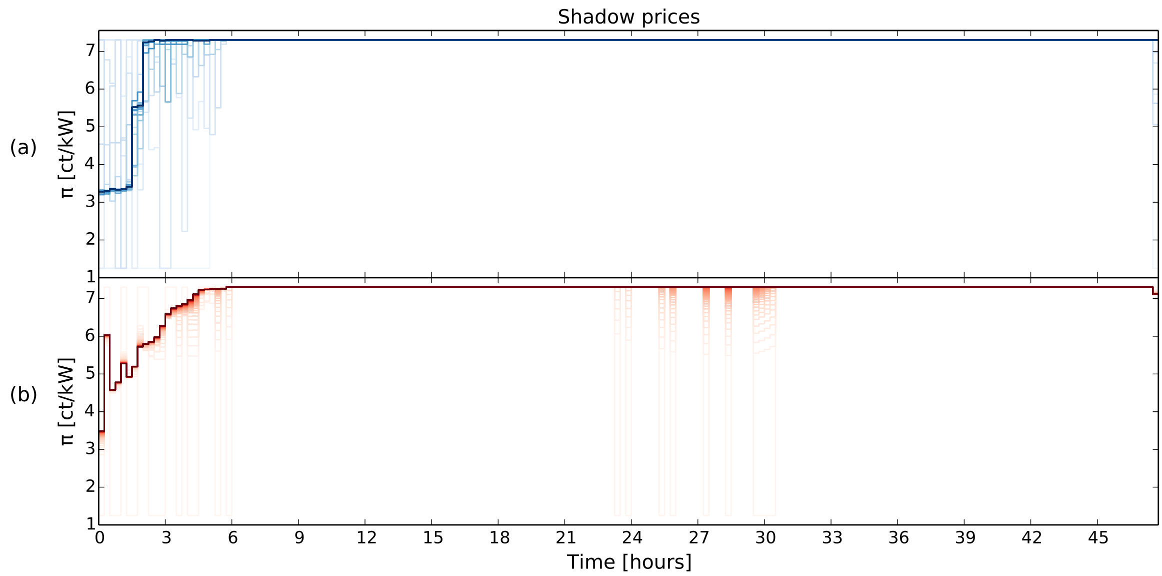

Figure B.11: Convergence of the shadow prices for a day in January; (a) CG: Conventional column generation algorithm; (b) LR: Combined algorithm

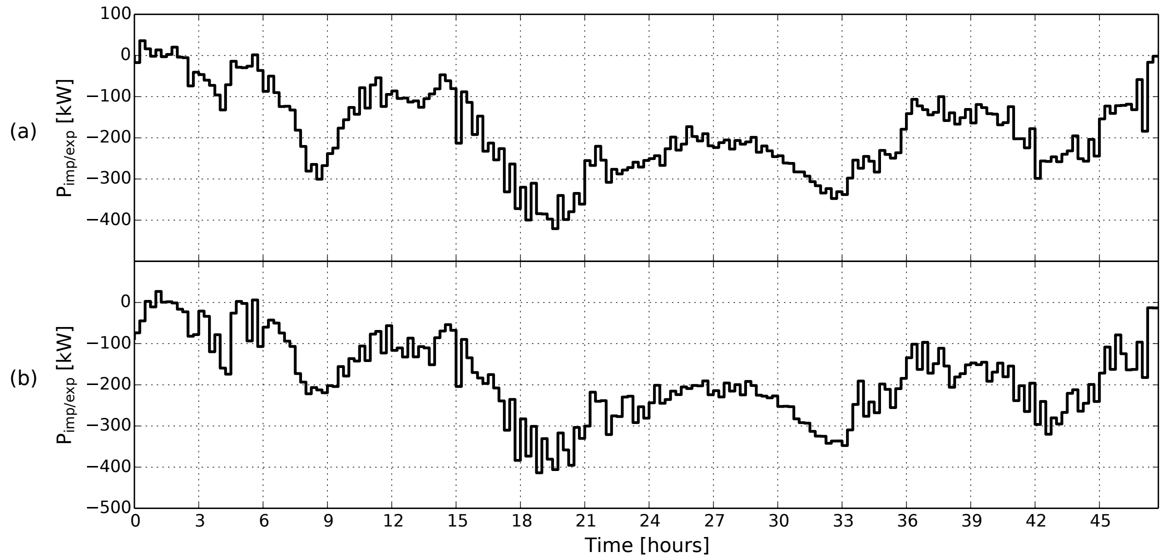

Figure B.12: Electricity exchange with macrogrid for a day in January; (a) Integering step with final proposals; (b) Integering step with pricing proposals

Declaration of Originality

I hereby declare that this thesis and the work reported herein was composed by and originated entirely from me. Information derived from the published and unpublished work of others has been acknowledged in the text and references are given in the list of sources. This thesis has not been submitted as exam work in neither the same nor a similar form. I agree that this thesis may be stored in the institutes library and database. This work may also be copied for internal use.

Aachen, March 27, 2015

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