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Non-line-of-sight Surface Reconstruction Using the Directional Light-cone Transform

Anonymous CVPR Submission

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Abstract

We propose a new, filtering approach for solving a large number of regularized inverse problems commonly found in computer vision. Traditionally, such problems are solved by finding the solution to the system of equations that expresses the first-order optimality conditions of the problem. This can be slow if the system of equations is dense due to the use of nonlocal regularization, necessitating iterative solvers such as successive over-relaxation or conjugate gradients. In this paper, we show that similar solutions can be obtained more easily via filtering, obviating the need to solve a potentially dense system of equations using slow iterative methods. Our filtered solutions are very similar to the true ones, but often up to 10 times faster to compute.

1. Introduction

Inverse problems are mathematical problems where one's objective is to recover a latent variable given observed input data. In computer vision, a classic inverse problem is that of estimating the optical flow [1], where the goal is to recover the apparent motion between an image pair. The problems of image super-resolution, denoising, deblurring, disparity and illumination estimation are examples of inverse problems in imaging and computer vision [2]–[5]. The ubiquity of these inverse problems for real-time computer vision applications places significant importance on efficient numerical solvers for such inverse problems. Traditionally, an inverse problem is formulated as a regularized optimization problem and the optimization problem then solved by finding the solution to its first-order optimality conditions, which can be expressed as a system of linear (or linearized) equations.

Recently, edge-preserving regularizers based on bilateral or nonlocal means weighting have found use in many vision problems [5]–[7]. Whereas such nonlocal regularizers often produce better solutions than local ones, they generate dense systems of equations that in practice can only be solved via slow numerical methods like successive over-relaxation and conjugate gradients. Such numerical methods are inherently iterative, and are sensitive to the conditioning of the overall problem. Iterative methods such as conjugate gradients also require the problem to be symmetric (and semi-definite).

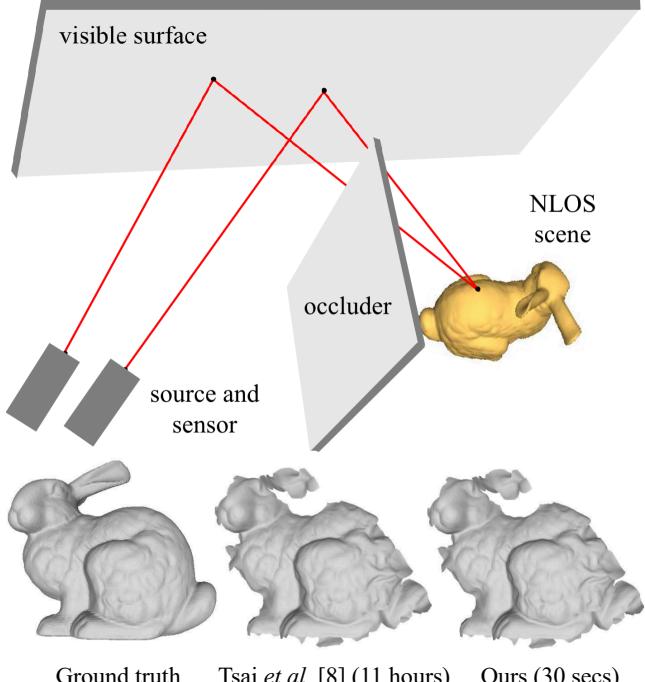


Figure 1. **Non-line-of-sight surface reconstruction:** Existing non-line-of-sight (NLOS) imaging methods for surface reconstruction can be time-consuming. Using our Directional LCT approach, we recover object surfaces of similar quality, 1000× faster.

In this work, we solve regularized optimization problems of the form

$$\text{minimize } f(\mathbf{u}) = \|\mathbf{H}\mathbf{u} - \mathbf{z}\|_2^2 + \lambda \mathbf{u}^* \mathbf{L} \mathbf{u} \quad (1)$$

using fast non-iterative filtering, obviating the need to solve dense systems of linear equations produced by geodesic and bilateral regularizers for example. We validate our approach on three classic vision problems: optical flow (and disparity) estimation, depth superresolution, and image deblurring and denoising, all of which are expressible in the form (1). Our filtered solutions to such problems are all very similar to the true ones as seen in Figure 1, but 10× faster to compute in some cases. Compared to the fast bilateral solver [5], our formalism is not specific to the bilateral regularizer, and can solve more advanced inverse problems such as the disparity and the optical flow estimation problems.

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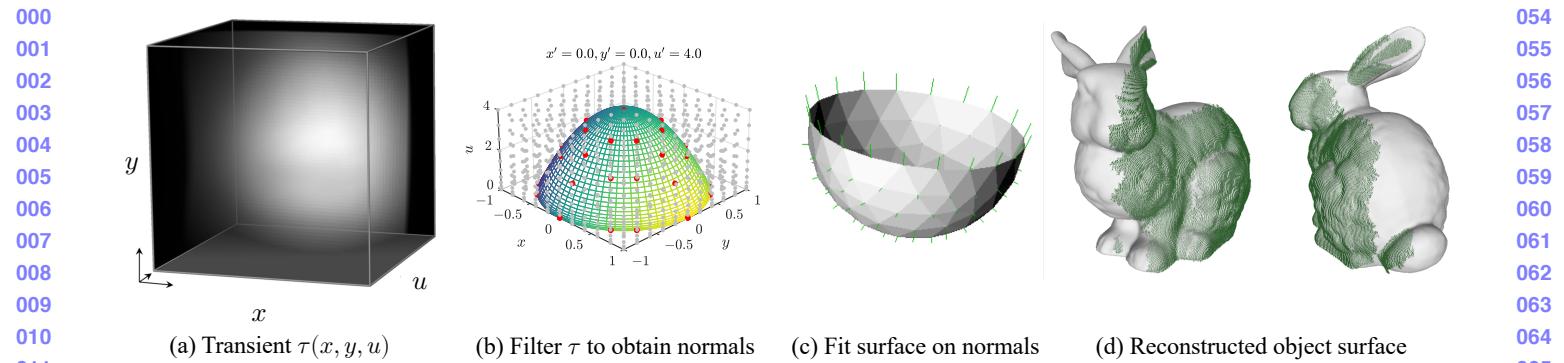


Figure 2. **Pipeline overview:** Transient τ (a) is filtered using the directional light-cone transform (b) to obtain surface normals. We perform surface fitting on the normals (c) to obtain the final reconstructed surface (d).

1.1. Contributions

The contributions of this work are:

- **Directional LCT:** We express non-line-of-sight surface reconstruction as a vectorial deconvolution problem via the Directional Light-cone Transform (D-LCT).
- **Cholesky-Wiener Solver:** We solve the above vectorial deconvolution problem efficiently in the Fourier domain to recover the surface normals.
- **Surface Fitting:** We reconstruct highly-accurate object surface descriptions by fitting surface parameters on the recovered normals.
- **Nonlinear Model:** We extend the baseline linear model to a nonlinear one and exploit the D-LCT in an iterative framework to improve the quality of obtained surfaces.

2. Related Work

3. Volumetric Albedo Models

4. Directional Light-cone Transform

5. Cholesky-Wiener Decomposition

6. Parametric Surface Fitting

7. Extensions to Nonlinear Models

8. Discussion

9. Conclusion

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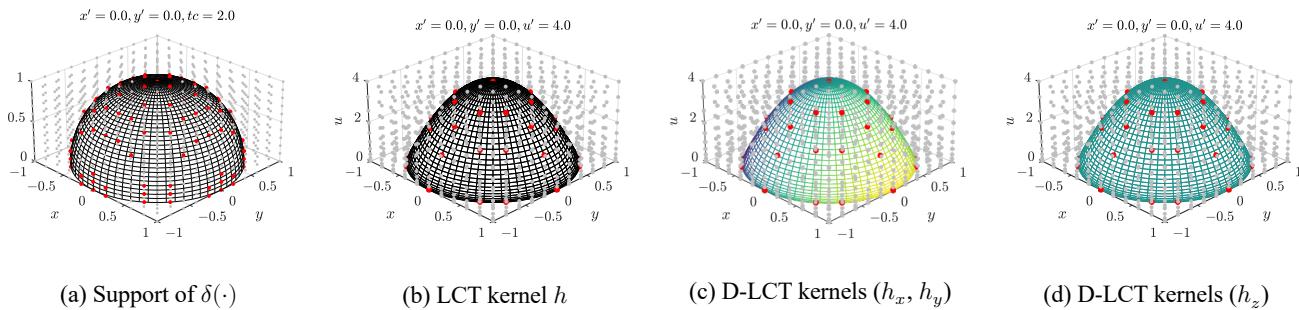


Figure 3. **Constructing the Directional LCT:** the original problem (1) cannot be posed as deconvolution since $\delta(\cdot)$ is not shift-invariant in the z direction (a). The light-cone transform produces a 3-dimensional, shift-invariant kernel (b). The D-LCT uses 3 shift-invariant kernels to extract the normals (c), (d).

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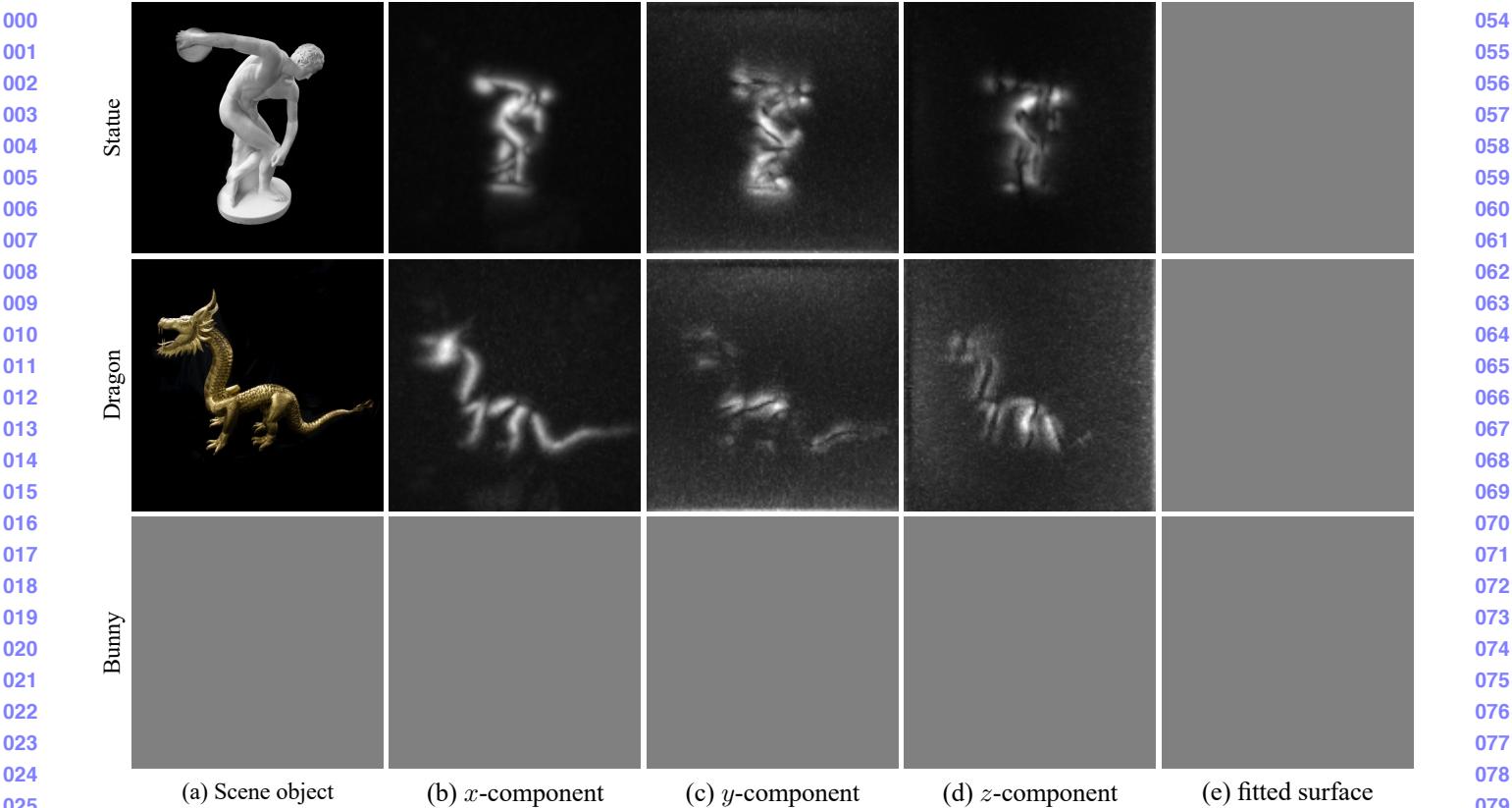


Figure 4. **Recovered normals and the fitted surfaces:** For the hidden scene objects (a), we visualize the estimated normal components (b)–(d) and the fitted surfaces (e).

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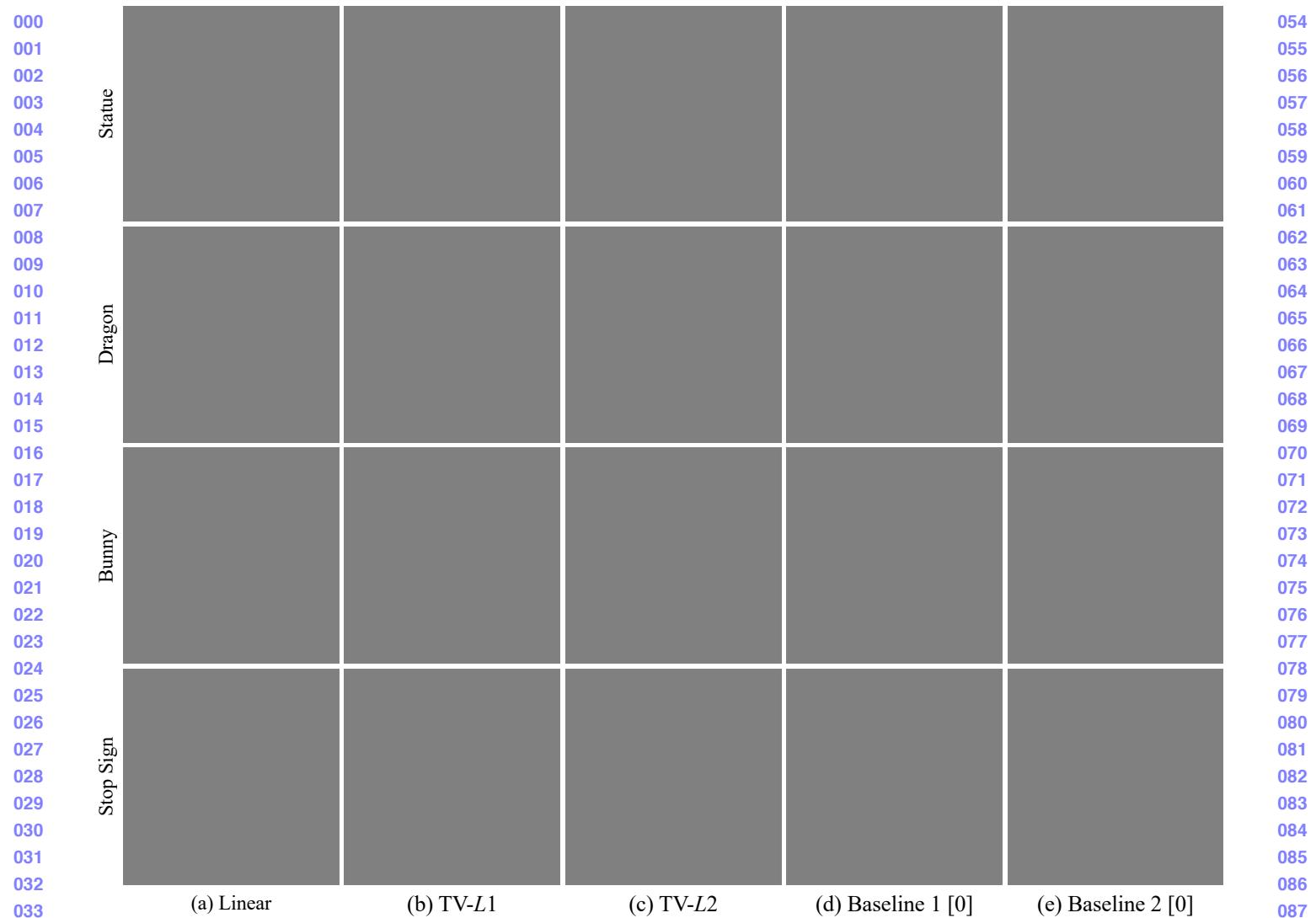


Figure 5. **Reconstructed surfaces using nonlinear models:** For the hidden scene objects (a), we estimate the surface normals using different nonlinear models and fit the surfaces as in Sec 2. For comparison we include the surfaces reconstructed using baseline methods [0] and [0].

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