

1

4

TTEST and ANOVA the theory

Probability and Statistics

COMS10011

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before we start a
quick reminder

I'M a DR

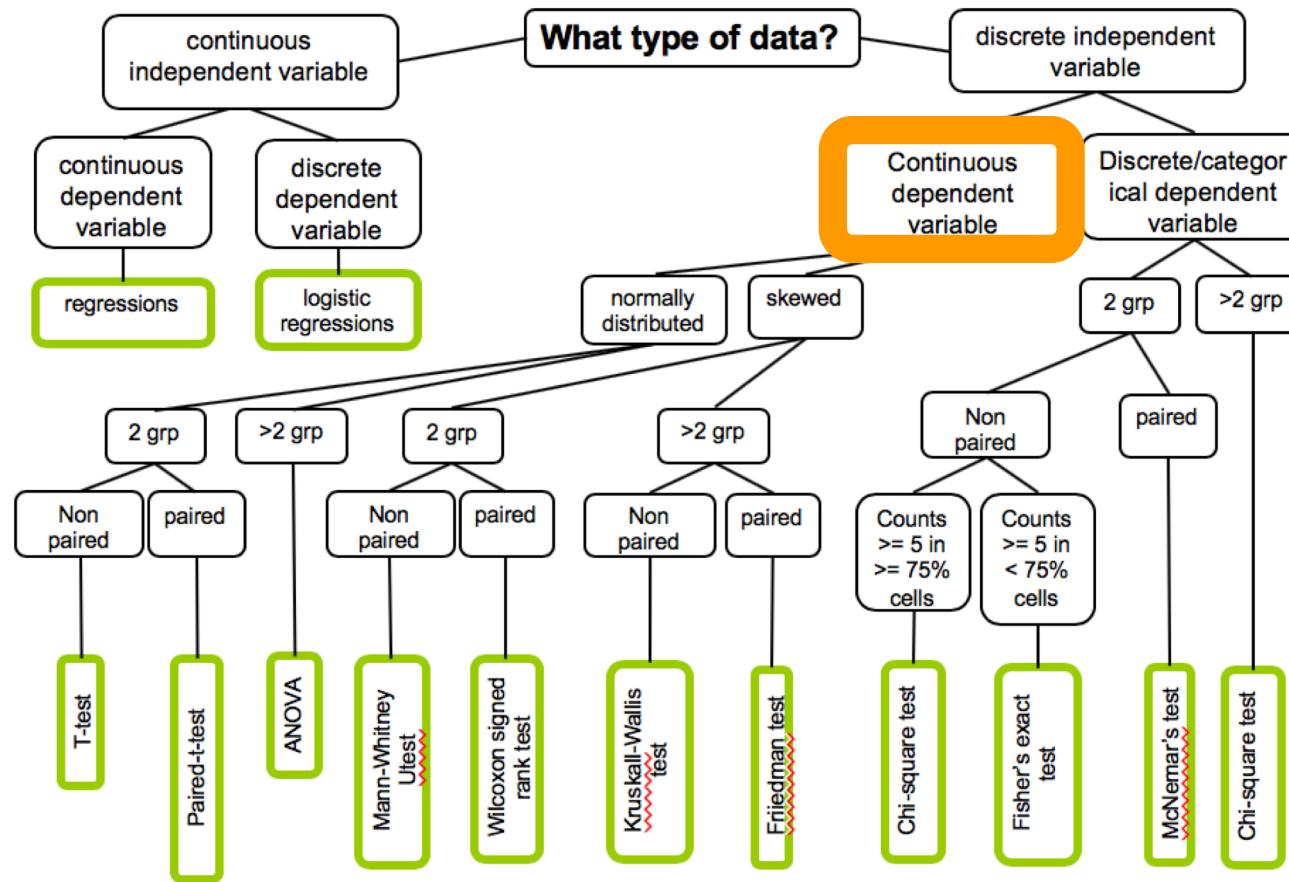
Independent

Manipulated

Responses

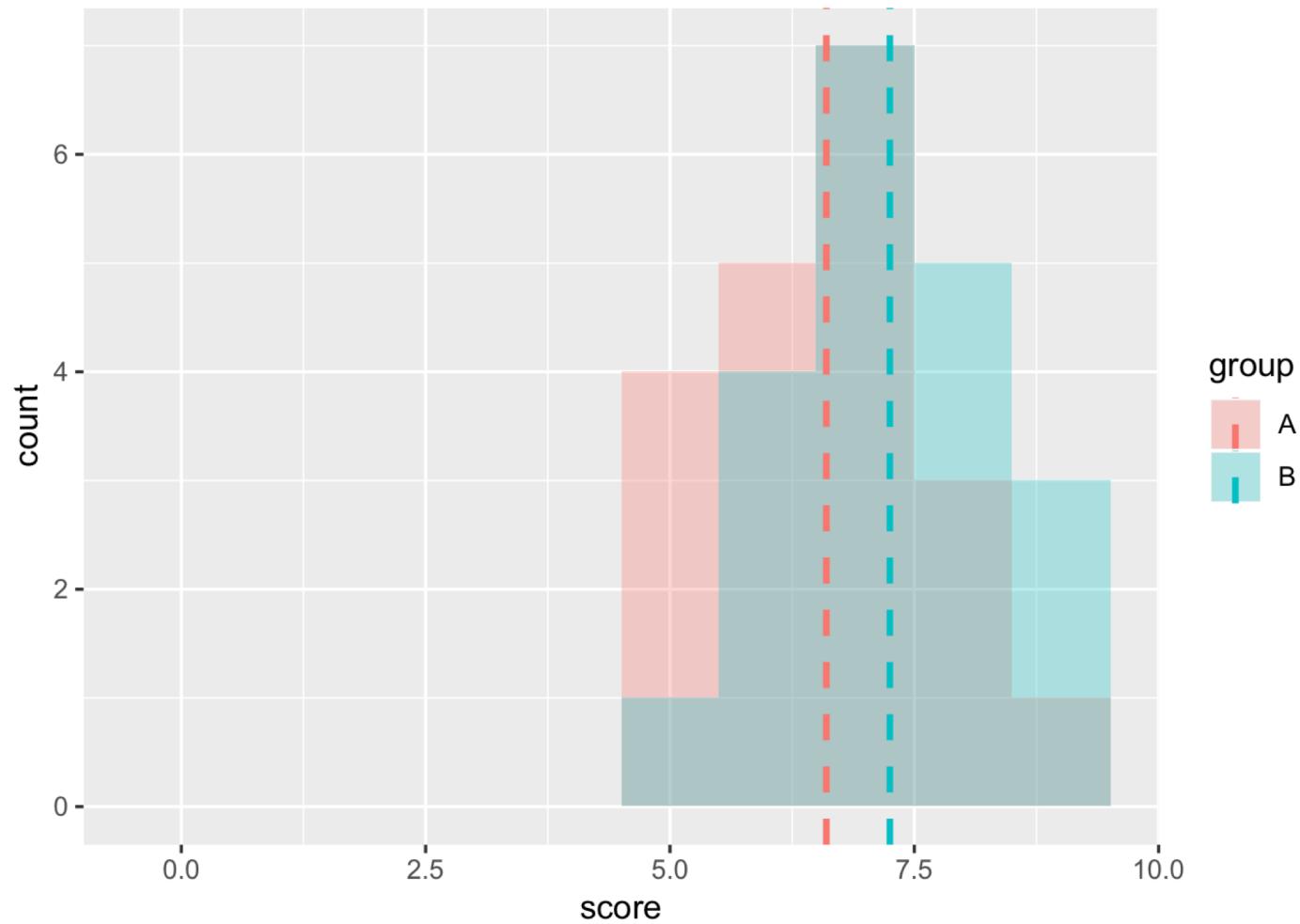
Dependent

Dependent vs Independent variables?

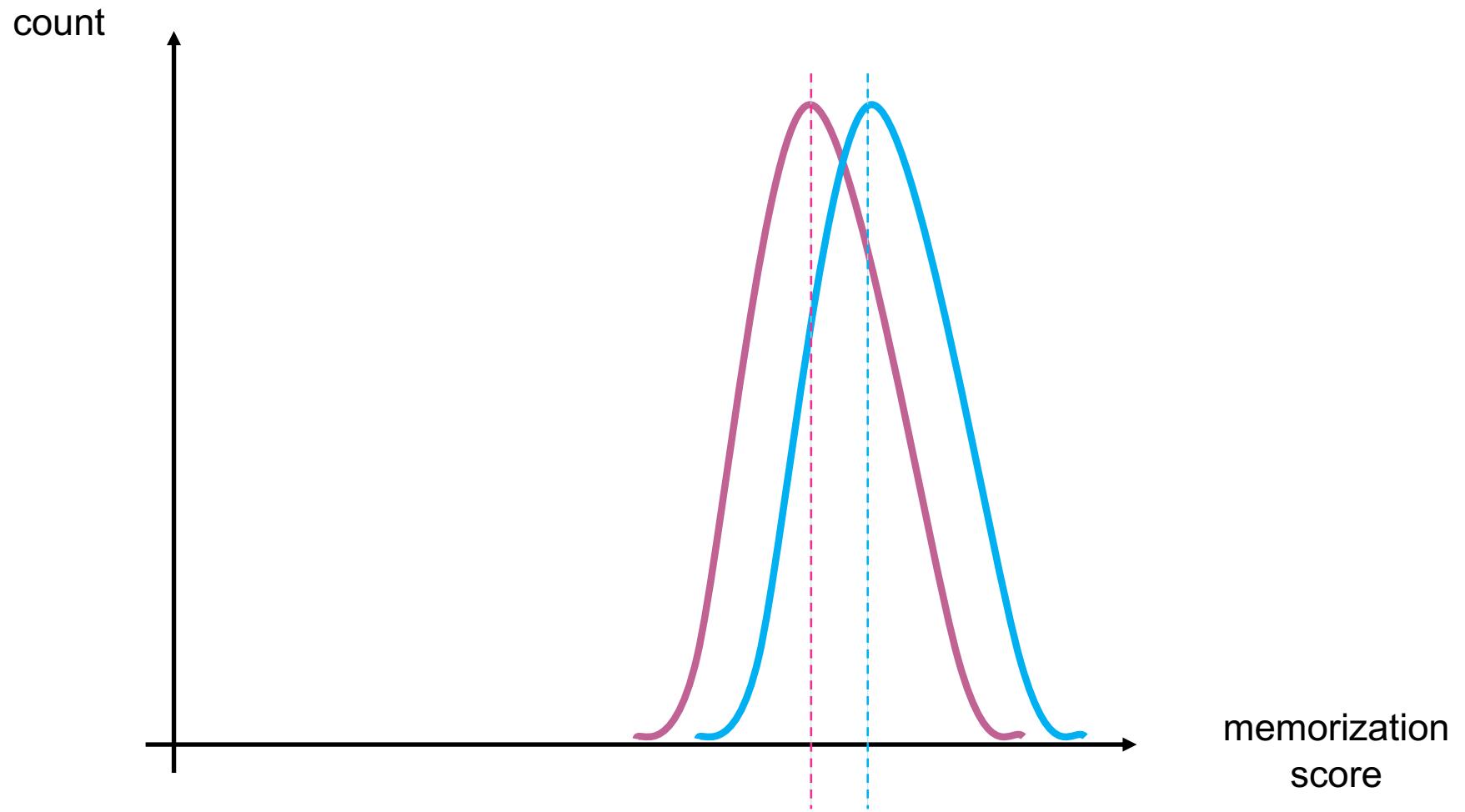


Likert are Ordinal but can be treated as
Continuous

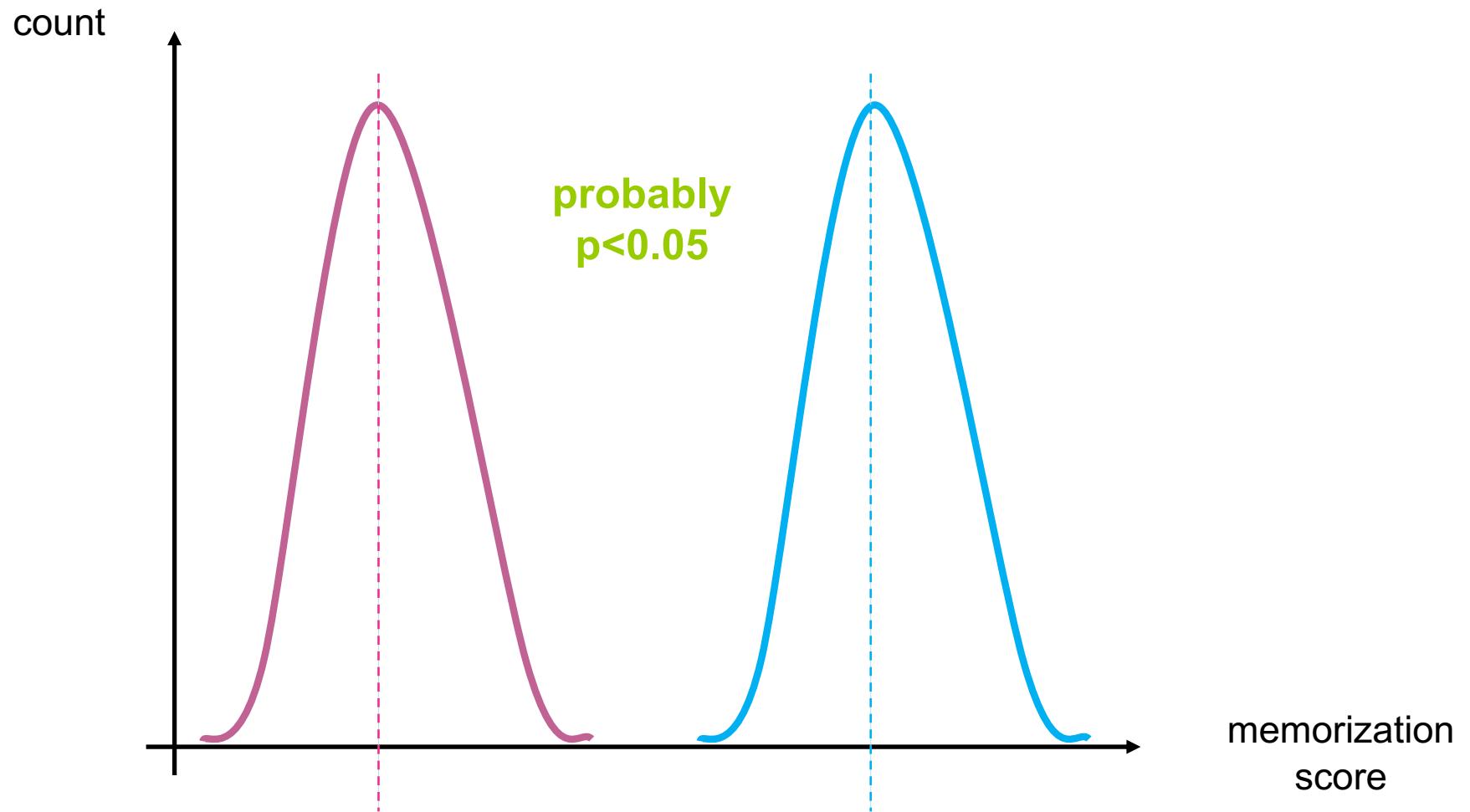
lets go back to
our memorization
experience



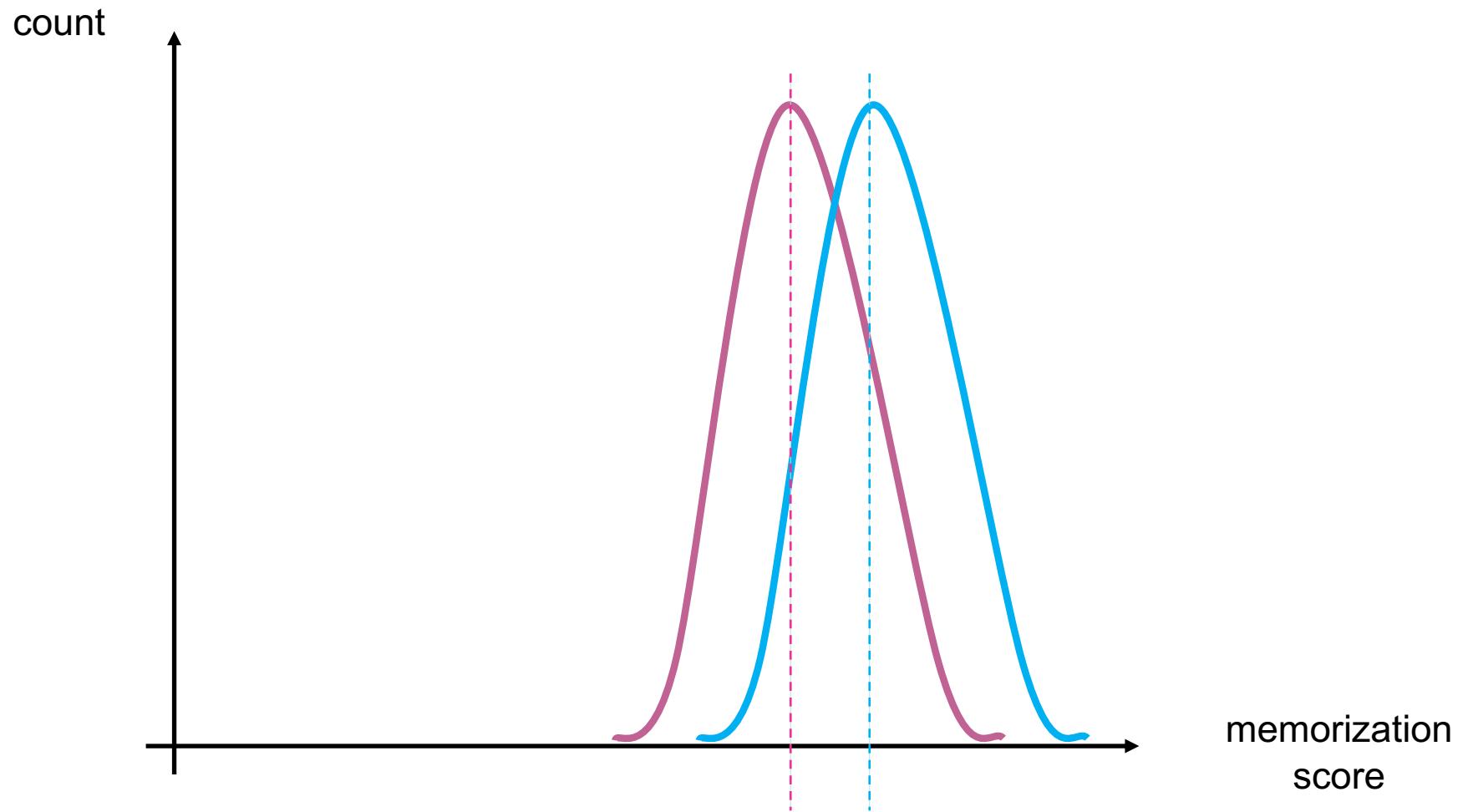
no evidences for chocolate vs. baseline ($p>0.05$)



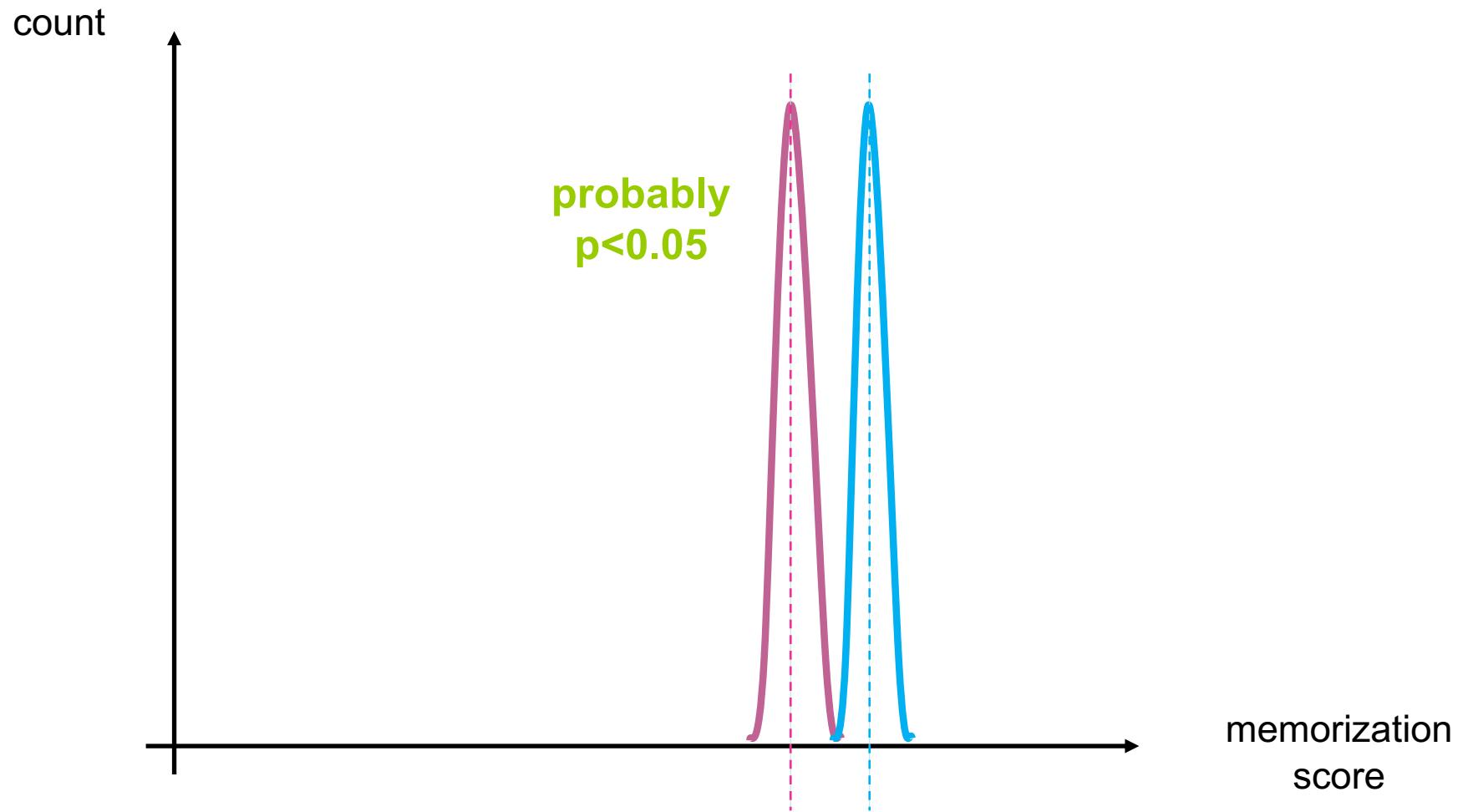
no evidences for chocolate vs. baseline ($p>0.05$)
(let's just assume these are normally distributed)



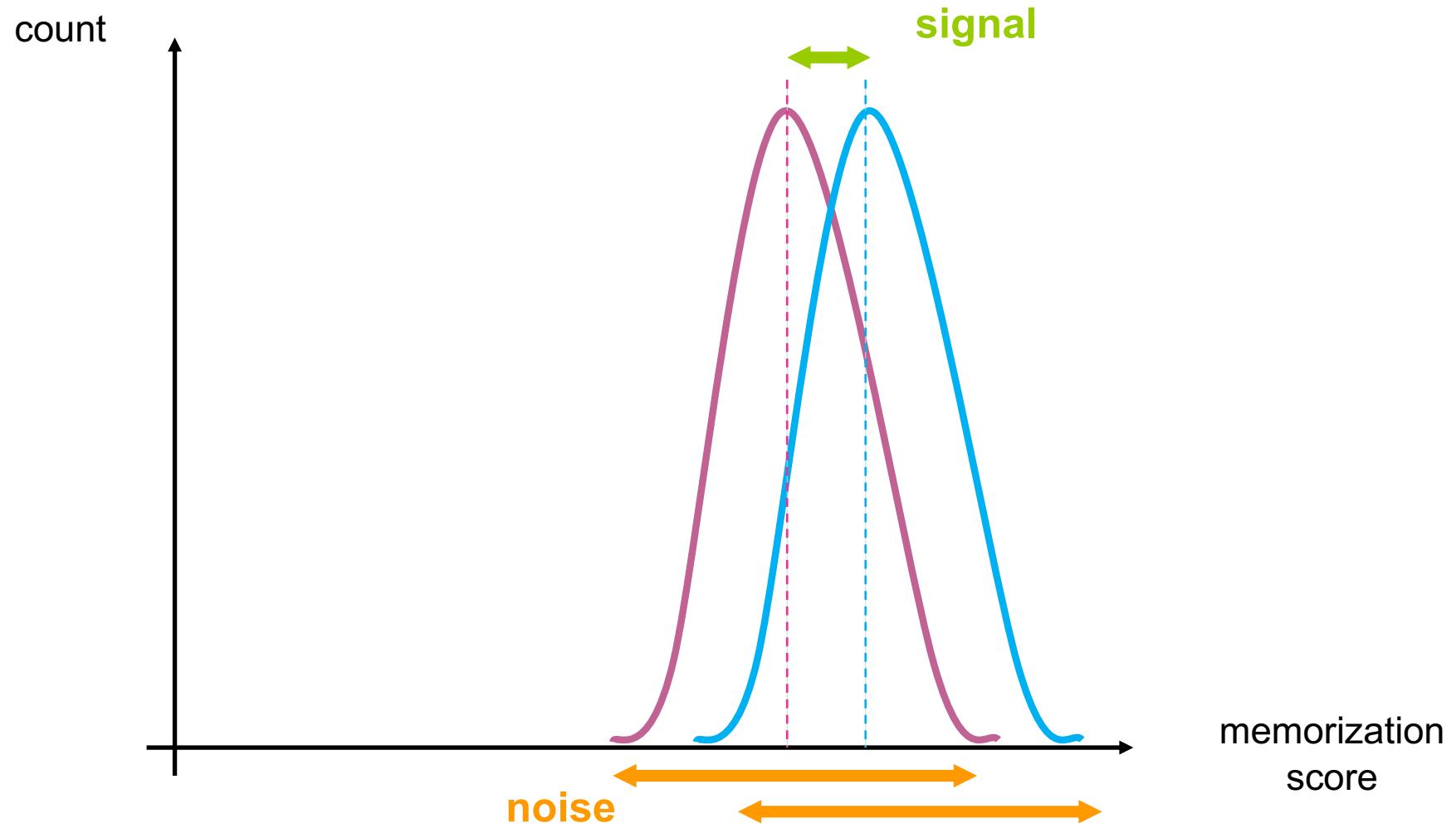
if the mean were further apart, it would increase our chances to have a $p < 0.05$



<without changing the means, what else can we do to these data to make some more different?>



if the distributions were less spread out, it would increase our chances to have a $p < 0.05$



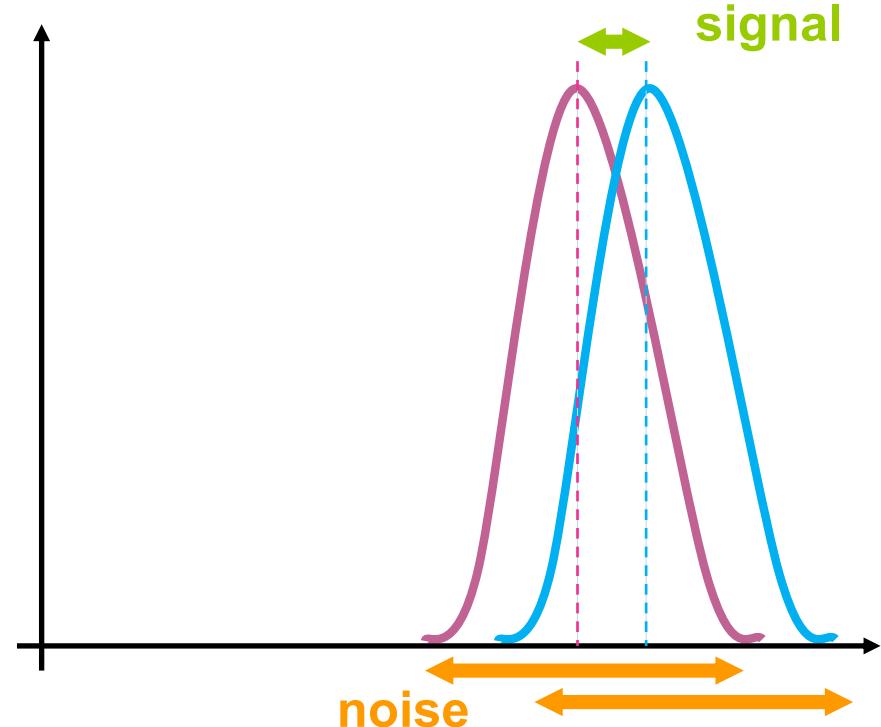
the goal of a study is to find
a signal in a lot of noise

any statistical tests ::

signal

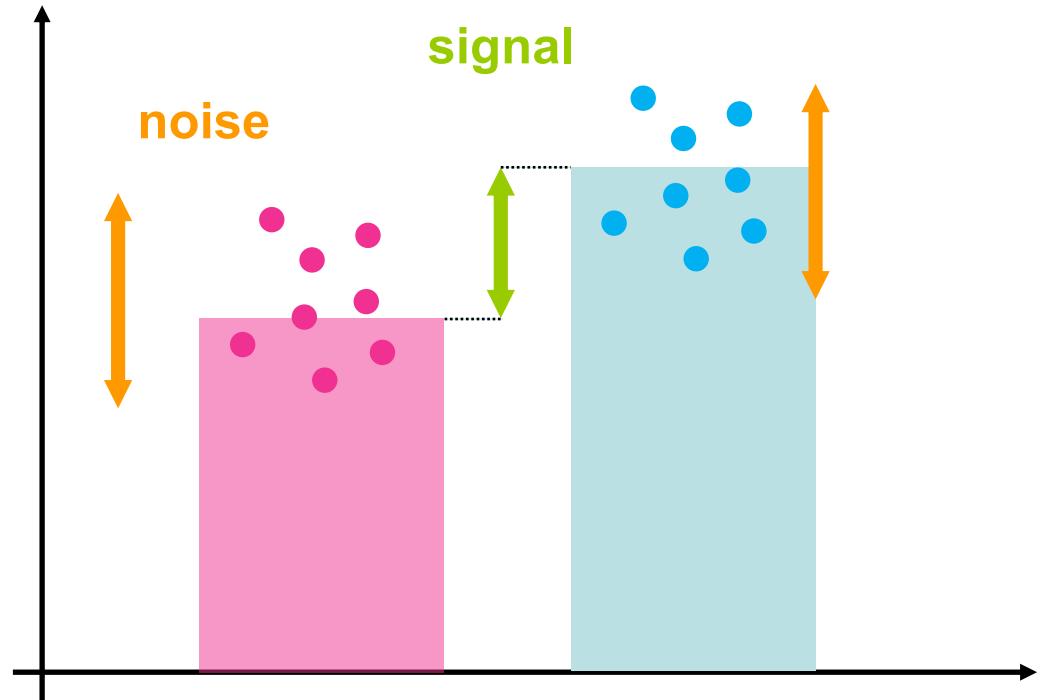
noise

T-tests ::



difference between group means
variability of groups

T-tests ::



difference between group means
variability of groups

T-tests ::

$$\text{Paired } t = \frac{\bar{x}_1 - \bar{x}_2}{s/\sqrt{n}}$$

$$\text{Unpaired } t = \frac{\bar{x}_1 - \bar{x}_2}{s \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

paired t-test

different between
group means
(to maximize)

$$t = \frac{\overline{x_1} - \overline{x_2}}{s / \sqrt{n}}$$

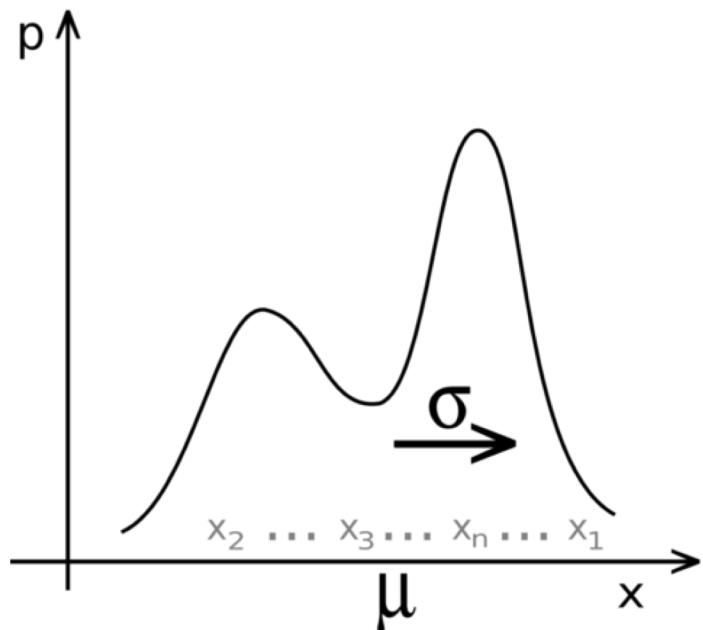
standard deviation
of the differences
(to minimize)

different between
group means
(to maximize)

$$t = \frac{\overline{x_1} - \overline{x_2}}{s / \sqrt{n}}$$

standard error of the mean
(to minimize)

but why do we need to divide by \sqrt{n} (n = sample size)?

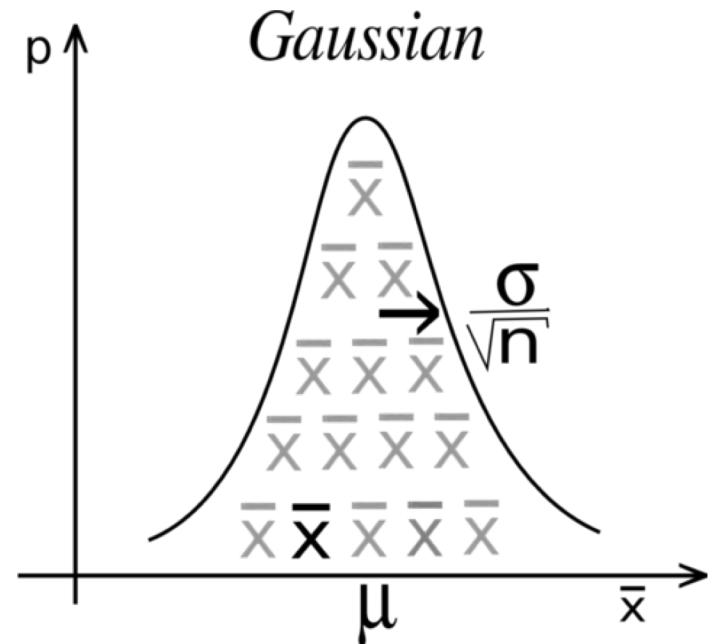


population
distribution

samples
of size n

\bar{x}

\bar{x}



Gaussian

sampling distribution
of the mean

this comes from the **central limit theorem**
you have seen before (lecture 10)

by dividing by \sqrt{n} , we add a “**penalty**” for using a sample instead of the entire population

penalty is large when sample is very small

as sample size increases, penalty diminishes ...

... infinitely approaching point where
sample = the population itself

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s/\sqrt{n}}$$

we get a t-value

(to maximize)

both signal and noise are in the units of your data

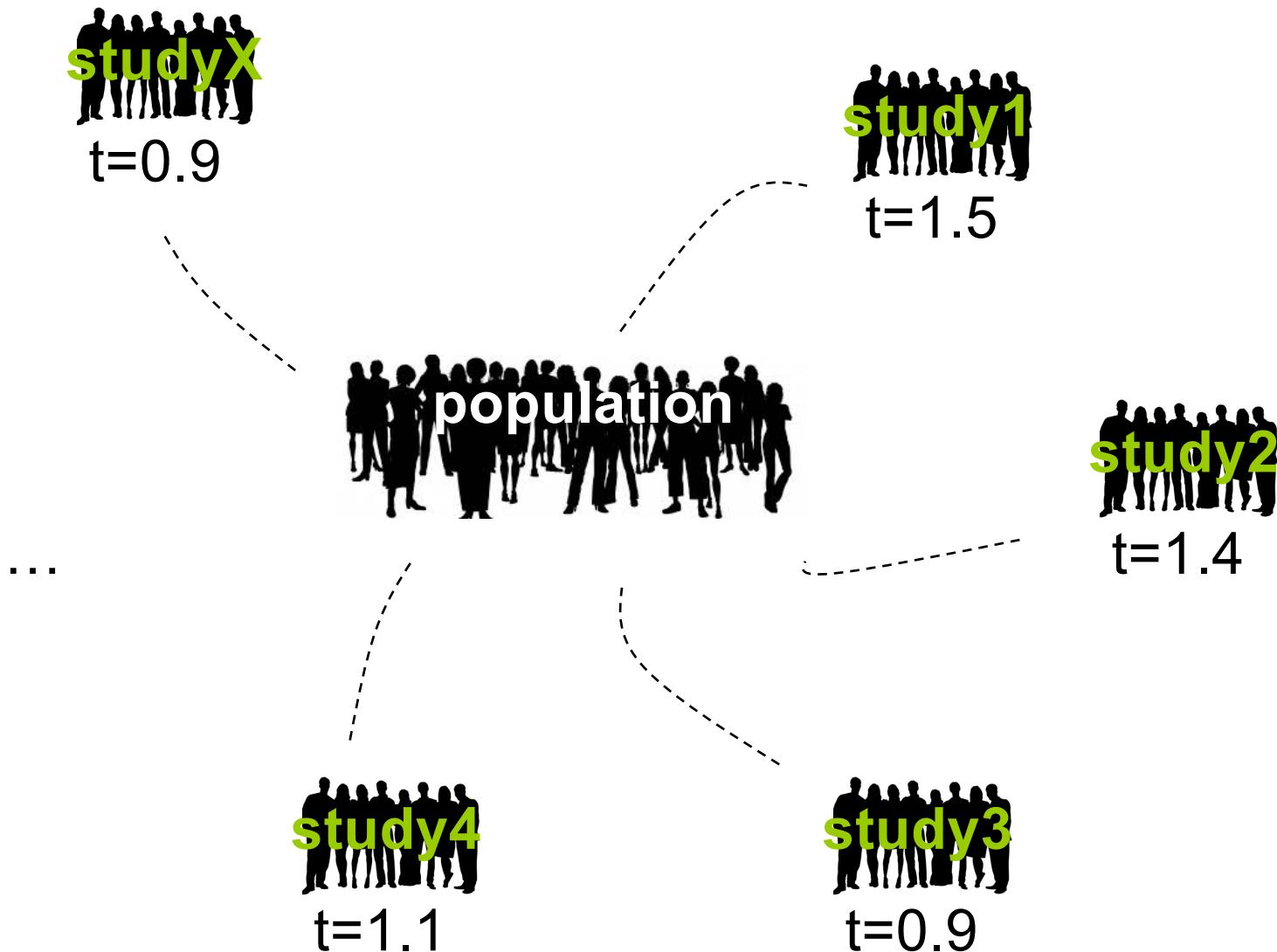
If signal = 6 and noise = 2, your t-value = 3, so the difference is 3 times the size of the standard error

If signal = 6 and noise = 6, your t-value = 1, the signal is at the same scale as the noise

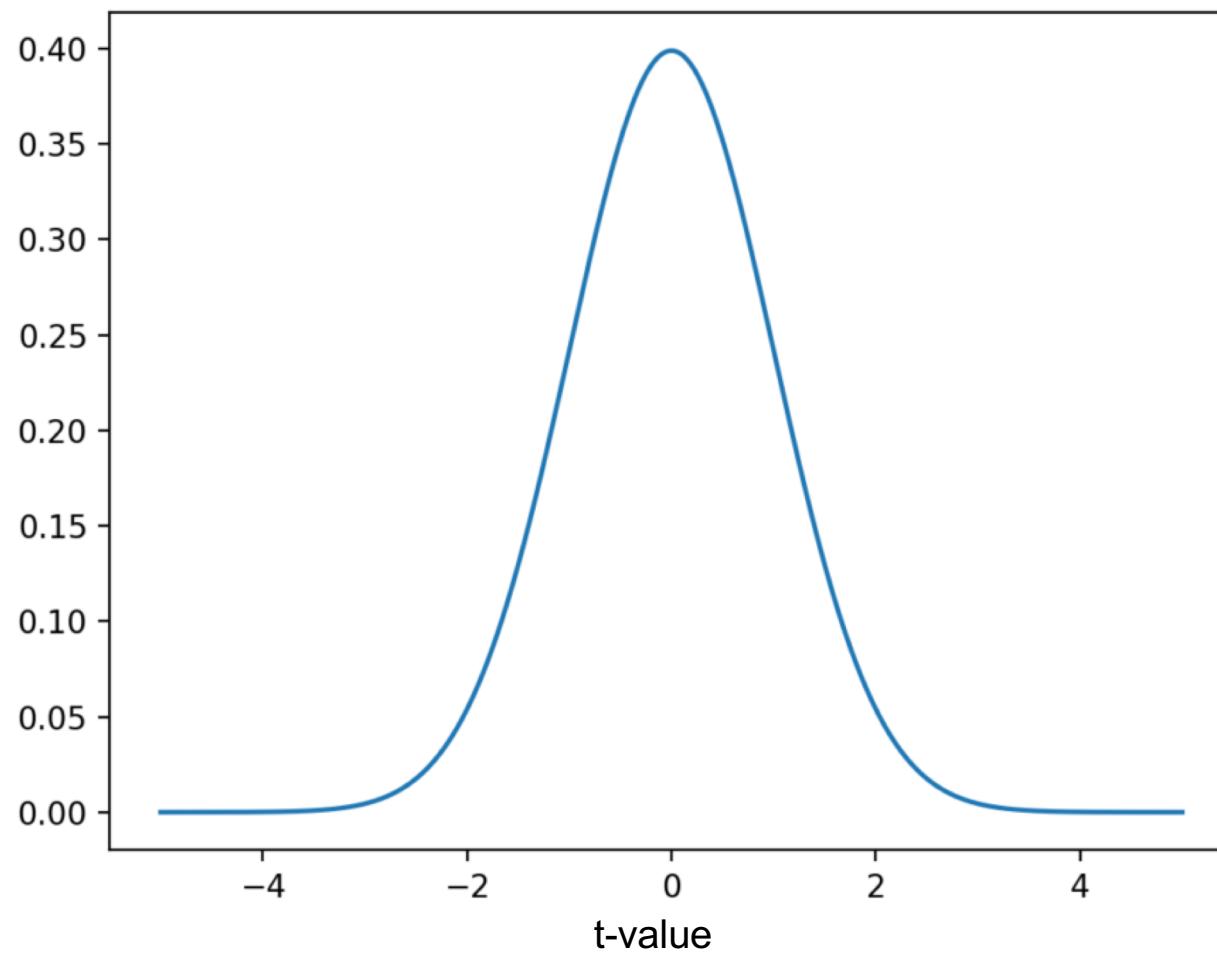
t-values = how distinguishable signal from noise

how do we know our t-value is any good, and how does this related to p-value?

this is where **t-distributions** come in



now let's take all these possible values and ...



... plot a distribution of them

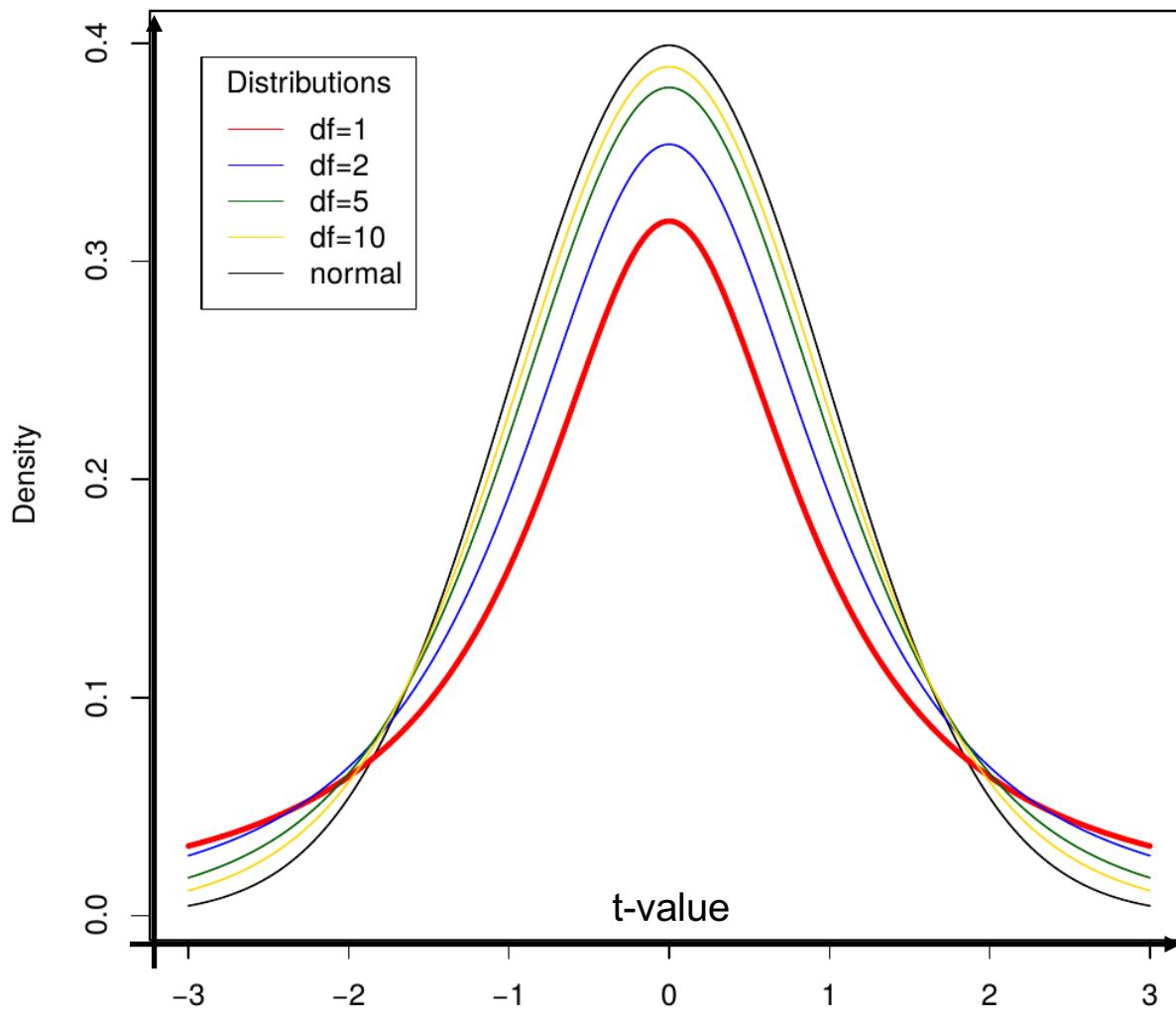
this type of distribution is a **sampling distribution**

fortunately, the properties of t-distributions are well understood in statistics, so we can plot them without having to collect many samples!

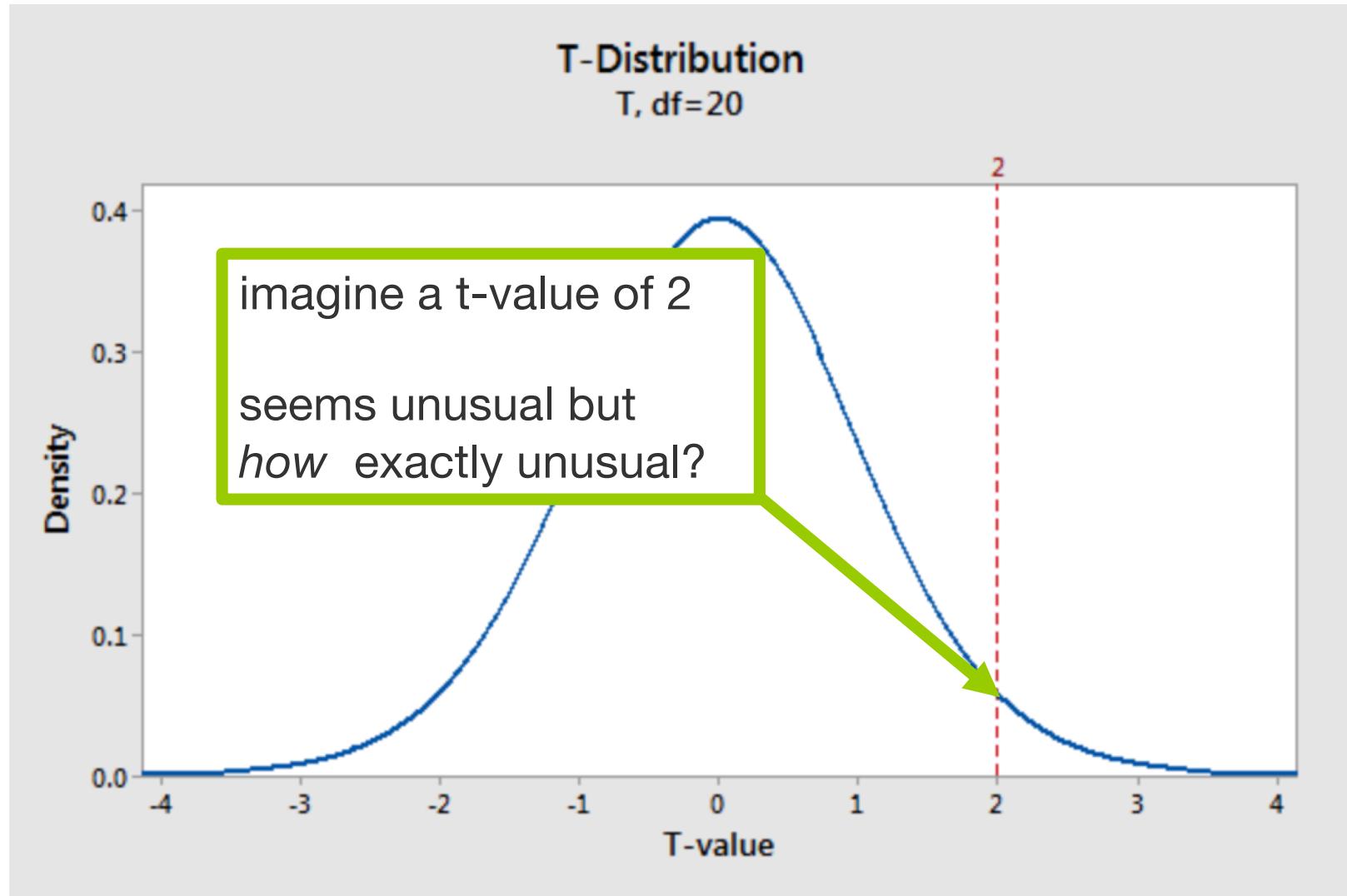
a specific t-distribution is defined by its **degrees of freedom** (DF), a value closely related to sample size (here $n-1$)

different t-distributions exist for every sample size

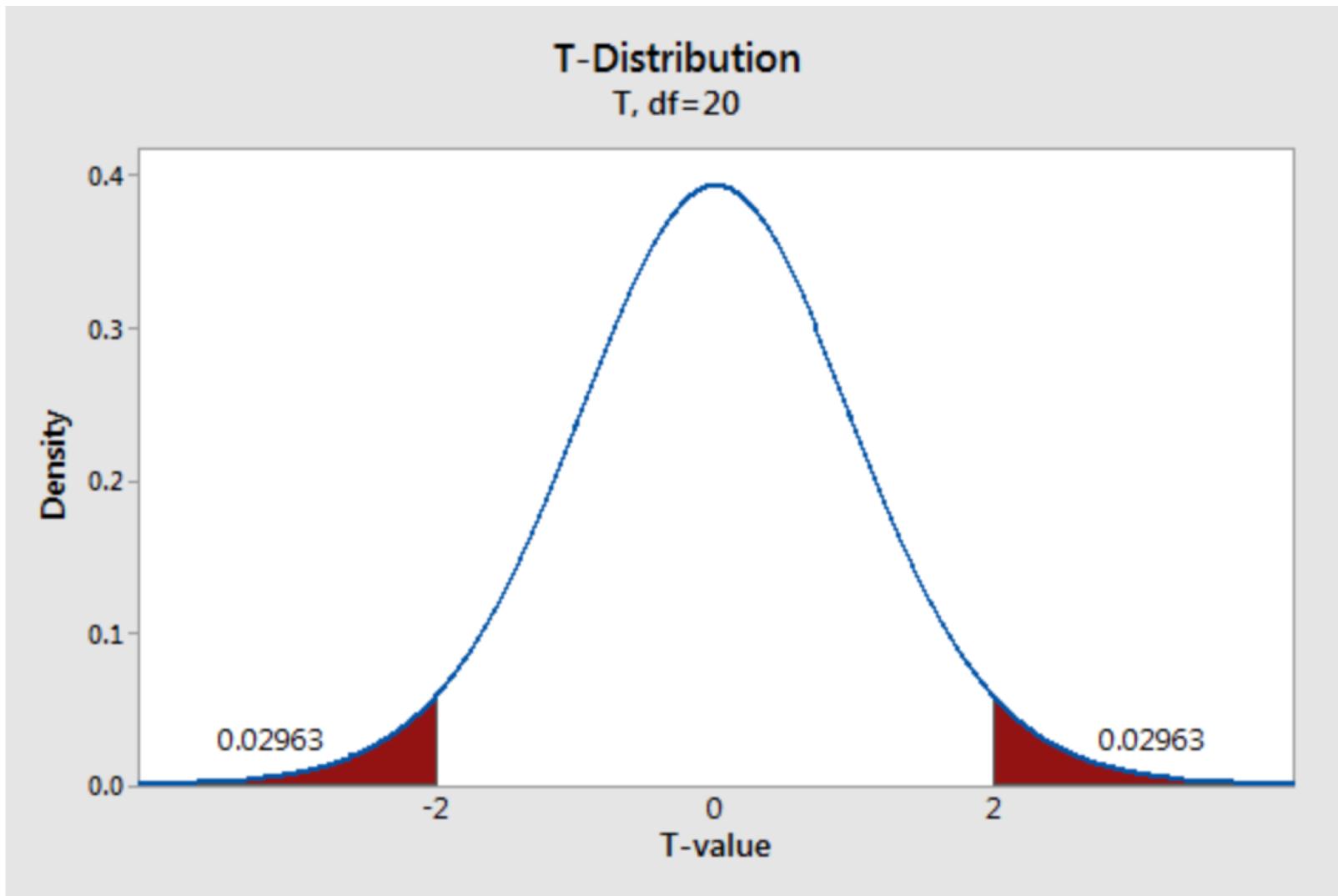
Comparison of t Distributions



t-distributions assume that you draw repeated random samples from a population where the null hypothesis is true. You place the t-value from your study in the t-distribution to determine how consistent your results are with the null hypothesis.



e.g. here a t-distribution (DF =20 which means a sample size of 21). It plots the probability density function (PDF), which describes the likelihood of each t-value.



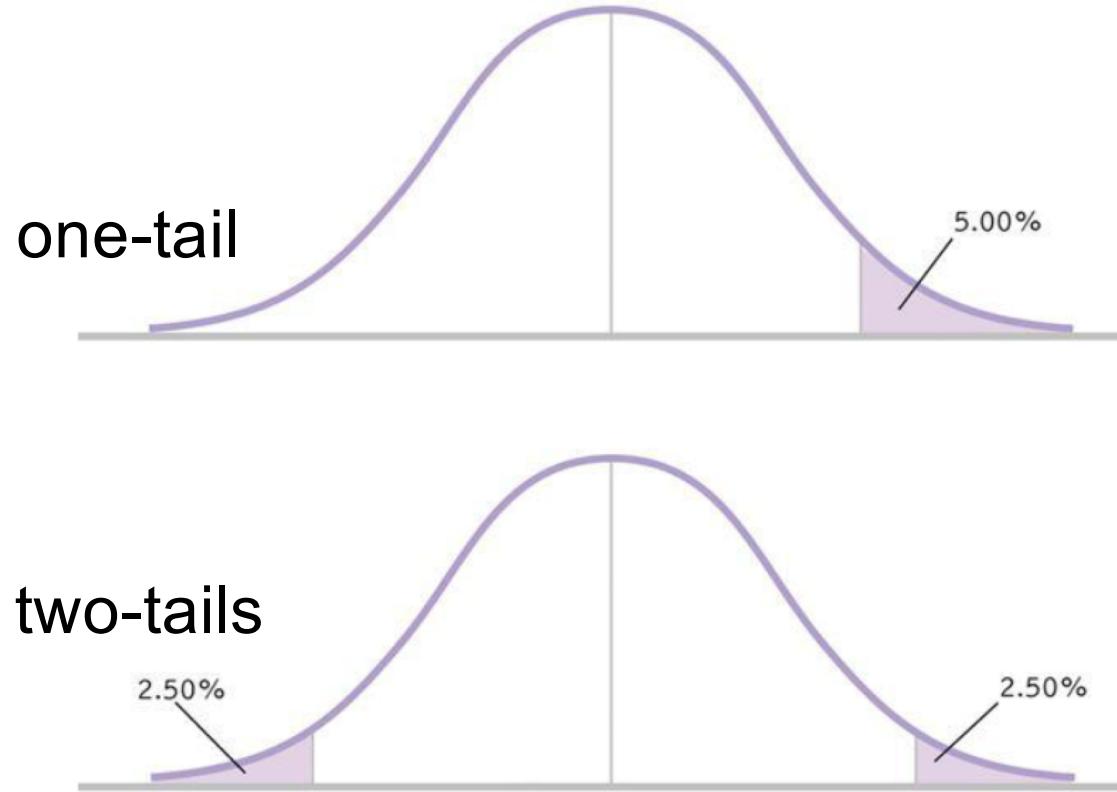
shade the area of the curve with t-values >2 and <-2

each regions has a probability of 0.02963, which sums to a total probability of 0.05926.

when the null hypothesis is true, the t-value falls within these regions nearly 5.9% of the time ...

... this is our pvalue!

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s/\sqrt{n}}$$



it also does explain **one-tail vs. two tails** t-tests: one-tail only case about $t=2$ (not -2 or oppositely), so multiply pvalue by two.

at this point you understand more the **three reasons** of a low p_value (or t_value)

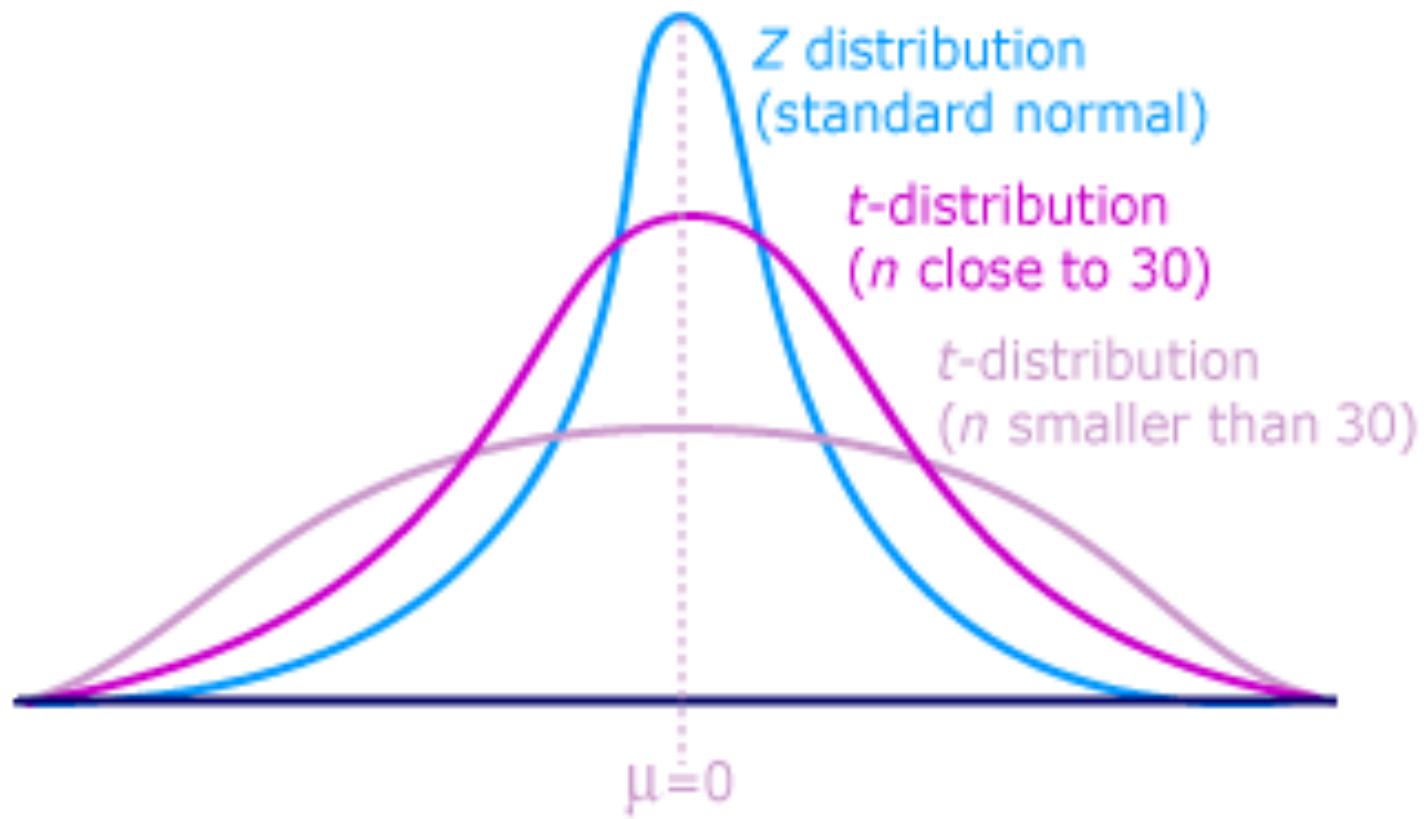
$$t = \frac{\overline{x_1} - \overline{x_2}}{s / \sqrt{n}}$$

1. difference not large enough
(what you are searching for,
your signal is weak)

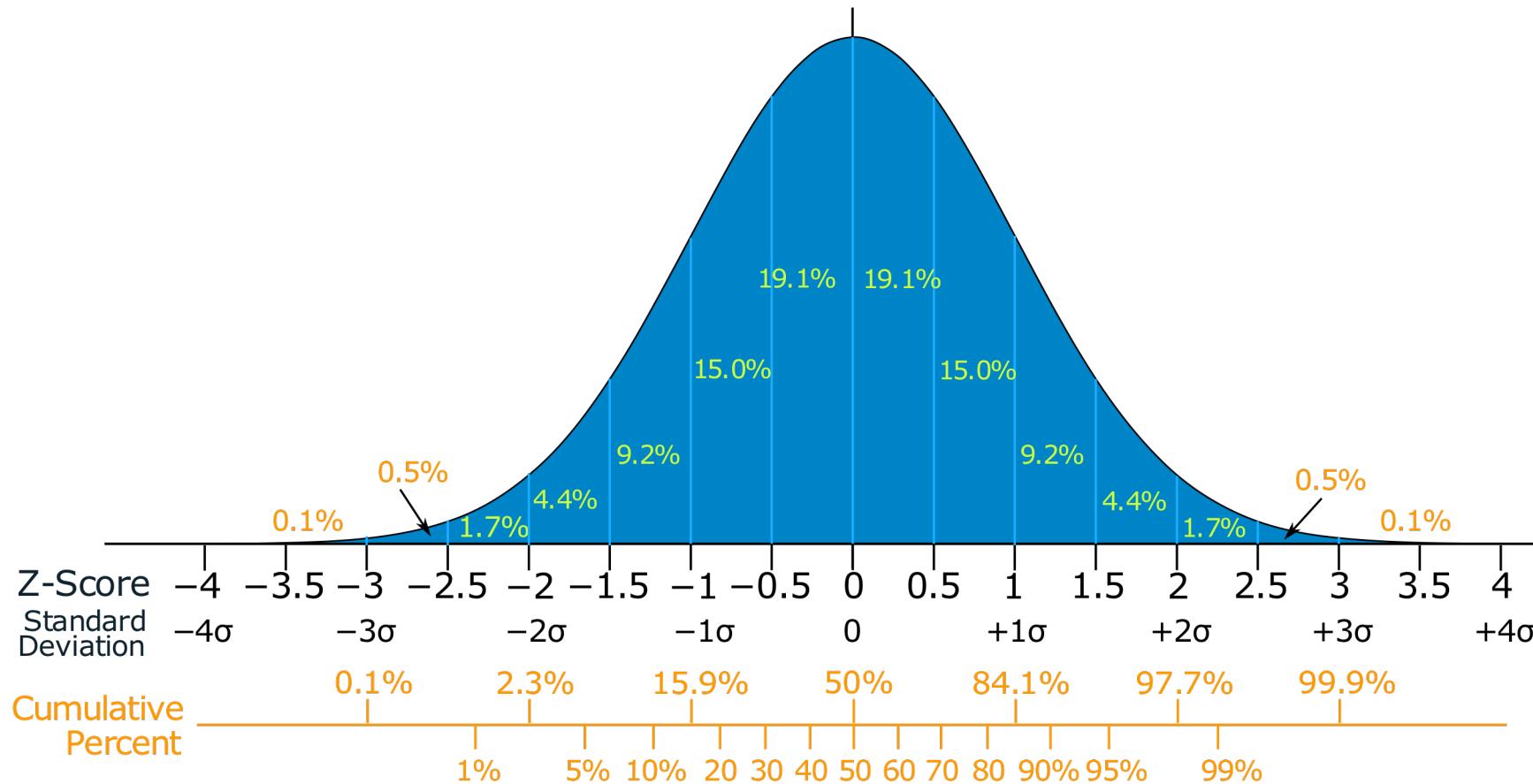
2. too much noise
(could your experimental
design introduce noise?
Check it)

3. not enough data
(run more participants,
gather more trials)

how much data is enough?



the larger the sample size, the more t-distributions become a z-distribution (at around $n=30$), the less the area under the curve to reach a low p_value .



back to using a Z-score (i.e. number of standard deviations from the mean on normal distribution)

so why using a t-test then?

well there are cases when we want to use less sample to speed up the evaluation

this was the case of **William Sealy Gosset** ...



Employee of Guinness, Gosset developed a **small sample** method to select the best yielding varieties of barley.

Biometricalians like Pearson typically had hundreds of observations.

Guinness allowed him to publish his method under the name “Student” to prevent disclosure of confidential information.

Where do t-distrubitions came from?
<https://www.youtube.com/watch?v=NvUDvmrd6fo&feature=youtu.be>

now it does not mean that 2 sample is enough ... a rule of thumb for a simple within experiment is 12-16 participants for example (and twice more for between experiment).

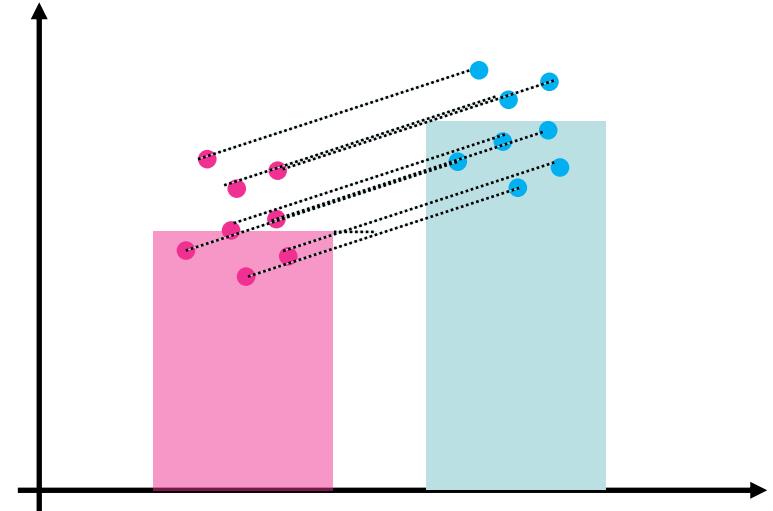
... we will actually look at how to know if your sample size is good in the lecture 18

unpaired t-test

T-tests ::

Paired $t = \frac{\bar{x}_1 - \bar{x}_2}{s/\sqrt{n}}$

Unpaired $t = \frac{\bar{x}_1 - \bar{x}_2}{s \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ similar than paired t-test except this



paired t-test :: divide by \sqrt{n} because data point paired

unpaired t-test :: multiply by $\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

you can do the math: unpaired t-test the denominator (noise) is larger because we add $n_1 + n_2$

... thus why harder to reach low pvalue with unpaired t-test

practically

A	B	C	D	E	F	G
IDs	Before	After	1. difference	Paired T-test example		
1	312	300	12	(In green the initial data, in blue the computation)		
2	242	201	41			
3	340	232	108	steps by steps		
4	388	312	76			
5	296	220	76	1. add new colum to compute the differences between conditions for each participants		
6	254	256	-2	2. compute the mean of the differences (use excel formula =AVERAGE(new column))	56.1111111	
7	391	328	63	3. compute the standard deviation of the differences (use formula =STDEV(new column))	34.173983	
8	402	330	72	4. compute de standard error of the mean difference (divide 3. by SQRT(n))	11.3913277	
9	290	231	59	5. compute t_value, i.e. step 2. divided by step. 4	4.92577449	

(excel file in the git hub repository)

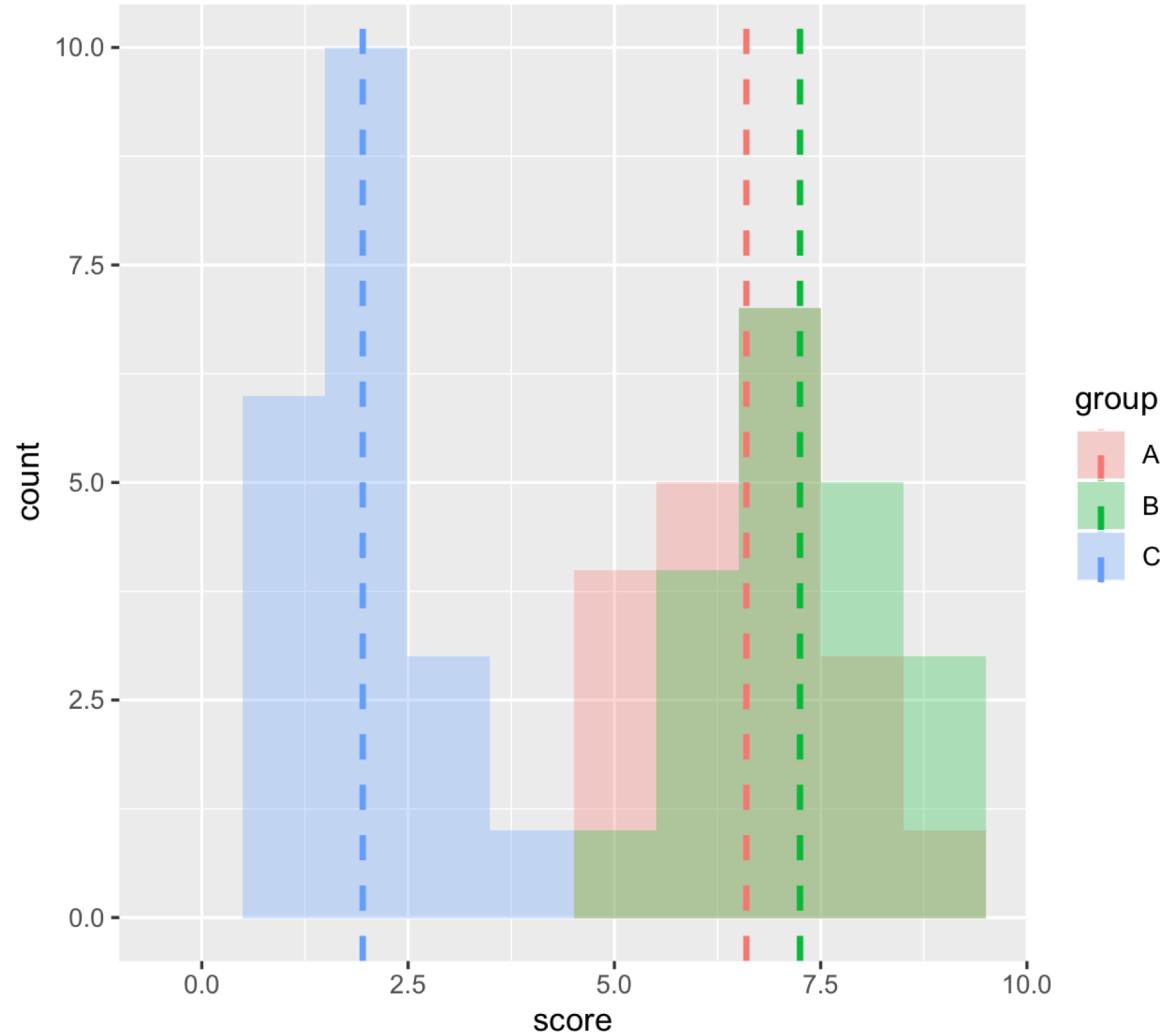
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

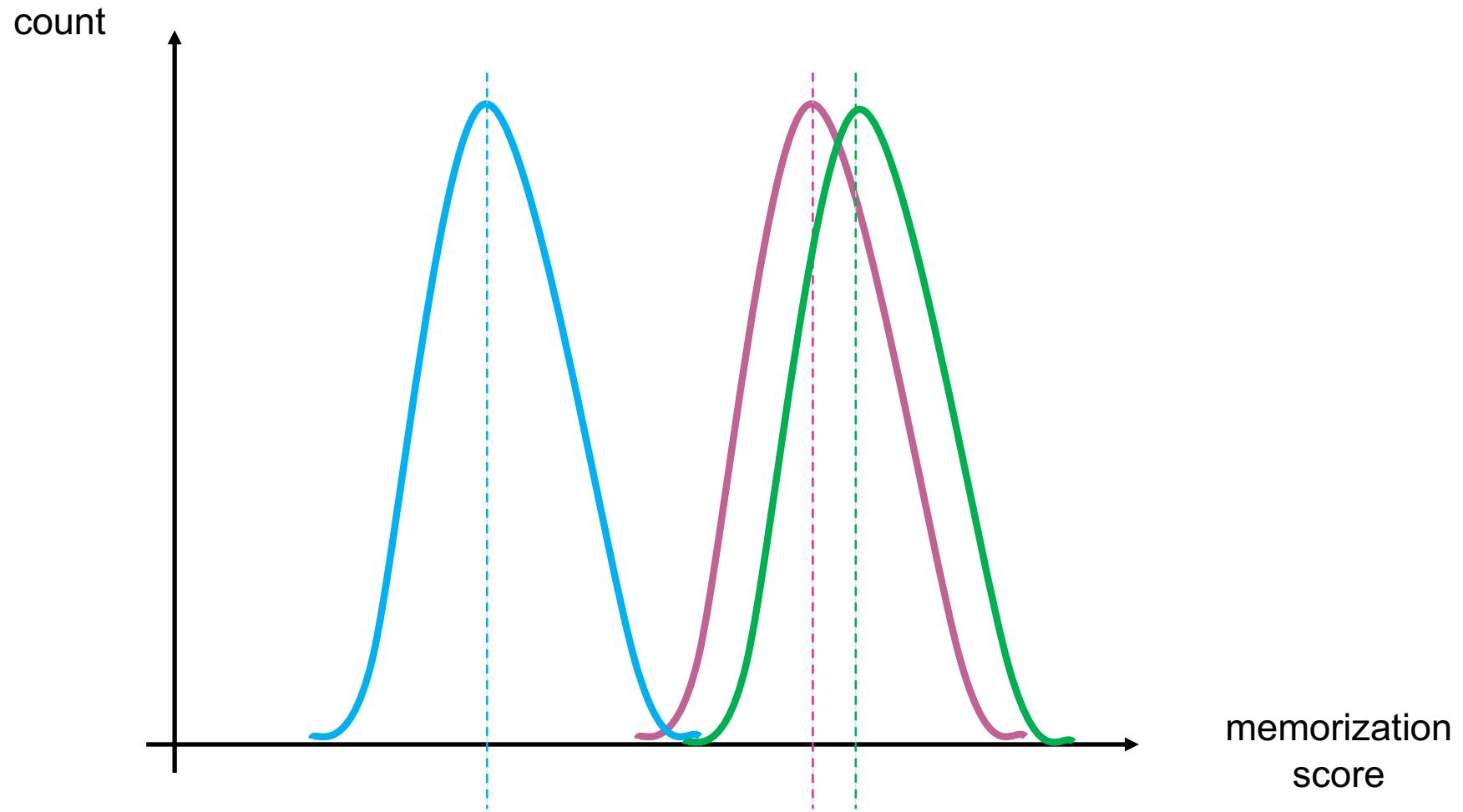
$$s^2 = \frac{\sum_{i=1}^{n_1} (x_i - \bar{x}_1)^2 + \sum_{j=1}^{n_2} (x_j - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

condA	condB	(xa-meanxa) ²	(xb-meanxb) ²	Paired T-test example (In green the initial data, in blue the computation)
134	70	196	961	
146	118	676	289	
104	101	256	0	step by step
119	85	1	256	1. compute the mean of condition A (=AVERAGE) 120
124	107	16	36	2. compute the mean of condition B (=AVERAGE) 101
161	132	1681	961	3. compute the sample size in condition A (=COUNT) 12
107	94	169	49	4. compute the sample size in condition B (=COUNT) 7
83		1369		5. add columns to compute square difference of each x in condition A minus mean condition A
113		49		6. add columns to compute square difference of each x in condition B minus mean condition B
129		81		7. compute the sum of 5. (=SUM) 5032
97		529		8. compute the sum of 6. (=SUM) 2552
123		9		9. compute the nominator of the tvalue (1. minus 2.) 19
				10. compute the variance (square of standard variation of the difference) = 7. + 8. divided by (3. + 4. - 2) 446.11765
				11. compute the denominator of the tvalue (SQRT(10. * (1/n1 + 1/n2)) 10.045276
				12. compute the tvalue 1.8914364

(excel file in the git hub repository)

anovas





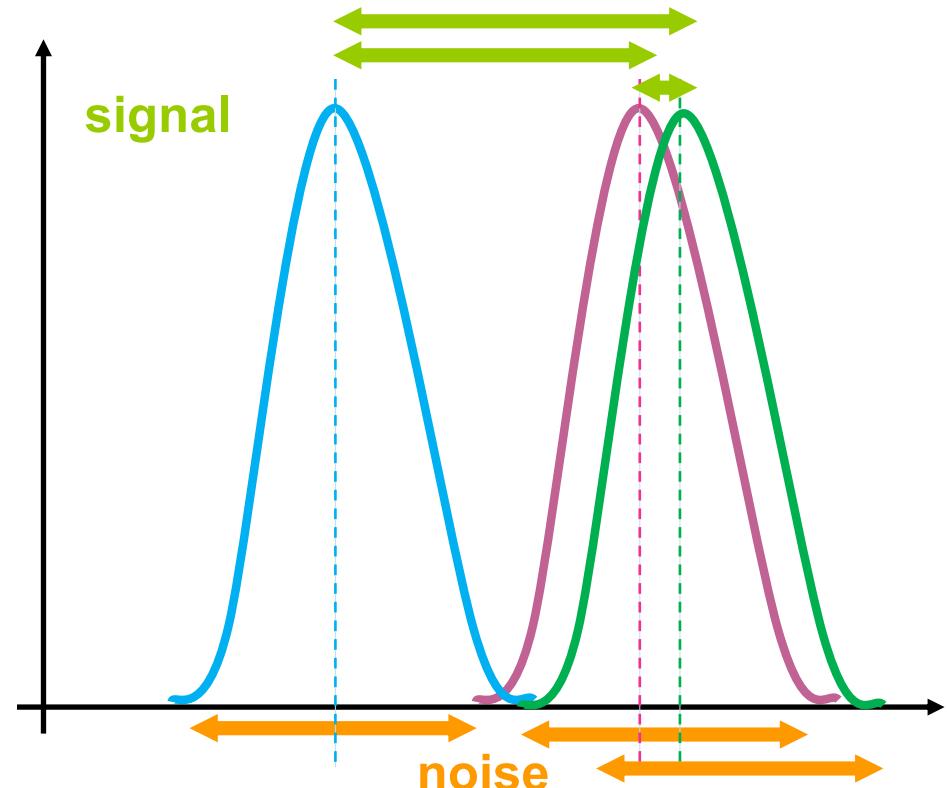
(let's assume again these are normally distributed)

any statistical tests ::

signal

noise

ANOVA ::



difference between group means
variability of groups

ANOVA ::

$$F = \frac{MS_{between}}{MS_{within}}$$

$$MS_{between} = \frac{SS_{between}}{df_{between}}$$

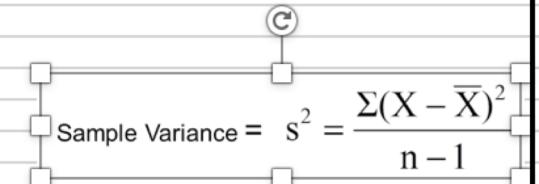
$$MS_{within} = \frac{SS_{within}}{df_{within}}$$

$$SS_{between} = \sum_{j=1}^p n_j (\bar{x}_j - \bar{x})^2$$

$$SS_{within} = \sum_{j=1}^p \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2$$

don't be afraid by this

	Group 1	Group 2	Group 3	Anova by hand (step by step)
n (sample size)	70	70	70	
M (mean)	4	3.7	3.4	1. compute the combined sample size N 210
s^2 (variance)	4.4	5.2	6.1	2. compute the degrees of freedom between (dfbetween) 2 (number of groups - 1)
				3. compute the degrees of freedom within (dfwithin) 207 (n1-1)+(n2-1)+(n3-1)
				the nominator
				4. compute the average mean 3.7
				5. compute the SSbetween 12.6
				6. compute the MSbetween (divide by dfbetween) 6.3
				the denominator
				7. compute the SSwithin 1083.3 (I multiply by ni here as the variance formula has a divisor which we don't need here)
				8. Compute Ms within (divide by dfwithin) 5.233333
				9. compute F 1.203822
				10. find p_value 0.302137 (p value NOT < alpha so DO NOT reject H0)

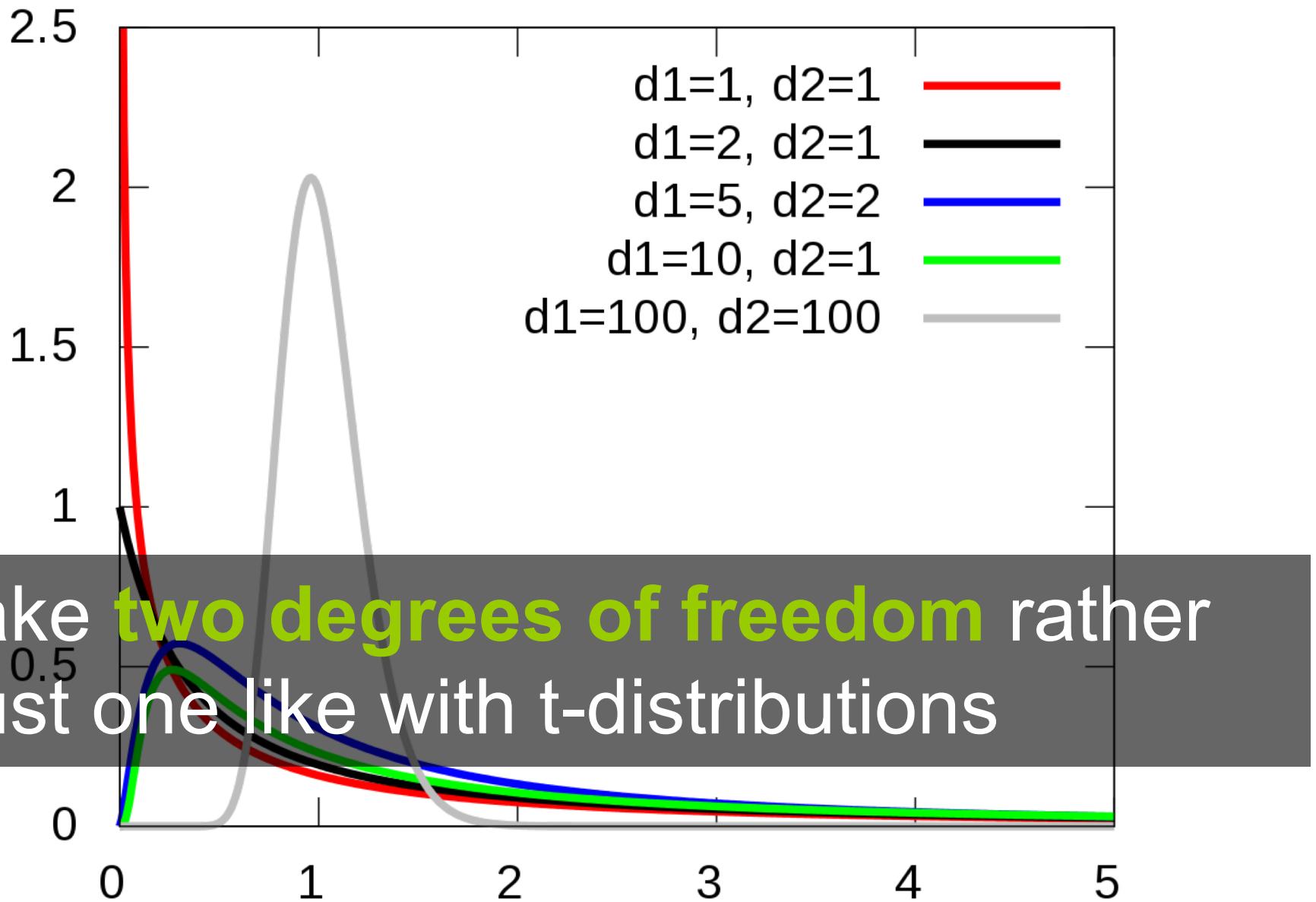


$$\text{Sample Variance} = S^2 = \frac{\sum(X - \bar{X})^2}{n-1}$$

(excel file in the git hub repository)

how do we know if the Fvalue is anygood?

... this is where **F-distributions** come in



F Distribution critical values for P=0.05

↓ Denominator (the within df – also called the error)

Numerator DF (the between df)

DF	1	2	3	4	5	7	10	15	20	30	60	120	500	1000
1	161.45	199.50	215.71	224.58	230.16	236.77	241.88	245.95	248.01	250.10	252.20	253.25	254.06	254.19
2	18.513	19.000	19.164	19.247	19.296	19.353	19.396	19.429	19.446	19.462	19.479	19.487	19.494	19.495
3	10.128	9.5522	9.2766	9.1172	9.0135	8.8867	8.7855	8.7028	8.6602	8.6165	8.5720	8.5493	8.5320	8.5292
4	7.7086	6.9443	6.5915	6.3882	6.2560	6.0942	5.9644	5.8579	5.8026	5.7458	5.6877	5.6580	5.6352	5.6317
5	6.6078	5.7862	5.4095	5.1922	5.0504	4.8759	4.7351	4.6187	4.5582	4.4958	4.4314	4.3985	4.3731	4.3691
7	5.5914	4.7375	4.3469	4.1202	3.9715	3.7871	3.6366	3.5108	3.4445	3.3758	3.3043	3.2675	3.2388	3.2344
10	4.9645	4.1028	3.7082	3.4780	3.3259	3.1354	2.9782	2.8450	2.7741	2.6996	2.6210	2.5801	2.5482	2.5430
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7066	2.5437	2.4035	2.3275	2.2467	2.1601	2.1141	2.0776	2.0718
20	4.3512	3.4928	3.0983	2.8660	2.7109	2.5140	2.3479	2.2032	2.1241	2.0391	1.9463	1.8962	1.8563	1.8498
30	4.1709	3.3159	2.9223	2.6896	2.5336	2.3343	2.1646	2.0149	1.9317	1.8408	1.7396	1.6835	1.6376	1.6300
60	4.0012	3.1505	2.7581	2.5252	2.3683	2.1666	1.9927	1.8365	1.7480	1.6492	1.5343	1.4672	1.4093	1.3994
120	3.9201	3.0718	2.6802	2.4473	2.2898	2.0868	1.9104	1.7505	1.6587	1.5544	1.4289	1.3519	1.2804	1.2674
500	3.8601	3.0137	2.6227	2.3898	2.2320	2.0278	1.8496	1.6864	1.5917	1.4820	1.3455	1.2552	1.1586	1.1378
1000	3.8508	3.0047	2.6137	2.3808	2.2230	2.0187	1.8402	1.6765	1.5811	1.4705	1.3318	1.2385	1.1342	1.1096

Example: F for df = 2,207 is 3.0718

also in form of table (here for the dfwithin
and dfbetween of our excel example)



```
# first we run the one-way anova  
library(ez)  
ezANOVA(dat,id,between=group,dv=score)
```

Effect	DFn	DFd	F	p	p<.05
group	2	57	154.8886	9.056612e-24	

ges
* 0.8445923

ok something is going
to be interesting here

```
# second, run the pairwise comparison
```

```
pairwise.t.test(dat$score,dat$group, paired=FALSE,  
p.adjust.method="bonferroni")
```

A	B
B	0.16
C	<2e-16

here are significant differences

and we don't need to do the Bonferroni
correction (already included)

this was from last week, R gives us all this
numbers

degrees of
freedom

degrees of freedom::

the number of values in the final calculation of
a **statistic** that are free to vary

a complex notion but here is the intuition ...

you have 7 hats, you want to wear one different every day for a week

Monday: 7 choices

Tuesday: 6 choices

Wednesday: 5 choices

Thursday: 4 choices

Friday: 3 choices

Saturday: 2 choices

Sunday: **NO choice**



degrees of freedom is $7-1$

if we have 7 observations and the mean of these observation (that you need to do t-test or Anova) your degree of freedom is 7-1

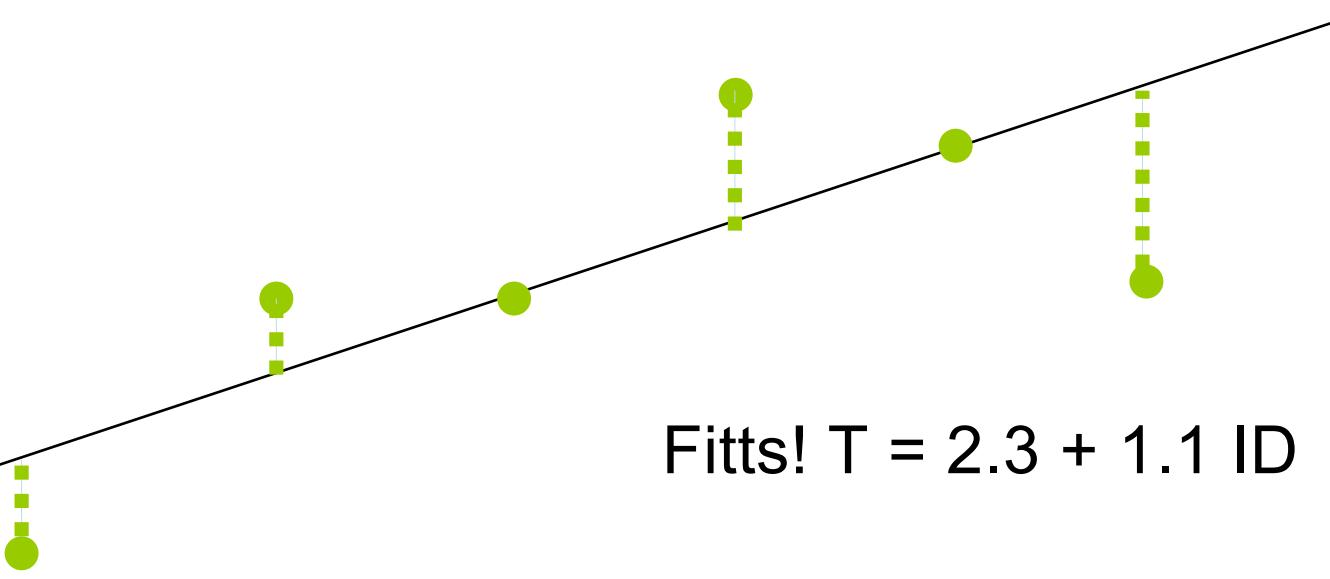
because if you know 6 observations you automatically know the 7th one (thanks to the mean)

remember the lecture on regression?

standard error of the estimate



T time
(ms)



$$\text{Fitts! } T = 2.3 + 1.1 \text{ ID}$$

$$\sqrt{\frac{\sum (\text{estimated } \hat{y} - \text{actual } y)^2}{(\text{sample size}-2)}}$$

ID index of
difficulty

also called degree of freedom

a simple (approximate) way to understand this is that we have two variables, the slope and the intercept of the regression line

that give us extra information, thus the minus 2

summary

1. Explain the general equation of a statistical test (signal/noise)
2. Explain how a t-test is computed and be able to do it by hand
3. Explain the 3 reasons why a t-value can be low (signal too low, noise too high, small sample)
4. Explain how a Anova is computed, and be able to do it by hand
5. Explain what are t-distributions and f-distributions

take away

end