Problem Sheet 2

Useful facts

- **Expected value**. For a discrete random variable with probability $p(x)$ this is

$$\langle g(X)\rangle = \sum_x p(x)g(x) \tag{1}$$

For a continuous random variable with density $f(x)$ this is

$$\langle g(X)\rangle = \int_{-\infty}^{\infty} f(x)g(x)dx \tag{2}$$

- **Mean and variance**. The mean is $\mu = \langle X\rangle$ and the variance is $\sigma^2 = \langle (X-\mu)^2\rangle = \langle X^2\rangle - \mu^2$.

- **Binomial distibution**. For n independent trials each with p change of success and $q = 1-p$ of failure, the probability of r successes is

$$p(r) = \begin{pmatrix} n \\ r \end{pmatrix} p^r q^{n-r} \tag{3}$$

and $\mu = pn$, $\sigma^2 = pqn$.

- **Poisson distribution**. This has

$$p(r) = \frac{\lambda^r}{r!}e^{-\lambda} \tag{4}$$

where $\mu = \lambda$ and $\sigma^2 = \lambda$.

- **Integrating a polnomial**

$$\int x^n dx = \frac{x^{n+1}}{n+1} \tag{5}$$

so the definite integral is

$$\int_a^b x^n dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1} \tag{6}$$

Questions

Four questions, each worth two marks with two marks for attendance.

1. The illusionist Derren Brown famously flipped a coin on camera so that it landed heads ten times in a row; he claimed that this was because of his mind powers, in fact it was because of his patience, he simply kept trying the trick again and again until it worked. It took him nine hours. What is the probability of a coin landing heads ten times in a row? If you flip a coin ten times what is the probability of getting five heads and five tails?

2. A fisher catches on average one fish every 25 minutes. What is the probability that they catche no fish in an hour?

3. The distribution of tree heights in a christmas tree forest is

$$p(h) = \begin{cases} 0.3 & 0 \leq h < 2 \\ 0.2 & 2 \leq h < 4 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

What is the mean height of trees in the forest?

4. Like the binomial distribution the geometric probability distribution is related to a series of independent trials where each trial has probability p of success and $q = 1 - p$ of failure. The geometric probability $p(r)$ is the probability that the rth trial is the first success. It is

$$p(r) = q^{r-1}p \tag{8}$$

It can be shown that

$$\sum_{r=1}^{\infty} p(r) = 1 \tag{9}$$

as it must be. You can assume that here. What is the mean of the geometric probability?

Extra questions

These are for you to do on your own, not for handing up. Solutions will be included in the solutions section. I haven't added these question yet, but they will be added to the online version of this problem sheet over the next couple of days. I have also added extra questions to ps1.