16

Non-Parametric

Statistical tests

Probability and Statistics

COMS10011

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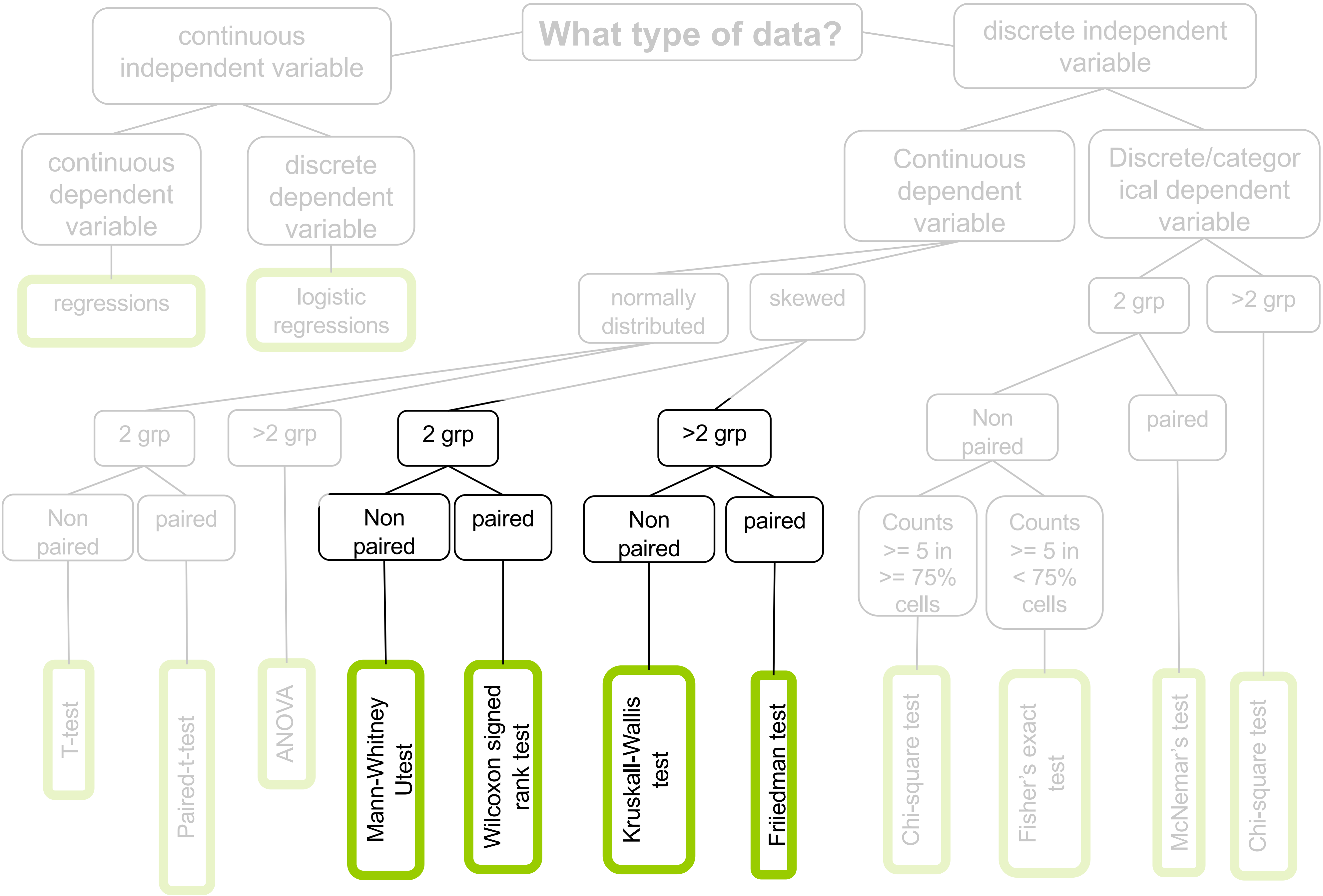
(Thanks S. Massa, Oxford)



What type of data?

continuous independent variable

discrete independent variable

continuous dependent variable

discrete dependent variable

Continuous dependent variable

Discrete/categorical dependent variable

regressions

logistic regressions

normally distributed

skewed

2 grp

>2 grp

2 grp

>2 grp

2 grp

>2 grp

Non paired

paired

Non paired

paired

Non paired

paired

Non paired

paired

Counts >= 5 in >= 75% cells

Counts >= 5 in < 75% cells

T-test

Paired-t-test

ANOVA

Mann-Whitney Utest

Wilcoxon signed rank test

Kruskall-Wallis test

Friiedman test

Chi-square test

Fisher's exact test

McNemar's test

Chi-square test

today::

we will look at **four non-parametric tests**

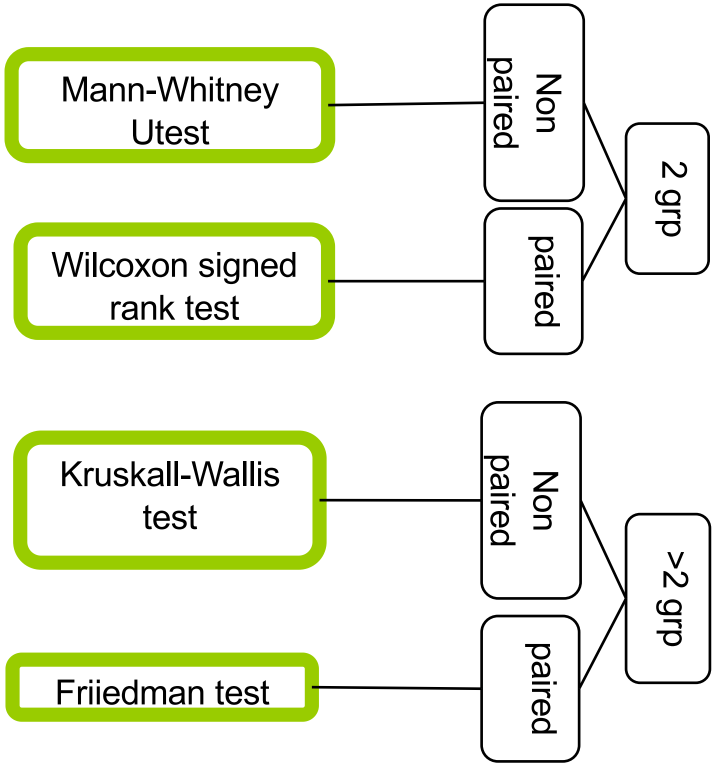
do some recap games about **what tests to use and when** comparing the means of two populations is very important

in the last lecture we saw what we can do if we assume that the samples are normally distributed

for large sample sizes, we can invoke the **central limit theorem** to claim that data are approximately normal

but in some cases the data are **NOT normal**, and

the sample size is too small to invoke the CLT four non-parametric tests are very robust: the significance level is known regardless of the distribution of the data, but nothing is perfect: what **you gain in robustness** **you lose in power**.



Mann Whitney by hand
pdf in GitHub repository
http://www.real-statistics.com/non-parametric-tests/mann-whitney-test/

unpaired t-test equivalent

rank sum test (Mann Whitney)

received drug A	9	9.50	9.75	10	13	9.50
received drug B	11.50	12	9	11.50	13.25	13

(different sets of participants for each)

1. rank the observations according to their size relative to the whole sample.

	9	**9**	**9.50**	**9.50**	**9.75**	**10**	**11.50**	**11.50**	**12**	**13**	**13**	**13.25**
rank	1	2	3	4	5	6	7	8	9	10	11	12
modified rank	1.5	1.5	3.5	3.5	5	6	7.5	7.5	9	10.5	10.5	12

(when ties – average the rank)

2. add up the ranks for the observations which came from smaller group. The sum of ranks in sample 2 is now determinate, since the sum of all the ranks equals N(N + 1)/2 where N is the total number of observations.

our statistic R is

$$R_1 - \frac{n_1(n_1+1)}{2}$$

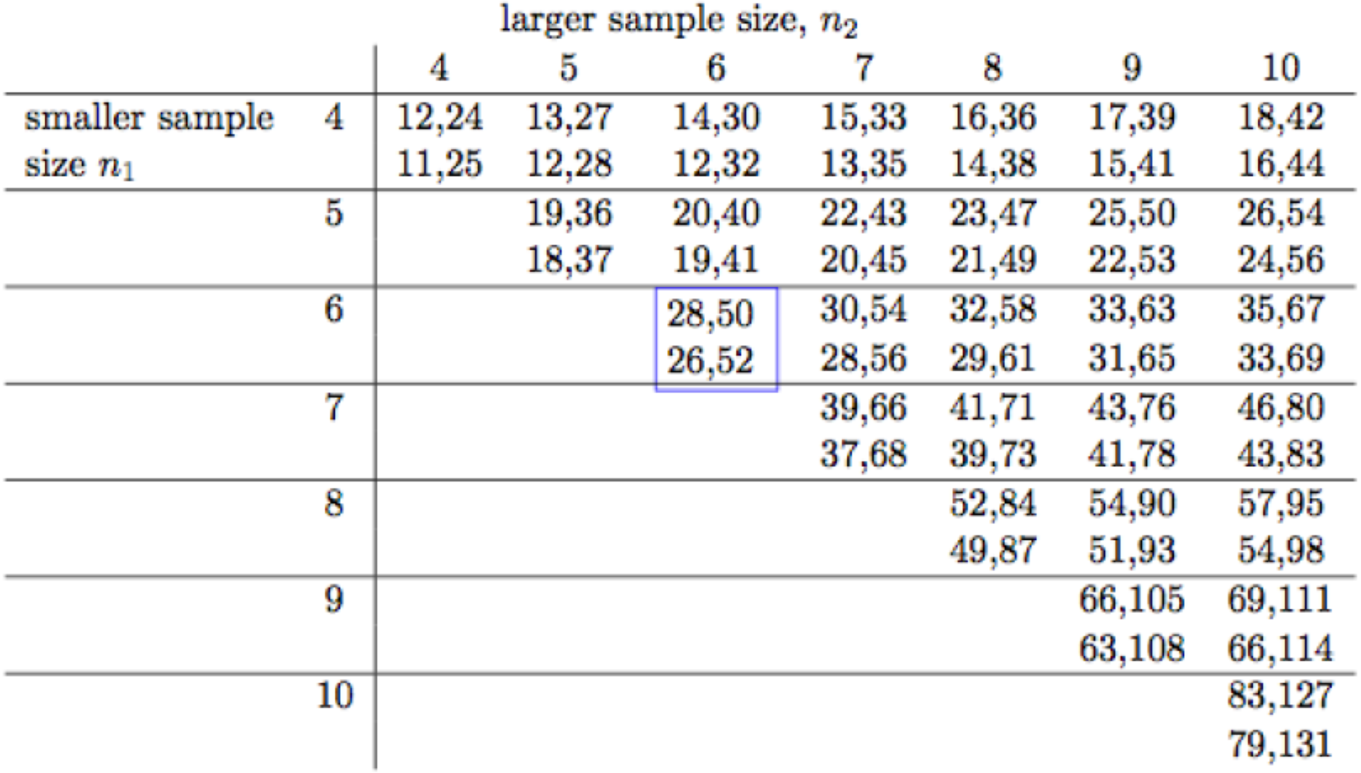
	9	**9**	**9.50**	**9.50**	**9.75**	**10**	**11.50**	**11.50**	**12**	**13**	**13**	**13.25**
modified rank	1.5	1.5	3.5	3.5	5	6	7.5	7.5	9	10.5	10.5	12

here we have the same sample size for each group so we can take any, e.g. **R (drug B) = 9**

```
#wilcox.test do both paired (Mann whitney test)
and unpaired, so paired = TRUE would run the
Wilcoxon sign rank test, otherwise the Mann
Whitney (sometime called Wilcoxon sum rank test)

y1<- c(9,9.50, 9.75, 10,13, 9.50)
y2<- c(11.50,12,9,11.50,13.25, 13)
wilcox.test(y1,y2,paired=FALSE)
```

```
data:  y1 and y2
W = 9, p-value = 0.1705
alternative hypothesis: true location shift is not
equal to 0
```

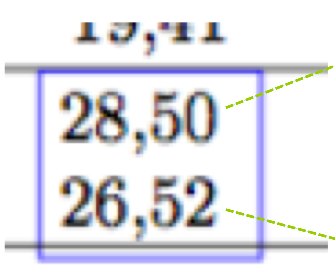
3. we then look in the critical table



		larger sample size, n_2						
		4	5	6	7	8	9	10
smaller sample size n_1	4	12,24 11,25	13,27 12,28	14,30 12,32	15,33 13,35	16,36 14,38	17,39 15,41	18,42 16,44
	5		19,36 18,37	20,40 19,41	22,43 20,45	23,47 21,49	25,50 22,53	26,54 24,56
	6			28,50 26,52	30,54 28,56	32,58 29,61	33,63 31,65	35,67 33,69
	7				39,66 37,68	41,71 39,73	43,76 41,78	46,80 43,83
	8					52,84 49,87	54,90 51,93	57,95 54,98
	9						66,105 63,108	69,111 66,114
	10							83,127 79,131

rows and columns correspond to the sizes of the smaller and larger samples, respectively.

… why two values?



19,41
28,50
26,52

the top gives the 10% critical values = **one-tail test**

the bottom the 5% ones = **two-tail test**

R = 9 < 26.52 (let’s say we do a two tails)

so we **reject the null hypothesis** and conclude that the two groups are significantly different note that the critical value table only goes up to n =10

for larger samples we can use normal approximation

$$
\begin{aligned}
z &= \frac{R-\mu}{\sigma}, \\
\mu &= \frac{1}{2} n_x (n_x + n_y + 1), \\
\sigma &= \sqrt{\frac{n_x n_y (n_x + n_y + 1)}{12}}.
\end{aligned}
$$

we then compare with the normal table, e.g. for two-tailed test at 0.05 reject null if $|z|$ > 1.96

Wilcoxon by hand
pdf in GitHub repository
http://www.real-statistics.com/non-parametric-tests/wilcoxon-signed-ranks-test/

paired t-test equivalent

signed rank test (Wilcoxon)

very quite similar but this time our data are paired (each participants made the two conditions so we have two data points per participants)

example: we measured the effect of two car seats on level of discomfort, here are the differences for 19 participants

-0.525, 0.172, -0.577, 0.200, 0.040, -0.143, 0.043, 0.010, 0.000, -0.522, 0.007, -0.122, -0.040, 0.000, -0.100, 0.050, -0.575, 0.031, -0.060

1. rank the observations **by absolute values** and removing the zeros

0.007	0.010	0.031	0.040	-0.040	0.043	0.050	-0.060	-0.100
1	2	3	4.5	4.5	6	7	8	9

-0.122	-0.143	0.172	0.200	-0.522	-0.525	-0.575	-0.577
10	11	12	13	14	15	16	17

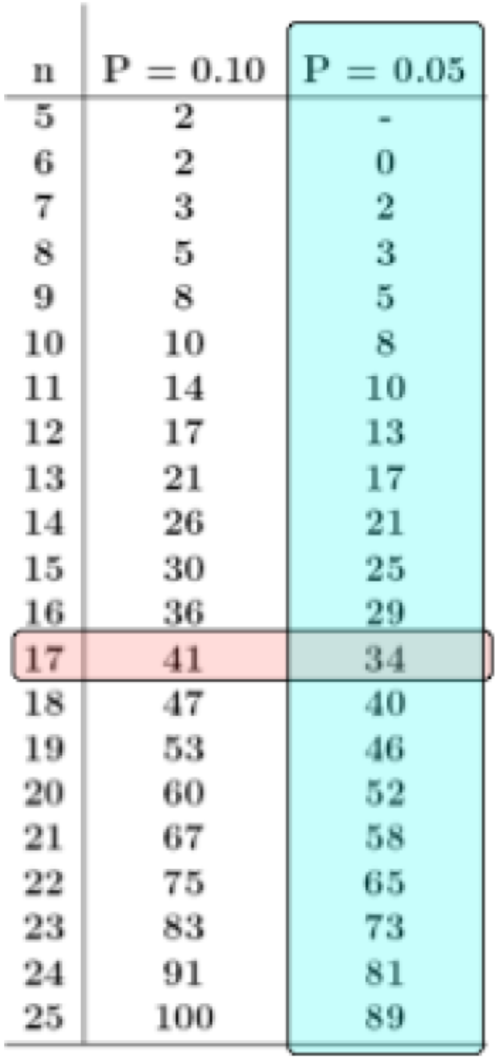
2. we then compute R+ (sum of ranks for only positive differences) and R- (sum of ranks for negative differences)

R+ = 48.5
R- = 104.5

3. We take the min of the two (call this T)

T = 48.5

4. we then compare with appropriate table



n	P = 0.10	P = 0.05
5	2	-
6	2	0
7	3	2
8	5	3
9	8	5
10	10	8
11	14	10
12	17	13
13	21	17
14	26	21
15	30	25
16	36	29
17	41	34
18	47	40
19	53	46
20	60	52
21	67	58
22	75	65
23	83	73
24	91	81
25	100	89

we computed T = 48:5

since we dropped two values (zeros) our sample size is 19-2=17.

we found the critical value of 34 at the 5% level.

since 48.5 > Tcric of 34, we can’t reject the null hypothesis, therefore **effect of these seats are not significantly different** rather simple no?

Kruskal Wallis and Friedman, which are the non-parametric ANOVA equivalent, work on a very similar principles but for more groups depending if they are paired or not (within or between)

Kruskal Wallis by hand
pdf in GitHub repository
http://www.real-statistics.com/one-way-analysis-of-variance-anova/kruskal-wallis-test/

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{g} \frac{\bar{r}_{i\cdot}^{2}}{n_i} - 3(N+1)$$

ANOVA between subject equivalent

Kruskal Wallis

Friedman by hand
pdf in GitHub repository
http://www.real-statistics.com/anova-repeated-measures/friedman-test/

$$Q = \frac{12n}{k(k+1)} \sum_{j=1}^{k} \left(\bar{r}_{\cdot j} - \frac{k+1}{2} \right)^2$$

ANOVA within subject (also called repeated measure ANOVA) equivalent

Friedman

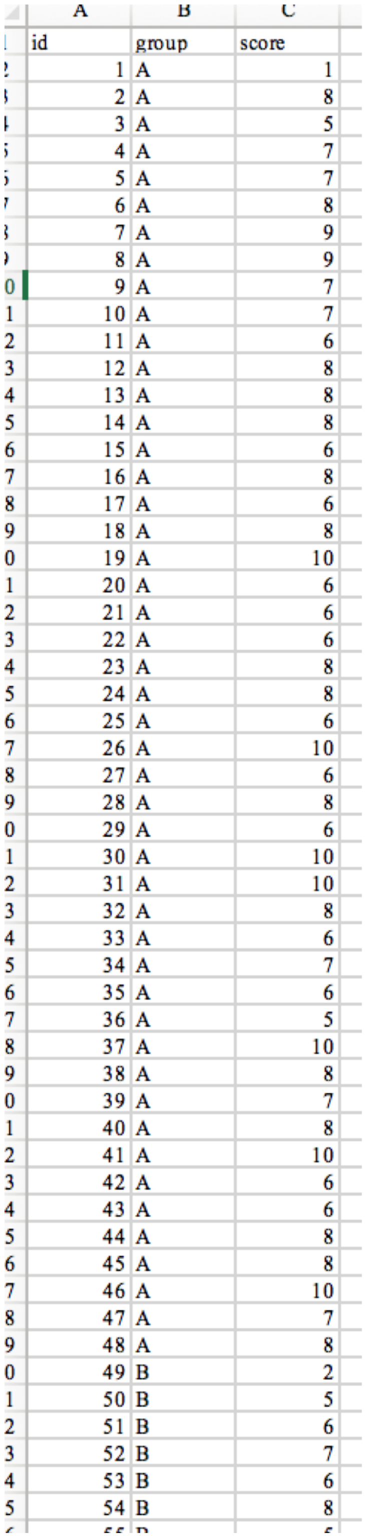
practically

and as data we will take one set we know well: **our experiment on reward vs. punishment**

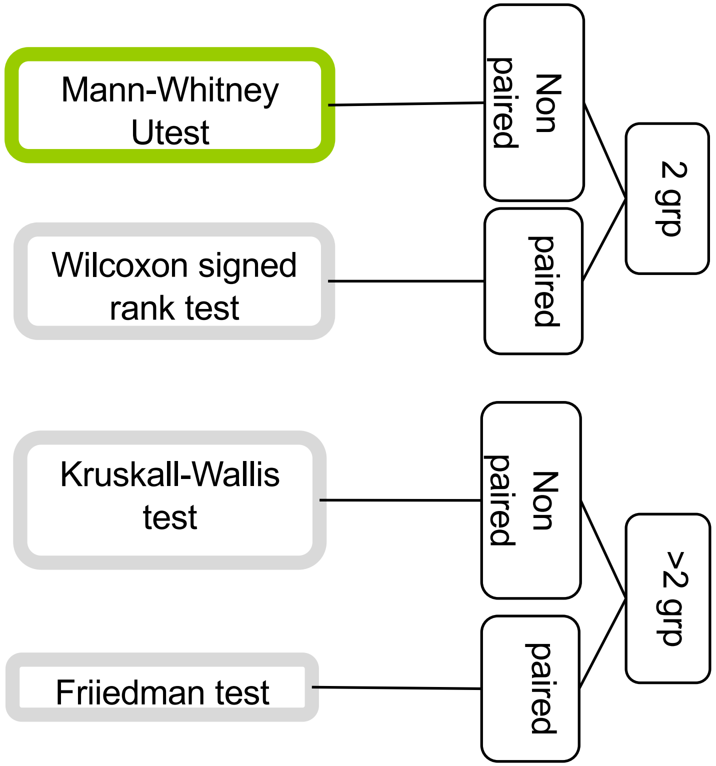
remember we assume the data was normal but it was absolutely not!

so now we will finally be able to conclude!

here is our data (chocolate vs. baseline)



A	B	C
id	group	score
1	A	1
2	A	8
3	A	5
4	A	7
5	A	7
6	A	8
7	A	9
8	A	9
9	A	7
10	A	7
11	A	6
12	A	8
13	A	8
14	A	8
15	A	6
16	A	8
17	A	6
18	A	8
19	A	10
20	A	6
21	A	6
22	A	6
23	A	8
24	A	8
25	A	6
26	A	10
27	A	6
28	A	8
29	A	6
30	A	10
31	A	10
32	A	8
33	A	6
34	A	7
35	A	6
36	A	5
37	A	10
38	A	8
39	A	7
40	A	8
41	A	10
42	A	6
43	A	6
44	A	8
45	A	8
46	A	10
47	A	7
48	A	8
49	B	2
50	B	5
51	B	6
52	B	7
53	B	6
54	B	8



2 grp

Non paired

Mann-Whitney Utest

paired

Wilcoxon signed rank test

>2 grp

Non paired

Kruskall-Wallis test

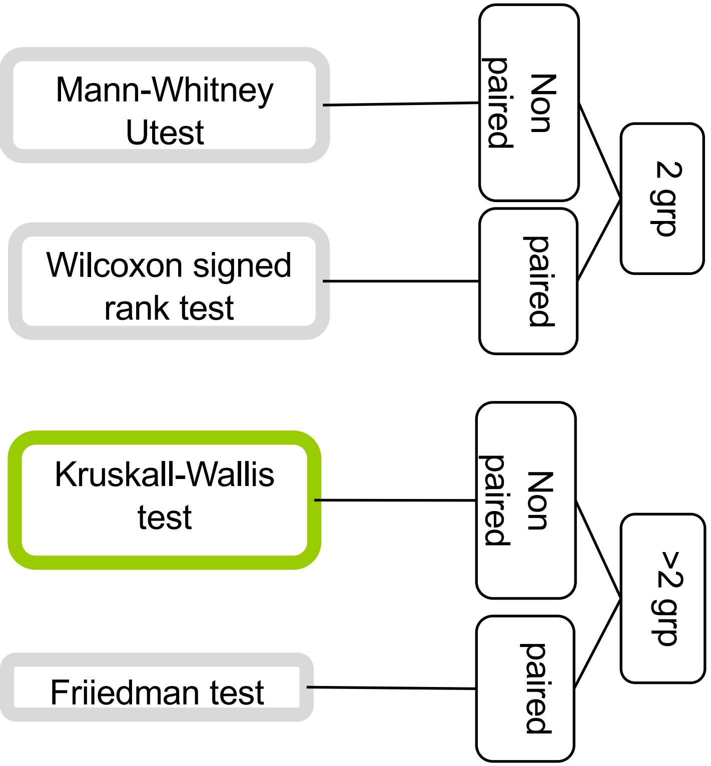
paired

Friiedman test

```
#wilcox.test do both paired (Mann whitney test)
and unpaired

dat = read.csv("HCI2018results.csv", header =
TRUE)


wilcox.test(dat$score[dat$group == "A"],
dat$score[dat$group =="B"],paired=FALSE)
```

```
Wilcoxon rank sum test with continuity correction


data:  dat$score[dat$group == "A"] and
dat$score[dat$group == "B"]
W = 1290, p-value = 0.6408
alternative hypothesis: true location shift is not
equal to 0
```

now let’s add the hypothetical group (punishment)



Mann-Whitney Utest

Non paired

Wilcoxon signed rank test

paired

2 grp

Kruskall-Wallis test

Non paired

Friiedman test

paired

>2 grp

```
dat = read.csv("HCI2018results.csv", header =
TRUE)
kruskal.test(score ~ group, data = dat)


data:  score by group
Kruskal-Wallis chi-squared = 44.77,
df = 2, p-value = 1.898e-10

pairwise.wilcox.test(dat$score, dat$group,
p.adjust.method = "bonferroni")


  A       B
B 1       -
C 1.6e-09 2.6e-09
```

here turns out we get the same tendencies than with parametric tests, i.e. there is no evidences of significant effect of chocolate reward on memorization

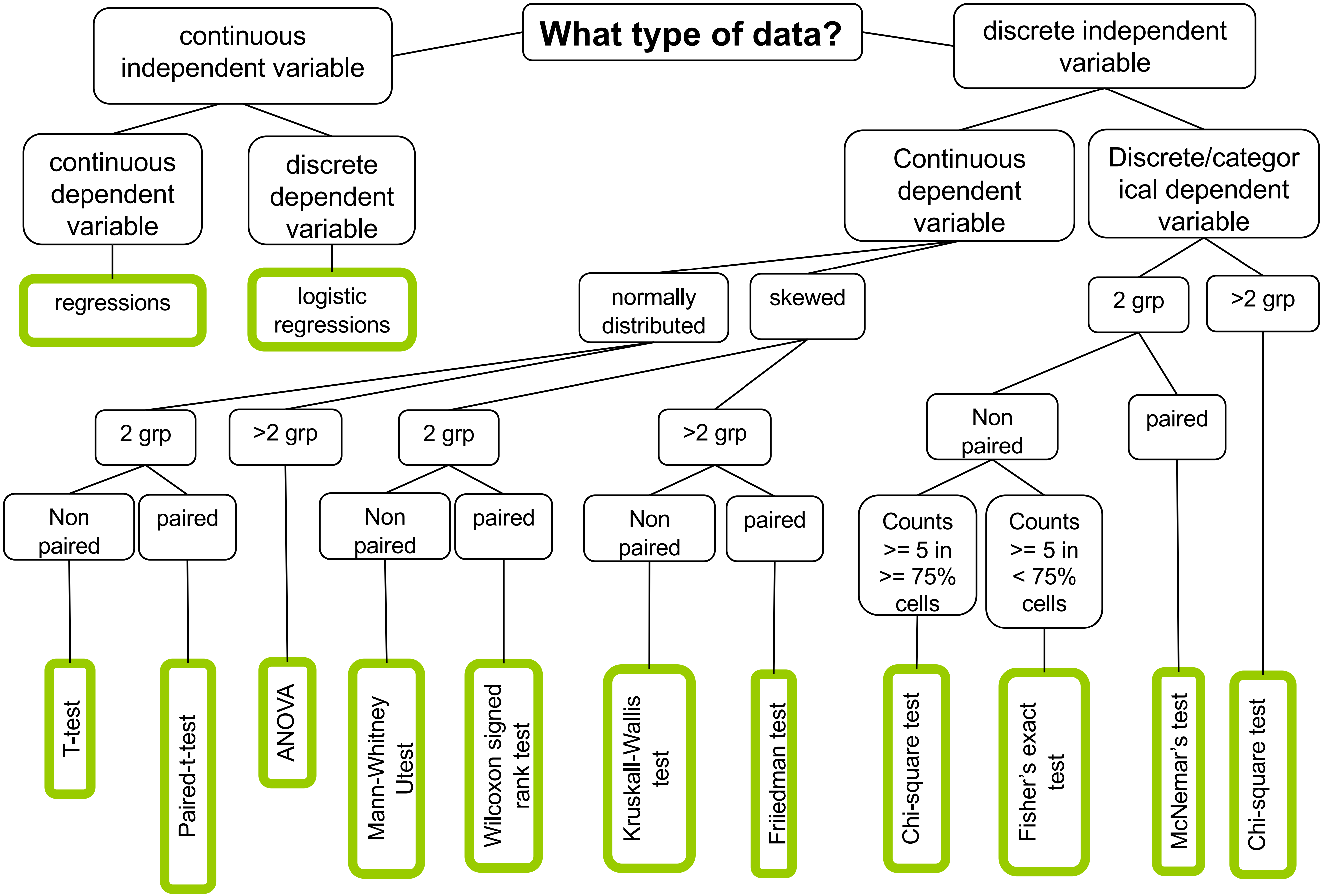
but there is an effect of punishment

```
#for friedman test (source in GitHub)
dat = read.csv("friedmanExample.csv", header =
TRUE)
friedman.test(dat$count, dat$year, dat$month)
```

```
data:  dat$count, dat$year and dat$month
Friedman chi-squared = 7.6, df = 2, p-value =
0.02237
```

```
# note there is a real drop in statistical power
when using a Friedman test. There are methods that
enable post-hoc tests but the power is such that
obtaining significance is well nigh impossible.
The best you can do is to present a boxplot of the
data (dependent ~ group).
```

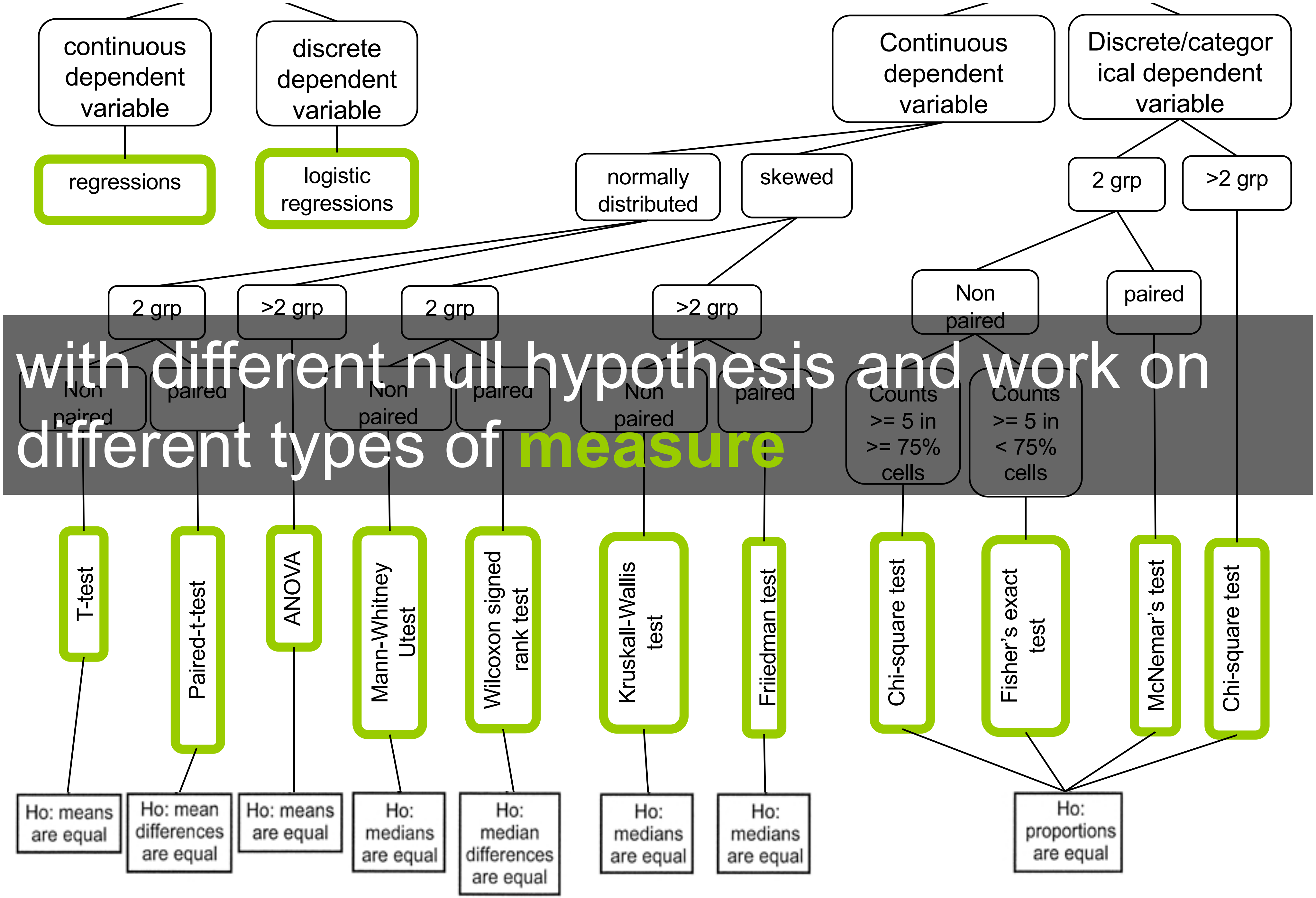
ok so now you know almost all the tests needed!



What type of data?

- continuous independent variable
  - continuous dependent variable
    - regressions
  - discrete dependent variable
    - logistic regressions
- discrete independent variable
  - Continuous dependent variable
    - normally distributed
      - 2 grp
        - Non paired
          - T-test
        - paired
          - Paired-t-test
      - >2 grp
        - ANOVA
    - skewed
      - 2 grp
        - Non paired
          - Mann-Whitney Utest
        - paired
          - Wilcoxon signed rank test
      - >2 grp
        - Non paired
          - Kruskall-Wallis test
        - paired
          - Friiedman test
  - Discrete/categorical dependent variable
    - 2 grp
      - Non paired
        - Counts >= 5 in >= 75% cells
          - Chi-square test
        - Counts >= 5 in < 75% cells
          - Fisher’s exact test
      - paired
        - McNemar’s test
    - >2 grp
      - Chi-square test

with different null hypothesis and work on different types of **measure**



continuous dependent variable

regressions

discrete dependent variable

logistic regressions

Continuous dependent variable

normally distributed

2 grp

Non paired

T-test

Ho: means are equal

paired

Paired-t-test

Ho: mean differences are equal

>2 grp

ANOVA

Ho: means are equal

skewed

2 grp

Non paired

Mann-Whitney Utest

Ho: medians are equal

paired

Wilcoxon signed rank test

Ho: median differences are equal

>2 grp

Non paired

Kruskall-Wallis test

Ho: medians are equal

paired

Friiedman test

Ho: medians are equal

Discrete/categor ical dependent variable

2 grp

Non paired

Counts >= 5 in >= 75% cells

Chi-square test

Counts >= 5 in < 75% cells

Fisher's exact test

paired

McNemar's test

>2 grp

Chi-square test

Ho: proportions are equal

the most important is not that you do these by hands but that you understand the intuition behind them and more importantly **when to use them** quiz

20 participants were asked to write text using two different keyboard layouts (A and B). Half of the participants started the task on the A layout and then the B and the other half of the participants started the task on the B layout and then the A. The number of words typed per minute was collected for each participant and layout. Choose the most appropriate procedure to decide which layout allow participants to type the fastest. Assumption normality and homogeneity are verified.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

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40 participants were randomized to two groups. One group received a drug to decrease hair loss and the other group received a placebo (a pill of sugar). At the end of the program, the percentage hair loss for each patient was recorded. Choose the most appropriate procedure to decide if there is a relationship between the use of the drug and the percentage of hair loss. Assumption normality and homogeneity are verified.

Paired T-test
Unpaired T-test
One-Way Anova (between)
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A study attempted to find out if the age of an animal had any relationship to their athletic ability. The researchers took the data of 104 cheetahs, calculating their age and running a test to measure their speed. Choose the most appropriate procedure to decide if the age has any relationship with the run speed.

Paired T-test
Unpaired T-test
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A study attempted to find out if the age of an animal had any relationship to their athletic ability. The researchers took the data of 104 cheetahs, calculating their age and running a test to measure their speed. Choose the most appropriate procedure to decide if the age has any relationship with the run speed.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

20 participants were asked to type of their phone touchscreen in four different postures (sitting, lying down, standing and running). The number of words typed per minute was collected for each participant and postures. Choose the most appropriate procedure to decide which posture allow participants to type the fastest. Assumption normality and homogeneity are verified.

Paired T-test
Unpaired T-test
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Linear regression
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20 participants were asked to run as fast as possible using two different pairs of shoes. Their speed was collected for each pairs of shoes. Choose the most appropriate procedure to decide which shoes allow participants to run the fastest. Assumption normality is verified but not the assumption of homogeneity.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

20 participants were asked to type of their phone touchscreen in four different postures (sitting, lying down, standing and running). They were asked to rate their comfort for each posture using a Likert Scale questionnaire. Choose the most appropriate procedure to decide which posture was most comfortable.

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A study has gathered 10000 observations of computer performances (speed) in three different room of varying temperature (15, 25 and 35 degrees Celsius). Choose the most appropriate procedure to decide if the data follows a normal distribution.

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Repeated Anova (within)

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A study has gathered 10000 observations of computer performances (speed) in three different room of varying temperature (15, 25 and 35 degrees Celsius). Choose the most appropriate procedure to decide if the data follows a normal distribution.

(because more than 50 observations!)

Paired T-test
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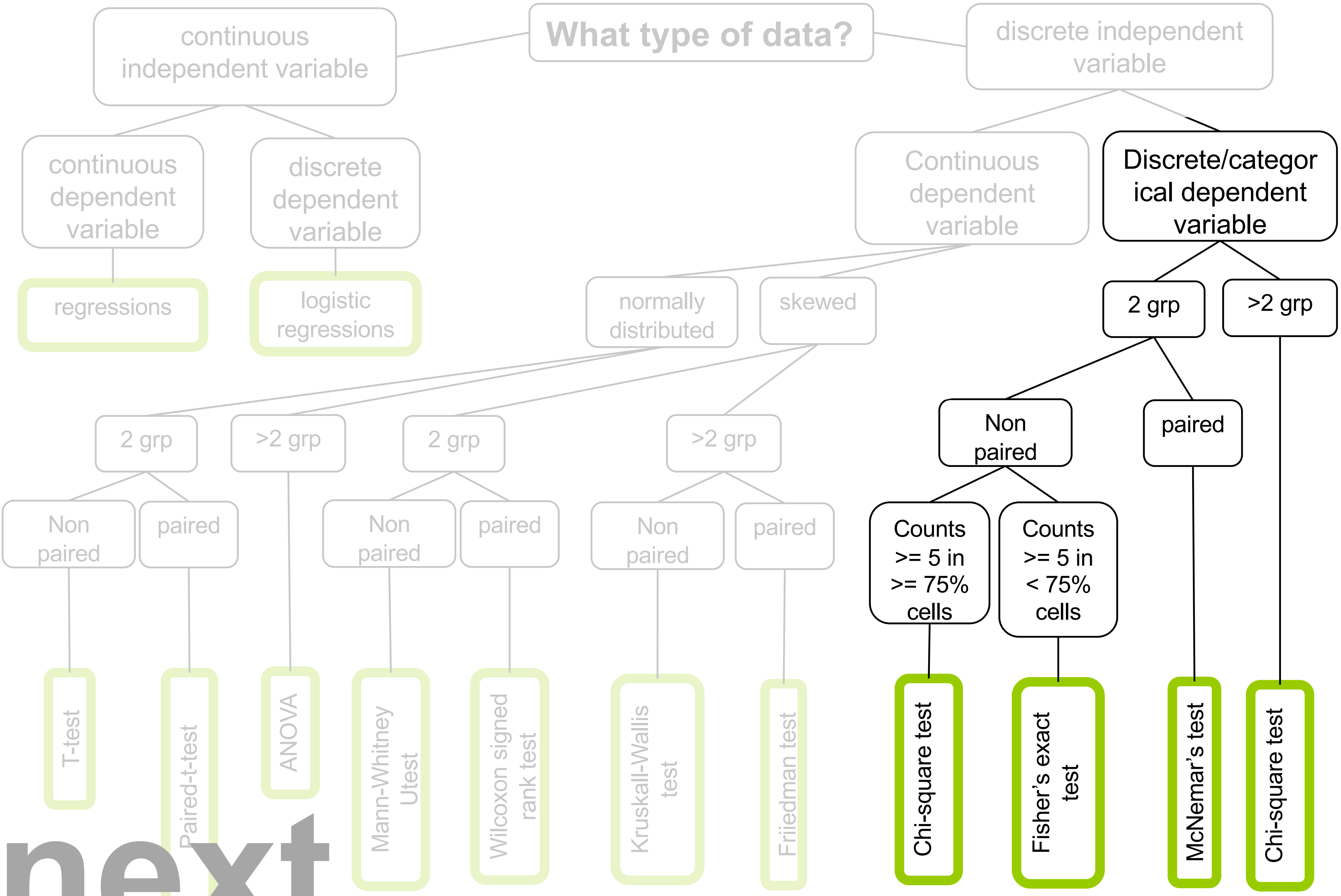
summary

1. Give the four non parametric tests seen today
2. Explain the basis of Mann Whitney and Wilcoxon test, aka that they use ranks rather than mean
3. I won’t ask you to do it by hand in the exam
4. Be able to read a design protocol and figure out what statistic tests to use

take away



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Counts >= 5 in >= 75% cells

Chi-square test

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McNemar's test

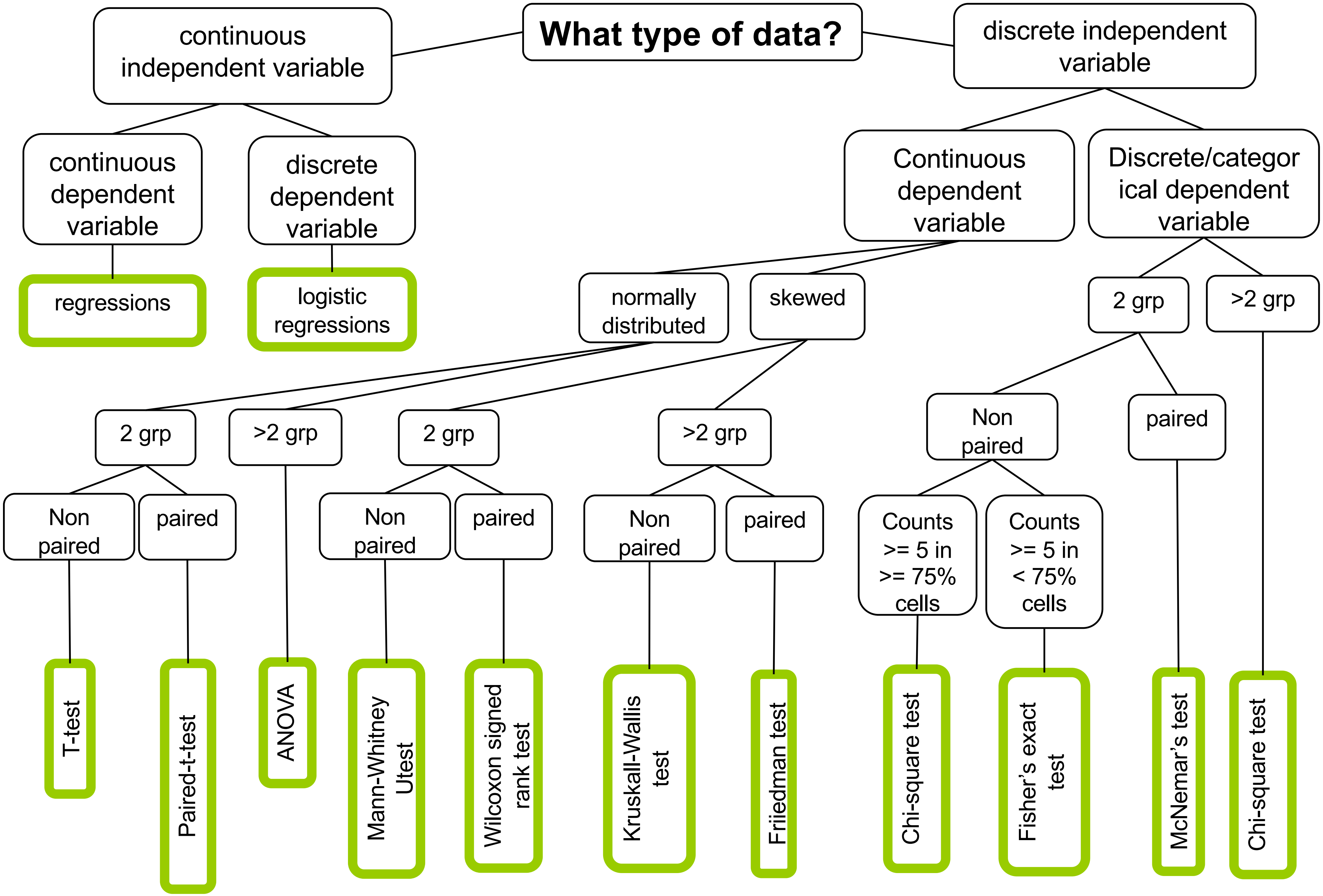
>2 grp

Chi-square test

next

end

quiz to print



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Kolmogorov-Smirnov
Shapiro-Wilk

40 participants were randomized to two groups. One group received a drug to decrease hair loss and the other group received a placebo (a pill of sugar). At the end of the program, the percentage hair loss for each patient was recorded. Choose the most appropriate procedure to decide if there is a relationship between the use of the drug and the percentage of hair loss. Assumption normality and homogeneity are verified.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

A study attempted to find out if the age of an animal had any relationship to their athletic ability. The researchers took the data of 104 cheetahs, calculating their age and running a test to measure their speed. Choose the most appropriate procedure to decide if the age has any relationship with the run speed.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

20 participants were asked to type of their phone touchscreen in four different postures (sitting, lying down, standing and running). The number of words typed per minute was collected for each participant and postures. Choose the most appropriate procedure to decide which posture allow participants to type the fastest. Assumption normality and homogeneity are verified.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

20 participants were asked to run as fast as possible using two different pairs of shoes. Their speed was collected for each pairs of shoes. Choose the most appropriate procedure to decide which shoes allow participants to run the fastest. Assumption normality is verified but not the assumption of homogeneity.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

20 participants were asked to run as fast as possible using two different pairs of shoes. Their speed was collected for each pairs of shoes. Choose the most appropriate procedure to decide which shoes allow participants to run the fastest. Assumption normality is verified but not the assumption of homogeneity.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk

A study has gathered 10000 observations of computer performances (speed) in three different room of varying temperature (15, 25 and 35 degrees Celsius). Choose the most appropriate procedure to decide if the data follows a normal distribution.

Paired T-test
Unpaired T-test
One-Way Anova (between)
Repeated Anova (within)

Mann Whitney
Wilcoxon
Kruskal Wallis
Friedman

Linear regression
Kolmogorov-Smirnov
Shapiro-Wilk