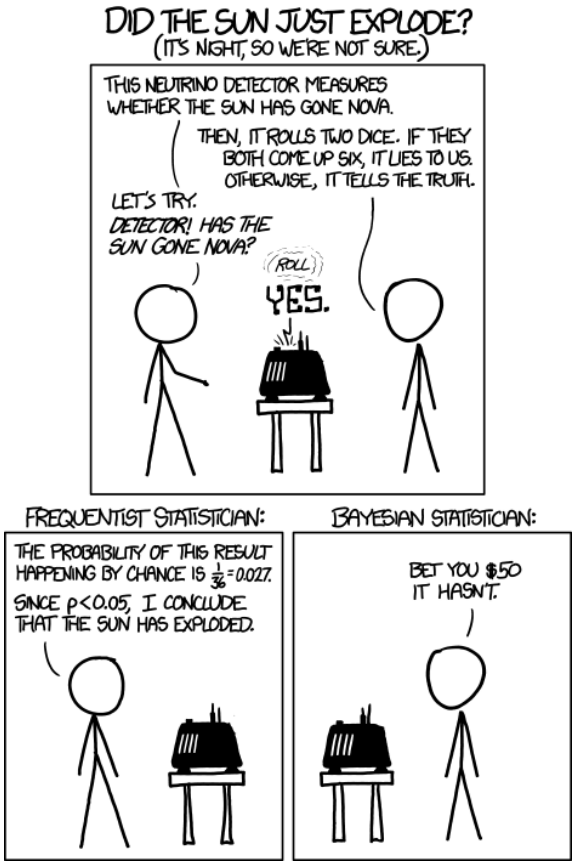
Problem Sheet 1



This is an xkcd cartoon that someone posted to the unit reddit, it is `https://xkcd.com/1132/`. It is also worth reading Randall Monroe's comment on the response to this cartoon:

> Hey! I was kinda blindsided by the response to this comic.
>
> Im in the middle of reading a series of books about forecasting errors (including Nate Silvers book, which I really enjoyed), and again and again kept hitting examples of mistakes caused by blind application of the textbook confidence interval approach.
>
> Someone asked me to explain it in simple terms, but I realized that in the common examples used to illustrate this sort of error, like the cancer screening/drug test false positive ones, the correct result is surprising or unintuitive. So I came up with the sun-explosion example, to illustrate a case where nave application of that significance test can give a result thats obviously nonsense.
>
> I seem to have stepped on a hornets nest, though, by adding Frequentist and Bayesian titles to the panels. This came as a surprise to me, in part because I actually added them as an afterthought, along with the final punchline. (I originally had the guy on the right making some other cross-panel comment, but I thought the bet thing was cuter.)
>
> The truth is, I genuinely didnt realize Frequentists and Bayesians were actual camps of peopleall of whom are now emailing me. I thought they were loosely-applied la-

> belsperhaps just labels appropriated by the books I had happened to read recently-for the standard textbook approach we learned in science class versus an approach which more carefully incorporates the ideas of prior probabilities.
>
> I meant this as a jab at the kind of shoddy misapplications of statistics I keep running into in things like cancer screening (which is an emotionally wrenching subject full of poorly-applied probability) and political forecasting. I wasnt intending to characterize the merits of the two sides of what turns out to be a much more involved and ongoing academic debate than I realized.
>
> A sincere thank you for the gentle corrections; Ive taken them to heart, and you can be confident I will avoid such mischaracterizations in the future!
>
> At least, 95.45% confident.

Thanks again to the person who posted this.

Useful facts

- **Combinations** The number of ways of choosing r items out of n is

$$\left(\begin{array}{c} n \\ r \end{array} \right) = \frac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots 1} = \frac{n!}{r!(n-r)!} \tag{1}$$

- **Bayes's rule**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2}$$

- **Set notation**: If C is a subset, the complement of C, that is the set of all the elements not in C, is written $\bar{C}$. For an event C, $\bar{C}$ is the event of C not happening.
- **Cards**: 52 cards made up of four suits; in each suit there are 13 values, ace, two through to ten and the jack, queen, kind.
- **Poker hands**: the number of poker hands is

$$\left(\begin{array}{c} 52 \\ 5 \end{array} \right) = 2598960 \tag{3}$$

Questions

Four questions, each worth two marks with two marks for attendance.

1. In the poker hand *two pair* there are two pairs of cards with each card in the pair matched by value; the fifth card has a different value to either pair. What is the probability of two pairs when five cards are drawn randomly. In a *full house* there is one pair and one triple, what is the probability of getting a full house?

2. A student answers a multiple choice question with four options, one of which is correct. 80% of students know the answer, 20% of students guess and choose randomly. If a student gets the answer correct what is the chance they knew the answer.

3. In the xkcd cartoon above, what is the chance the Bayesian will win his or her bet if the chance the sun has exploded is one in a million? In reality the chance is, of course, much less than one in a million! Show the answer to six decimal places.

4. A three-sided dice is rolled three times. X is the sum of the largest two values. Write down the probability distribution for X.