11

Intro and regressions

# Probability and Statistics

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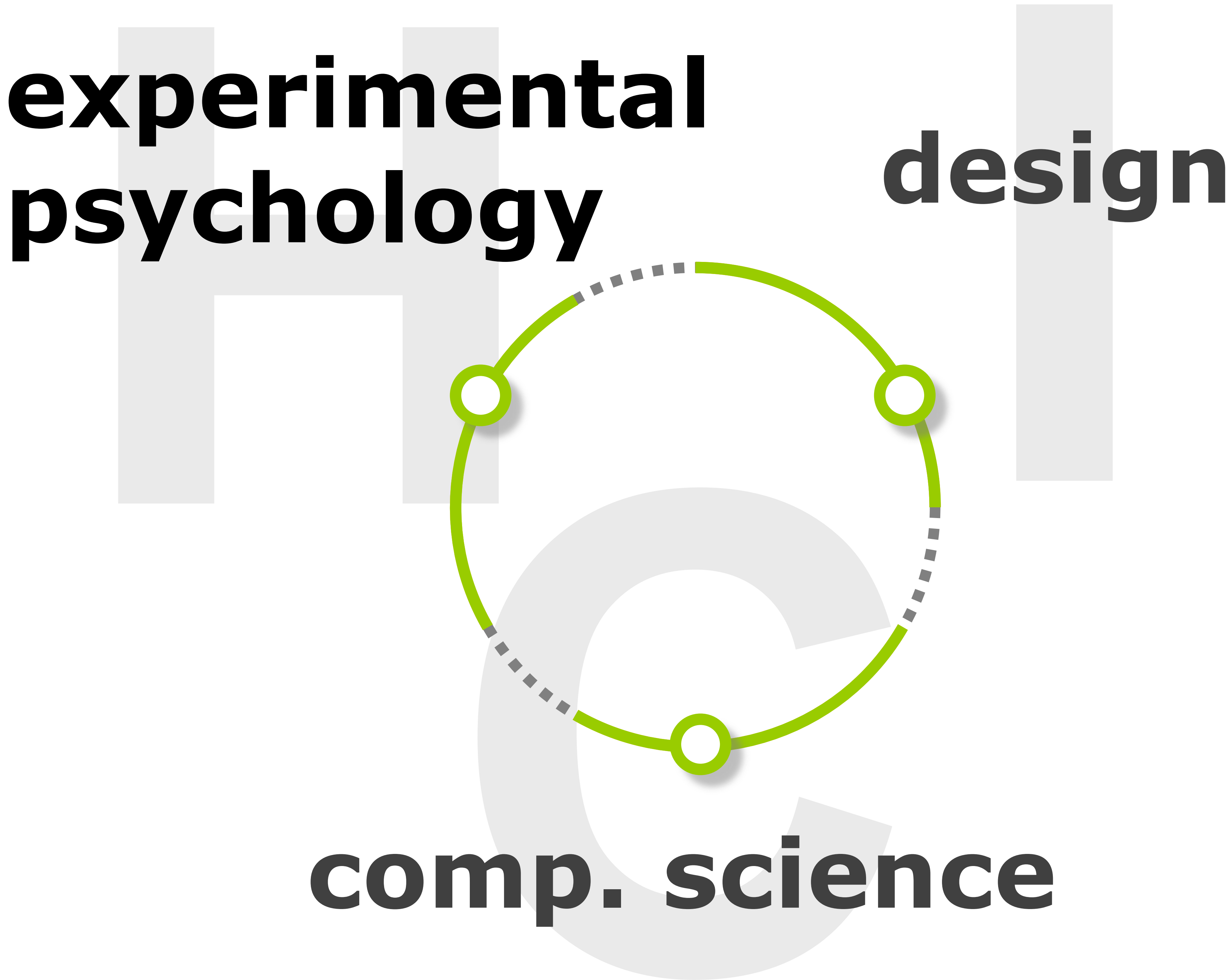
# who am i?

# Human Computer Interaction (HCI)::

a multidisciplinary field of study focusing on the design of computer technology and, in particular, the interaction between humans (the users) and computers


experimental psychology
design
comp. science

**experimental psychology::**

the branch of psychology concerned with the scientific investigation of the responses of individuals to stimuli in controlled situations


e.g. bandwagon effect (one of our many cognitive biases)

what is the link with statistics?

like in many fields, statistics is the main tool to **analyse, demonstrate, evaluate or predict**

**let's start with an example**

imagine you are designing a graphical interface for a new application on a laptop

how big should the buttons/icons be?

# Fitts' law ::

the time required to **acquire a target** of size *w* at distance *d* can be described as $T = a + b \log (1 + d/w)$


smaller bin = harder and further = harder

# Fitts' law ::

the time required to **acquire a target** of size *w* at distance *d* can be described as T = a + b log (1 + d/w)

$$T = a + b \; I_{ndex}D_{ifficulty}$$

(depends on input device)

Word 2000

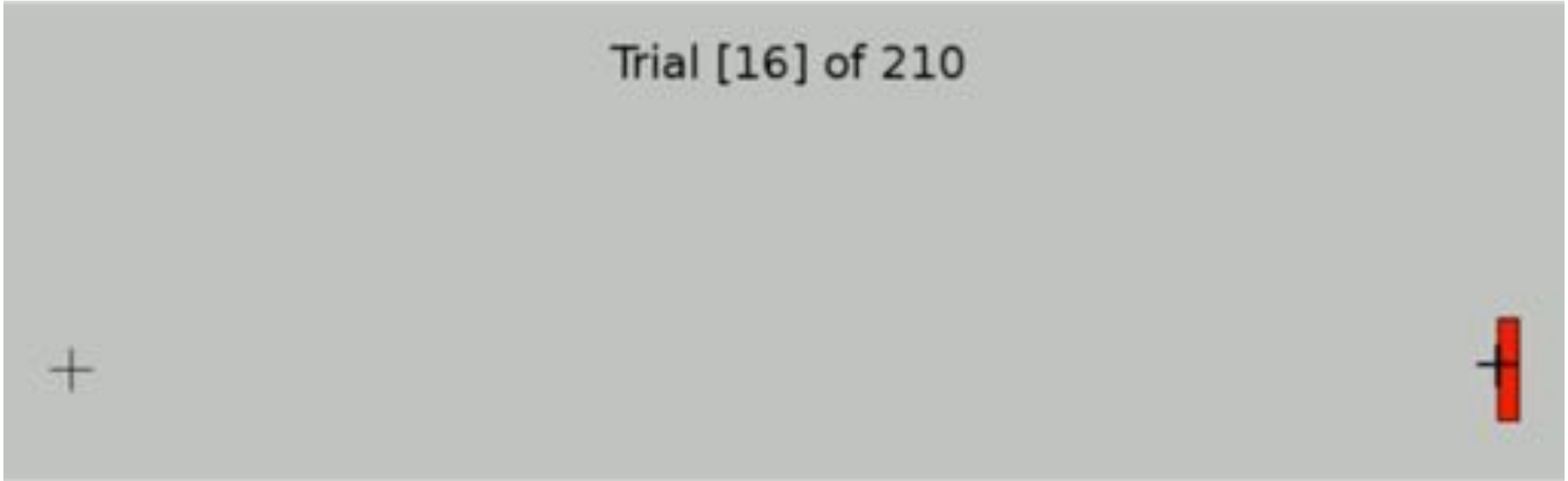
Word today


e.g. one reason why we have ribbons in Word now

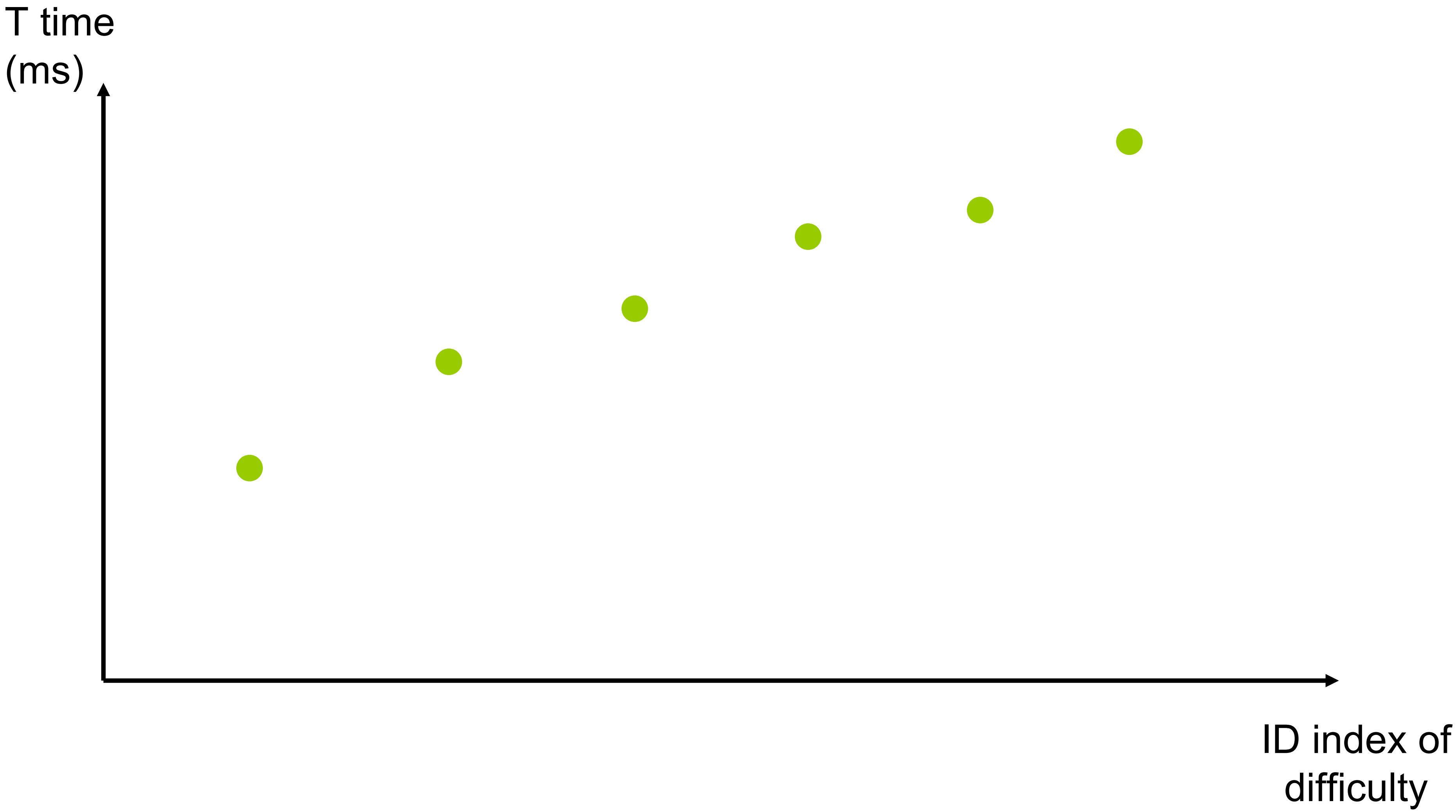
# Fitts’ law :: T = a + b ID

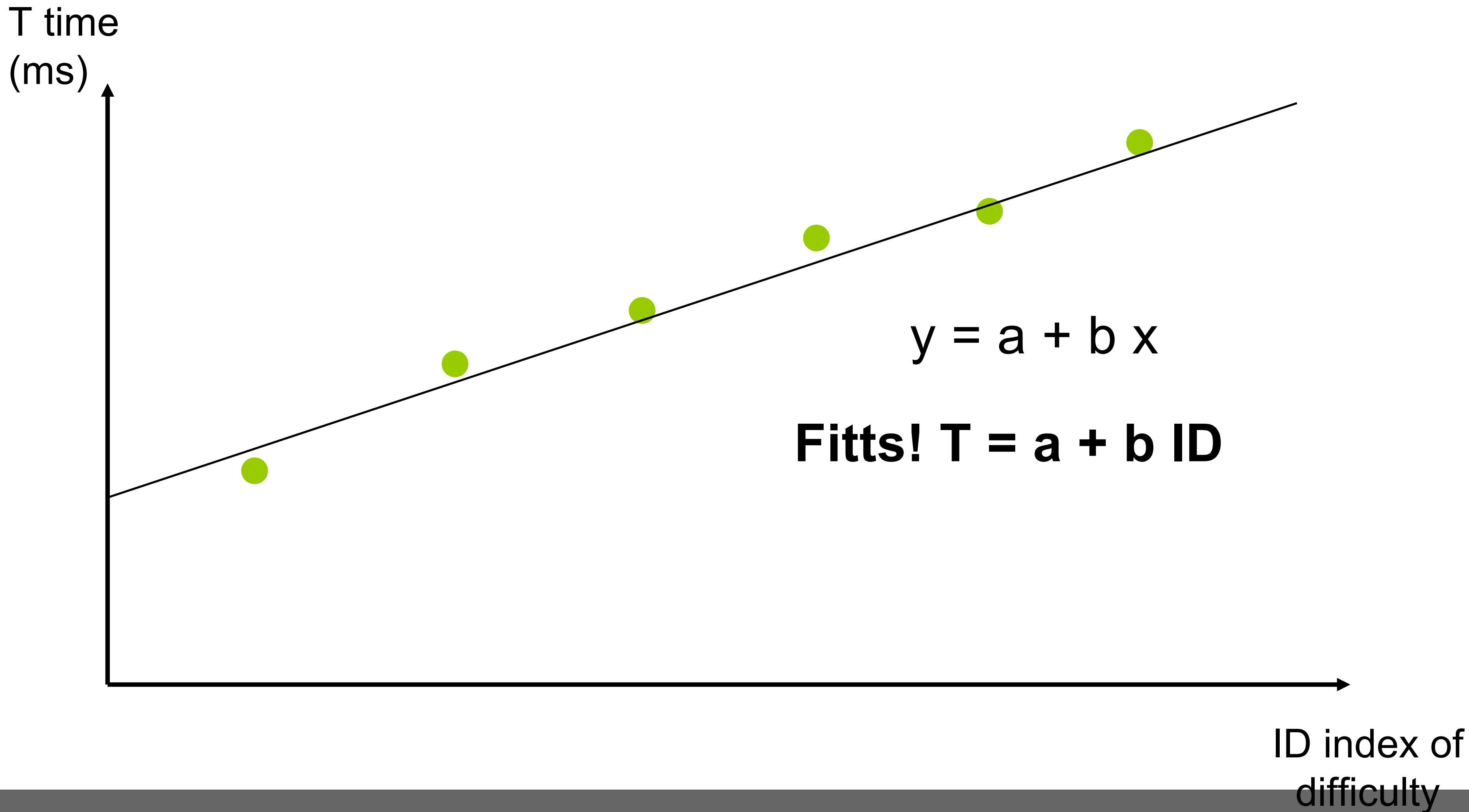
time required to **acquire a target**

but where does this equation come from?




let’s run an experiment and ask one participant to click on targets of different IDs


T time
(ms)
ID index of
difficulty



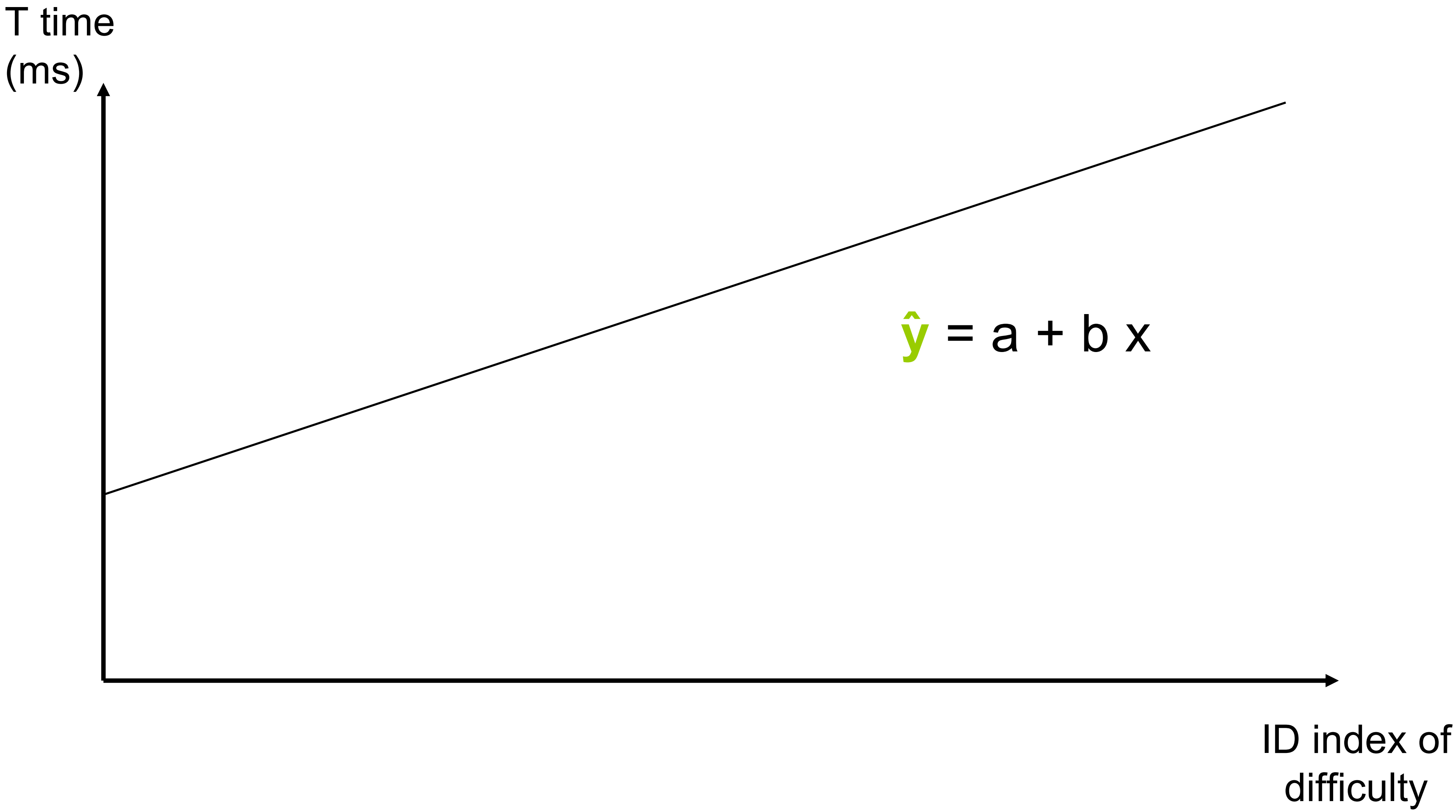

this a regression line

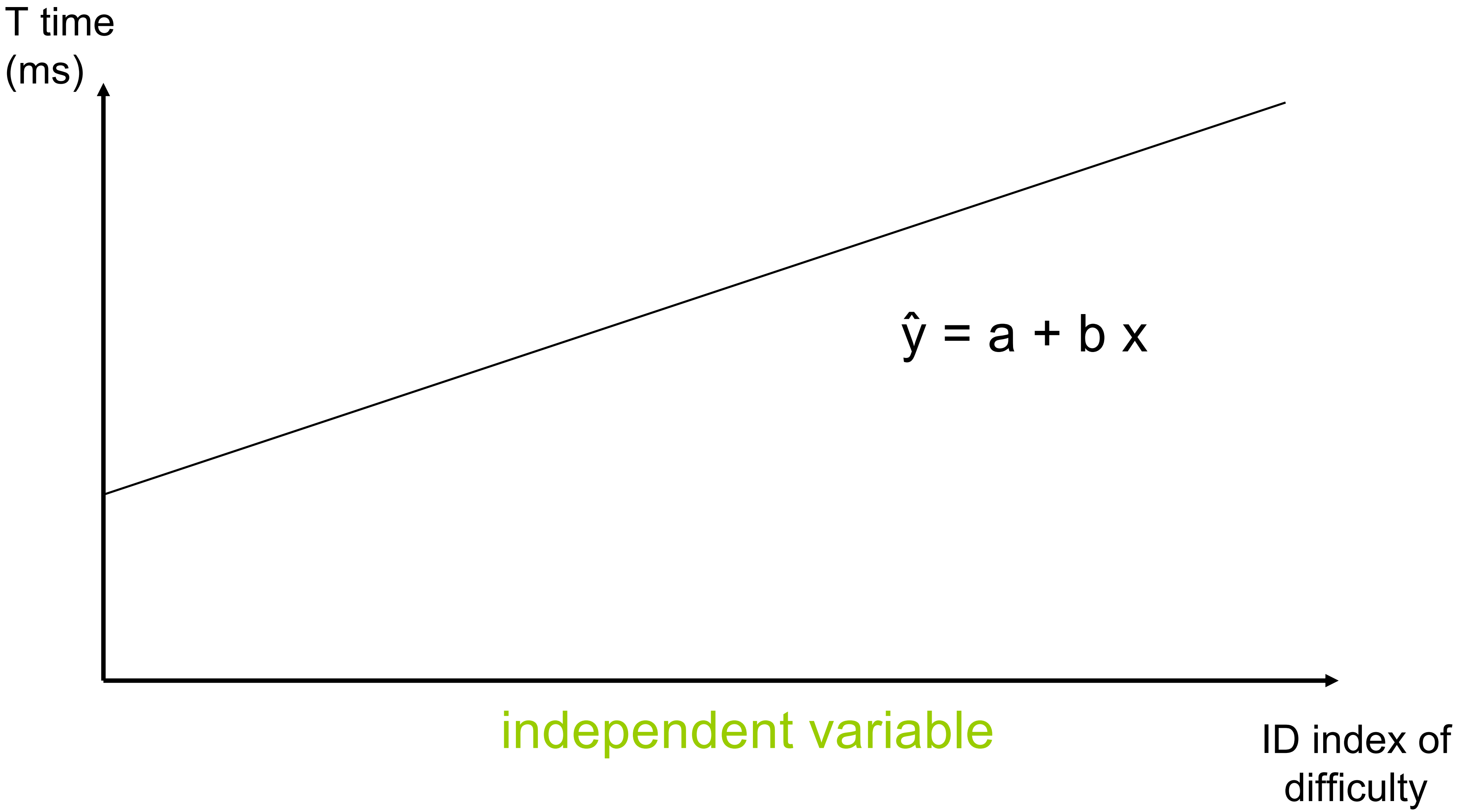
# regression ::

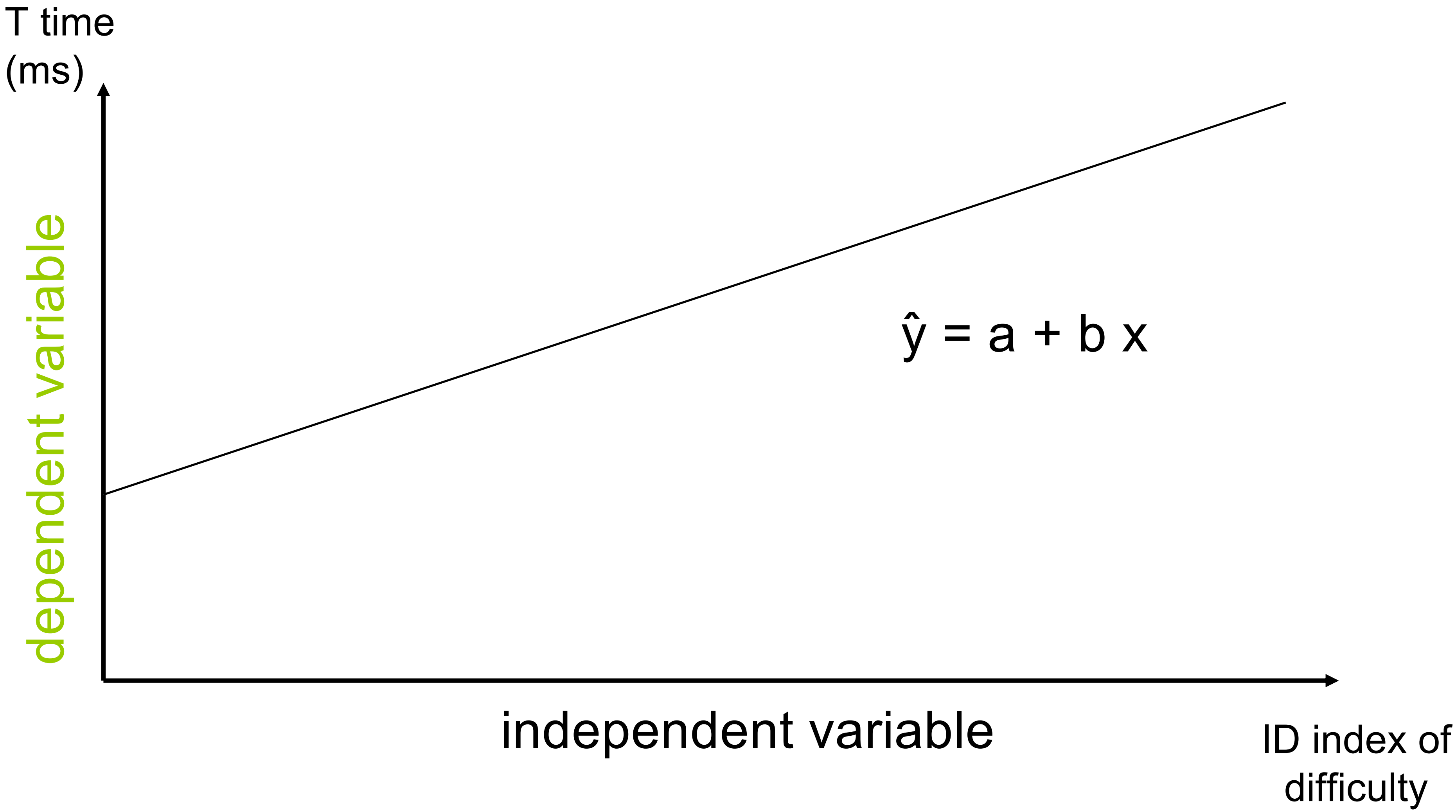
a technique for determining the statistical relationship between two or more variables where a change in a **dependent variable** is associated with, and depends on, a change in one or more **independent variables**

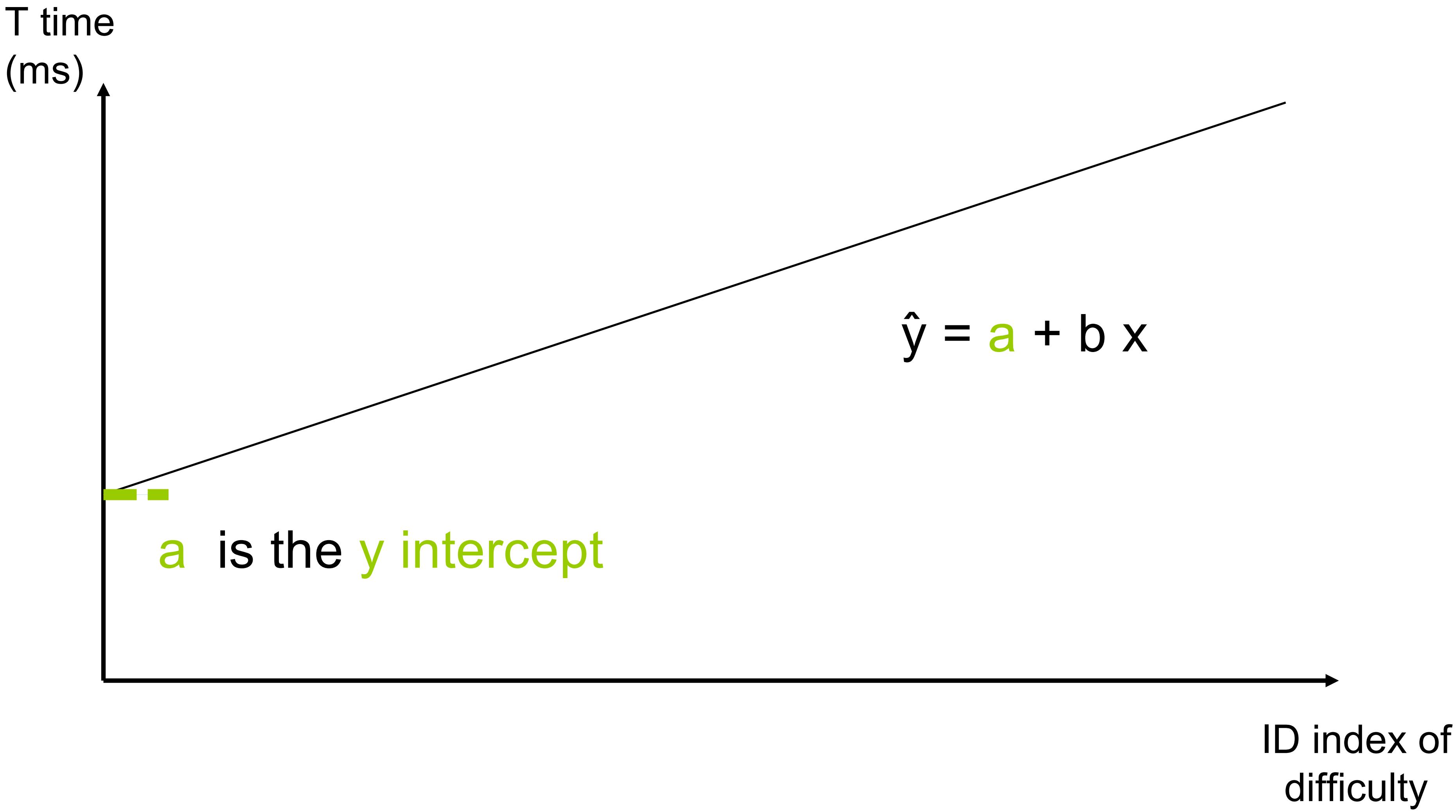
arguably the most basic technique for **machine learning**

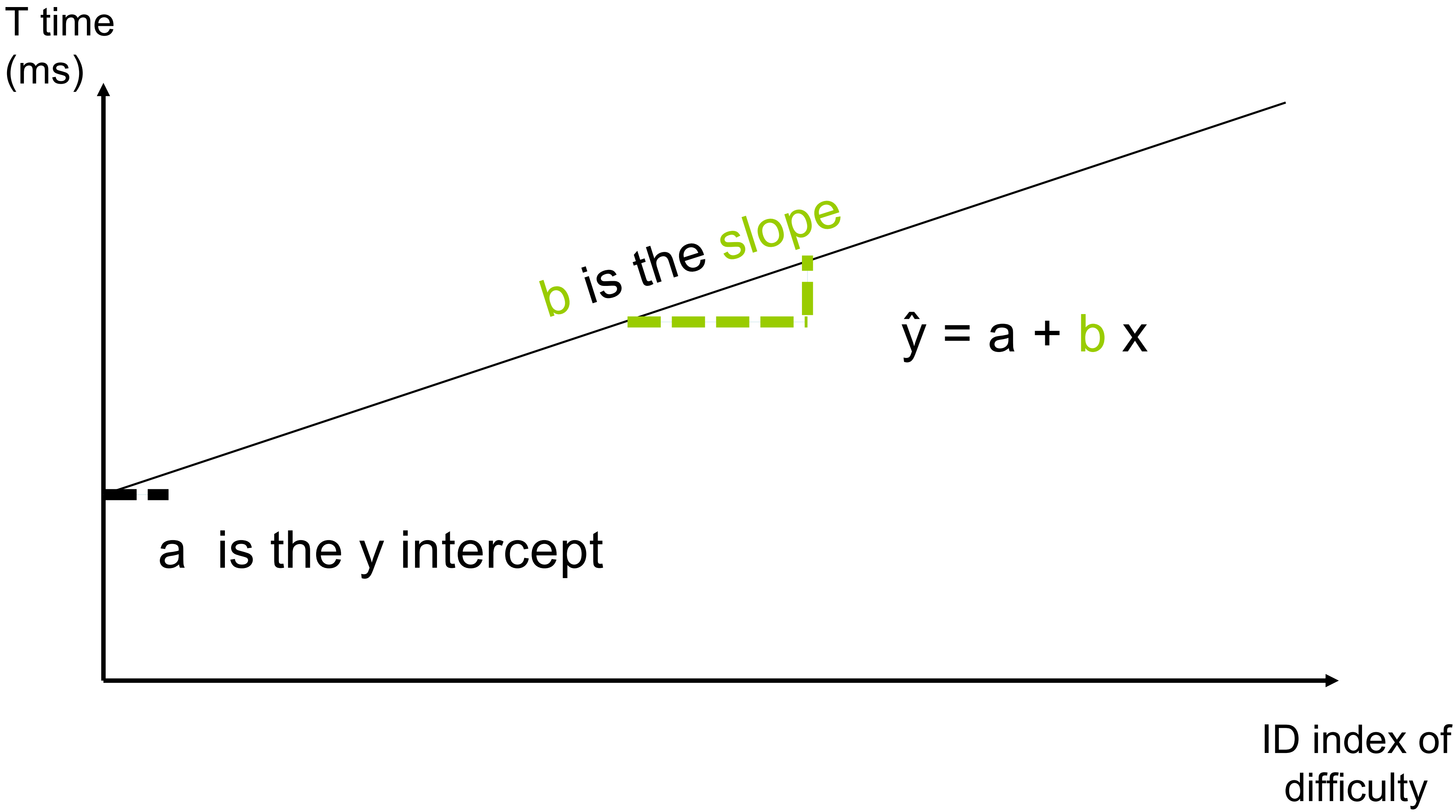
# quick terminology of regressions


T time
(ms)
ŷ = a + b x
ID index of
difficulty

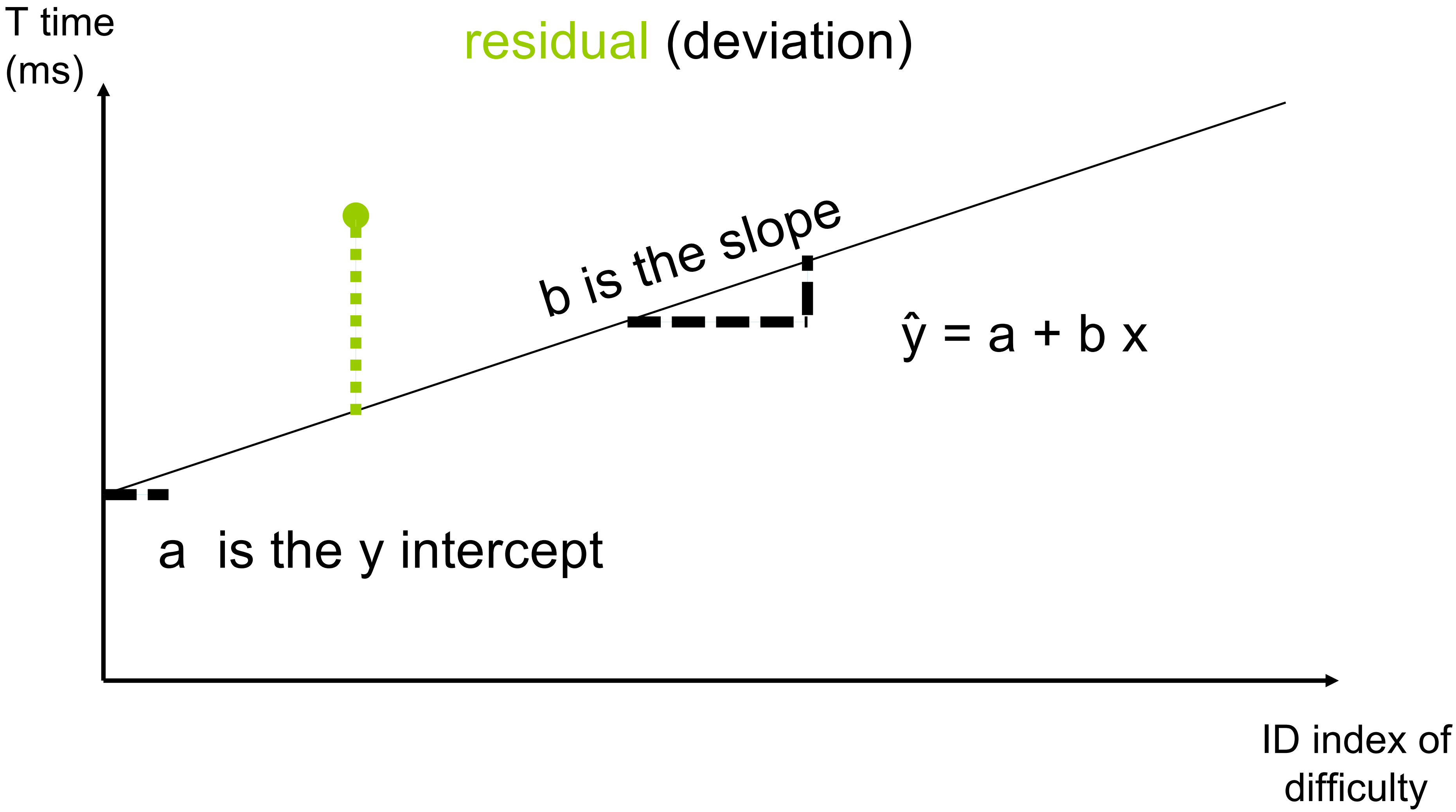

T time
(ms)
ŷ = a + b x
independent variable
ID index of
difficulty


T time
(ms)
dependent variable
ŷ = a + b x
independent variable
ID index of
difficulty

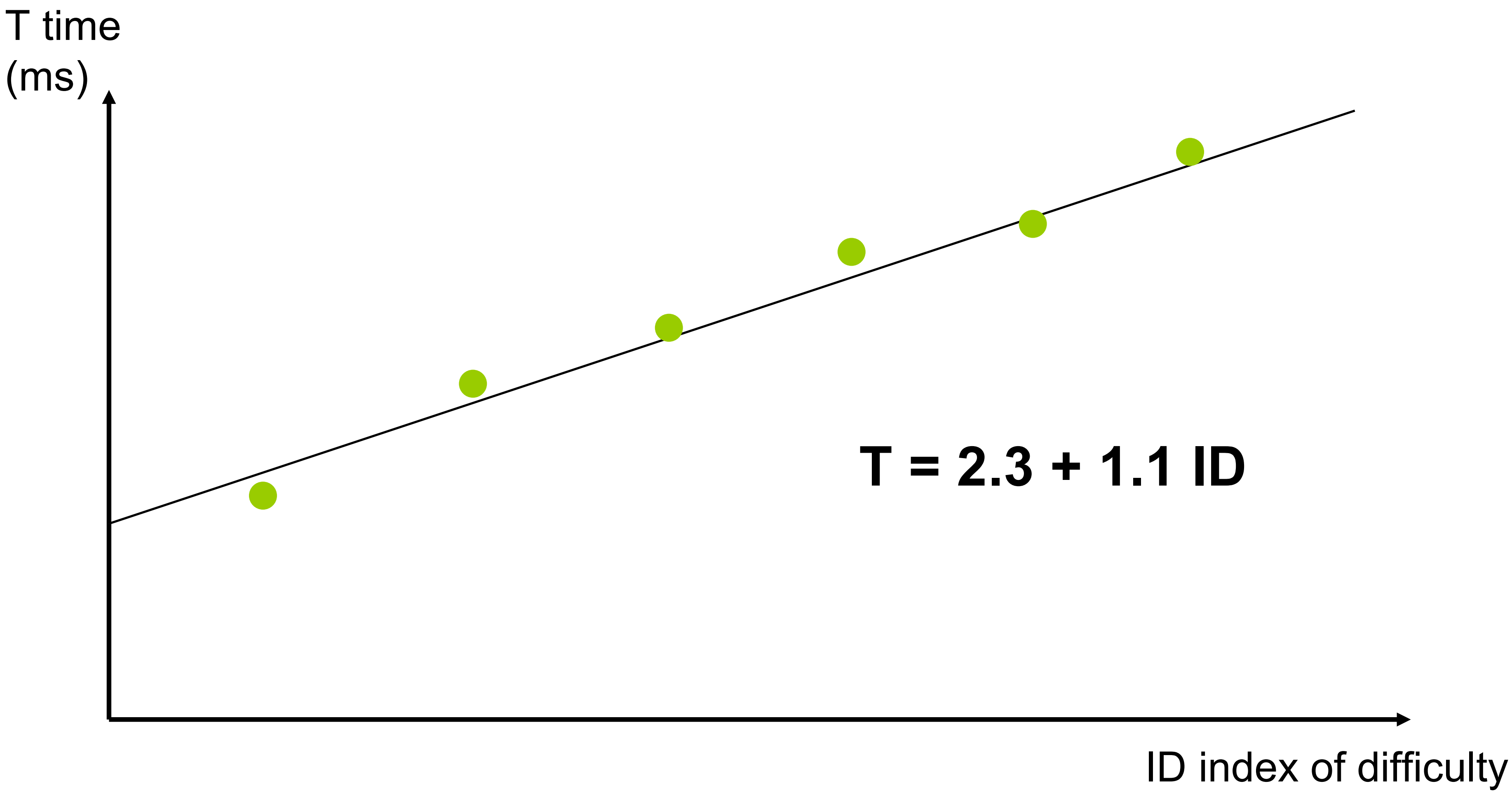

T time
(ms)
ŷ = a + b x
a is the y intercept
ID index of
difficulty



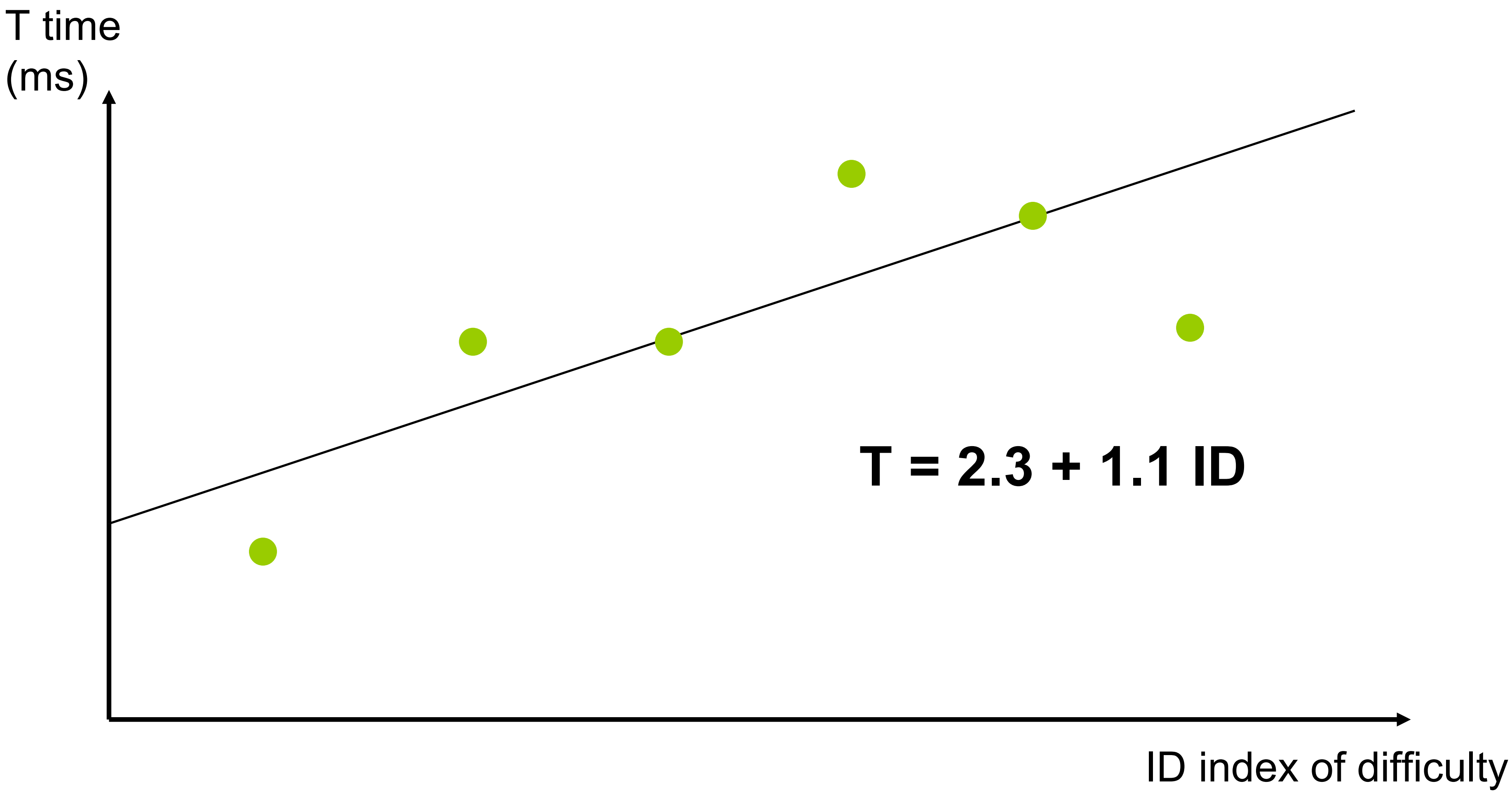
T time
(ms)
b is the slope
ŷ = a + b x
a is the y intercept
ID index of
difficulty


T time (ms)
residual (deviation)
b is the slope
ŷ = a + b x
a is the y intercept
ID index of difficulty

# goodness of fit




how can you be sure this line is a good fit to our data?



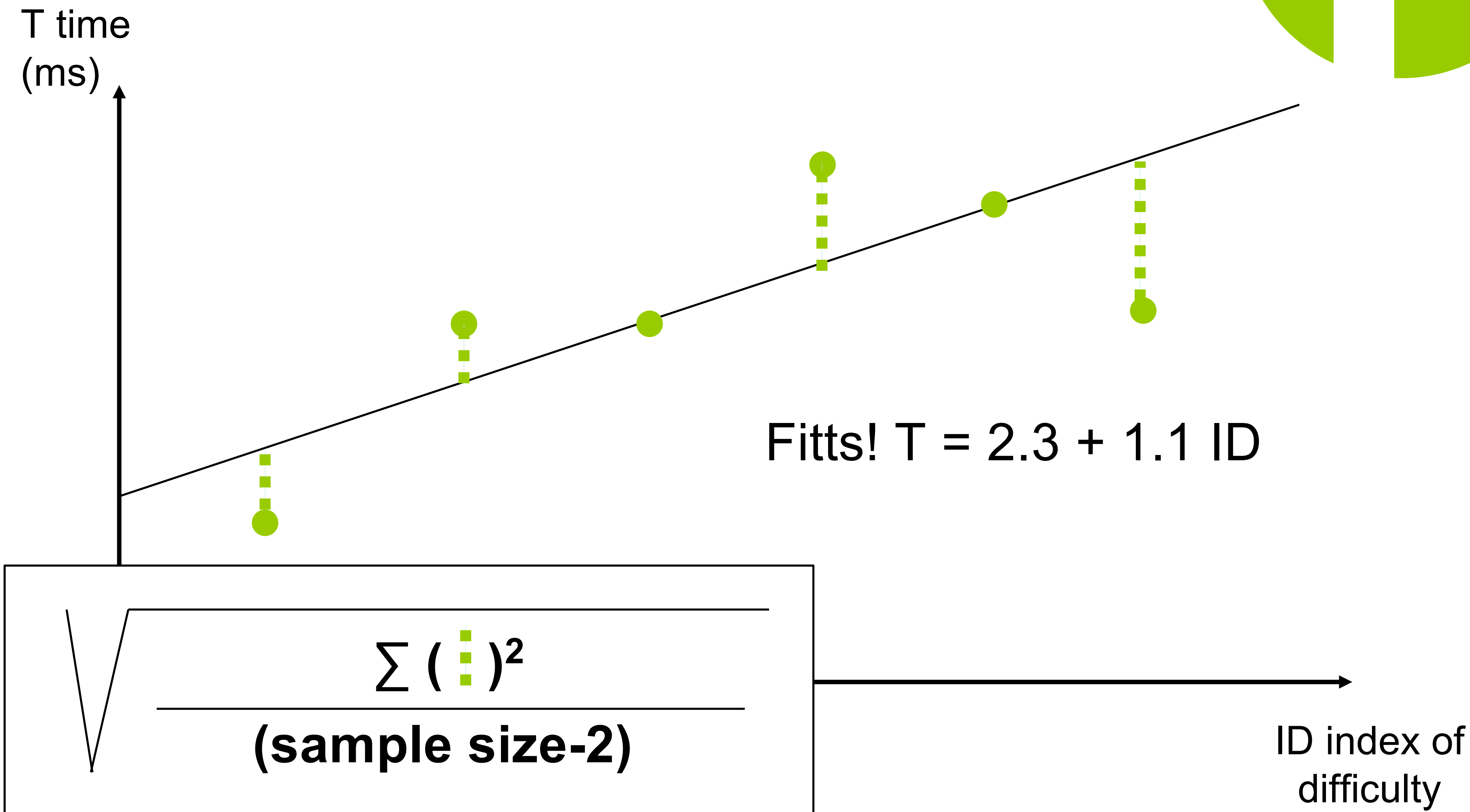

how can you be sure this line is a good fit to our data? what about now?

<brainstorming with your neighbor>

we can compute the **goodness of fit** with several methods

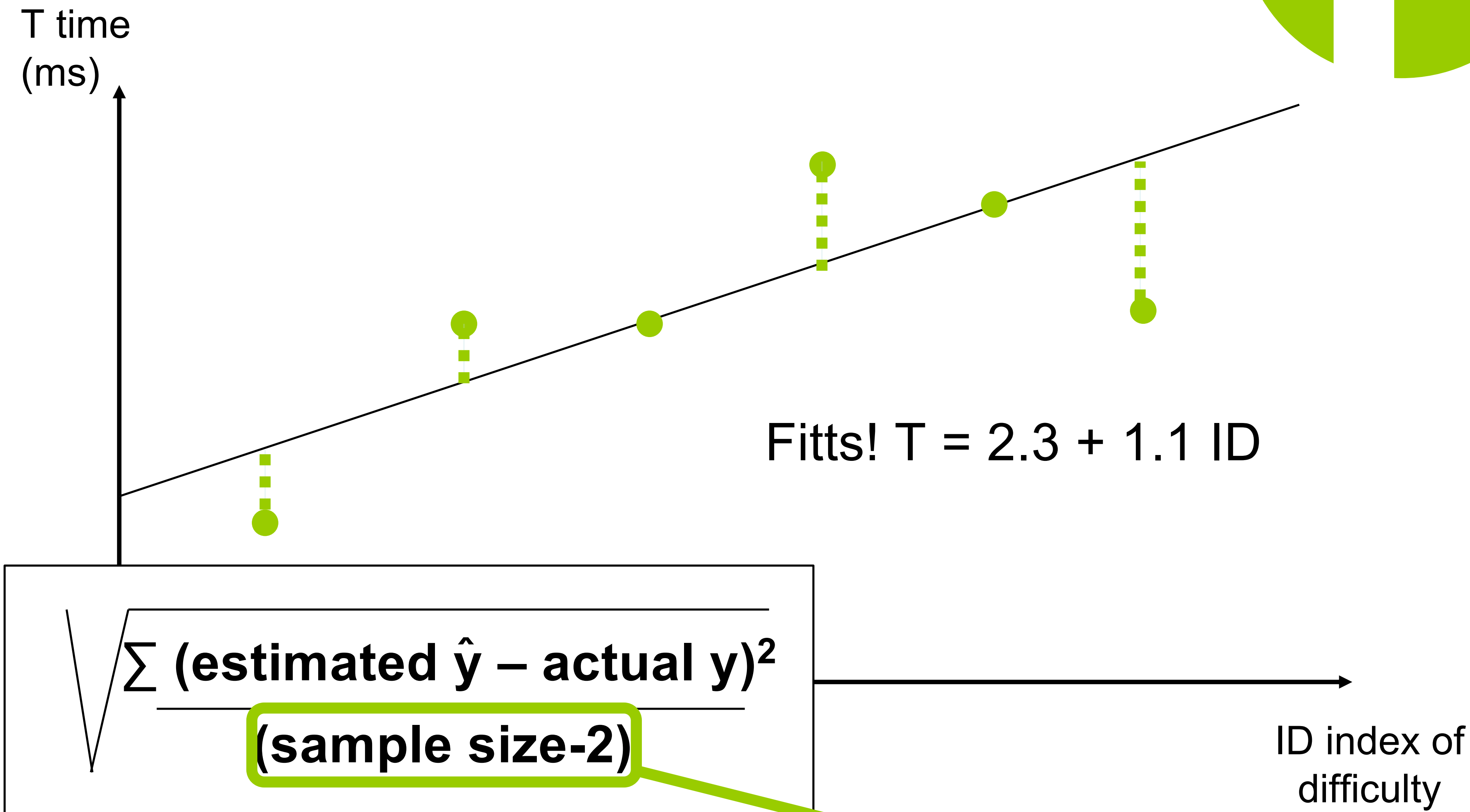
e.g. standard error of the estimate
or R squared

# standard error of the estimate




$$\sqrt{\frac{\sum (\;)^2}{\textbf{(sample size-2)}}}$$

# standard error of the estimate

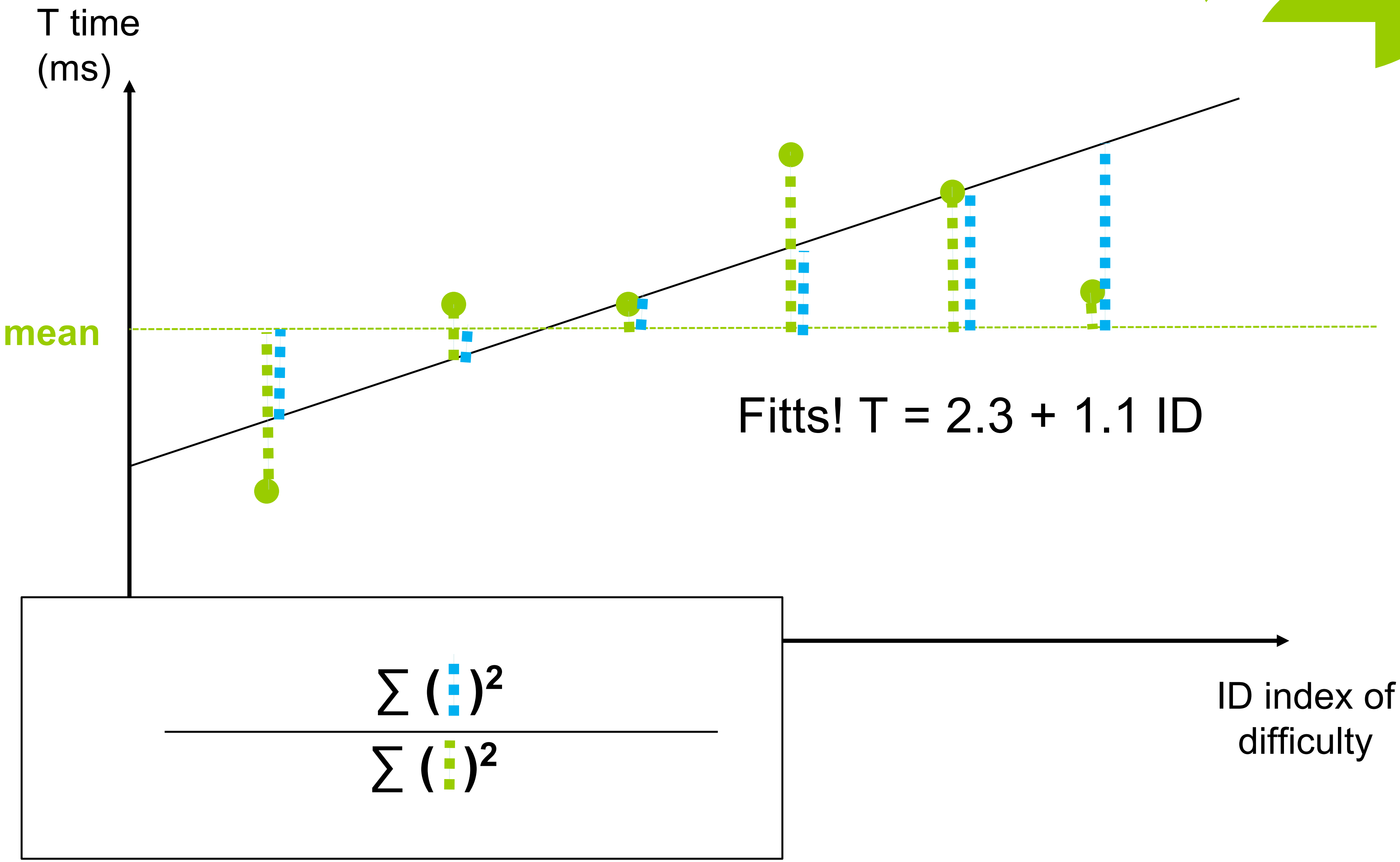



$$\sqrt{\frac{\sum (\textbf{estimated } \hat{y} - \textbf{actual } y)^2}{(\textbf{sample size-2})}}$$

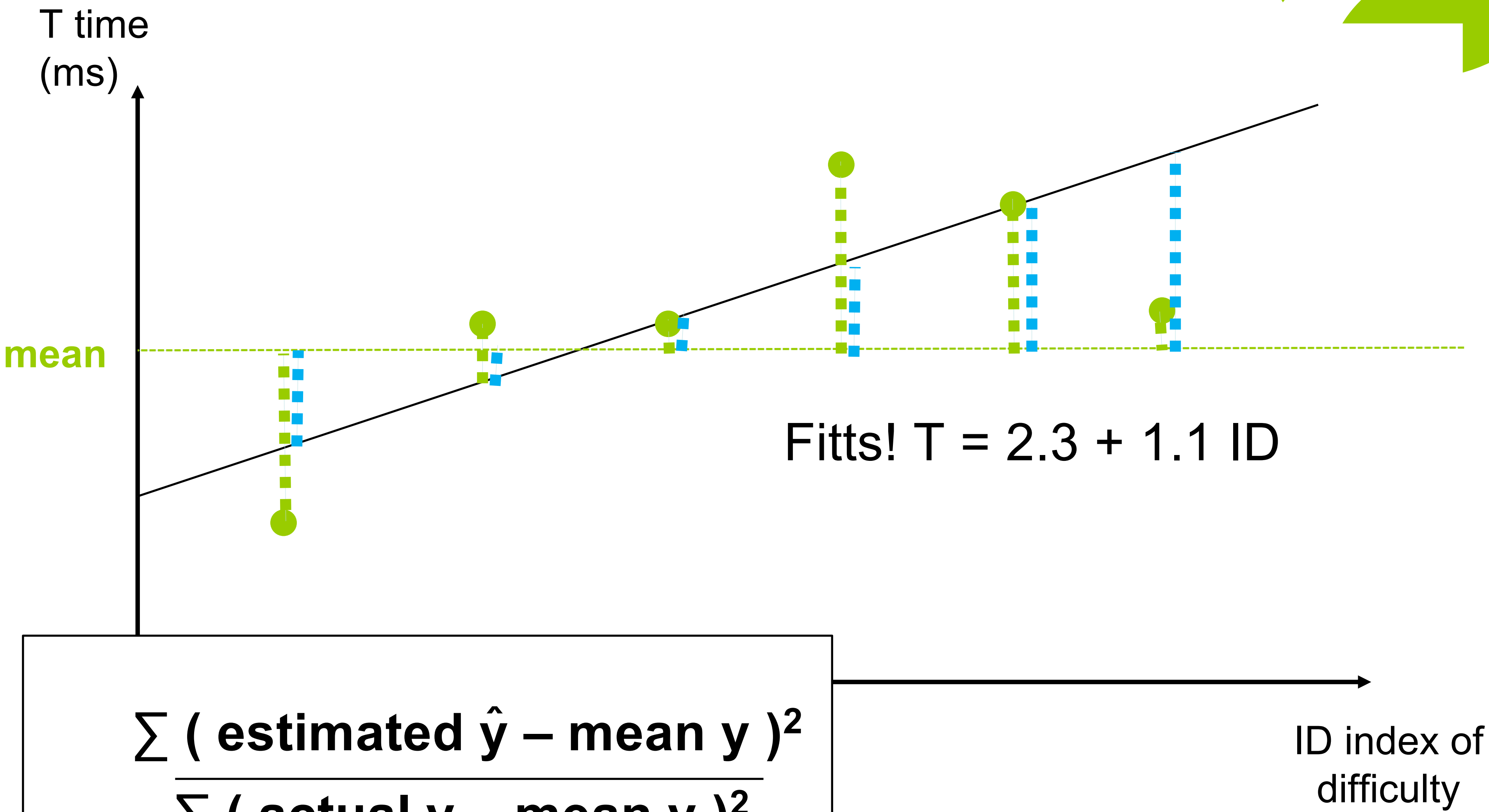
**also called degree of freedom**

S gives a standard error in the metric of the data (the less the better)

# R squared

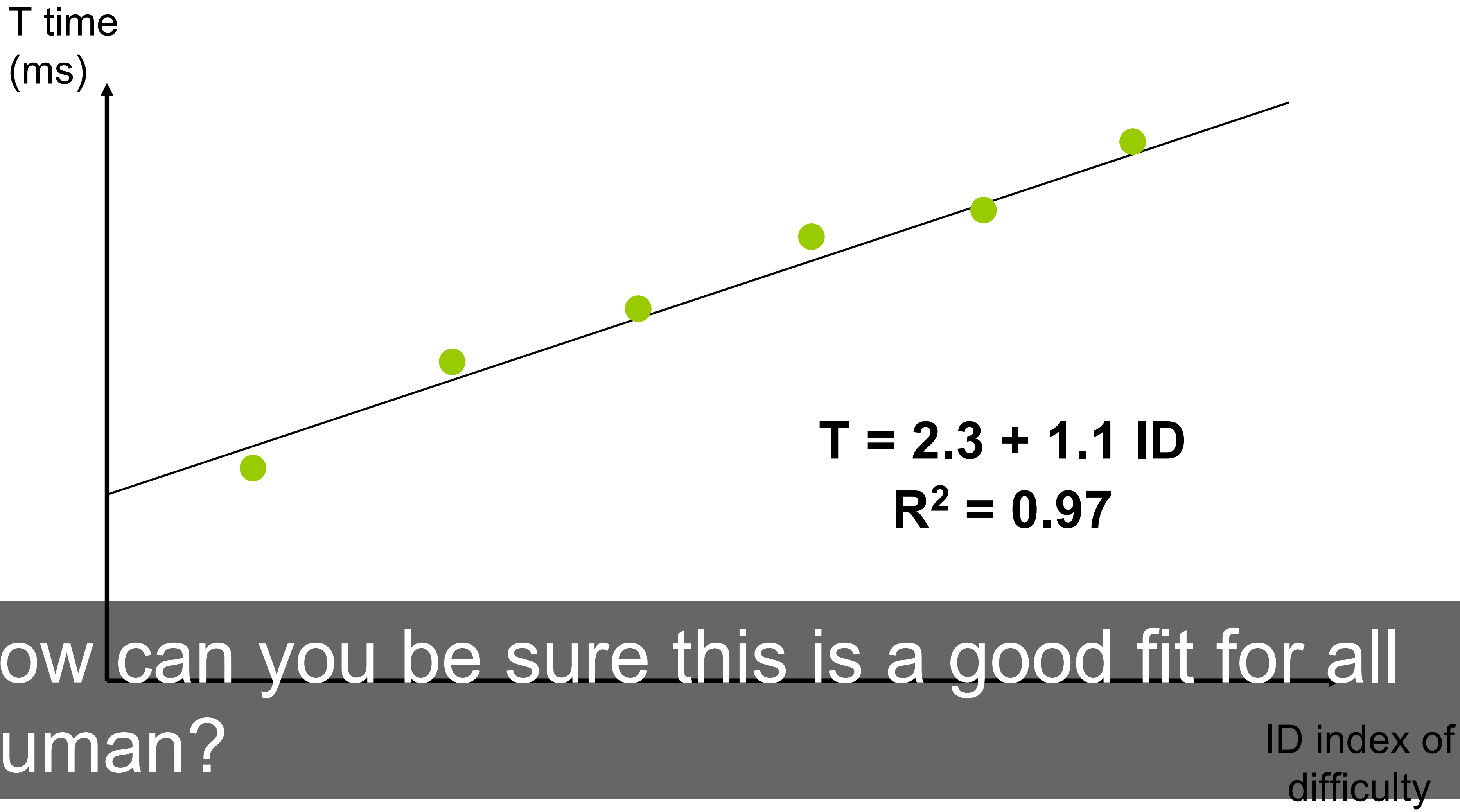


# R squared




$$\frac{\sum (\text{ estimated } \hat{y} - \text{mean } y\ )^2}{\sum (\text{ actual } y - \text{mean } y\ )^2}$$

$R^2$ gives a percentage and 100% means perfect fit (>70% is better)




how can you be sure this is a good fit for all human?

to **gain additional confidence** we **repeat**


we gain trust in a model if it fits the data with **little error** when

1. it is verified with a **lot of data**
2. it holds across very **different people**
3. it is verified in **independent studies...**

The information capacity of the human motor system in controlling the amplitude of movement.

PM Fitts - Journal of experimental psychology, 1954 - psycnet.apa.org

Reports of 3 experiments testing the hypothesis that the average duration of responses is directly proportional to the minimum average amount of information per response. The results show that the rate of performance is approximately constant over a wide range of movement amplitude and tolerance limits. This supports the thesis that" the performance capacity of the human motor system plus its associated visual and proprioceptive feedback mechanisms, when measured in information units, is relatively constant over a considerable ...

☆ 99 Cited by 7707 Related articles All 18 versions Web of Science: 3367

Fitts's paper probably most cited in HCI, studies done and redone many times practically

```
[vpn-user-244-044:~ nenisea$ R
```

Install at
https://www.r-project.org/

```
[> print ("hello world!")
[1] "hello world!"
>
```

we will be using **R** and I will try to give you as much as possible of examples

## in your terminal

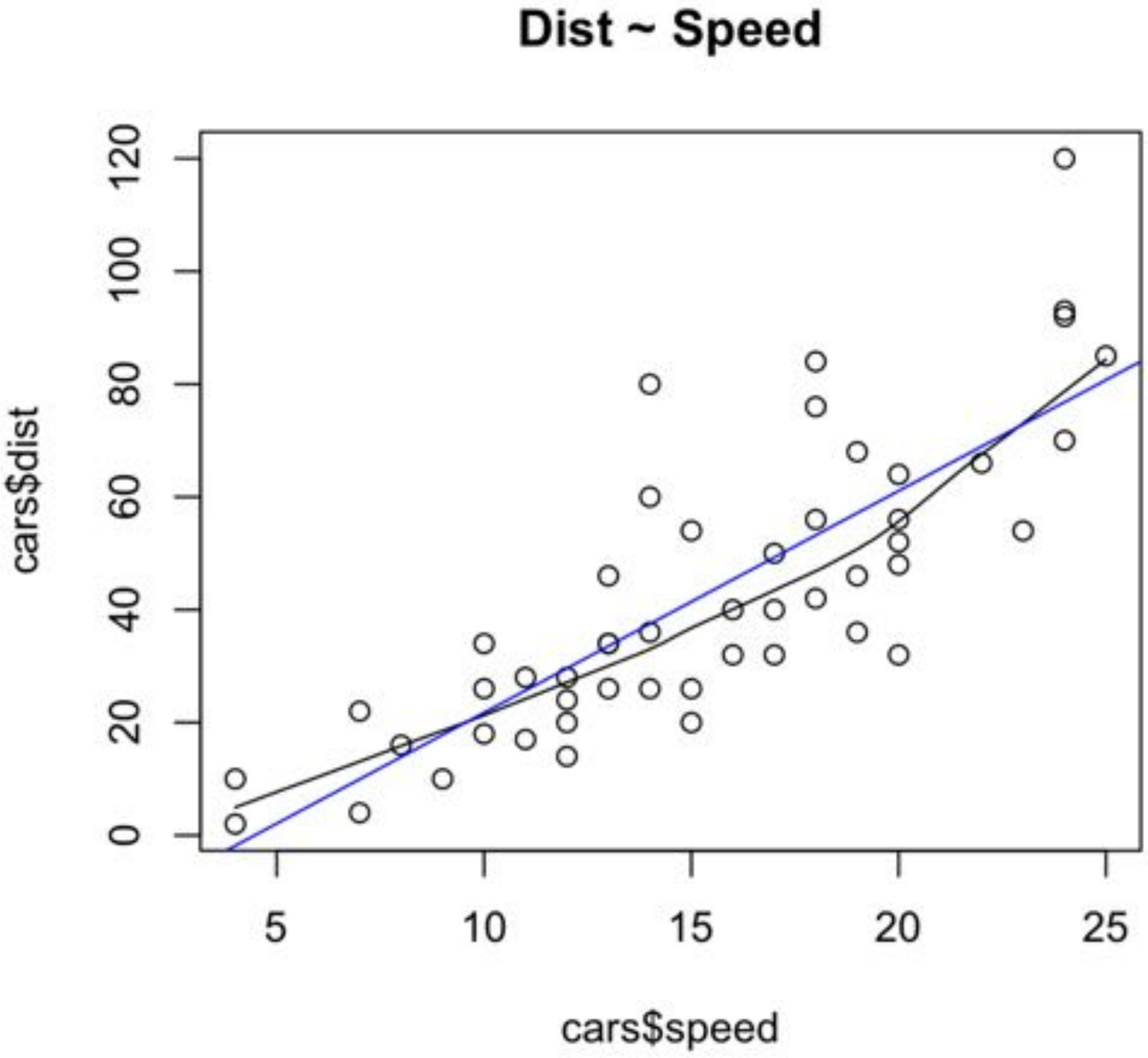
```
head(cars) # cars is a table that already comes with R and
contain 50 observations of speed and distance in two rows

scatter.smooth(x=cars$speed, y=cars$dist, main="Dist ~
Speed")

linearMod <- lm(dist ~ speed, data=cars) # build linear
regression model

abline(linearMod, col="blue") # draw the regression line

summary(linearMod) # goodness of fit
```


Dist ~ Speed
cars$dist
cars$speed
0
20
40
60
80
100
120
5
10
15
20
25

**dist = -17.5791 + 3.9324 * speed**

```
Call:
lm(formula = dist ~ speed, data = cars)

Residuals:
    Min      1Q  Median      3Q     Max 
-29.069  -9.525  -2.272   9.215  43.201 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -17.5791     6.7584  -2.601   0.0123 *  
speed         3.9324     0.4155   9.464 1.49e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 15.38 on 48 degrees of freedom
Multiple R-squared:  0.6511, Adjusted R-squared:  0.6438 
F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

```
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---
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F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

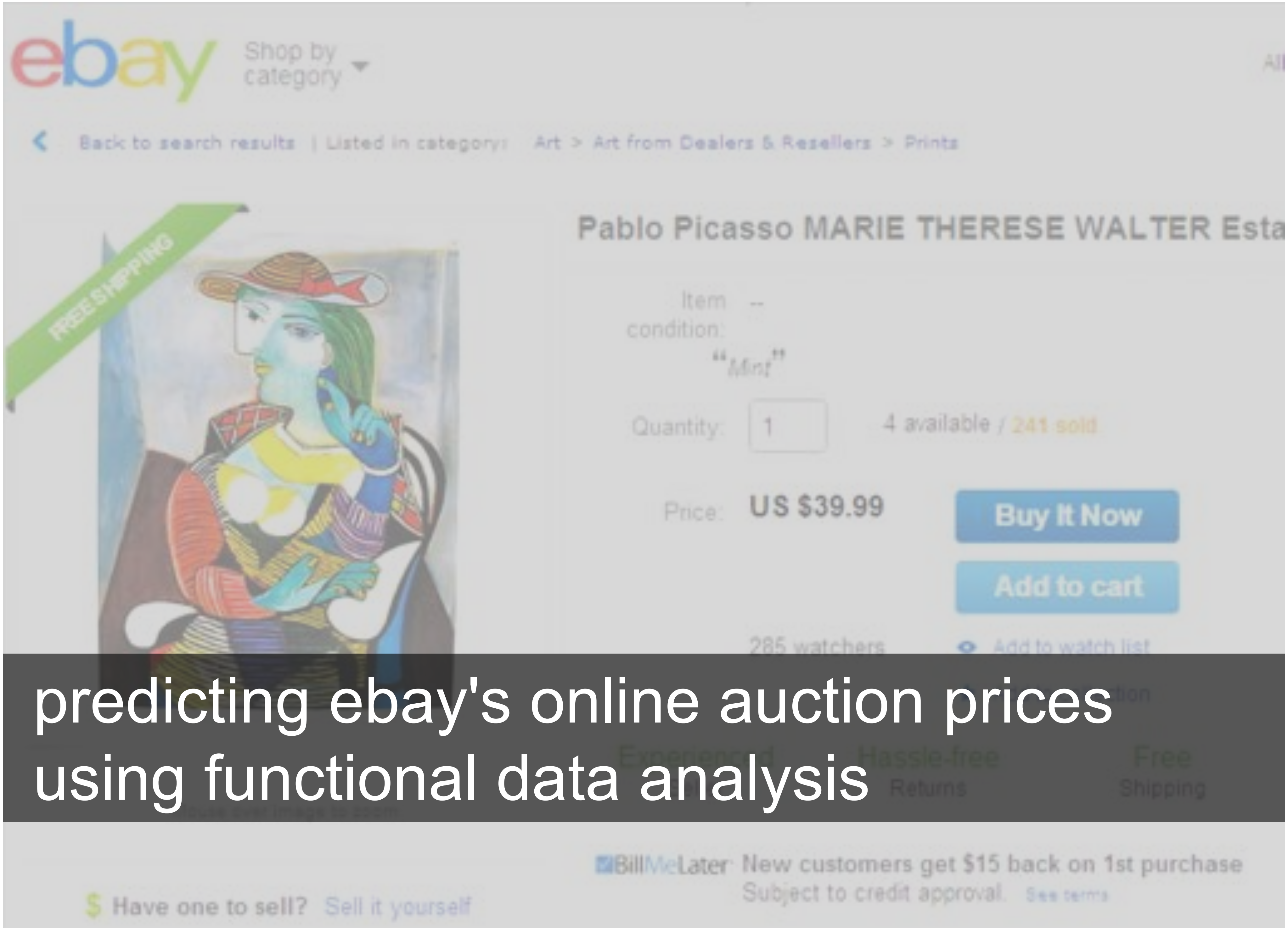
**also note this**

**pvalue<0.01**

**pvalue <0**

**(will explain next week)**

# usages of regressions




# predicting ebay's online auction prices using functional data analysis


predicting the number of passerby who will pass in front of a public display and use the data for choosing advertisement prices


predicting the number of claim in a given time period (used by insurance companies)


understand the impact of a machine (on a production line) on the quality of the products created


understand the relationship between wait times of callers and number of complaints in a call centre


retail store wants to extend shopping hours to increase sales, but regression indicates that increase in revenue not sufficient to support rise in operating expenses

you can also fit a curve = polynomial fitting

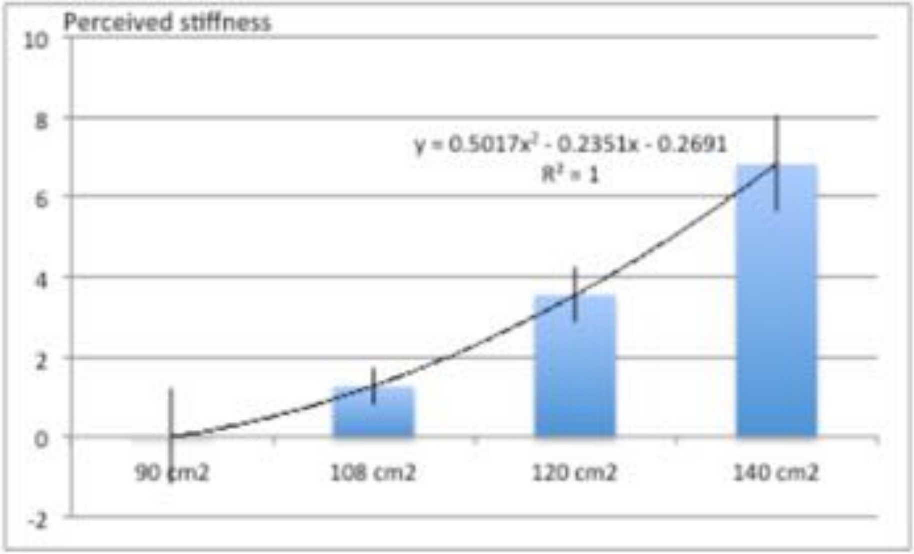



**Figure 11. Bradley-Terry-Luce model output as well as a polynomial regression.**

Our results are illustrated in Figure 11. We observed a clear distinction between the perceived stiffness of the 4 patches, the size of the patch increasing the perceived stiffness. In particular A is the least restrictive, followed by B, then C and then D is the most restrictive. We found that each paired comparison was significant ($p<0.0125$). This thus allows us to compare the different patches and conclude that D is the most efficient patch. We also performed a polynomial regression on our data and found a very accurate fit: y=0.5017x2-0.2351x-0.2691 ($R^2=1$). This suggests a quadratic correlation between the area of the patch and the perceived stiffness, which allows us to imagine bigger patches in order to restrict movements of the knee, which would require more stiffness. Of course further investigations need to be done to confirm this.

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jam th

**Prelin**

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First c
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hands
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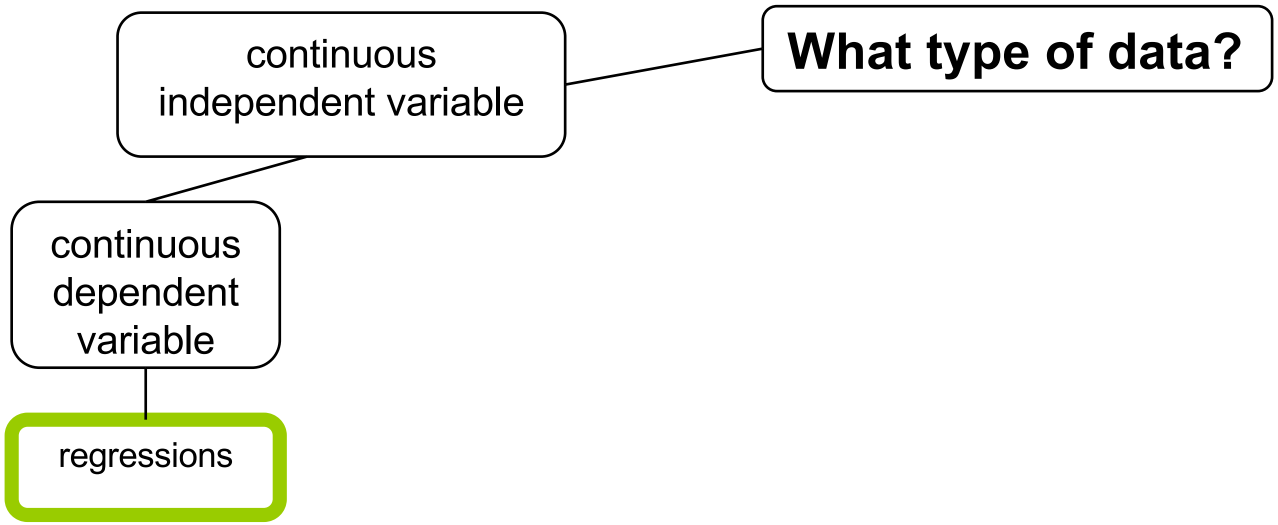
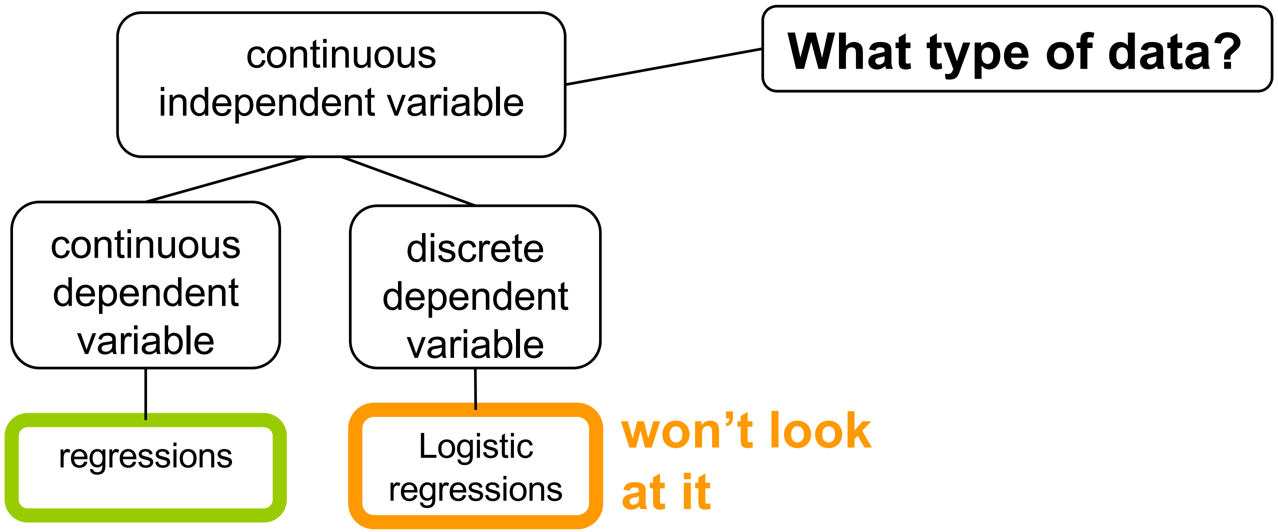
**Patch**

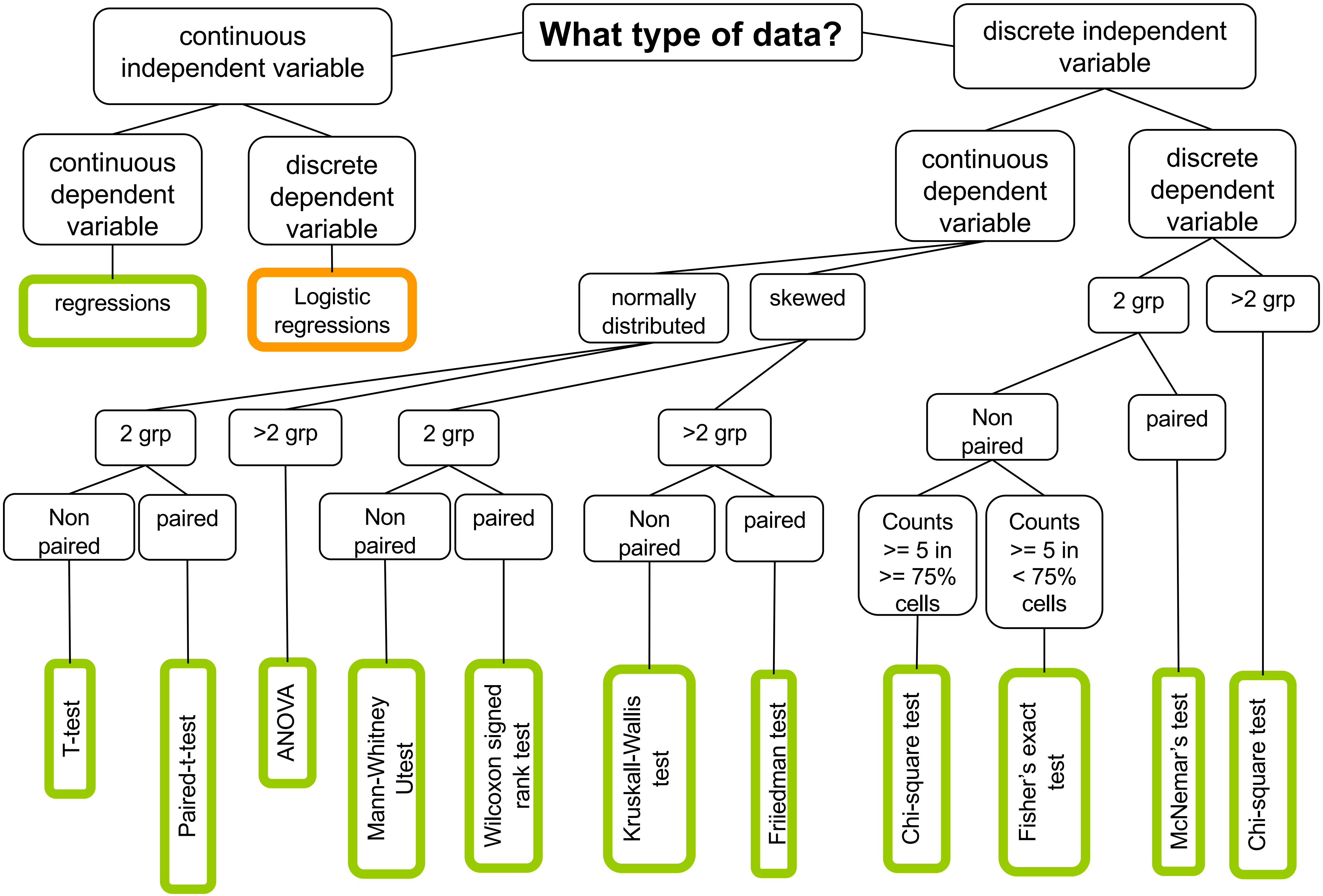
## polynomial regression model

```
Mod2 <- lm(dist~poly(speed,2,raw=TRUE), data=cars)

Mod3 <- lm(dist~poly(speed,3,raw=TRUE), data=cars)

Mod4 <- lm(dist~poly(speed,4,raw=TRUE), data=cars)
```

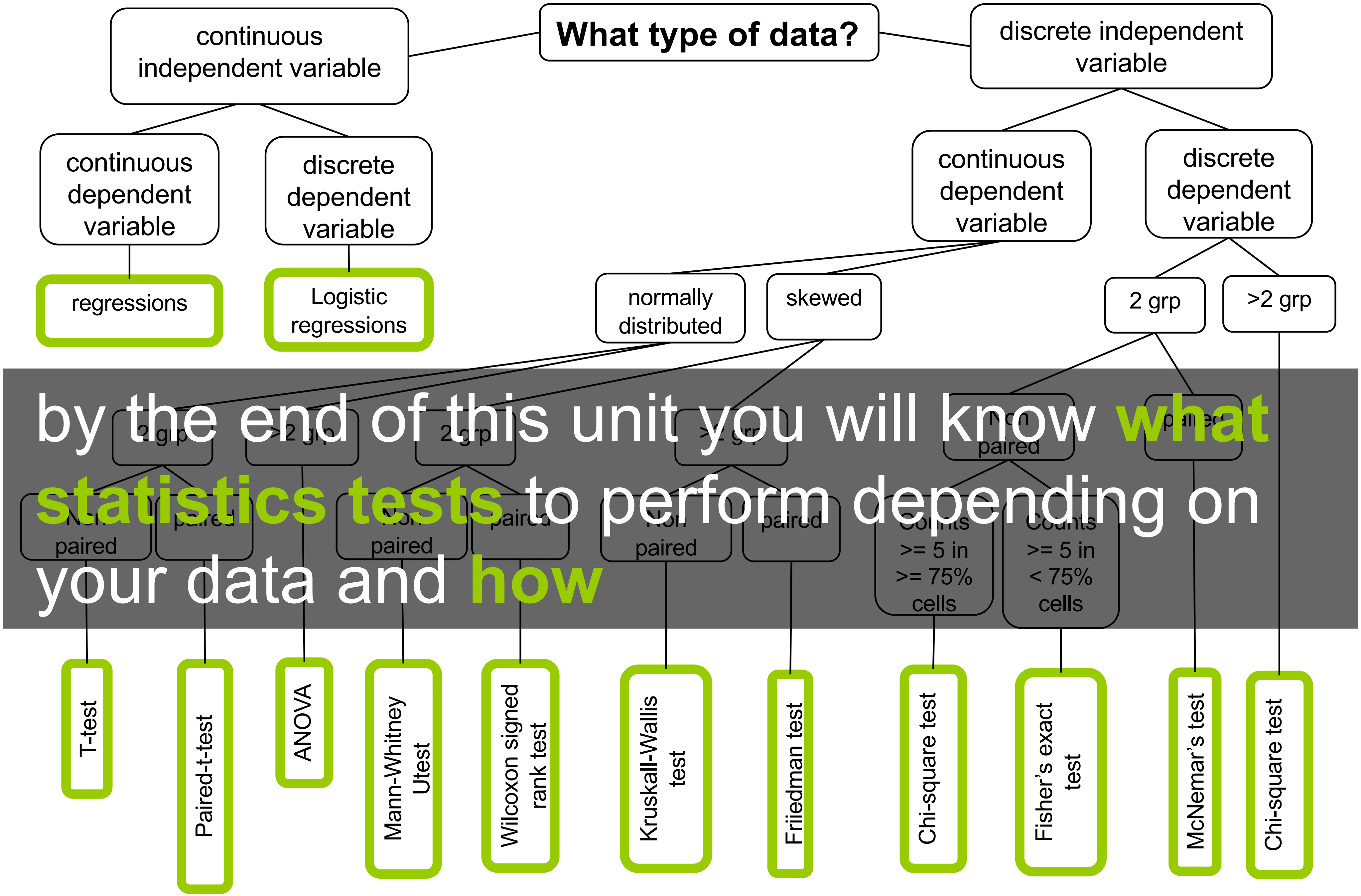
# summary

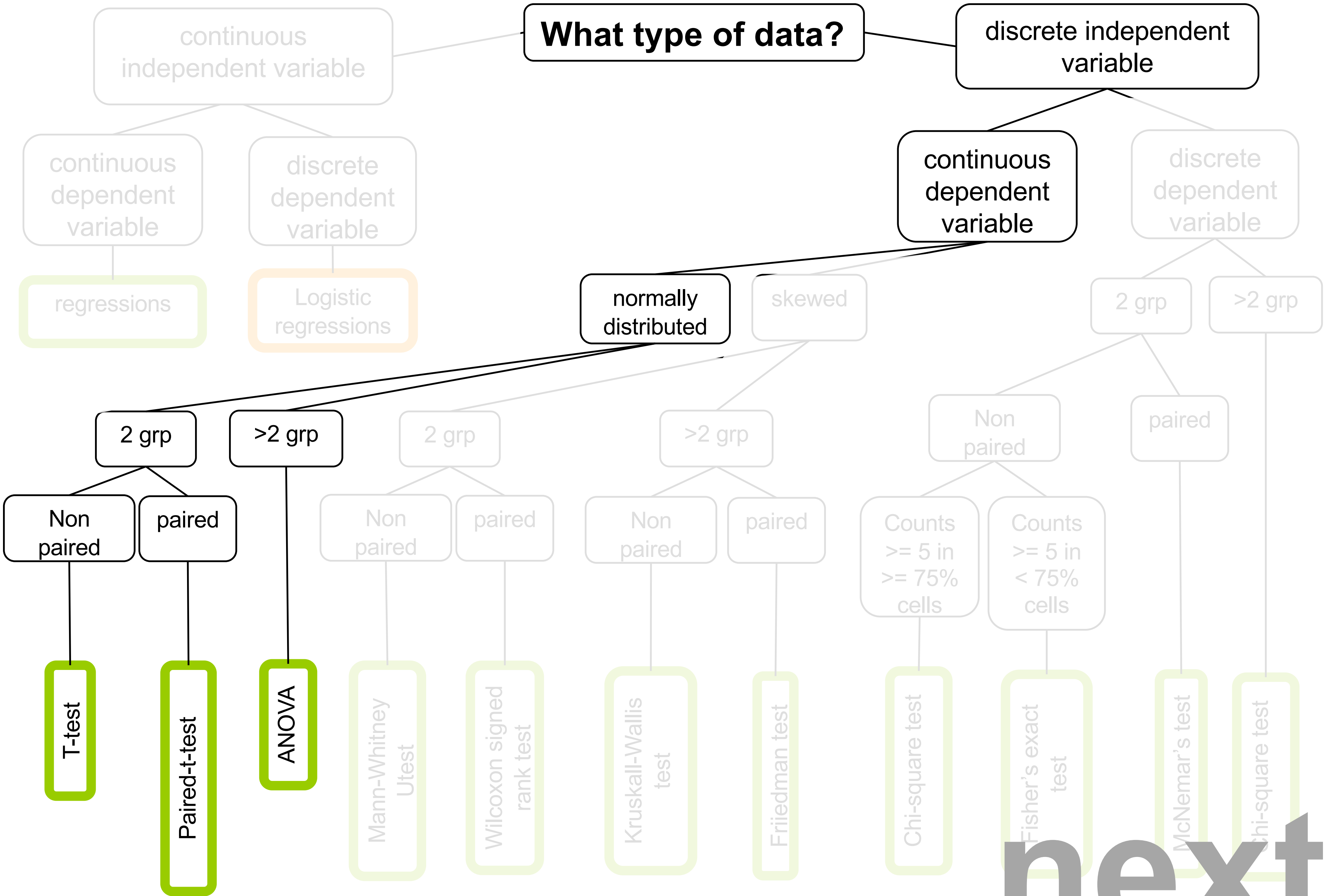

What type of data?
continuous independent variable
continuous dependent variable
regressions


What type of data?
continuous independent variable
continuous dependent variable
discrete dependent variable
regressions
Logistic regressions
won't look at it



What type of data?
continuous independent variable
discrete independent variable
continuous dependent variable
discrete dependent variable
regressions
Logistic regressions
continuous dependent variable
discrete dependent variable
normally distributed
skewed
2 grp
>2 grp
2 grp
>2 grp
2 grp
>2 grp
Non paired
paired
Non paired
paired
Non paired
paired
Non paired
paired
Counts >= 5 in >= 75% cells
Counts >= 5 in < 75% cells
T-test
Paired-t-test
ANOVA
Mann-Whitney Utest
Wilcoxon signed rank test
Kruskall-Wallis test
Friiedman test
Chi-square test
Fisher's exact test
McNemar's test
Chi-square test

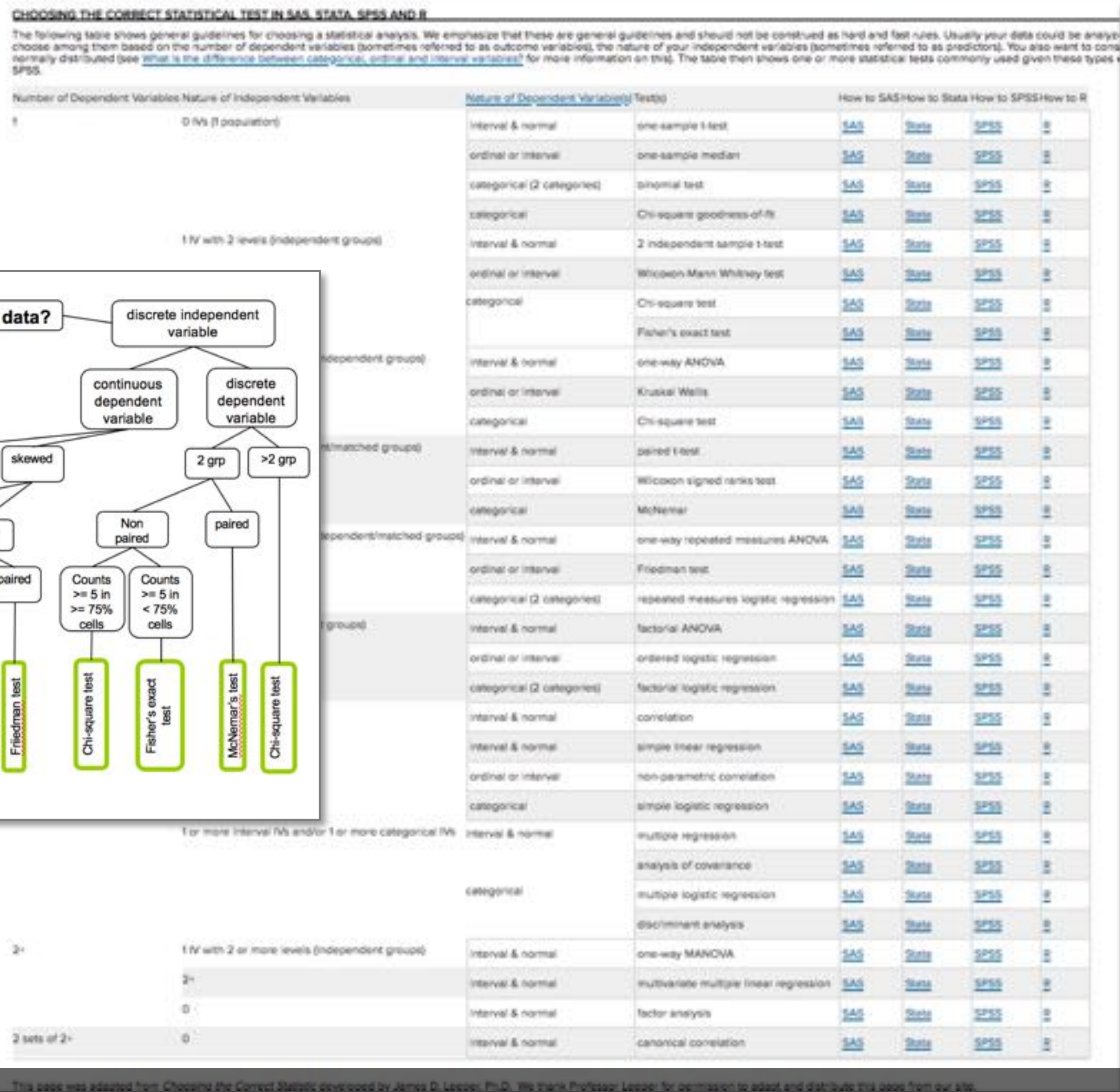

What type of data?
continuous independent variable
discrete independent variable
continuous dependent variable
discrete dependent variable
regressions
Logistic regressions
continuous dependent variable
discrete dependent variable
normally distributed
skewed
2 grp
>2 grp
by the end of this unit you will know **what statistics tests** to perform depending on your data and **how**
paired
paired
paired
>= 5 in >= 75% cells
>= 5 in < 75% cells
T-test
Paired-t-test
ANOVA
Mann-Whitney Utest
Wilcoxon signed rank test
Kruskall-Wallis test
Friiedman test
Chi-square test
Fisher's exact test
McNemar's test
Chi-square test


What type of data?
continuous independent variable
discrete independent variable
continuous dependent variable
discrete dependent variable
regressions
Logistic regressions
continuous dependent variable
discrete dependent variable
normally distributed
skewed
2 grp
>2 grp
2 grp
>2 grp
Non paired
paired
Non paired
paired
Non paired
paired
Non paired
paired
Counts >= 5 in >= 75% cells
Counts >= 5 in < 75% cells
T-test
Paired-t-test
ANOVA
Mann-Whitney Utest
Wilcoxon signed rank test
Kruskall-Wallis test
Friedman test
Chi-square test
Fisher's exact test
McNemar's test
Chi-square test
next

1. Linear regression
2. Hypothesis testing, comparing things
3. Experimental design a: T-test
4. Experimental design b: ANOVA
5. How T-test and ANOVA work
6. Non-parametric tests a, normality tests
7. Non-parametric tests b
8. Categorical data: Chi-square
9. Sample size, power and effect size
10. Alternatives to p value testing
11. Questions before exam

# unit menu

# resources

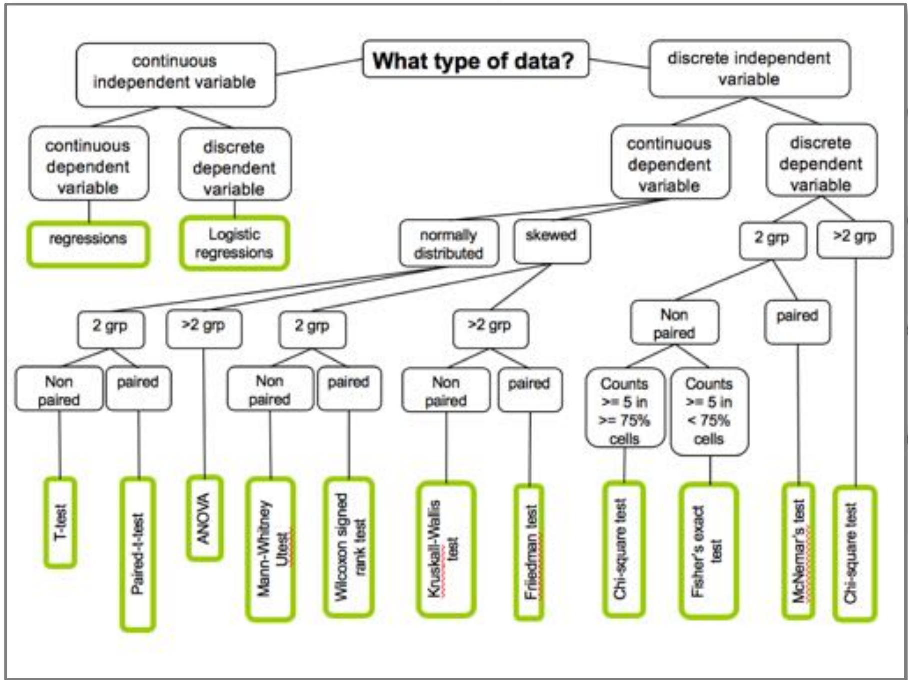


CHOOSING THE CORRECT STATISTICAL TEST IN SAS, STATA, SPSS AND R

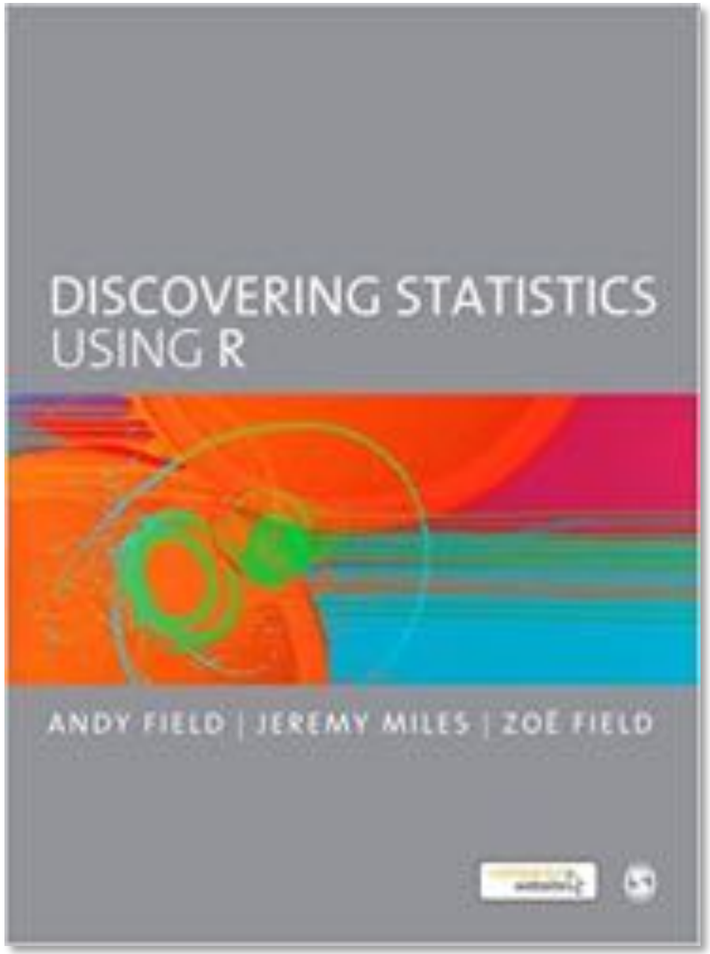
The following table shows general guidelines for choosing a statistical analysis. We emphasize that these are general guidelines and should not be construed as hard and fast rules. Usually your data could be analyz... choose among them based on the number of dependent variables (sometimes referred to as outcome variables), the nature of your independent variables (sometimes referred to as predictors). You also want to cons... normally distributed (see What is the difference between categorical, ordinal and interval variables? for more information on this). The table then shows one or more statistical tests commonly used given these types o... SPSS.

| Number of Dependent Variables | Nature of Independent Variables | Nature of Dependent Variable(s) | Test(s) | How to SAS | How to Stata | How to SPSS | How to R |
|---|---|---|---|---|---|---|---|
| 1 | 0 IVs (1 population) | interval & normal | one-sample t-test | SAS | Stata | SPSS | R |
| | | ordinal or interval | one-sample median | SAS | Stata | SPSS | R |
| | | categorical (2 categories) | binomial test | SAS | Stata | SPSS | R |
| | | categorical | Chi-square goodness-of-fit | SAS | Stata | SPSS | R |
| | 1 IV with 2 levels (independent groups) | interval & normal | 2 independent sample t-test | SAS | Stata | SPSS | R |
| | | ordinal or interval | Wilcoxon-Mann Whitney test | SAS | Stata | SPSS | R |
| | | categorical | Chi-square test | SAS | Stata | SPSS | R |
| | | | Fisher's exact test | SAS | Stata | SPSS | R |
| | ...independent groups) | interval & normal | one-way ANOVA | SAS | Stata | SPSS | R |
| | | ordinal or interval | Kruskal Wallis | SAS | Stata | SPSS | R |
| | | categorical | Chi-square test | SAS | Stata | SPSS | R |
| | ...t/matched groups) | interval & normal | paired t-test | SAS | Stata | SPSS | R |
| | | ordinal or interval | Wilcoxon signed ranks test | SAS | Stata | SPSS | R |
| | | categorical | McNemar | SAS | Stata | SPSS | R |
| | ...ependent/matched groups) | interval & normal | one-way repeated measures ANOVA | SAS | Stata | SPSS | R |
| | | ordinal or interval | Friedman test | SAS | Stata | SPSS | R |
| | | categorical (2 categories) | repeated measures logistic regression | SAS | Stata | SPSS | R |
| | ...groups) | interval & normal | factorial ANOVA | SAS | Stata | SPSS | R |
| | | ordinal or interval | ordered logistic regression | SAS | Stata | SPSS | R |
| | | categorical (2 categories) | factorial logistic regression | SAS | Stata | SPSS | R |
| | | interval & normal | correlation | SAS | Stata | SPSS | R |
| | | interval & normal | simple linear regression | SAS | Stata | SPSS | R |
| | | ordinal or interval | non-parametric correlation | SAS | Stata | SPSS | R |
| | | categorical | simple logistic regression | SAS | Stata | SPSS | R |
| | 1 or more interval IVs and/or 1 or more categorical IVs | interval & normal | multiple regression | SAS | Stata | SPSS | R |
| | | | analysis of covariance | SAS | Stata | SPSS | R |
| | | categorical | multiple logistic regression | SAS | Stata | SPSS | R |
| | | | discriminant analysis | SAS | Stata | SPSS | R |
| 2+ | 1 IV with 2 or more levels (independent groups) | interval & normal | one-way MANOVA | SAS | Stata | SPSS | R |
| | 2+ | interval & normal | multivariate multiple linear regression | SAS | Stata | SPSS | R |
| | 0 | interval & normal | factor analysis | SAS | Stata | SPSS | R |
| 2 sets of 2+ | 0 | interval & normal | canonical correlation | SAS | Stata | SPSS | R |

This page was adapted from Choosing the Correct Statistic developed by James D. Leeper, Ph.D. We thank Professor Leeper for permission to adapt and distribute this page from our site.




https://stats.idre.ucla.edu/other/mult-pkg/whatstat/





the **text book** I am using and a suggestion of **YouTube video channel**

## videos on regressions

https://www.youtube.com/watch?v=WWqE7YHR4Jc

https://www.youtube.com/playlist?list=PLF596A4043DBEAE9C

1. Explain what is a linear regression
2. Give the terminology of a regression line
3. Give the two formulas of goodness of fit
4. Be able to compute the two formulas given a few observations

**exam**

end