Comparing things
and hypothesis testing

12

Probability and Statistics

COMS10011

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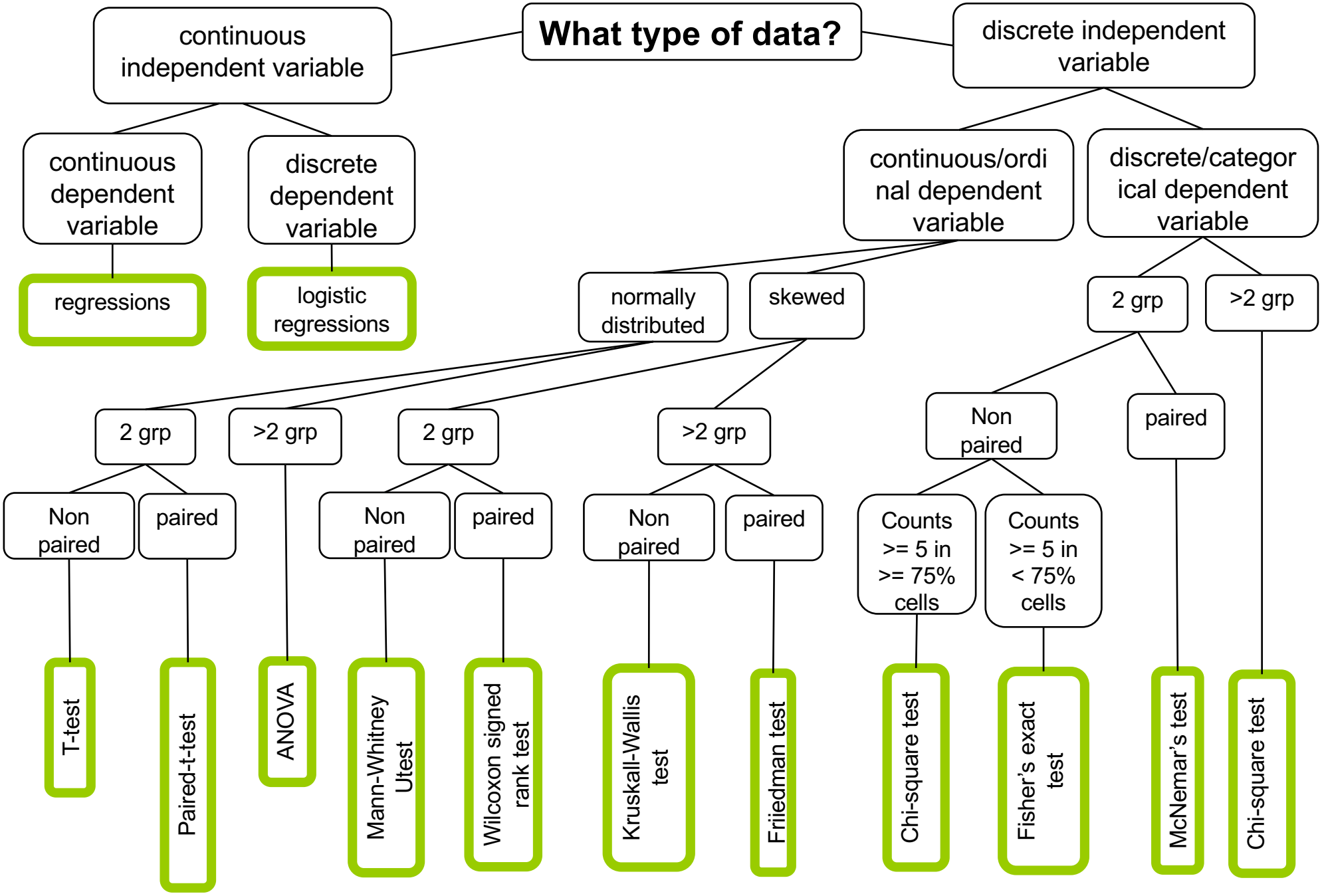
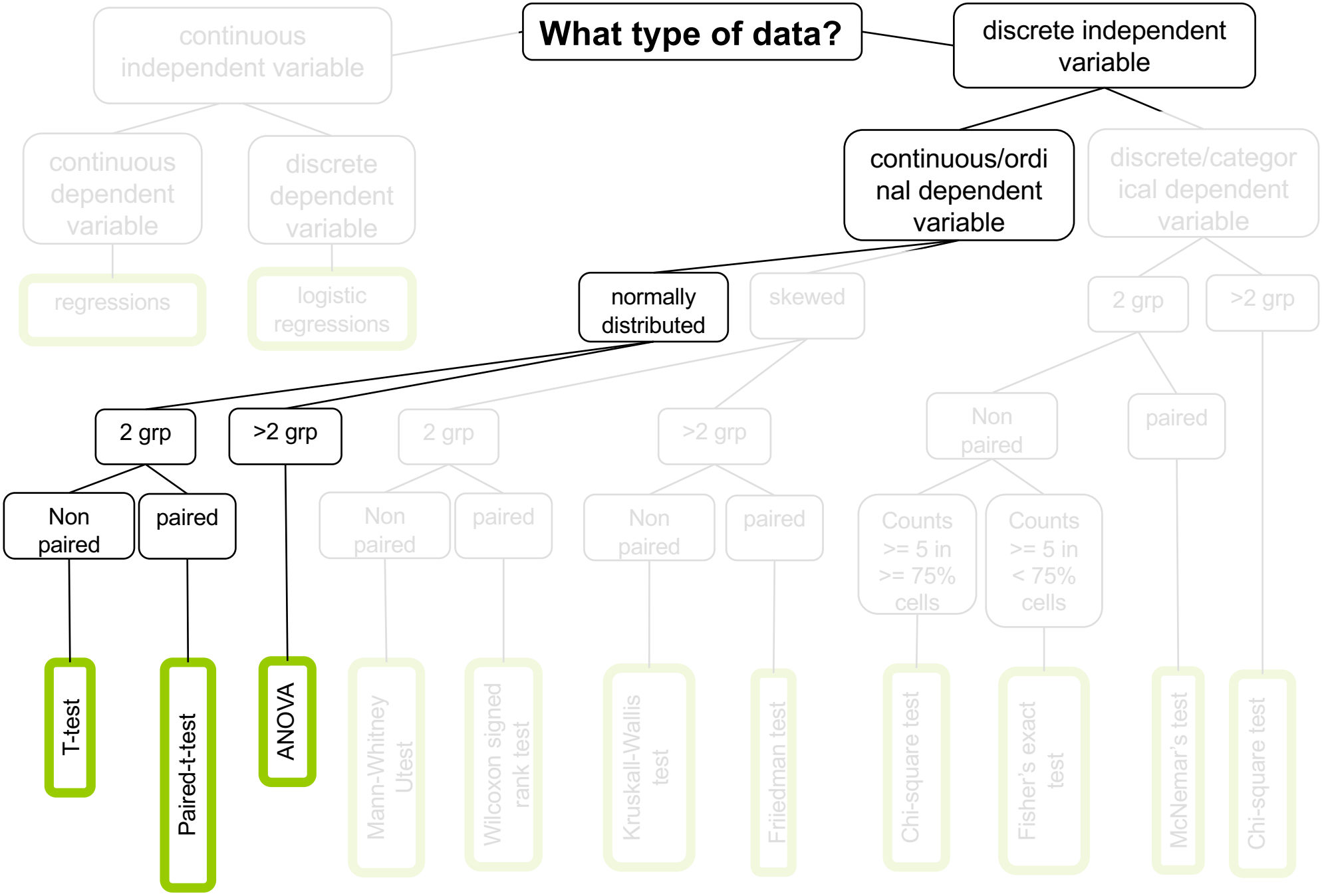

What type of data?
continuous independent variable
discrete independent variable
continuous dependent variable
discrete dependent variable
continuous/ordinal dependent variable
discrete/categorical dependent variable
regressions
logistic regressions
normally distributed
skewed
2 grp
>2 grp
Non paired
paired
T-test
Paired-t-test
ANOVA
Mann-Whitney Utest
Wilcoxon signed rank test
Kruskall-Wallis test
Friedman test
Counts >= 5 in >= 75% cells
Counts >= 5 in < 75% cells
Chi-square test
Fisher's exact test
McNemar's test
Chi-square test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Fisher's exact test
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Chi-square test


let’s start with a contrived example…

you own a magic pair of shoes. You have run the 10 meters with it a lot and you **have a long log of the time you have run with it.**

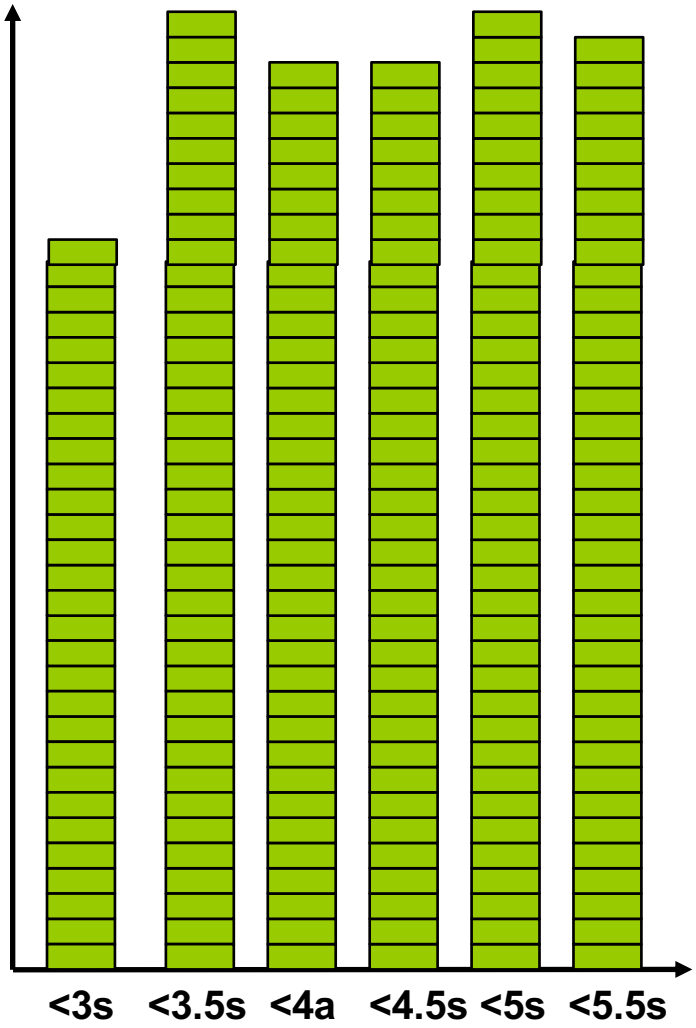
one day, you come home and it seems like someone has moved your shoes. You get concerned that someone might have taken your (beloved) shoes and instead **replaced it with identical looking shoes.**

you inspect the shoes long and hard and they *look* the same. But still worried. **How do you verify that they are the same?**

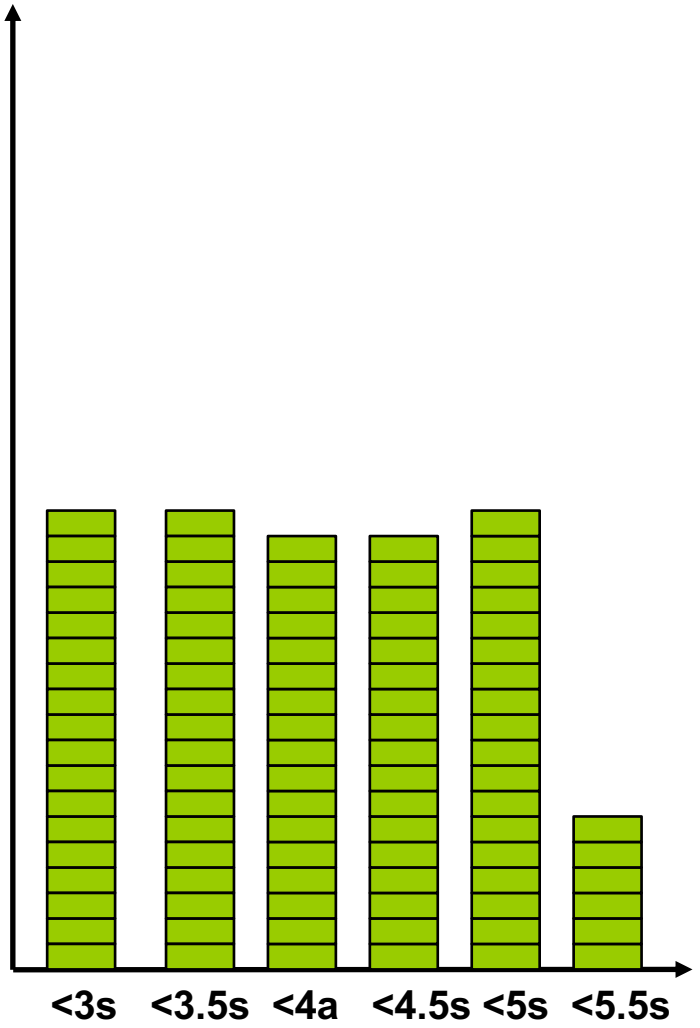
<30 sec brainstorming>

ok, so **you run the 10 meters with the “questionable” shoes** a bunch of times.

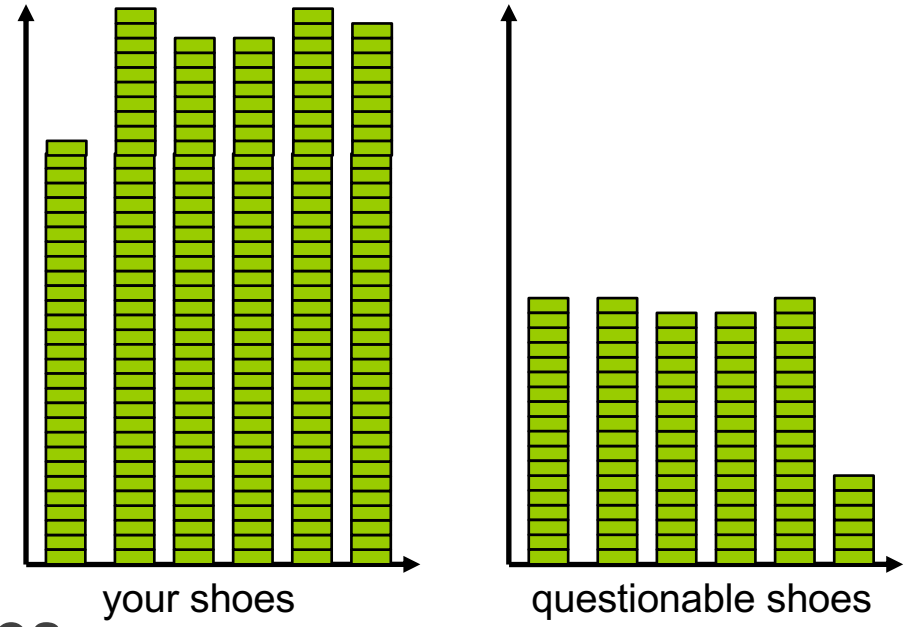
here is what you see…



your shoes



questionable shoes

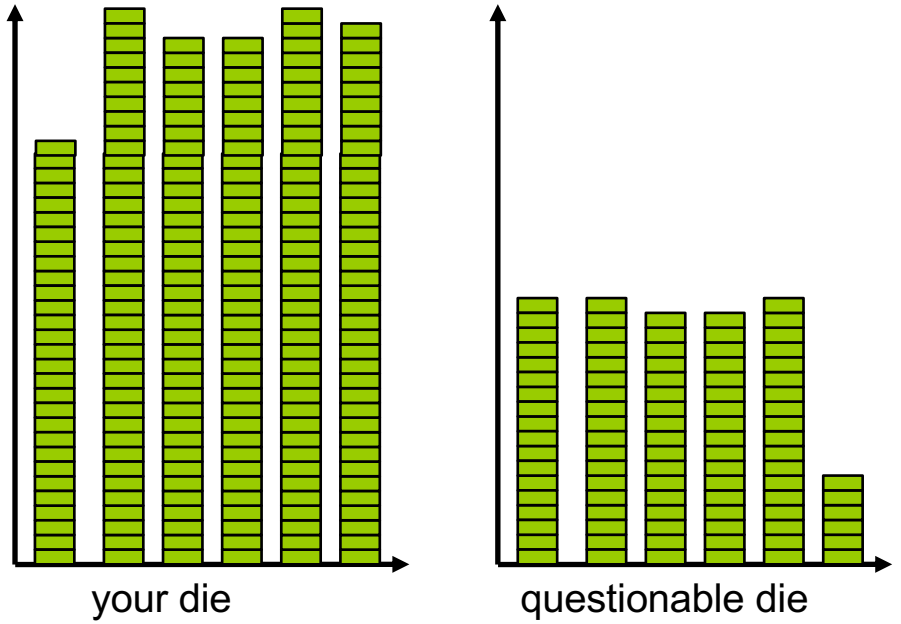


[ ] this is still the original shoes

[ ] someone has replaced my shoes

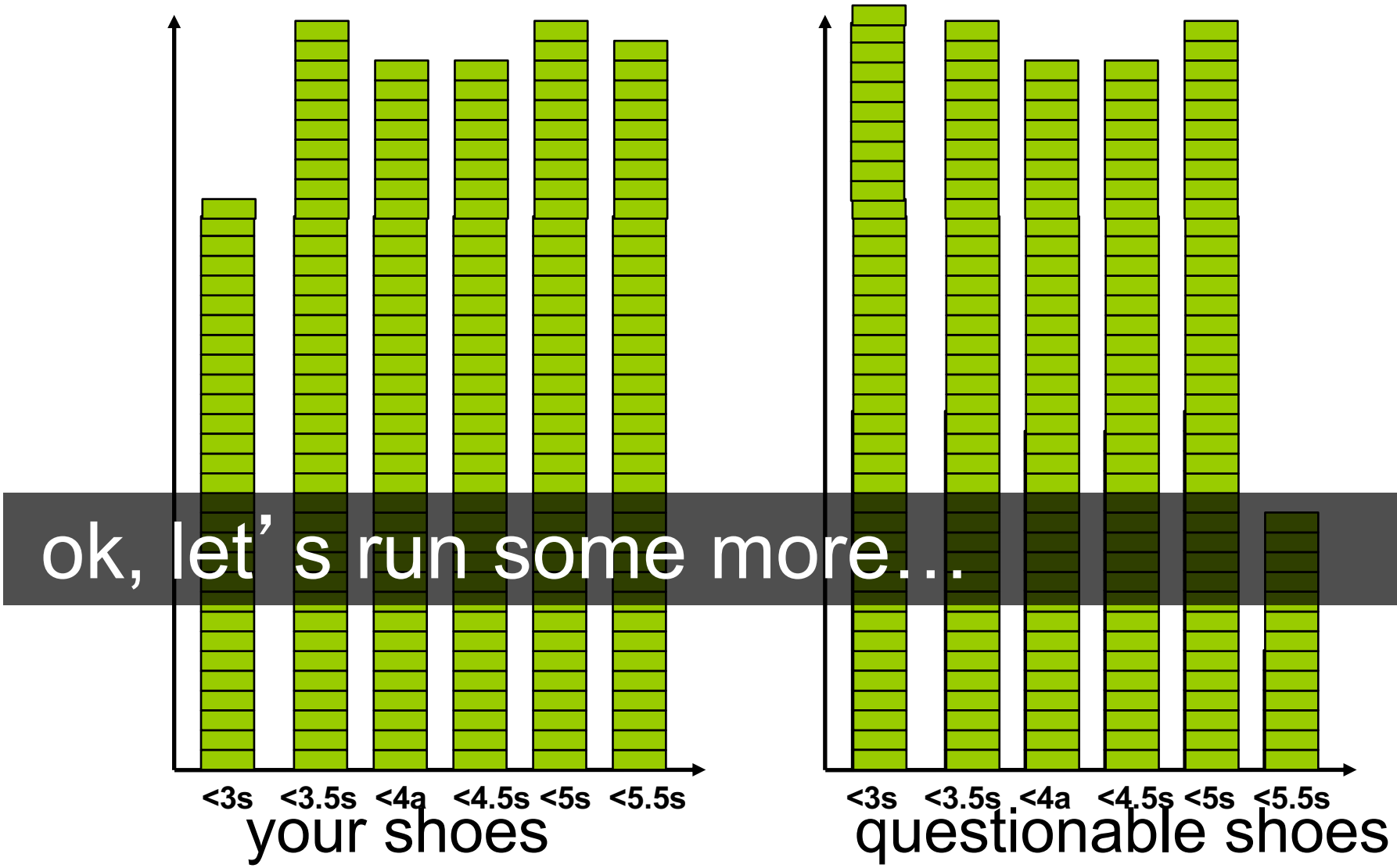
[x] could be the original or replacement shoes

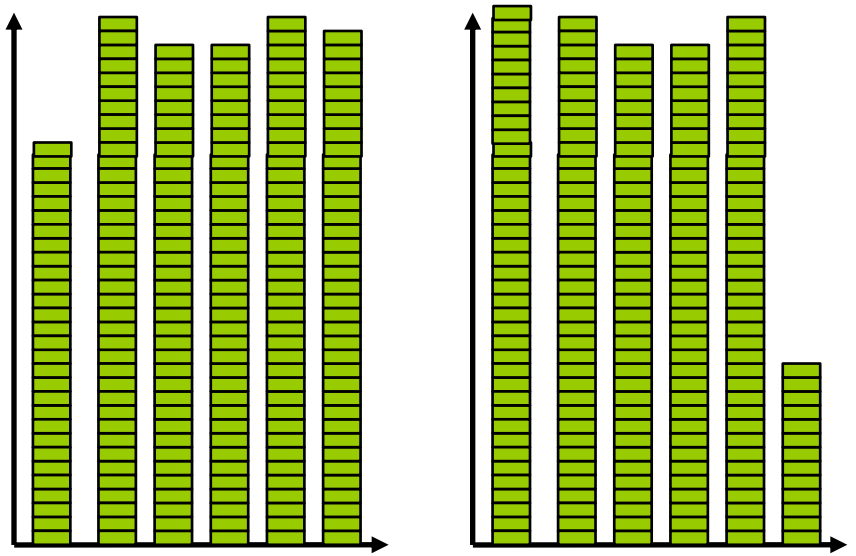
<let's vote>



the distribution looks different from the shoes you know

while it is **possible** that it is the same shoes, it seems somewhat **unlikely**


ok, let’s run some more…
<3s
<3.5s
<4a
<4.5s
<5s
<5.5s
your shoes
<3s
<3.5s
<4a
<4.5s
<5s
<5.5s
questionable shoes

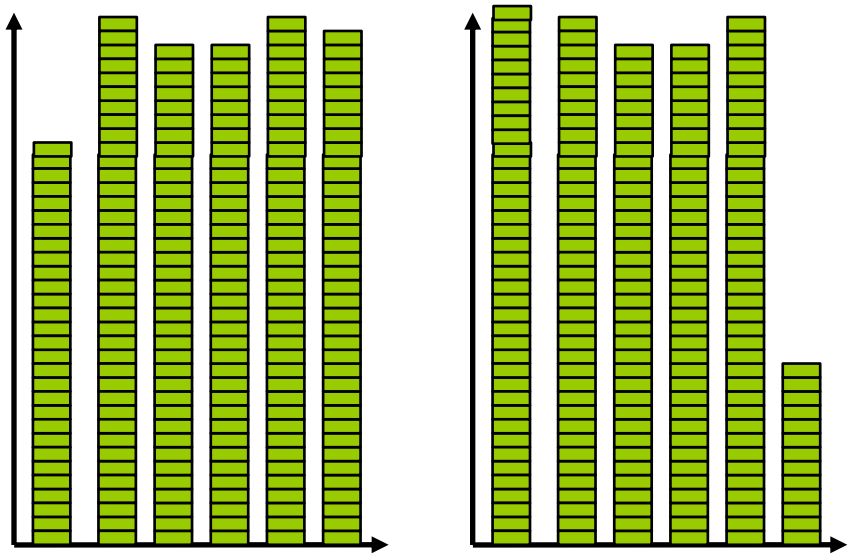


[ ] this is probably still the original shoes

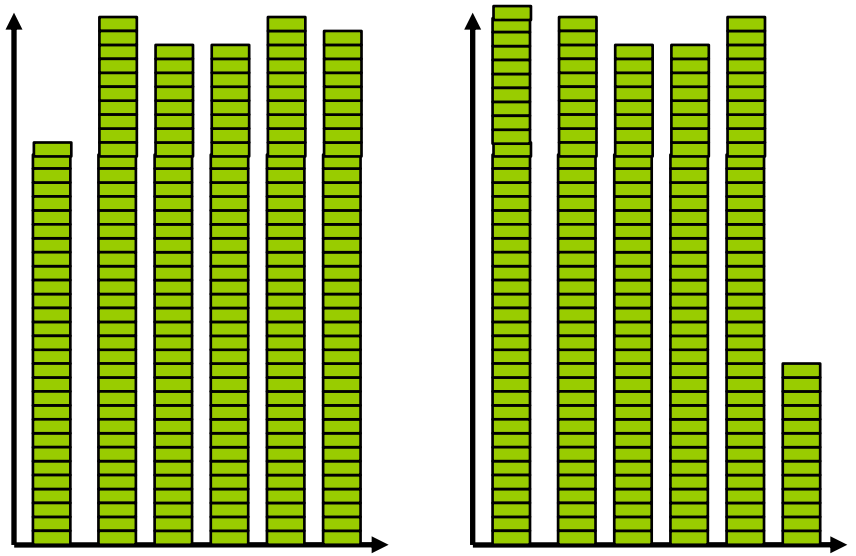
[x] someone probably has replaced my shoes

[x] could be the original or replacement shoes

<let’s vote>



again, the distribution could have happened by chance, but it seems **even more unlikely.** This is **probably not** your shoes



again, the distribution could have happened by chance, but it seems **even more unlikely.** This is **probably not** your shoes

are you **sure** this it is not your shoes?

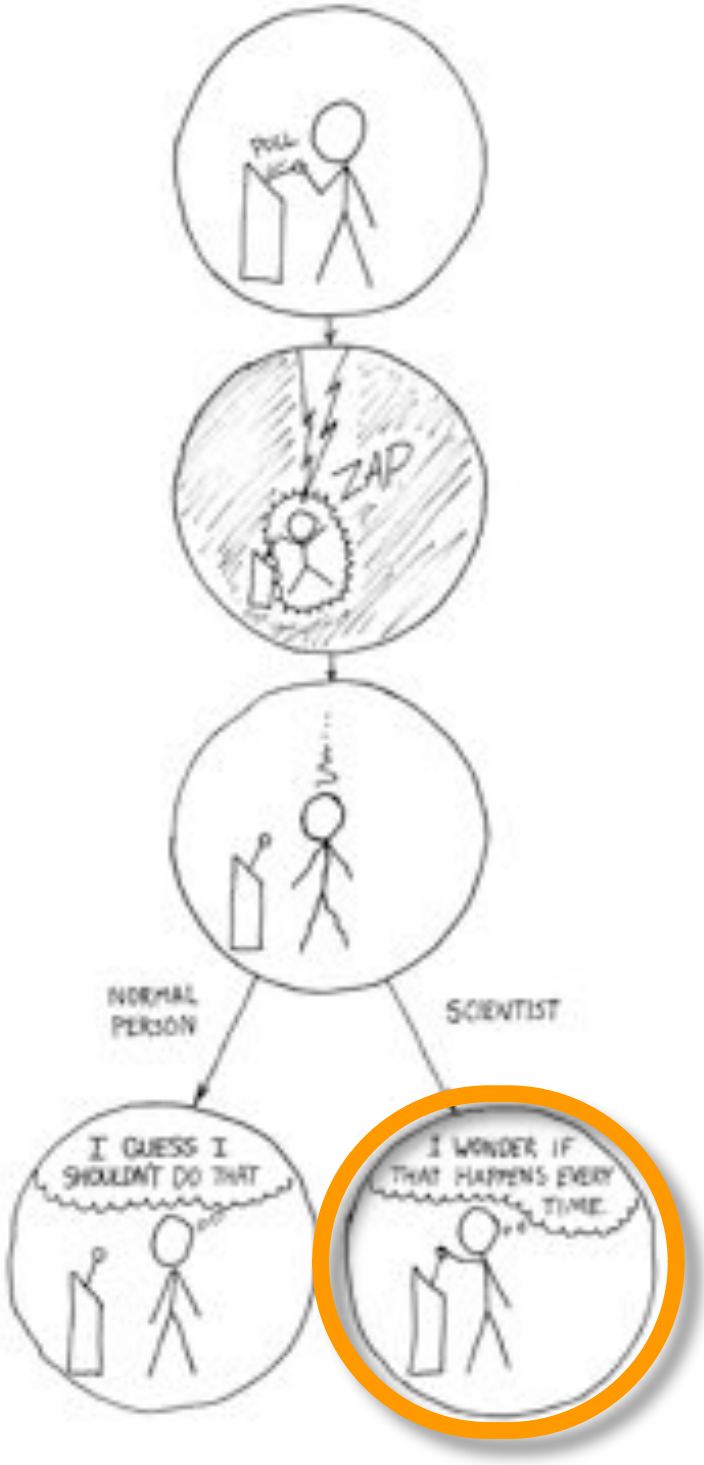
you are **not sure.**

what can you do **to be sure?**

there is **nothing** you can do,
you can **never be sure**

it is a limitation of science:

no matter how often you pull the lever, it could **always** be chance



the good news:

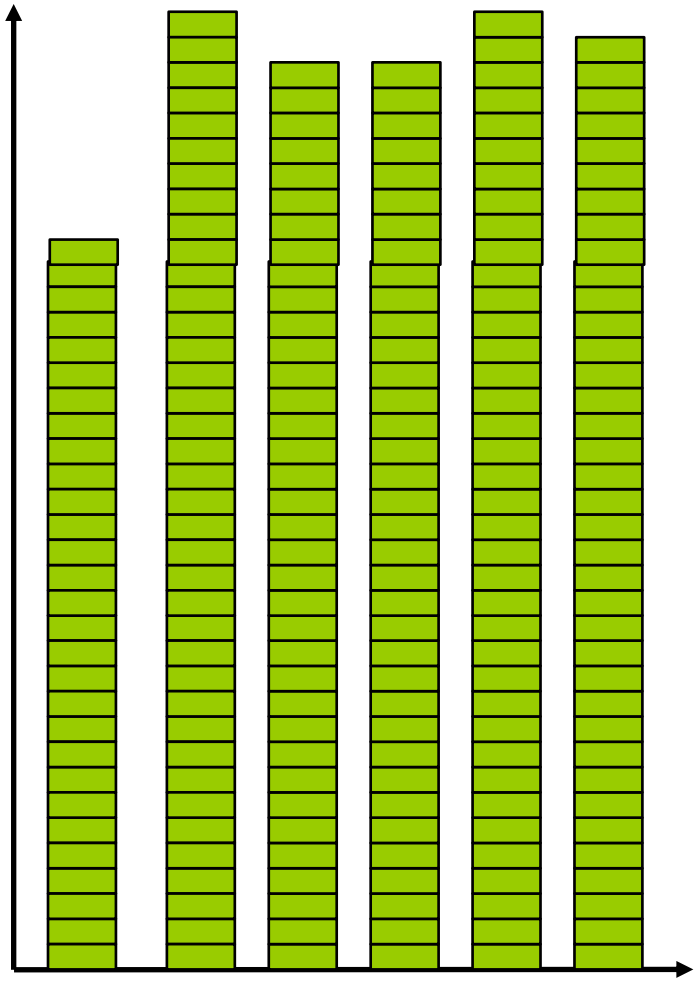
the more sample (# of runs), the more your confidence increases

→ you can be **arbitrarily sure**

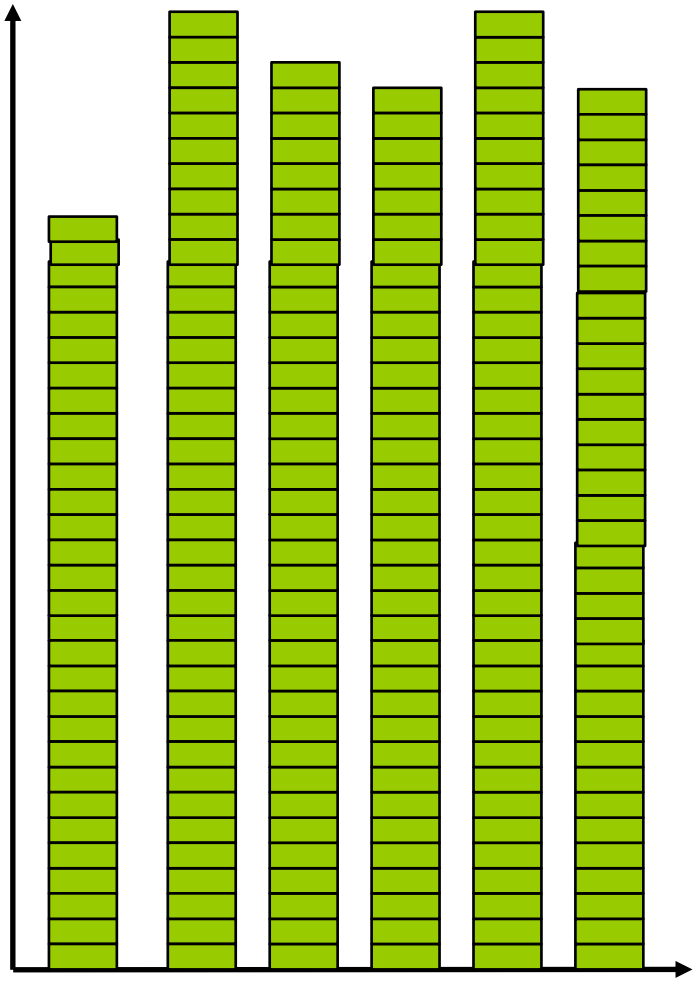
another round

ok, you get your original shoes back

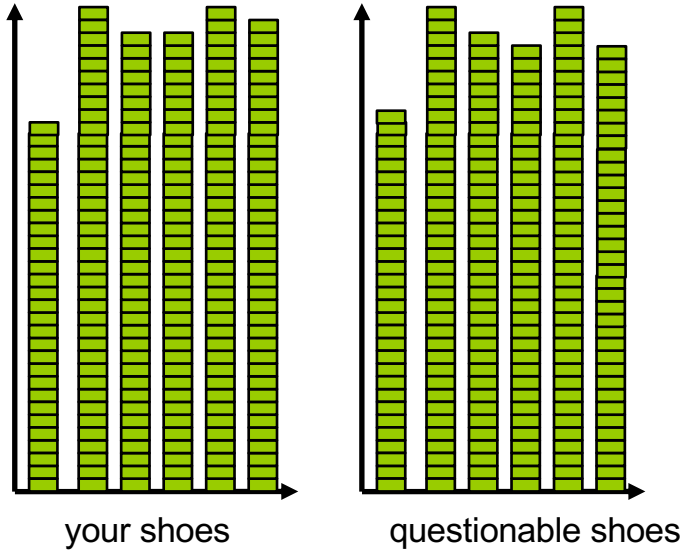
a week later the same thing **happens again.**
again, **you run 10 meters a few times** with the
questionable shoes …



your shoes



questionable shoes

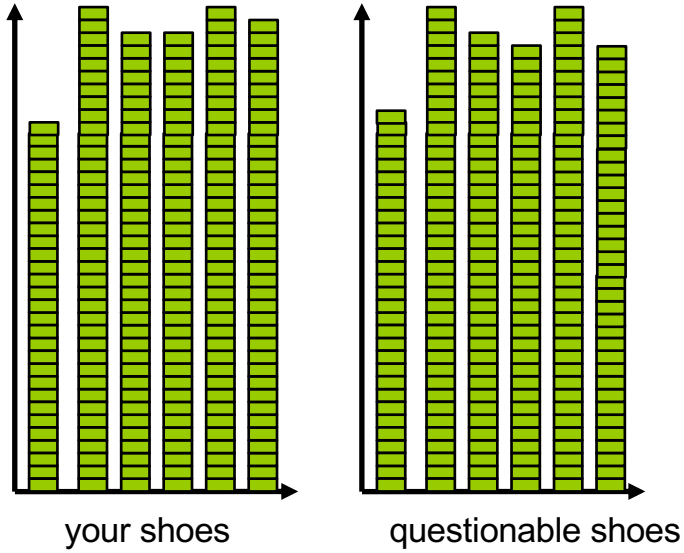


[ ] this is still the original shoes

[ ] someone has replaced my shoes

[ ] could be the original or replacement shoes

<let’s vote>



[ ] this is still the original shoes

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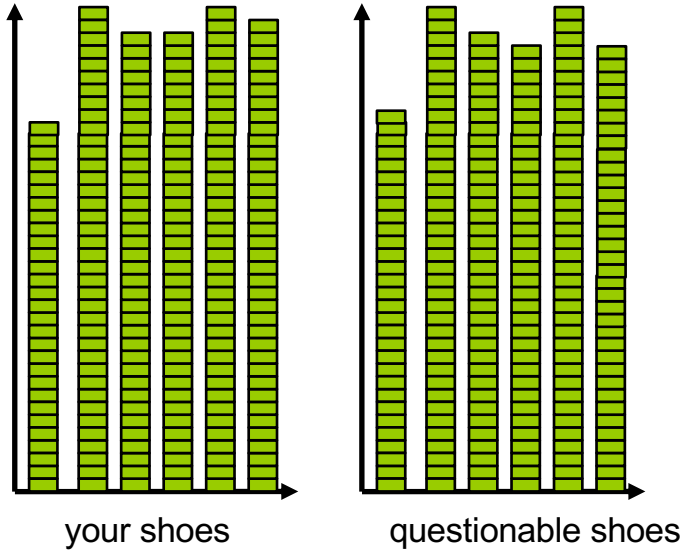
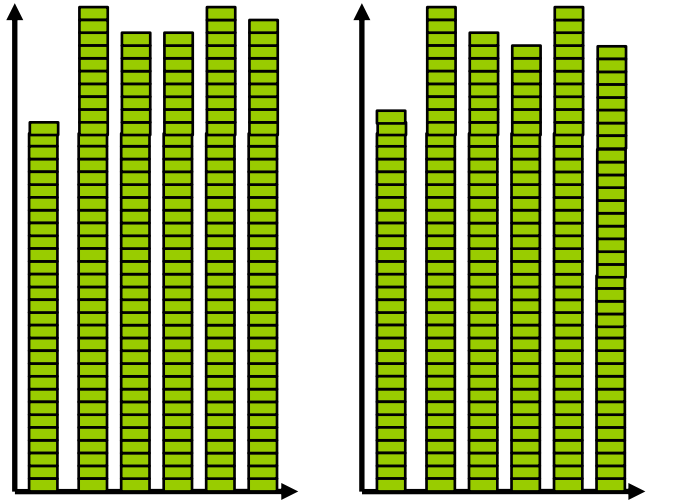


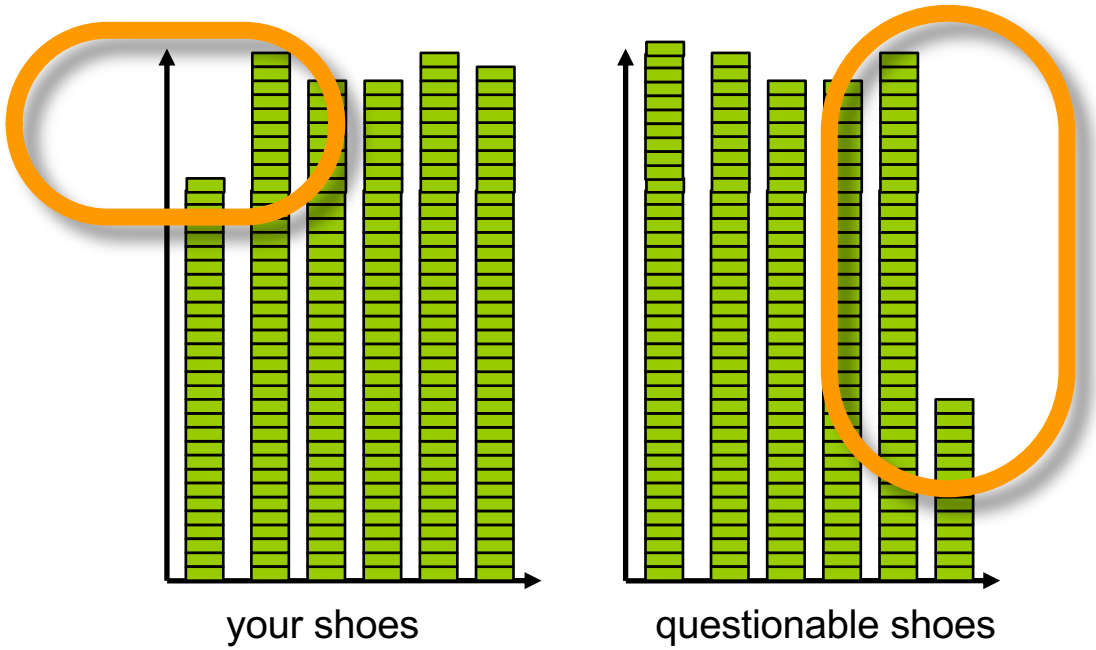
it could be your original shoes, or one that just happens to **behave the same.** a very, very, good copy maybe.



what are **the odds** of this being a different shoes?

you **cannot compute** the odds.

that’s strange! why not?



this seems **unlikely…**

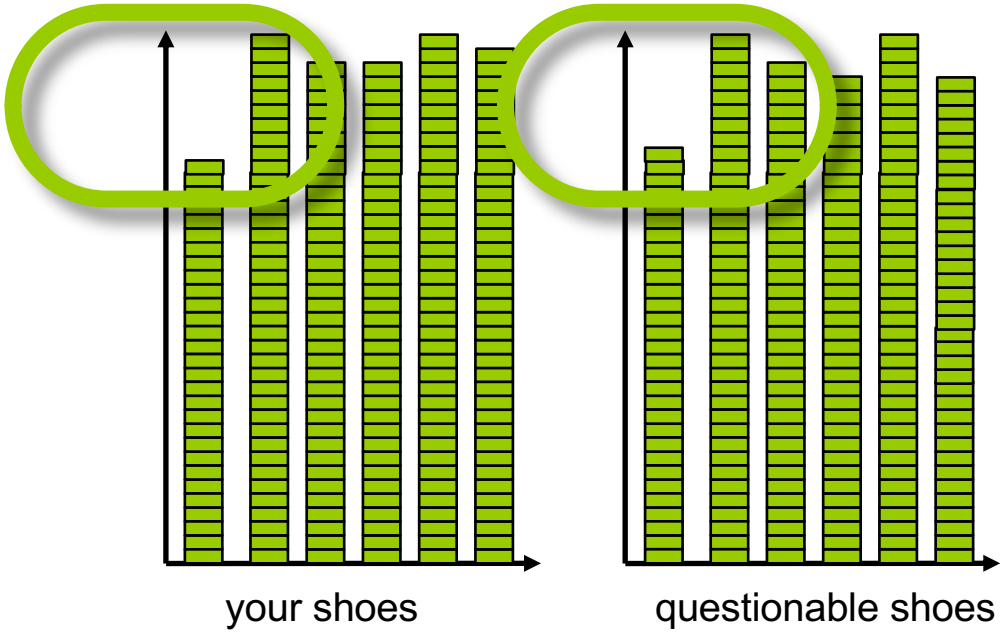
in both cases, there are two explanations

1.same shoes

2.different shoes

…thus this **must be true**

…now, in the other case



this **does seem not unlikely…**

in both cases, there are two explanations

1.same shoes

2.different shoes

…thus…

no thus

we still have **two possible explanations**

→ we **cannot conclude** anything

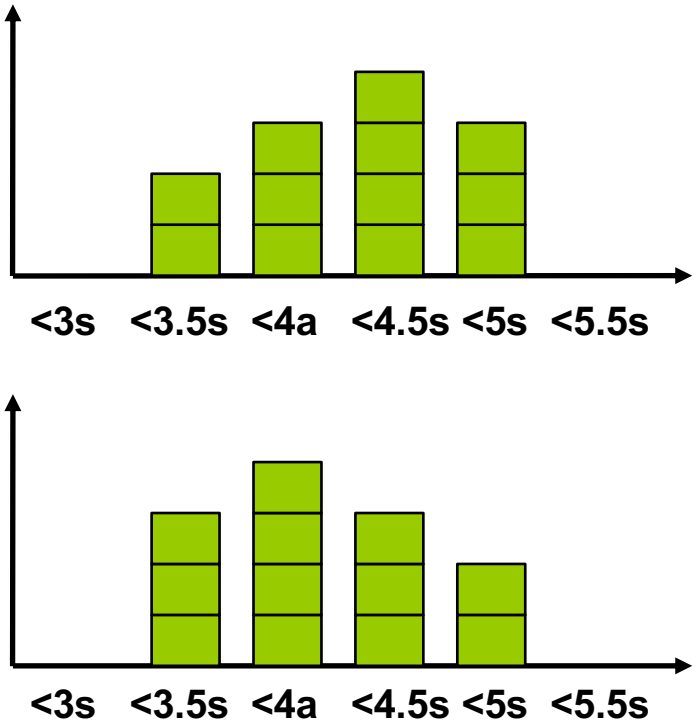
let’s use stats

statistical significance ::

a result is called statistically significant if it is **unlikely to have occurred by chance**

before I show you how to compute, let's test our **intuition**

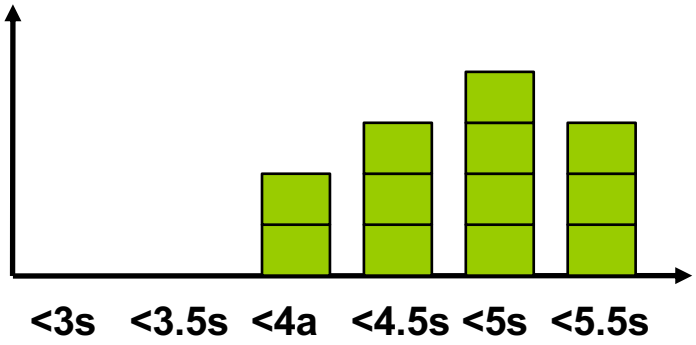
I show you pairs of distributions,
you tell me if the differences are **"statistically different"**

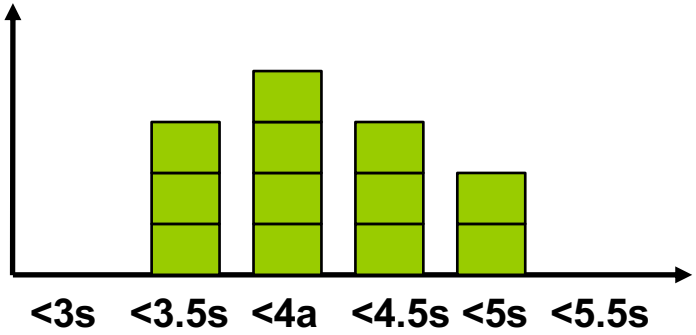


could have happened **by chance**
(45% dissimilar)

<30sec brainstorming>

TTEST pvalue = 0.4548

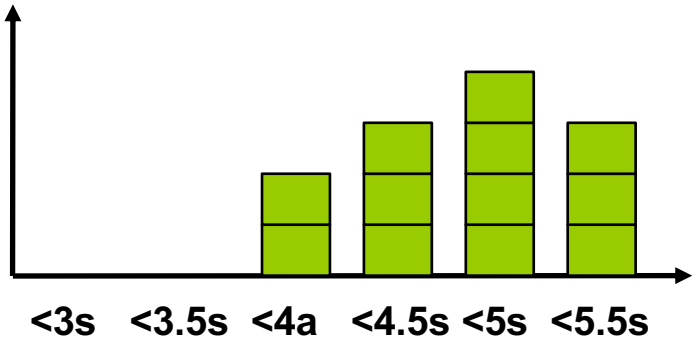


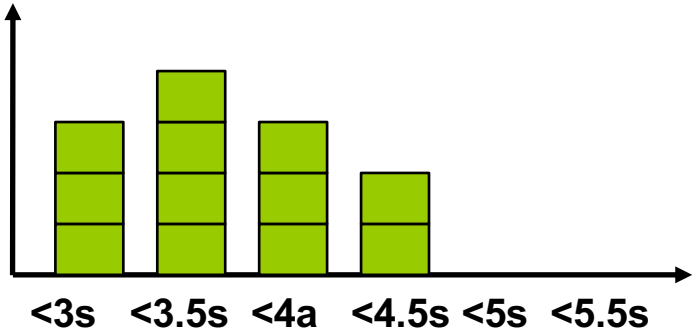


still could have happened **by chance**
(14% dissimilar)

<30sec brainstorming>

TTEST pvalue = 0.1423





unlikely to have happened **by chance**
(0.1% dissimilar)

<30sec brainstorming>

TTEST pvalue = 0.00097

(student's) t-test

return a **p-value**

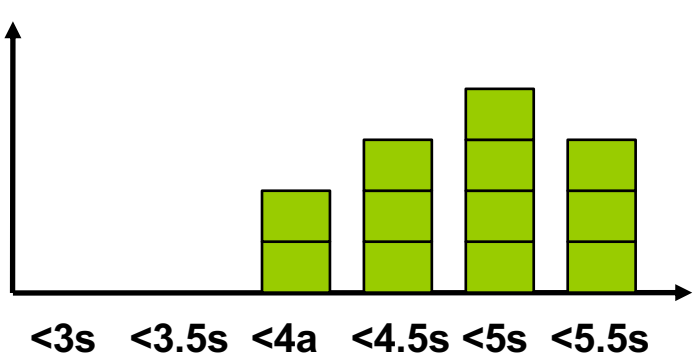
want to verify this: **run a t-test**

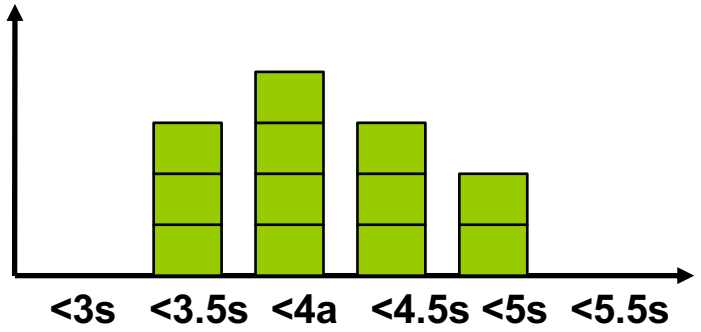
significance level ::

If a test of significance gives a **p-value lower** than the significance level, such results are informally referred to as 'statistically significant’ .

Popular levels of significance are 10% (0.1), 5% (0.05), 1% (0.01), 0.5% (0.005), and 0.1% (0.001).

i.e., oddly, when we want to prove that they are different,
we ask **whether they are the same…**





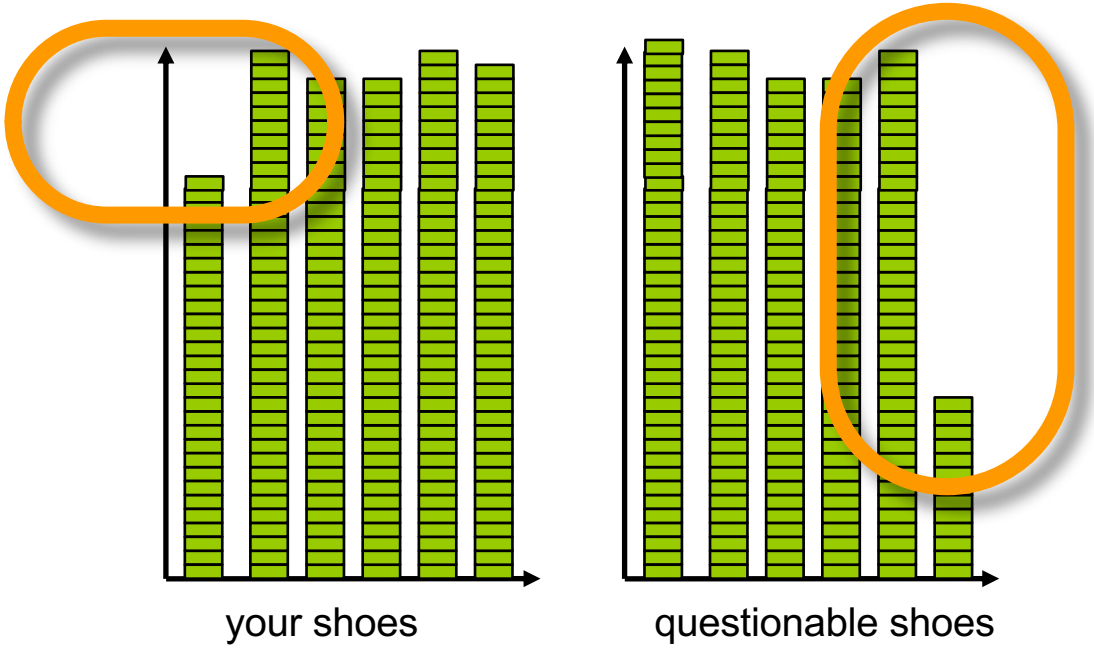
VS

null hypothesis: both data sets are from same mechanism

we are running stats in the hope that we will be able to **reject** the null hypothesis

→ if comparison of two groups reveals no statistically significant difference between the two, it does not mean **that there is no difference in reality.**

It only means that there is not enough evidence to reject the null hypothesis (it **fails to reject the null hypothesis).**



this seems **unlikely…**

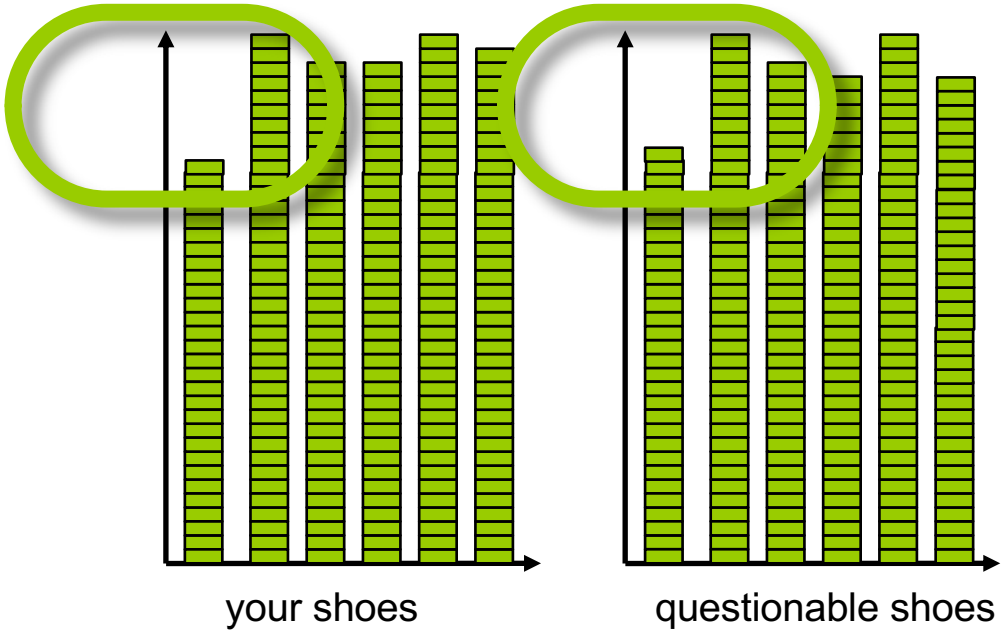
in both cases, there are two explanations

1.same shoes

2.different shoes

…thus this **must be true**

…now, in the other case



this **does seem not unlikely…**

in both cases, there are two explanations

1.same shoes

2.different shoes

we still have **two possible explanations**

→ we **cannot conclude** anything

…thus… **no thus**

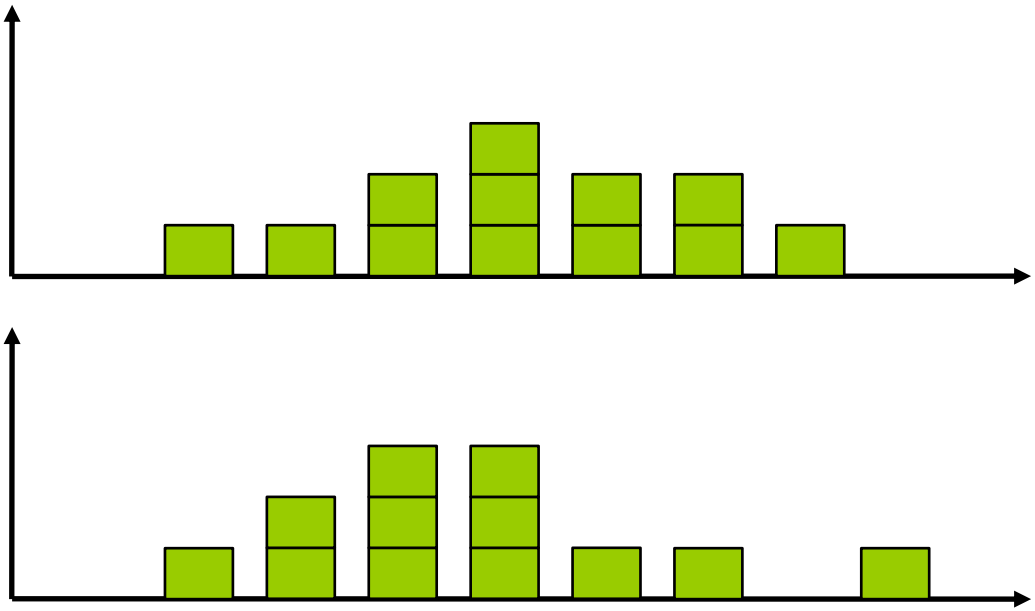
a classic screw-ups

you are making a new input device. You know that it cannot be better than a mouse, but you want to show that it is **as good as the mouse.**

how do you proceed?

<30sec brainstorming

how about you run a test and if stats come out non-significant you write “our tests showed that there was **no difference**”?

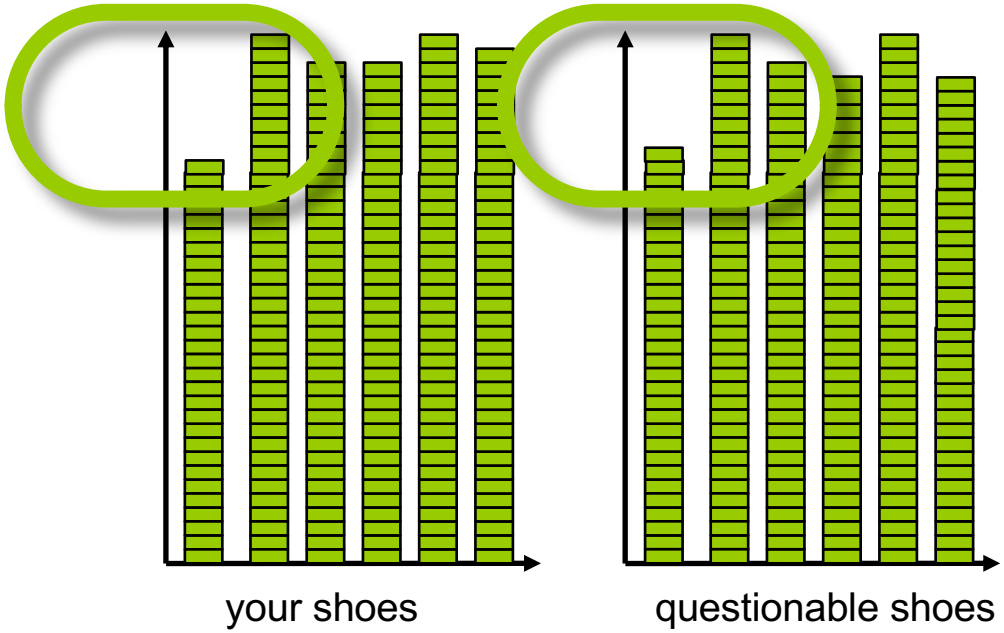


nope!

significant difference → mechanisms different

no significant difference → **nothing**

so how do you prove that two mechanisms
are the same?



this **does seem not unlikely…**

in both cases, there are two explanations

1.same shoes

2.different shoes

…thus… **no thus**

you cannot

how to report non-significant results:

“our test did not **find** a significant difference”

practically

you want to test the effect of **two soporific drugs (independent variable)** on **amount of sleep(dependent variable)**. You take 10 participants and make then sleep to get their basic (control) sleep time. Then you give them drug 1 and note the difference of sleep time. You do the same for drug 2.

	sleep extra drug 1
1	0.7
2	-1.6
3	-0.2
4	-1.2
5	-0.1
6	3.4
7	3.7
8	0.8
9	0.0
10	2.0

	sleep extra drug 2
1	1.9
2	0.8
3	1.1
4	0.1
5	-0.1
6	4.4
7	5.5
8	1.6
9	4.6
10	3.4

in your terminal

```
head(sleep) # sleep is the table that already comes with R
and contain 20 observations on 10 patients to show the
effect of two soporific drugs on the increase in hours of
sleep

plot(extra ~ group, data = sleep)
```

you then have two options for t-test: paired or unpaired

```
with(sleep, t.test(extra[group == 1], extra[group == 2]))#
unpaired

with(sleep, t.test(extra[group == 1], extra[group == 2],
paired = TRUE))# paired
```

you want to test the effect of **two soporific drugs (independent variable)** on **amount of sleep(dependent variable)**. You take 10 participants and make then sleep to get their basic (control) sleep time. Then you give them drug 1 and note the difference of sleep time. You do the same for drug 2.

all participants did both conditions, i.e. had both drugs

= within subject experiment so the data is **paired**

otherwise (e.g. take 10 new participants for drug 2)

= between subject experiment so the data is **unpaired**

between subject experiment

```
with(sleep, t.test(extra[group == 1], extra[group == 2]))#
unpaired
```

```
data:  extra[group == 1] and extra[group == 2]
t = -1.8608, df = 17.776, p-value = 0.07939
alternative hypothesis: true difference in means is not
equal to 0
95 percent confidence interval: -3.3654832  0.2054832
sample estimates: mean of x mean of y 0.75      2.33
```

what you would write

“An unpaired student t-test showed no significant difference between the two drugs.”

within subject experiment

```
with(sleep, t.test(extra[group == 1], extra[group == 2],
paired = TRUE))# paired
```

```
data:  extra[group == 1] and extra[group == 2]
t = -4.0621, df = 9  p-value = 0.002833
alternative hypothesis: true difference in means is not
equal to 0
95 percent confidence interval: -2.4598858 -0.7001142
sample estimates: mean of the differences -1.58
```

what you would write

“A paired student t-test showed significant difference

between the two drugs (two-tailed $t(9)=-4.0621$, $p < 0.05$)”

(note: try to design your studies within-subject as it will increase the chance to reach a smaller p-value

… otherwise need twice more participants!)

one vs. two tail?

```
with(sleep, t.test(extra[group == 1], extra[group == 2],
paired = TRUE, alternative="less")) # or "greater"
```

```
Paired t-test


data:  extra[group == 1] and extra[group == 2]
t = -4.0621, df = 9, p-value = 0.001416
alternative hypothesis: true difference in means is less
than 0
95 percent confidence interval: -Inf -0.8669947
sample estimates: mean of the differences      -1.58
```

what you would write

"A paired student t-test showed significant difference

between the two drugs (one-tailed $t(9)=-4.0621$, $p < 0.001$)."

two-tails: effect of drug 1 is > and/or < Drug 2

one-tail: only one side of the effect, i.e.

effect of Drug 1 > Drug 2 (less)

or

effect of Drug 1 < Drug 2 (greater)

e.g. created a shampoo for hair loss and want to know if better than concurrent one

(note: you will mostly use two-tails but if you can use a one-tail do it, it will increase the chance to reach a smaller p-value!)

multiple variables

what if we have more than two variables?



you are making two new input devices, a track pad and a stylus. You want to know which one is better and if they are also better than a mouse.

how do we proceed?

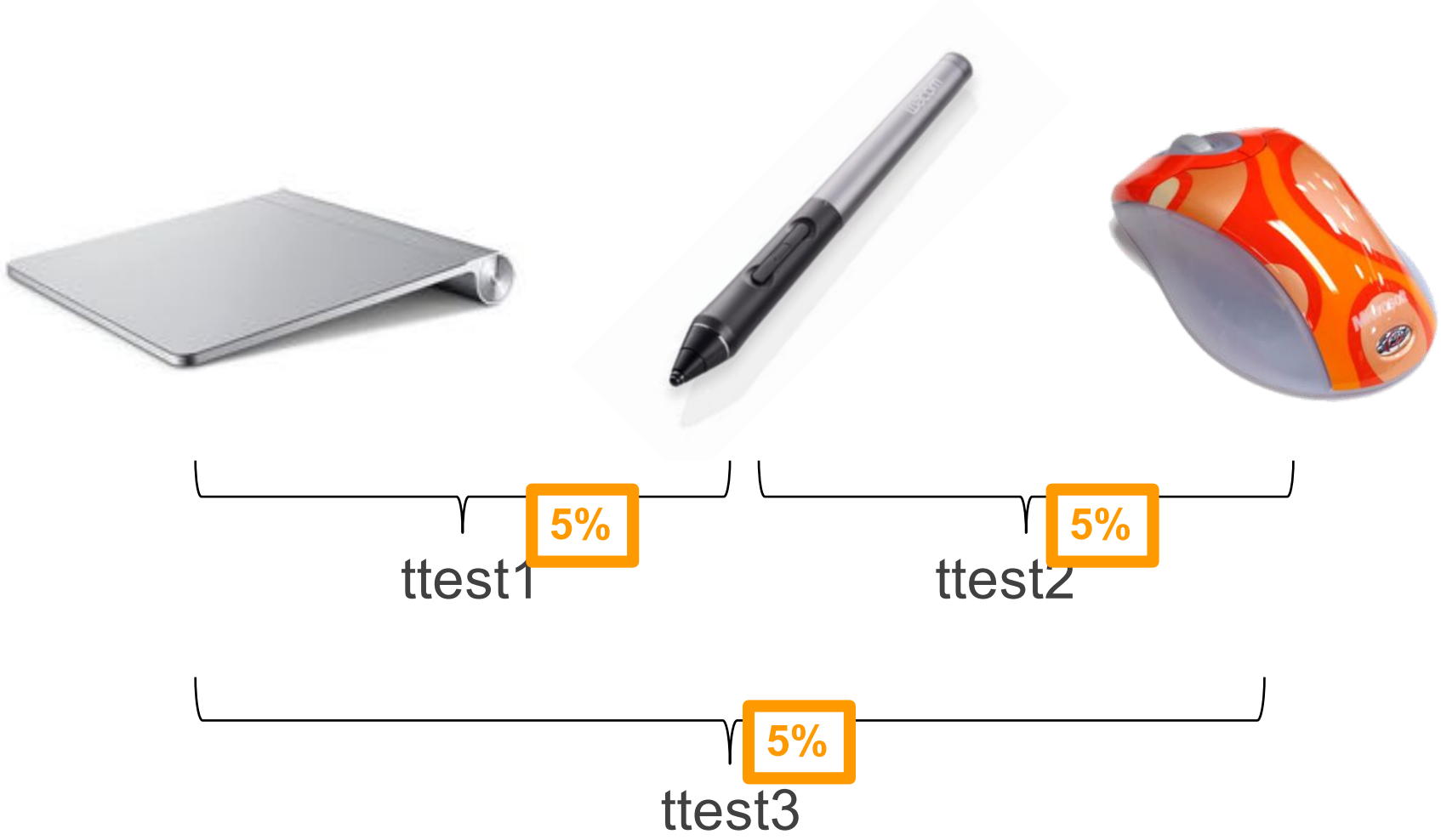
<30sec brainstorming>

ttest1

ttest2

ttest3

a simple solution would be to do this …



a simple solution would be to do this …

problem: any given test has a 5% chance of lying to you so when you use them multiple time you increase your risk of having errors (statisticians call this a “type I error”)

so there are two solutions to that:

bonferroni correction ::

when testing *n* hypotheses, test each one **against 0.05/n**

bonferroni correction ::

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in our example we would need to use **0.05/3** as a significant threshold instead of 0.05

or you could also use an

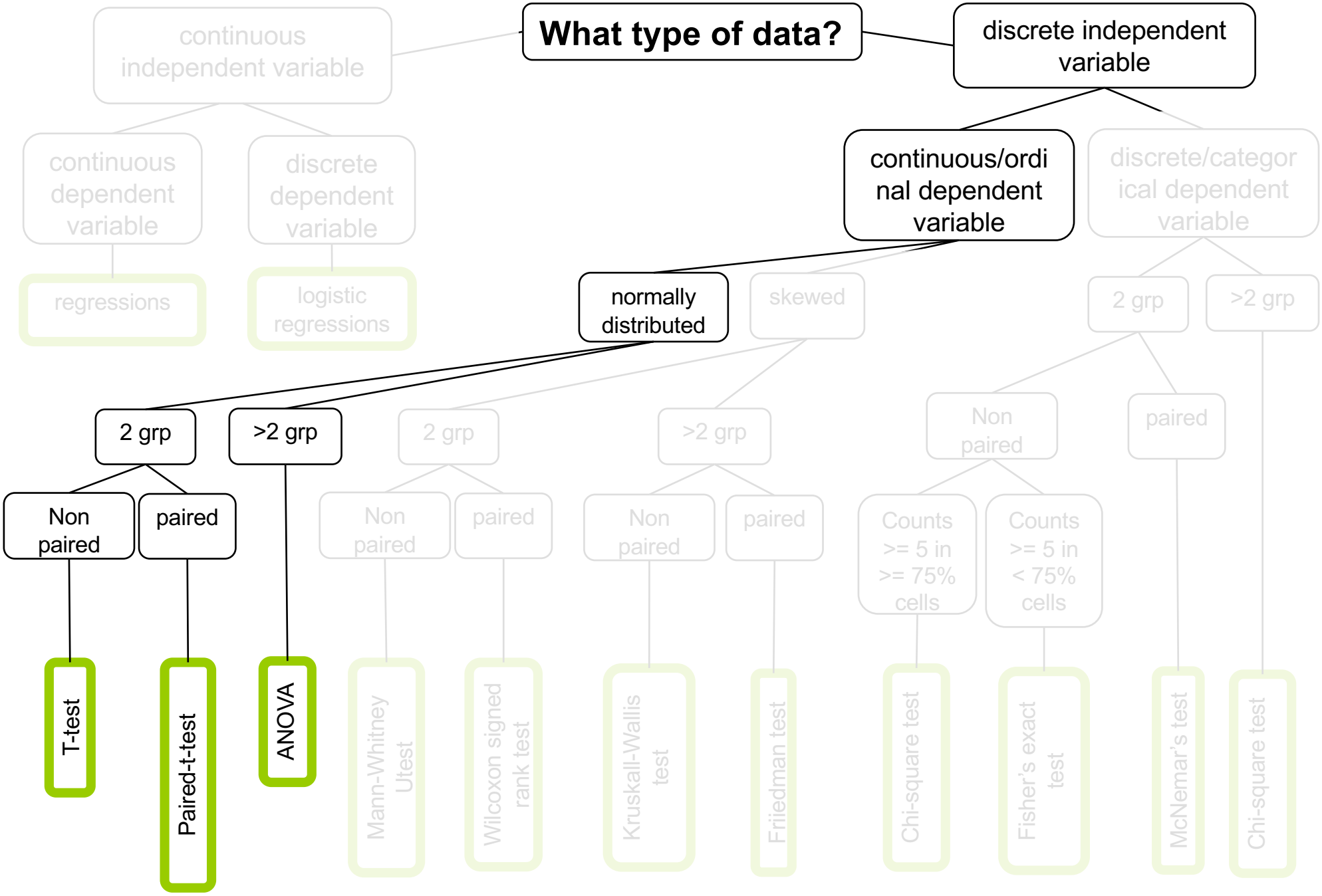
anova::

analyze of variance to compare multiple variables

one-way anova = one variable with multiple levels

two-way anova = two variables with multiple levels

… we will look at ANOVA in the next few weeks


What type of data?
continuous independent variable
discrete independent variable
continuous dependent variable
discrete dependent variable
continuous/ordinal dependent variable
discrete/categorical dependent variable
regressions
logistic regressions
normally distributed
skewed
2 grp
>2 grp
2 grp
>2 grp
Non paired
paired
Non paired
paired
Non paired
paired
Non paired
paired
Counts >= 5 in >= 75% cells
Counts >= 5 in < 75% cells
T-test
Paired-t-test
ANOVA
Mann-Whitney Utest
Wilcoxon signed rank test
Kruskall-Wallis test
Friedman test
Chi-square test
Fisher's exact test
McNemar's test
Chi-square test

1. Explain what is hypothesis testing
2. Identify the limit of hypothesis testing (we cannot prove that things are similar)
3. Explain what is a p value and a significance value
4. Explain what is a t-test and when to use it
5. Explain the difference between within and between subject studies
6. Explain what is a Bonferroni correction and find the new significance level given an experimental design

take away

end