dsc: Dynamic Synthetic Control for Time Series with Heterogeneous Adjustment Speeds

Jian Cao[1] **and Thomas Chadefaux**[1]

1 Department of Political Science, Trinity College Dublin, Ireland

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Software

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Summary

The dsc package introduces Dynamic Synthetic Control, a new approach for comparative case studies in time series settings. Synthetic control methods are widely used to estimate causal effects, but they often fail when treated and donor units react to shocks at different speeds. The dsc package addresses this by incorporating Dynamic Time Warping (DTW) to account for heterogeneous adjustment speeds across units, improving counterfactual estimation.

Implemented in R, dsc aligns donor units to the treated unit using speed-adjusted time series before estimating synthetic weights. This innovation helps avoid biases arising from asynchronous reactions and supports more accurate estimation of treatment effects. The package is useful for applications in economics, public policy, and political science.

Statement of Need

Synthetic control methods are now standard tools in causal inference, especially for policy evaluation. However, the standard approach assumes all units respond to shocks or treatments at a uniform rate. This is rarely true in practice: countries, regions, or institutions often differ in how quickly they adjust to change. If this heterogeneity is not accounted for, it may lead to incorrect counterfactuals and misleading conclusions.

The dsc package fills this gap by incorporating time-series alignment using DTW before the construction of the synthetic control. This ensures that donor units are temporally matched with the treated unit, even if their dynamics differ. Unlike some other synthetic control extensions that address poor pre-treatment fit by removing problematic donors or augmenting the model, DSC fixes the problem at the alignment stage without discarding information.

No other R package currently offers DTW-based alignment within the synthetic control framework in a fully automated and documented way. Thus, dsc represents a substantial contribution to applied research workflows.

Model Overview

The synthetic control estimator builds counterfactuals for treated units using weighted combinations of untreated donor units. Let y_1t denote the treated unit, and y_{jt} denote donors $j = 2, ..., J + 1$. The goal is to find weights w_j such that:

$$y_{1t} \approx \sum_{j=2}^{J+1} w_j y_{jt}$$

for the pre-treatment period $t < T$.

However, if donor series differ in response speed, direct comparison misaligns their dynamics. The DSC method introduces DTW-based warping functions $g_j(t)$ such that the warped donor series $y_{jt}^{(w)} = y_{j,g_j(t)}$ is aligned with the treated unit's trajectory in the pre-treatment window. Synthetic weights are then computed using these aligned donor series.

The warping is applied only to the pre-treatment period to preserve the post-treatment information. In the post-treatment window, the same transformation is propagated, ensuring comparability and interpretability.

Implementation

The core of the `dsc` method is a three-step process:

1. **Warping Pre-Treatment Series**: Use DTW to align each donor series y_j to the treated unit y_1 during the pre-treatment period.
2. **Propagate Speed Alignment**: Apply the inferred warping path to the post-treatment period of each donor unit.
3. **Construct Synthetic Control**: Estimate weights w_j to best fit the warped donor series y_j^w to y_1 before treatment.

This preserves any speed differences introduced by the treatment itself, while eliminating those inherited from structural or institutional differences.

The package also includes:

Diagnostic plots (e.g., observed vs synthetic, treatment gap)

Placebo tests

Parallel processing for faster estimation

Code Example

The `dsc` package (Cao and Chadefaux, 2024) can be installed from GitHub using:

```r
devtools::install_github("conflictlab/dsc")
```

Here's a representative use case based on the Basque Country dataset:

```r
library(dsc)
library(Synth)

# Load dataset
data(basque, package = "Synth")
data <- basque

# Prepare data
colnames(data)[1:4] <- c("id", "unit", "time", "value")
data$invest_ratio <- data$invest / data$value

# Specify special predictors
special_preds <- expression(list(
  list(dep.var, 1960:1969, c("mean")),
  list("invest_ratio", 1964:1969, c("mean")),
  list("popdens", 1969, c("mean")),
  list("sec.agriculture", 1961:1969, c("mean")),
  list("sec.energy", 1961:1969, c("mean")),
  list("sec.industry", 1961:1969, c("mean")),
```

```r
  list("sec.construction", 1961:1969, c("mean")),
  list("sec.services.venta", 1961:1969, c("mean")),
  list("sec.services.nonventa", 1961:1969, c("mean")),
  list("school.illit", 1964:1969, c("mean")),
  list("school.prim", 1964:1969, c("mean")),
  list("school.med", 1964:1969, c("mean")),
  list("school.high", 1964:1969, c("mean")),
  list("school.post.high", 1964:1969, c("mean"))
))

# Run DSC
result <- dsc(
  data = data,
  start.time = 1955,
  end.time = 1997,
  treat.time = 1970,
  dependent = "Basque Country (Pais Vasco)",
  predictors = NULL,
  parallel = TRUE,
  special.predictors = special_preds,
  time.predictors.prior = 1955:1969,
  time.optimize.ssr = 1955:1969,
  plot.figures=TRUE
)
```

Empirical Applications

Terrorism and GDP in the Basque Country

We replicate Abadie and Gardeazabal (2003), estimating the effect of terrorism on GDP using DSC. Compared to traditional synthetic control, DSC shows a closer match for placebo units and reduced mean squared error.

Proposition 99: Tobacco Control in California

We revisit the effect of California's anti-smoking policy, Proposition 99. DSC estimates a larger reduction in cigarette consumption and outperforms the original model on placebo test sharpness.

German Reunification

We assess the impact of reunification on West Germany's GDP. Again, DSC improves the counterfactual fit for placebo countries and yields more precise treatment estimates.

Monte Carlo Evaluation

To validate performance, we simulate data where units react to shocks at varying speeds. Across 100 replications, DSC consistently produces treatment effect estimates with lower variance and bias compared to standard synthetic control.

We define the relative improvement as:

$$r = \log\left(\frac{\mathrm{MSE}_{DSC}}{\mathrm{MSE}_{SC}}\right)$$

The average r is negative across all scenarios, indicating that DSC yields lower mean squared errors.

Discussion and Limitations

The dsc method improves synthetic control estimation by accounting for reaction speed heterogeneity. However, it assumes that speed differences are stable in the pre-treatment period and that no spillover effects contaminate donor units. Extensions to multi-treatment cases or endogenously determined timing remain areas for future work.

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