SVM Algorithm

based on Lagrangian multipliers and KKT conditions

Matteo Conti

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University of Bologna

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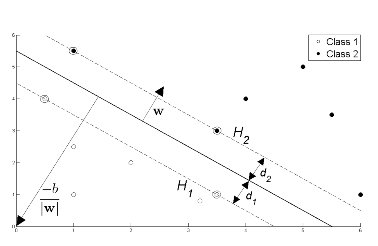
Problem Formulation

Our Problem

initial constrainted problem:

$$\min \quad \frac{1}{2}\left\|w\right\|^2$$
$$\text{s.t.} \;\; \forall_i \;\; y_i(x_i \cdot w + b) \;\geq\; 0$$



the points (circled) $H1$ and $H2$ that lie closest to the separating hyperplane, i.e. the **Support Vectors**, can be described by

$$x_i \cdot w + b = +1 \quad \text{for} \quad H1$$
$$x_i \cdot w + b = -1 \quad \text{for} \quad H2$$

Lagrangian Multipliers Strategy

we formulate our problem as:

$$L_P \equiv \frac{1}{2}\|w\|^2 - \alpha[\forall_i\ y_i(x_i \cdot w + b) - 1]$$

where α_i are the Lagrange **multipliers** such that:

$$\forall_i\ \alpha_i \geq 0$$

we want to **min** $L_P(w, b)$ and then **max** $L_P(\alpha)$.

Final Problem Formulation

Dual form of the primary Lagrangian problem:

$$\max_{\alpha} \sum_{i=1}^{L} \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j x_i \cdot x_j \equiv \sum_{i=1}^{L} \alpha_i - \frac{1}{2} \alpha^T H \alpha$$

where:

$$H \equiv y_i y_j x_i \cdot x_j$$

subject to:

$$\sum_{i=1}^{L} \alpha_i y_i = 0 \quad \text{and} \quad \forall_i \ \alpha_i \geq 0$$

Quadratic-Programming Optimization

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Soft-Margin Assumption

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Non-Linear Kernel

Non-Linear Kernel

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Non-Linear Kernel

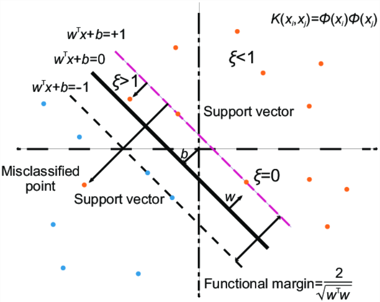

Figure 1: Second

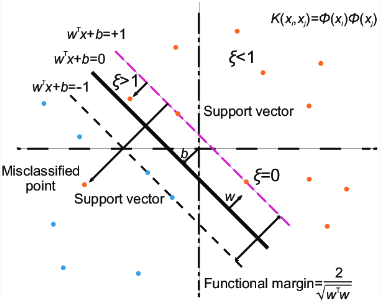

Figure 2: Third

https://github.com/contimatteo/SVM-from-scratch