University of Michigan – Shanghai Jiao Tong University Joint Institute
Center of Optics and Optoelectronics

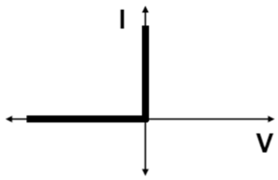
VE 320 – Summer 2012
Introduction to Semiconductor Device

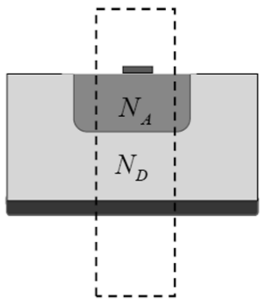
PN Junction Electrostatics

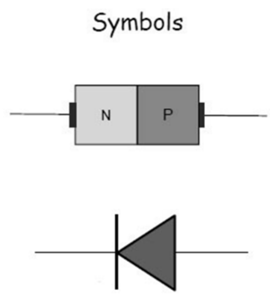
Instructor: Professor Hua Bao

p-n Junction Devices

One way current flow







Electrostatics

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

← **equilibrium**

DC dn/dt=0
Small signal dn/dt ~ jωt x n
Transient --- full solution

Electrostatics Review

Gauss's Law (1D)

$$\boxed{\frac{d(\varepsilon\mathcal{E})}{dx} = \rho}$$

ε – permittivity of material

ℰ – Electric field

ρ – charge density

q=1.6×10^{-19}C, elemental charge

$$F = -q\mathcal{E} \qquad dW = -q\mathcal{E}dx = qdV \qquad \boxed{\mathcal{E} = -\frac{dV}{dx}}$$

Poisson's Equation

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon}$$

Potential Energy

$$P.E. = -qV$$

$$\boxed{\mathcal{E} = \frac{1}{q}\frac{dE}{dx}}$$

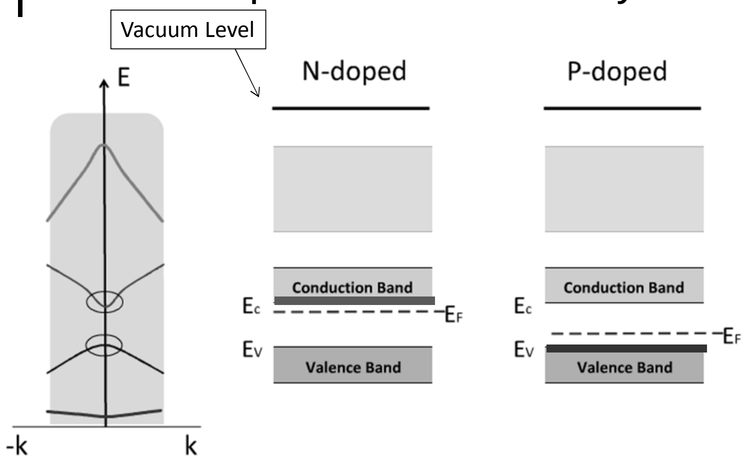
Poisson's Equation

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0 K_s}$$

Given charge density,

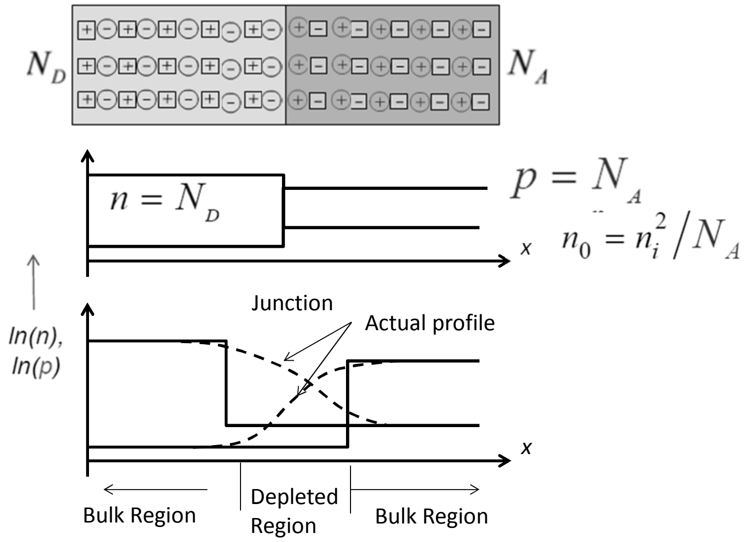
- Integrate once to get electric field
- Integrate twice to get electric potential (and P.E.)

P and N doped Material Side by Side

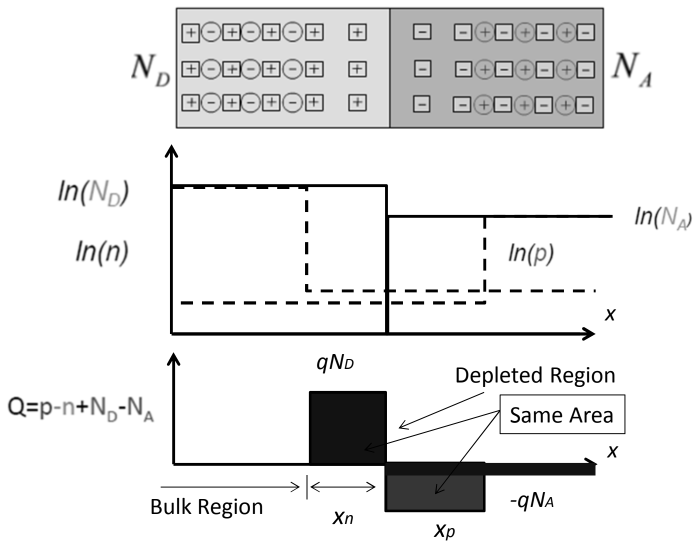


No electron transfer under such circumstances!

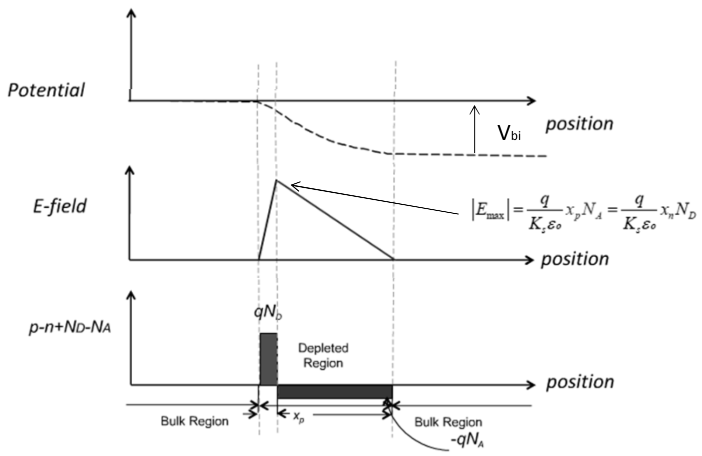
Forming a Junction



Formation of a Junction



Qualitative Analysis



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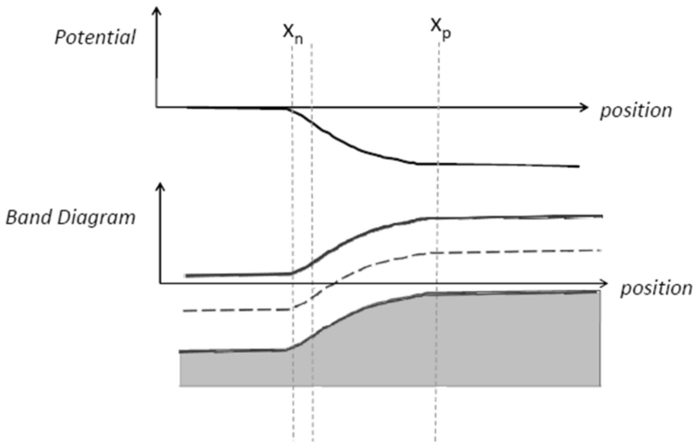
Sketch of Electrostatics



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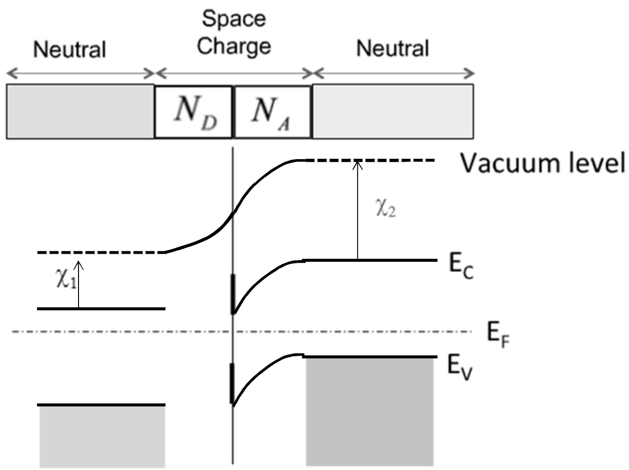
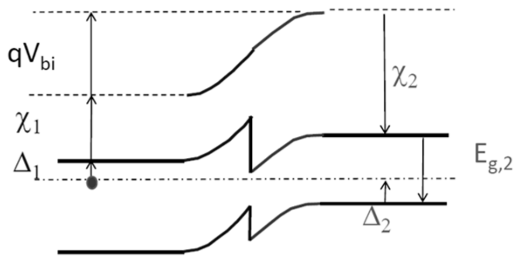
Band Diagram



Equivalent to solving Poisson Equation

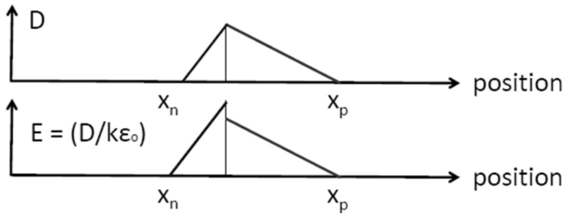
Built-in Potential



$$\Delta_1 + \chi_1 + qV_{bi} = \chi_2 + E_{g,2} - \Delta_2$$

$$\begin{aligned} qV_{bi} &= E_{g,2} - \Delta_2 - \Delta_1 + \chi_2 - \chi_1 \\ &= \left(E_{g,2} + k_B T \ln \frac{N_A}{N_{V,2}} \right) + k_B T \ln \frac{N_D}{N_{C,1}} + (\chi_2 - \chi_1) \\ &= k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + (\chi_2 - \chi_1) \end{aligned}$$

Interface Boundary Conditions

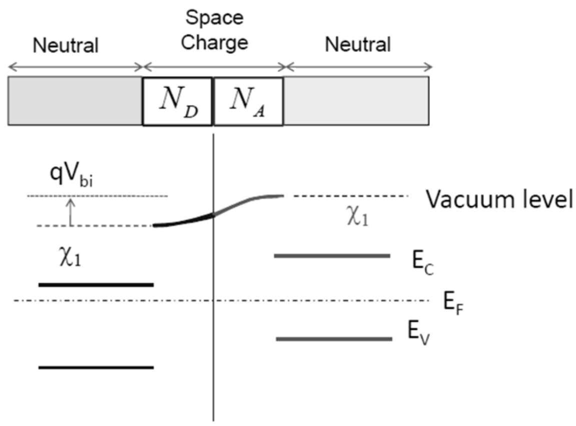


$$D_1 = K_1\varepsilon_0 E(0^-) = K_2\varepsilon_0 E(0^-) = D_2$$

$$E(0^-) = \frac{K_2}{K_1}E(0^+)$$

Displacement is continuous across the interface, field need not be ..

Built-in voltage for **Homo**-junctions



$$qV_{bi} = k_B T\, ln\frac{N_A N_D}{N_{V,2}N_{C,1}e^{-E_{g,2}/k_BT}} + (\chi_2 - \chi_1) = k_B T\, ln\frac{N_A N_D}{N_V N_C e^{-E_g/k_BT}} = k_B T\, ln\frac{N_A N_D}{n_i^2}$$

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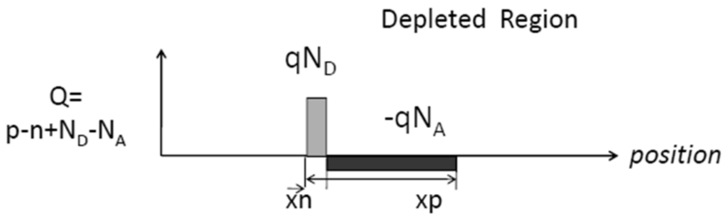
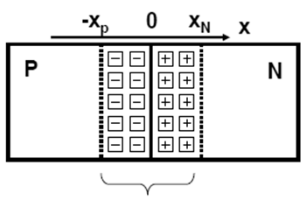
Now the Calculation



$$K_S \varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p - n + N_D^+ - N_A^-\right)$$

The Depletion Approximation

- Analyze p-n junction using electrostatics (Poisson's equation)
- No (or negligible) mobile charge in depletion region (depleted of mobile carriers), only fixed charge from the ionized impurities
- Regions outside the depletion region stay neutral



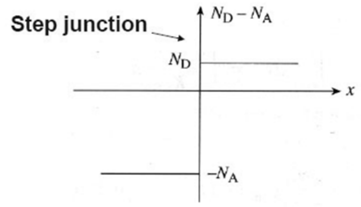
"space charge" region
(Depletion region)

$$\rho = \begin{cases} -qN_A & \text{for } -x_p \le x \le 0 \\ qN_D & \text{for } 0 \le x \le x_n \\ 0 & \text{for } x \le -x_p \text{ and } x \ge x_n \end{cases}$$

$$\rho = p - n + N_D - N_A$$

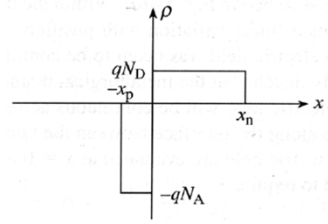
Step Junction

Doping Profile



Charge Density

$$\rho = \begin{cases} -qN_A & \text{for } -x_p \le x \le 0 \\ qN_D & \text{for } 0 \le x \le x_n \\ 0 & \text{for } x \le -x_p \text{ and } x \ge x_n \end{cases}$$



Electric Field

- Electric field

$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_s}$$

$$\mathscr{E}(x) = -\frac{qN_A}{\varepsilon_0 K_s}(x + x_p), -x_p < x < 0$$

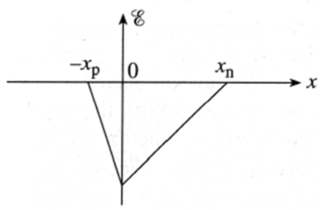
$$\mathscr{E}(x) = -\frac{qN_D}{\varepsilon_0 K_s}(x_n - x), 0 < x < x_n$$

$\mathscr{E}(x) = 0$ elsewhere

- At x=0, displacement vector must be continuous

$$\mathscr{E}(0) = -\frac{qN_D}{\varepsilon_0 K_s} \cdot x_n = -\frac{qN_A}{\varepsilon_0 K_s} \cdot x_p$$

$$N_D x_n = N_A x_p$$



Potential

$$\frac{dV}{dx} = -\mathscr{E}$$

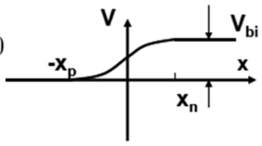
- Electrostatic potential, integrate electric field

$$V(x) = V_P \qquad -\infty < x < -x_p$$

$$V(x) = \frac{qN_A}{2\varepsilon_0 K_s}(x + x_p)^2 + V_p \qquad -x_p < x < 0$$

$$V(x) = -\frac{qN_D}{2\varepsilon_0 K_s}(x_n - x)^2 + V_n \qquad 0 < x < x_n$$

$$V(x) = V_N \qquad x_n < x < \infty$$



$$V(x_n) - V(0) = \frac{qN_D}{2\varepsilon_0 K_s}x_n^2$$

$$V(0) - V(-x_p) = \frac{qN_A}{2\varepsilon_0 K_s}x_p^2$$

$$V(x_n) - V(-x_p) = \frac{qN_D x_n^2}{2\varepsilon_0 K_s} + \frac{qN_A x_p^2}{2\varepsilon_0 K_s}$$

$V_{bi} =$ Built-in Potential

Built-In Potential In Depletion Region

$$\begin{cases} V_{bi} = V(x_n) - V(-x_p) = \dfrac{qN_D x_n^2}{2\varepsilon_0 K_s} + \dfrac{qN_A x_p^2}{2\varepsilon_0 K_s} \\ N_D x_n = N_A x_p \end{cases}$$

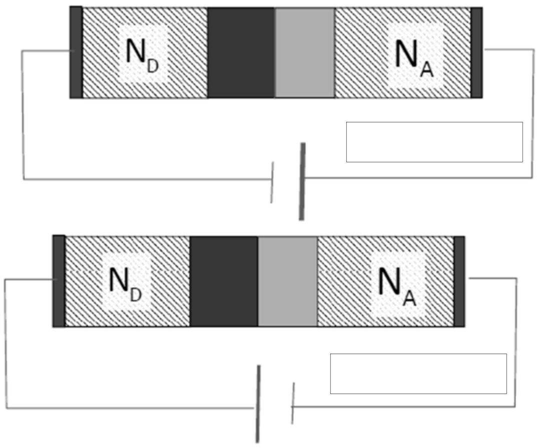
$$\Rightarrow \quad x_p(V_{bi}) = \sqrt{\frac{2\varepsilon_0 K_s V_{bi}}{q}\left[\frac{N_D}{N_A(N_A + N_D)}\right]}$$

$$x_n(V_{bi}) = \sqrt{\frac{2\varepsilon_0 K_s V_{bi}}{q}\left[\frac{N_A}{N_D(N_A + N_D)}\right]}$$

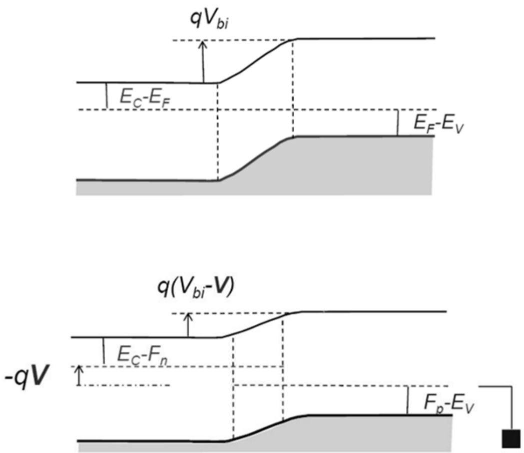
Depletion Width

$$\boxed{W(V_{bi}) = x_p(V_{bi}) + x_n(V_{bi}) = \sqrt{\frac{2\varepsilon_0 K_s V_{bi}}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}}$$

Forward and Reverse Bias



Applying a Bias: Poisson Equation

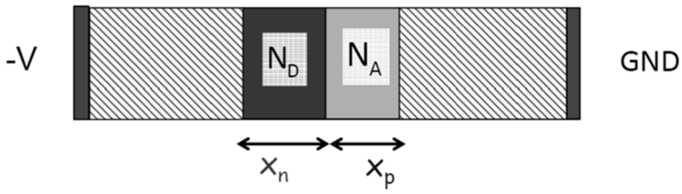


$$n(x) = n_i\, e^{(F_n - E_i)\beta}$$

$$p(x) = n_i\, e^{-(F_p - E_i)\beta}$$

$$n \times p = n_i^2\, e^{(F_n - F_p)\beta}$$

Depletion Widths



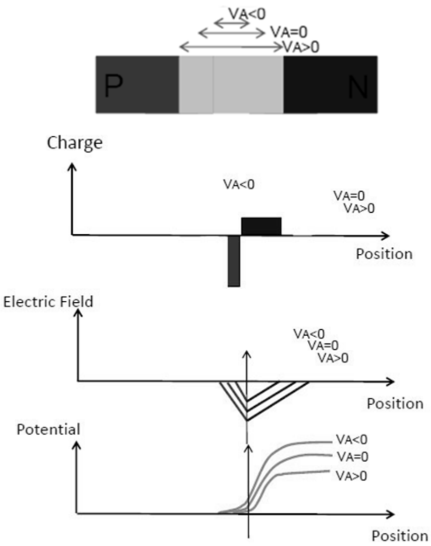
$$N_D x_n = N_A x_p$$

$$q\left(V_{bi} - V\right) = \frac{qN_D x_n^{\ 2}}{2k_s\varepsilon_0} + \frac{qN_A x_p^{\ 2}}{2k_s\varepsilon_0}$$

$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D\left(N_A + N_D\right)}(V_{bi} - V)}$$

$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A\left(N_A + N_D\right)}(V_{bi} - V)}$$

Fields and Depletion at Forward/Reverse Biases

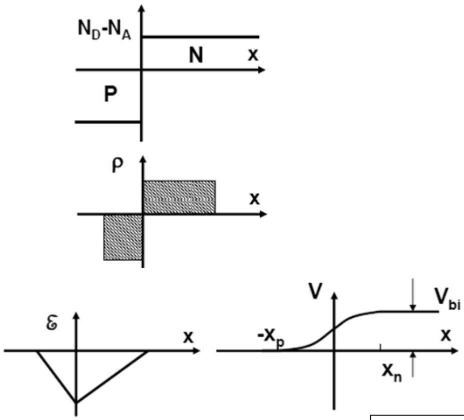


V_A : applied voltage

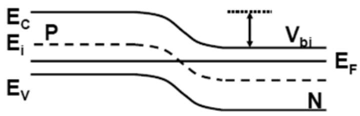
Barrier height is reduced at forward biases

Significant increase of peak field at reverse bias ...

Electrostatics Summary



Drawing band diagrams



$$\boxed{N_D x_n = N_A x_p}$$

$$\boxed{V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

$$\boxed{W = \sqrt{\frac{2K_s\varepsilon_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_A)}}$$