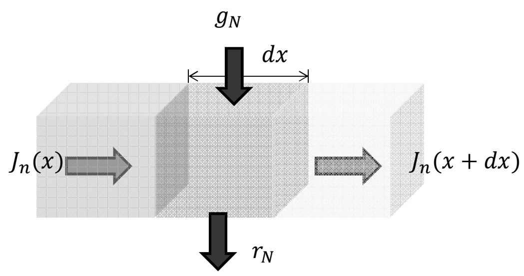
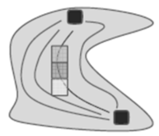
VE 320 – Summer 2012
Introduction to Semiconductor Device

Continuity Equations

Instructor: Professor Hua Bao

Continuity Equation

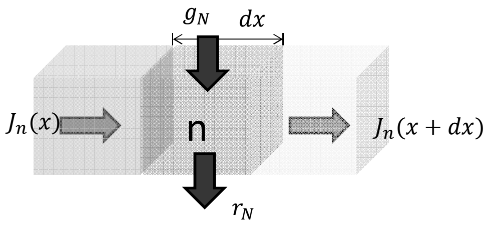




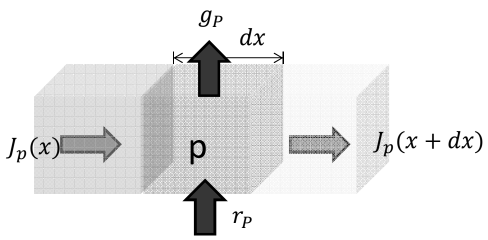
$$\frac{\partial\left(A\times\Delta x\times n\right)}{\partial t}=\frac{A\times J_n(x)-A\times J_n(x+dx)}{-q}+A\times g_N\Delta x-A\times r_N\Delta x$$

$$\frac{\partial n}{\partial t}=\frac{J_n(x)-J_n(x+dx)}{-q\Delta x}+g_N-r_N=\frac{1}{q}\nabla\bullet J_n+g_N-r_N$$

Continuity Equations for Electrons/Holes

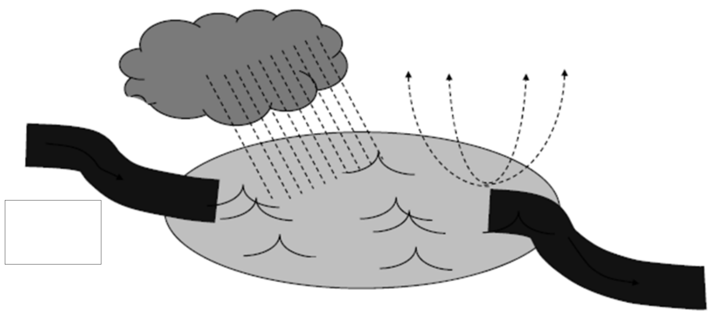


$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla\cdot J_n + \frac{\partial n}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial n}{\partial t}\Big|_{\substack{other\\process}}$$



$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla\cdot J_p + \frac{\partial p}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial p}{\partial t}\Big|_{\substack{other\\process}}$$

Continuity Equation: A Good Analogy



Rate of increase of water level in lake = (in flow - outflow) + rain - evaporation

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet J_n \quad + g_N \quad - r_N$$

Summary of All Equations

$$\nabla \cdot D = q(p - n + N_D^+ - N_A^-)$$

$$J_n = qn\mu_n \mathcal{E} + qD_N \nabla n$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_n + \frac{\partial n}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial n}{\partial t}\Big|_{\substack{other\\process}}$$

$$J_p = qp\mu_p \mathcal{E} - qD_P \nabla p$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_p + \frac{\partial p}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial p}{\partial t}\Big|_{\substack{other\\process}}$$

These are the equations needed for the device analysis!

Minority Carrier Diffusion Equation

Simplifying Assumptions for a common case:

- One dimensional.
- We will only consider minority carriers
- Electric field is approximately zero in regions subject to analysis, hence negligible drift.
- Low-level injection conditions apply.
- R-G center is the main recombination-generation mechanism.
- The only “other” mechanism is photogeneration.

Minority Carrier Diffusion Equation

Under these conditions, the continuity Eq. is simplified to

$$\boxed{\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G} \qquad \boxed{\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G}$$

n_p: electron (minority carrier) concentration in p-material
p_n: hole (minority carrier) concentration in n-material

Simplified equations useful for device analysis.

Common Simplifications

$$\boxed{\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G} \qquad \boxed{\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G}$$

Steady state (Different than Equilibrium Condition)

$$\frac{\partial \Delta n_p}{\partial t} \to 0 \text{ and } \frac{\partial \Delta p_n}{\partial t} \to 0$$

No minority carrier diffusion gradient

$$D_N \frac{\partial^2 \Delta n_p}{\partial x^2} \to 0 \text{ and } D_P \frac{\partial^2 \Delta p_n}{\partial x^2} \to 0$$

No thermal R-G

$$\frac{\Delta n_p}{\tau} \to 0 \text{ and } \frac{\Delta p_n}{\tau} \to 0$$

No light

$$G_L \to 0$$

Quasi-Fermi Levels

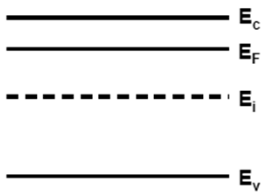
- Fermi level is a concept only exist in equilibrium conditions!
- Define "quasi-Fermi levels", which position is *by definition* determined by the carrier concentration.

$$n \equiv n_i \exp\left(\frac{F_n - E_i}{kT}\right) \qquad F_n \equiv E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$p \equiv n_i \exp\left(\frac{E_i - F_p}{kT}\right) \qquad F_p \equiv E_i - kT \ln\left(\frac{p}{n_i}\right)$$

Quasi-Fermi Levels

At t = 0, under equilibrium



At t > 0, photogeneration happen. Assuming low level injection and steady state, sketch the position of F_N, F_P.

$$F_n \equiv E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$F_p \equiv E_i - kT \ln\left(\frac{p}{n_i}\right)$$

Quasi-Fermi Levels

Total (net) electron current density:

$$J_n = J_{n,drift} + J_{n,diff} = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx}$$

$$n \equiv n_i \exp\left(\frac{F_n - E_i}{kT}\right)$$

$$\frac{dn}{dx} = \frac{n_i}{kT}\exp\left(\frac{F_n - E_i}{kT}\right)\left(\frac{dF_n}{dx} - \frac{dE_i}{dx}\right) = \frac{n}{kT}\left(\frac{dF_n}{dx} - \frac{dE_i}{dx}\right)$$

$$\mathcal{E} = \frac{1}{q}\frac{dE_i}{dx} \qquad D_n = \frac{kT}{q}\mu_n$$

$$\Rightarrow \boxed{J_n = J_{n,drift} + J_{n,diff} = \mu_n n \frac{dF_n}{dx}}$$

Quasi-Fermi Levels

Total electron current, including J_{drift} and J_{diff}

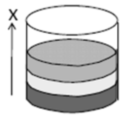
$$\boxed{J_n = \mu_n n \nabla F_n} \qquad F_n \equiv E_i + kT\ln\left(\frac{n}{n_i}\right)$$

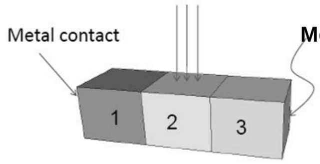
and similarly

$$\boxed{J_p = \mu_p p \nabla F_p} \qquad F_p \equiv E_i - kT\ln\left(\frac{p}{n_i}\right)$$

If there is a change in quasi-Fermi levels, current flows!

Consider a Complex System



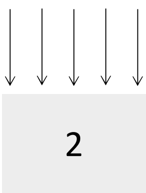


- Acceptor doped
- Light turned on in the middle section.
- The right region is full of mid-gap traps
- The left region is trap free.
- The left/right regions contacted by metal electrode.

Example: Transient, Uniform Illumination

uniform

$$\frac{\partial n}{\partial t} = \cancel{\frac{1}{q}\nabla \cdot J_N} - r_N + g_N \qquad J_N = qn\mu_N\mathcal{E} + qD_N\nabla n$$

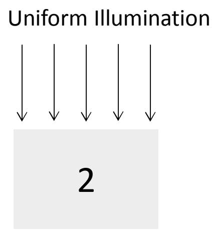


$$\frac{\partial(\cancel{n_0} + \Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$

$$\nabla \cdot D = q(p - n + N_D^+ - N_A^-) = q(p_0 + \Delta n - n_0 - \Delta p + N_D^+ - N_A^-) = 0$$

- Electrons and holes are balanced at any time.
- No electric field because 1) divergence is zero 2)there are no contacts (thus no input electric field).

Example: Transient, Uniform Illumination



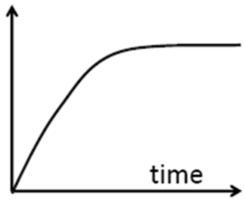
$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$

$$\Delta n(x,t) = A + Be^{-t/\tau_n}$$

$$t = 0, \Delta n(x,0) = 0 \Rightarrow A = -B$$

$$t \to \infty, \Delta n(x,\infty) = G\tau_n = A$$

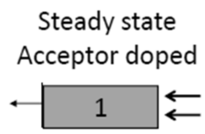
$$\Delta n(x,t) = G\tau_n(1 - e^{-t/\tau_n})$$

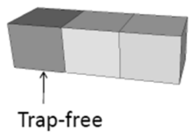


At steady state, $\Delta n(x,t) = G\tau_n$

number of carriers does not change with time or space.

Example: One sided Minority Diffusion





Trap free- no thermal R-G

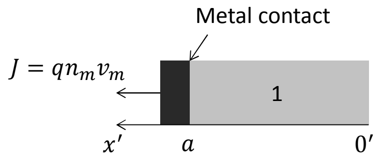
$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{r_N} + \cancel{g_N}$$

$$\Rightarrow \quad 0 = D_N \frac{d^2 n}{dx^2}$$

$$J_N = qn\cancel{\mu_N}\mathcal{E} + qD_N\frac{dn}{dx}$$

Example: One sided Minority Diffusion

$$0 = D_N \frac{d^2 n}{dx^2} = D_N \frac{d^2 \Delta n}{dx^2}$$



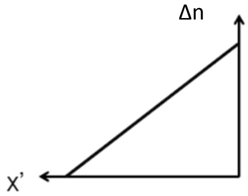
$$\Delta n(x', t) = C + Dx'$$

At metal contact, you can always assume Δn=0

$$\Delta n(x' = 0') = C$$

$$\Delta n(x' = a) = 0$$

$$\Delta n(x', t) = \Delta n(x' = 0')(1 - \frac{x'}{a})$$

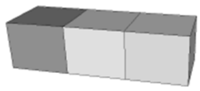


At steady state, Δn is linearly dependent on *x*. It is not a function of *t*.

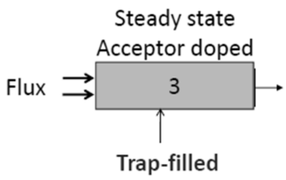
Example: Minority Diffusion with RG

Only thermal R-G

$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + \cancel{g_N}$$



$$J_N = qn\cancel{\mu_N}\mathcal{E} + qD_N \frac{dn}{dx}$$



$$0 = D_N \frac{d^2(\cancel{n_0} + \Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

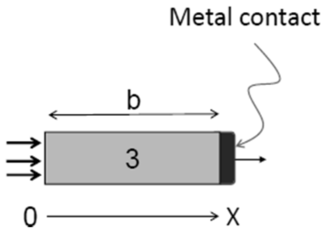
$$0 = D_N \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n}$$

Diffusion with Recombination …

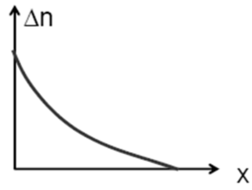
$$D_N \frac{d^2\Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0$$

Define diffusion length $L_N = \sqrt{D_N \tau_n}$

$$\Delta n(x,t) = Ee^{x/L_N} + Fe^{-x/L_N}$$



$$x = b, \Delta n(x = b) = 0$$
$$\Rightarrow F = -Ee^{2b/L_n}$$



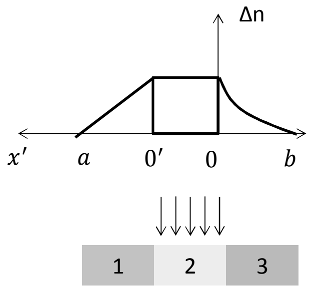
$$x = 0, \Delta n(x = 0) = E + F = \Delta n(0)$$

$$\Delta n(x,t) = \frac{\Delta n(0)}{(1 - e^{2b/L_N})}(e^{x/L_N} - e^{2b/L_N}e^{-x/L_N})$$

Combining them all ….

$$\Delta n_2(x) = G\tau_n = \Delta n_2(0) = \Delta n_2(0')$$

$$\Delta n_1(x') = \Delta n(0')\left(1 - \frac{x'}{a}\right)$$
$$= G\tau_n\left(1 - \frac{x'}{a}\right)$$



Match Boundary Condition

$$\Delta n_3(x) = \frac{\Delta n(0)}{(1 - e^{2b/L_n})}(e^{x/L_N} - e^{2b/L_N}e^{-x/L_N})$$
$$= \frac{G\tau_n(e^{x/L_N} - e^{2b/L_N}e^{-x/L_N})}{(1 - e^{2b/L_N})}$$

Once Δn is known, electron current can be calculated! $J_N = qn\mu_N \mathcal{E} + qD_N \frac{dn}{dx}$

Summary

1) Continuity Equations form the basis of analysis of all the devices we will study in this course.
2) Analytical solutions provide a great deal of insight into the key physical mechanism involved in the operation of a device.