University of Michigan – Shanghai Jiao Tong University Joint Institute
Center of Optics and Optoelectronics

VE 320 – Summer 2012
Introduction to Semiconductor Device

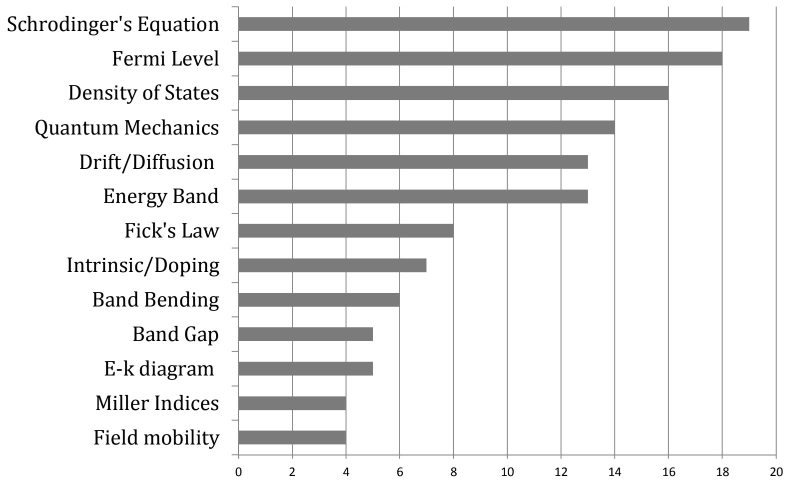
Mid-term Review

Instructor: Professor Hua Bao

NANO ENERGY LAB

Bao Ve320 S12 **1**

Confusing Topics/Concepts



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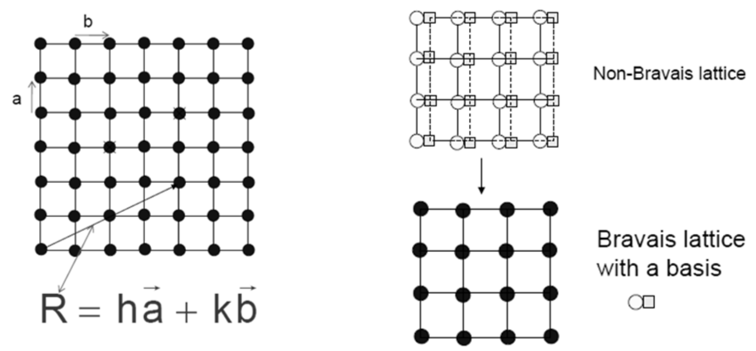
Bao Ve320 S12 **2**

Topics

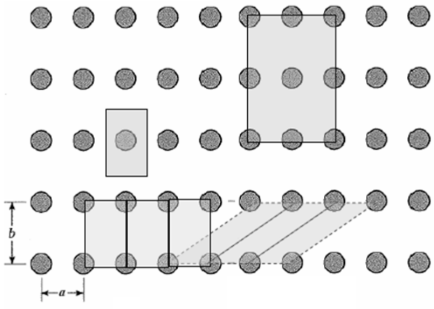
- Crystal Structure: Bravais lattice, Miller Indices
- Quantum Mechanics
- E-k Diagram (Energy Bands)
- Density of States
- Fermi-Dirac Statistics
- Carrier Statistics/Doping
- Drift and Diffusion
- Recombination - Generation
- Continuity Equation
- PN Junction – Electrostatics
- PN Junction – DC Response

Geometry of Lattice Points

- **Bravais lattice** is an infinite array of discrete points generated by a set of discrete translation operations
- Each lattice point has the same environment as any other point
- Crystal can be constructed by the Bravais lattice + basis

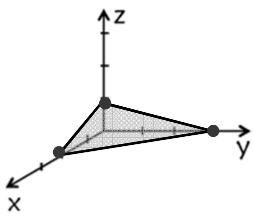


Unit Cell of a Periodic Lattice



- Unit cell is not unique
- Unit cell can be primitive or non-primitive
- Properties of one cell define the properties of the crystal

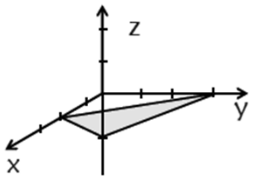
Miller Indices: Rules



1. Set up axis along the edges of the unit cell
2. Normalize intercepts … 2, 3, 1
3. Invert intercepts … 1/2, 1/3, 1
4. Rationalize (smallest common denominator) 3/6, 2/6, 6/6
5. Enclose the numbers in curvilinear brackets
   (3,2,6)

Few More Rules …

Negative Intercept



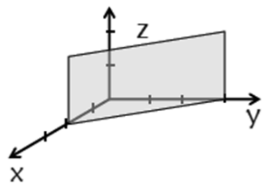
2, 3, -2

1/2, 1/3, -1/2

3, 2, -3

$(3\ 2\ \bar{3})$

Intercept at infinity



2, 3, ∞

1/2, 1/3, 0

3, 2, 0

(3 2 0)

Schrodinger's Equation

- Classical theory is not consistent with experimental observation. That's the origin of quantum mechanics
- We saw how Schrodinger equation can arise as a consequence of quantization and relativity, *but that was not a derivation.*

$$i\hbar\frac{d\Psi}{dt}=\left(-\frac{\hbar^2}{2m_0}\frac{d^2\Psi}{dx^2}\right)+V\Psi$$

This is the governing equation for electrons, just like Newton's law of motion, Maxwell's equation

Schrodinger's Equation

1) $\frac{d^2\psi}{dx^2}+k^2\psi=0$ ⟶ 2N unknowns for N regions

2) $\psi(x=-\infty)=0$, $\psi(x=+\infty)=0$ ⟶ Reduces 2 unknowns

3) $\psi|_{x=x_B^-}=\psi|_{x=x_B^+}$, $\left.\frac{d\psi}{dx}\right|_{x=x_B^-}=\left.\frac{d\psi}{dx}\right|_{x=x_B^+}$ ⟶ Set 2N-2 equations for 2N-2 unknowns (for continuous U)

4) Det (coefficient matrix)=0 And find E by graphical or numerical solution

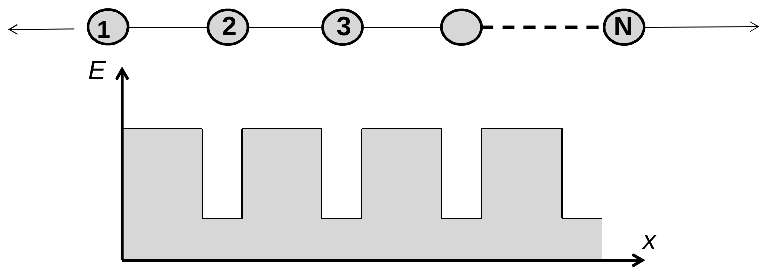
5) $\int_{-\infty}^{\infty}|\psi(x,E)|^2\,dx=1$ for wave function

Following the solution process is not necessary.

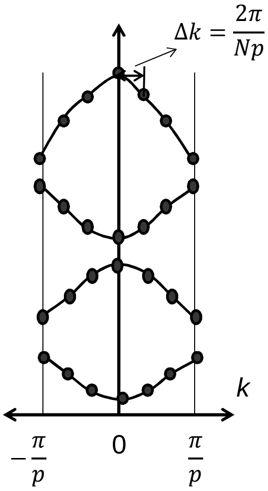
Solution of Schrodinger's equations gives you energy and wave function (where the electrons can stay).

E-k Diagram/Band Structure/Energy Bands

With Bloch theorem and periodic boundary condition, the Schrodinger's equation of a 1-D periodic system can be analytically solved.



E-k Diagram



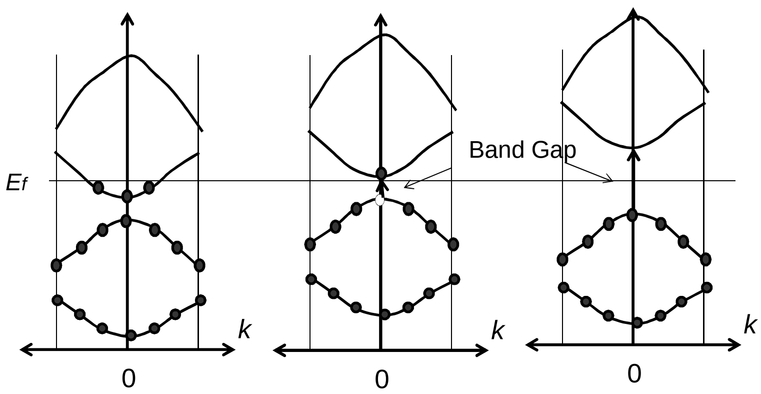
$$k = \pm \frac{2\pi n}{Np} \qquad n = -\frac{N}{2}, \ldots - 1,0,1, \ldots, \frac{N}{2}$$

E-k diagram tells you where the electrons can stay in a periodic crystal.

E ~ energy of the state
k ~ wave vector (related to the momentum)

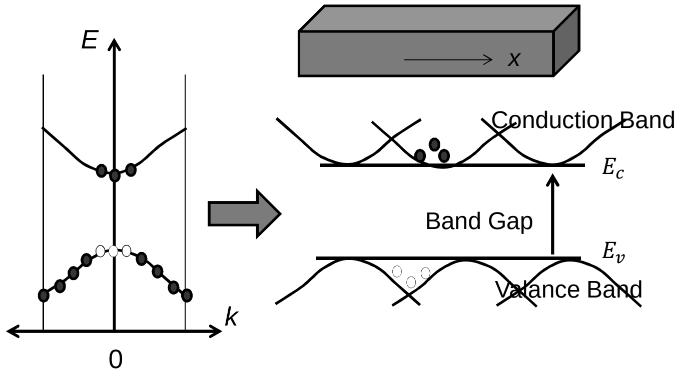
For an atom, there is only E. In periodic crystals, k is also needed to characterize the states.

Metal, Semiconductor, Insulator



- **Metal:** has partially filled energy bands at zero temperature.
- **Semiconductor:** does not have partially filled bands at zero temperature, but thermal effect can excite electrons into conduction bands.
- **Insulator:** does not have partially filled bands at zero temperature, but band gap energy is too large and thermal effect cannot excite electrons into conduction bands.

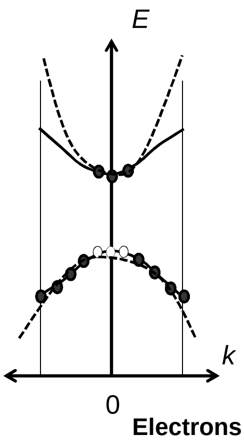
E-k Diagram and Band Diagram



E-k diagram/energy bands/band structure is NOT band diagram!

- E-k diagram tells you for which energy and wave vector electrons can stay in a crystal
- Band diagram is the Ec and Ev in a semiconductor as a function of location

Effective Mass



Wave-particle duality:

$$p = \hbar k, \qquad E \approx \frac{p^2}{2m^*}$$

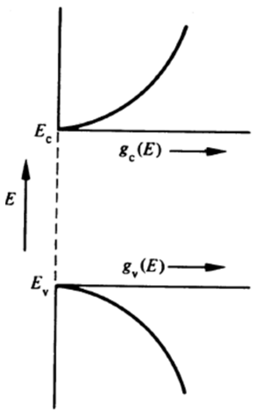
$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

Effective Mass is inversely proportional to the curvature of E-k diagram. Larger curvature, smaller effective mass!

Electrons moving in a solid: $F = -q\mathcal{E} = m_n{}^*\frac{dv}{dt}$

Similar equation for holes: $F = q\mathcal{E} = m_p{}^*\frac{dv}{dt}$

Density of States for Electrons and Holes



$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}, \qquad E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}, \qquad E \leq E_v$$

DOS tells you how many states are there at any given energy in the conduction and valance band!

g(E)dE represents the number of states per unit volume lying in the energy between *E* and *E+dE*

Fermi-Dirac Statistics

$$f(E) = \frac{N(E)}{g(E)} = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

k = Boltzmann Constant

$k = 8.617 \times 10^{-5} eV / K$

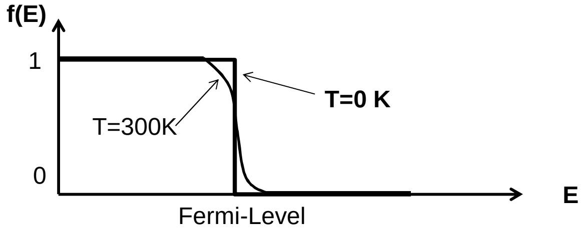
$kT = 0.026eV$ at T = 300K

Can you see f(E) is always smaller than one?

Fermi-Dirac Distribution

$$f(E) = \frac{N(E)}{g(E)} = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

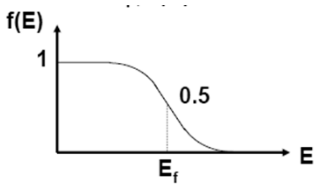
Assume arbitrary Fermi-level, plot f(E) at T=0 K and T= 300 K



Apply the Fermi-Dirac distribution function to electrons in a semiconductor. If g(E) is the density of states, what does f(E) tell us?

Fermi Energy

At *E_F*, *f(E)*=1/2



Note that the Fermi function is the probability of occupation, regardless of whether or not a state exists!

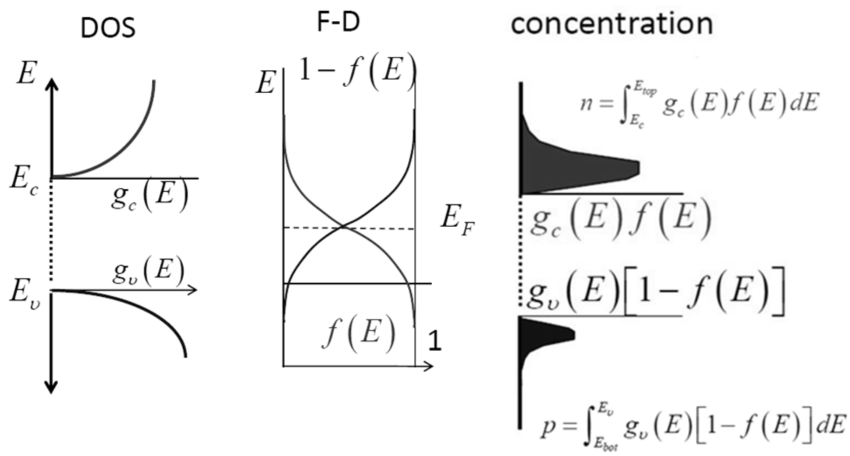
Boltzmann Approximation

For E-E$_F$ >> kT

$$\boxed{f(E) \approx \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)} = \exp\left(-\frac{E-E_F}{kT}\right)}$$

You can assume this approximation is valid for (E-Ef) > 3kT

Carrier Statistics



n or p =Number of electrons or holes.

Boltzmann vs. Fermi-Dirac Statistics

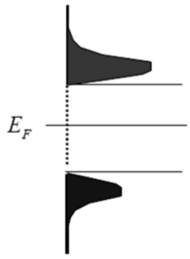
$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{\eta_c} \quad if \quad -\eta_c \equiv \beta(E_C - E_F) > 3$$

This approximation is equivalent to Boltzmann approximation!
The Boltzmann approximation can be invoked for non-degenerate semiconductors (Fermi-level is not too close to Ec or Ev).

$$n = N_C e^{-\beta(E_c - E_F)}$$

$$p = N_V e^{+\beta(E_v - E_F)}$$

$$n \times p = N_C N_V e^{-\beta(E_c - E_v)} = N_C N_V e^{-\beta E_g}$$



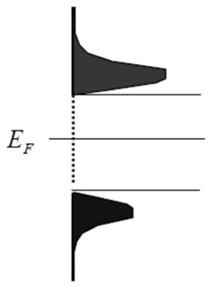
Intrinsic Semiconductors

$$n = p = n_i$$

$$\boxed{n_i^2 = N_C N_V e^{-\beta E_g}} \quad \boxed{np = n_i^2}$$

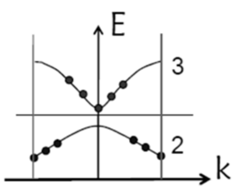
$$n_i = \sqrt{N_C N_V} e^{-\beta E_g/2}$$

$E_F \equiv E_i$ Fermi level for an intrinsic semiconductor

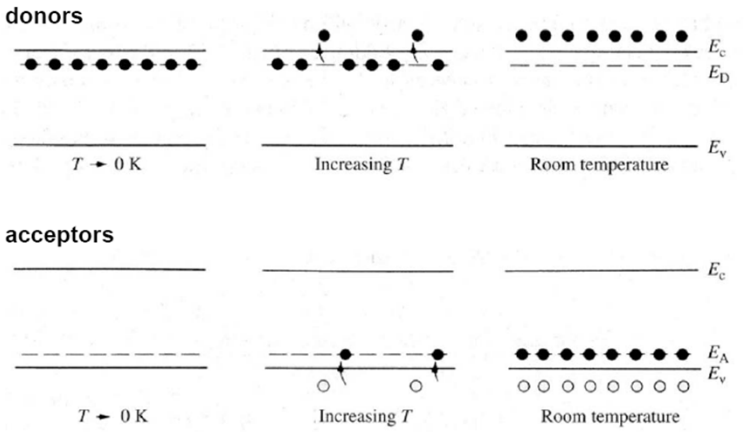


$$n = p \Rightarrow N_C e^{-\beta(E_c - E_i)} = N_V e^{+\beta(E_v - E_i)}$$

$$E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

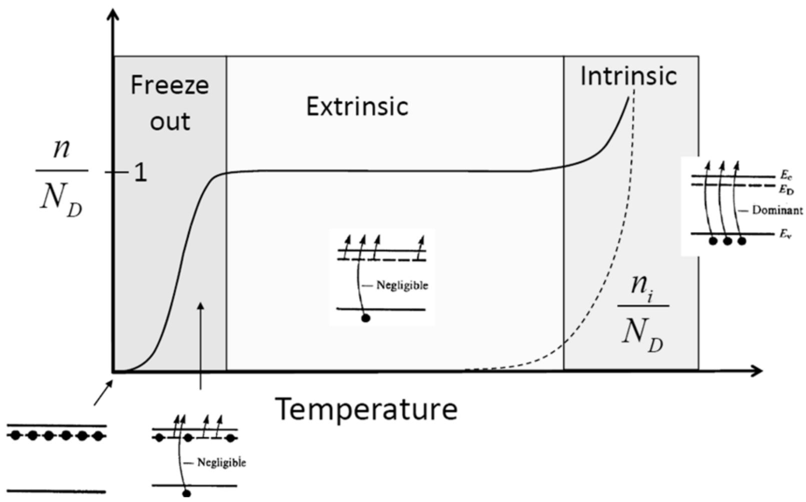


Doping: tuning the density of carriers



For "good dopant" majority of dopants ionized at room temperature

Majority Carrier Concentration



Fermi-Level for Doped Semiconductors

By doping we have added additional carriers into the semiconductors, the Fermi level will change accordingly.

$$n = n_i e^{(E_F - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_F)/k_B T}$$

Drift and Diffusion

- Drift: carrier motion when there is electric field.
- Diffusion: carrier motion when there is concentration gradient.

Drift Current: $\boxed{J = J_n + J_p = (nq\mu_n + pq\mu_p)\mathcal{E}}$

Conductivity: $\boxed{\sigma = nq\mu_n + pq\mu_p}$

Total Current:

$$J_N = J_{N|drift} + J_{N|diff} = qn\mu_n\mathcal{E} + qD_N\frac{dn}{dx}$$

$$J_P = J_{P|drift} + J_{P|diff} = qp\mu_p\mathcal{E} - qD_P\frac{dp}{dx}$$

Einstein's Relationship

Constants for diffusion and drift related (can calculate diffusion constants from μ)

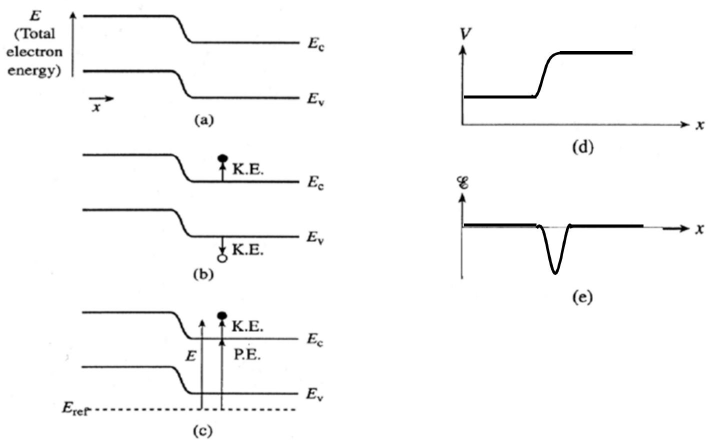
$$\boxed{\frac{D_n}{\mu_n} = \frac{kT}{q}} \qquad \boxed{\frac{D_p}{\mu_p} = \frac{kT}{q}}$$

Band Bending

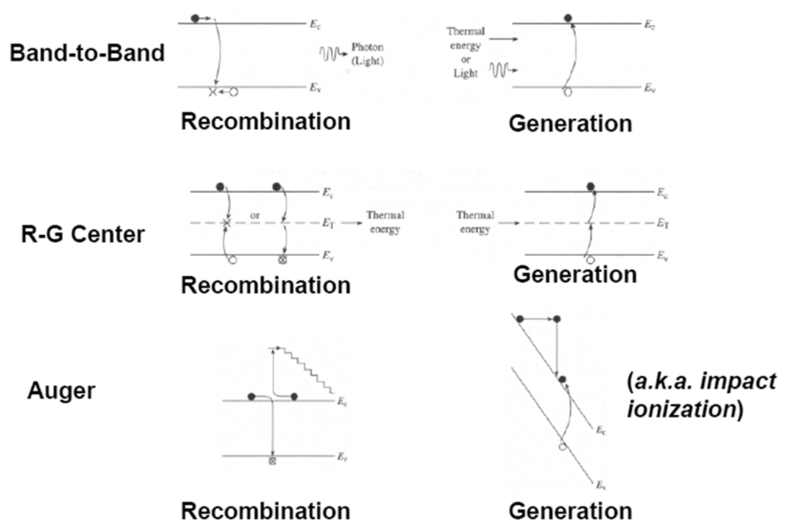
$$\boxed{V = \text{P.E.}/(-q) = (E_C - E_{ref})/(-q)}$$

$$\boxed{\mathcal{E} = -\nabla V}$$

If there is electric field inside the material….



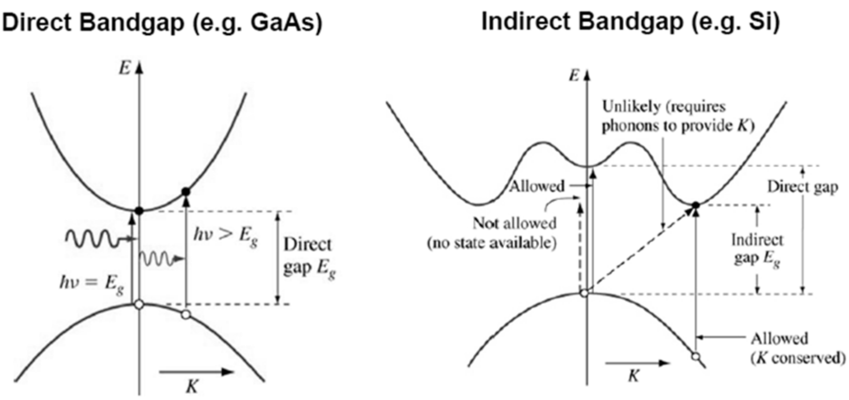
R-G Process



Optical Generation

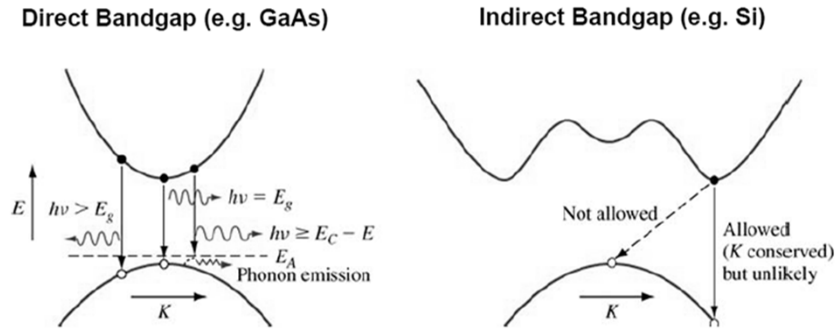
Both energy and momentum conservation needs to be satisfied.

The momentum of a photon is small



Optical Recombination

The momentum of a photon is small



Would you expect radiative recombination (light emission) to be significant in silicon?

Photo Generation and Indirect Thermal R-G

Photo Generation

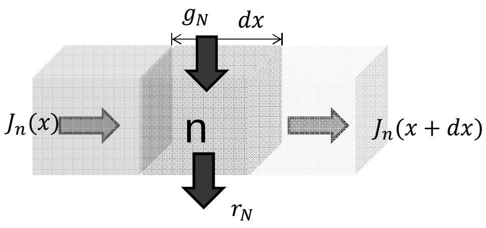
$$\frac{\partial n}{\partial t}\Big|_{light} = \frac{\partial n}{\partial t}\Big|_{light} = G_L(x,\lambda)$$

Thermal R-G

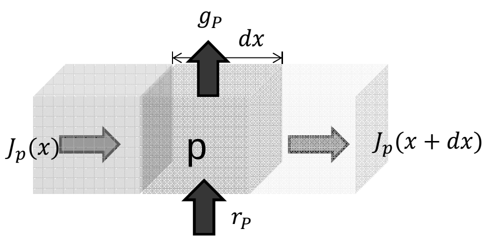
$$\frac{\partial p}{\partial t}\Big|_{\substack{i-thermal \\ R-G}} = -\frac{\Delta p}{\tau_p} \qquad \frac{\partial n}{\partial t}\Big|_{\substack{i-thermal \\ R-G}} = -\frac{\Delta n}{\tau_n}$$

Applicable only to **minority carriers and low-level injection** condition must be satisfied!

Continuity Equations for Electrons/Holes



$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_n + \frac{\partial n}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial n}{\partial t}\Big|_{\substack{other\\process}}$$



$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_p + \frac{\partial p}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial p}{\partial t}\Big|_{\substack{other\\process}}$$

Summary of All Equations

$$\nabla \cdot D = q(p - n + N_D^+ - N_A^-)$$

$$J_n = qn\mu_n \mathcal{E} + qD_N \nabla n$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_n + \frac{\partial n}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial n}{\partial t}\Big|_{\substack{other\\process}}$$

$$J_p = qp\mu_p \mathcal{E} - qD_P \nabla p$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_p + \frac{\partial p}{\partial t}\Big|_{\substack{thermal\\R-G}} + \frac{\partial p}{\partial t}\Big|_{\substack{other\\process}}$$

These are the equations needed for the device analysis!

Minority Carrier Diffusion Equation

Simplifying Assumptions for a common case:

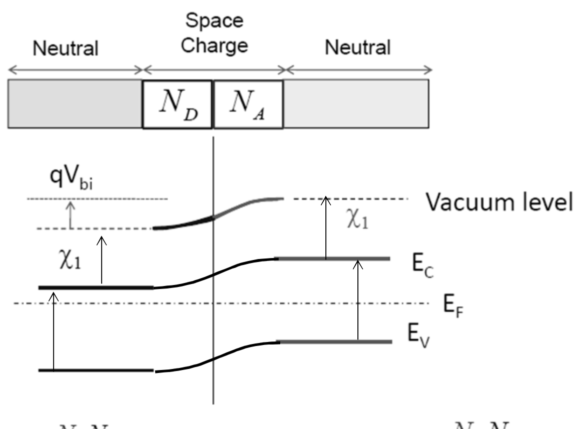
- One dimensional.
- We will only consider minority carriers
- Electric field is approximately zero in regions subject to analysis, hence negligible drift.
- Low-level injection conditions apply.
- R-G center is the main recombination-generation mechanism.
- The only "other" mechanism is photogeneration.

Under these conditions, the continuity Eq. is simplified to

$$\boxed{\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G}$$

$$\boxed{\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G}$$

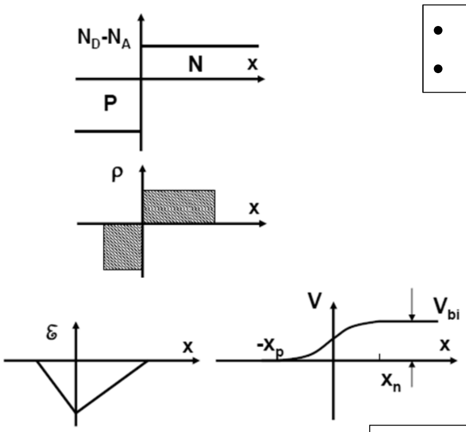
Built-in voltage for **Homo**-junctions



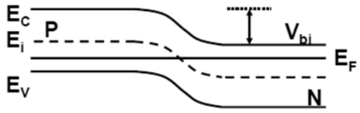
$$qV_{bi} = k_B T\, ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + (\chi_2 - \chi_1) = k_B T\, ln \frac{N_A N_D}{N_V N_C e^{-E_g/k_B T}} = k_B T\, ln \frac{N_A N_D}{n_i^2}$$

Check how this is derived in the textbook…

Electrostatics



- Depletion approximation
- Band diagram



$$\boxed{N_D x_n = N_A x_p}$$

$$\boxed{V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

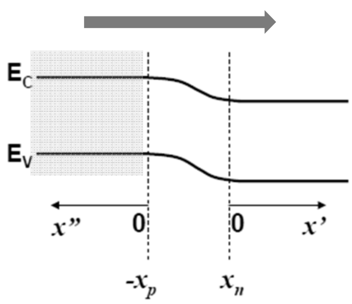
$$\boxed{W = \sqrt{\frac{2K_s\varepsilon_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_A)}}$$

Quantitative Studies

Minority carrier diffusion eqations apply in the “quasi-neutral” regions

In the quasi-neutral p-region

$$\cancel{\frac{\partial \Delta n_p}{\partial t}} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + \cancel{G_L}$$



- **Steady State:**
  Currents are the same everywhere.
- **No Light**

$$J = J_N(x) + J_P(x)$$

Minority Carrier Diffusion Current

In quasi-neutral P region

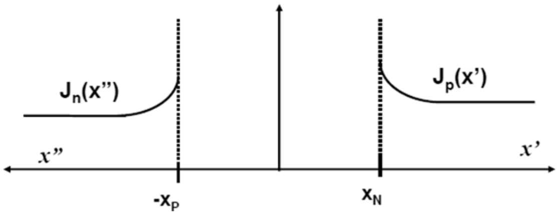
$$J_n = nq\mu_n \mathscr{E} + qD_n \frac{dn}{dx} \approx qD_n \frac{dn_p}{dx} = -qD_n \frac{d\Delta n_p}{dx''}$$

$$\Delta n_p(x) = \Delta n(x''=0)e^{-x''/L_N}$$

In quasi-neutral N region

$$J_p = pq\mu_p \mathscr{E} - qD_p \frac{dp}{dx} \approx -qD_p \frac{dp_n}{dx} = -qD_p \frac{d\Delta p_n}{dx'}$$

$$\Delta p_n(x) = \Delta p(x'=0)e^{-x'/L_P}$$



Ideal Diode

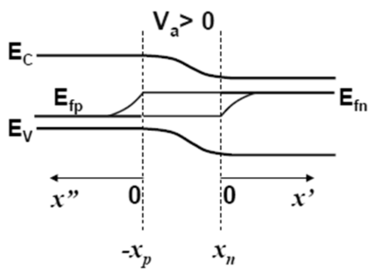
$\Delta n_p(-x_p)$, and $\Delta p_n(x_n)$ are still not known

By definition

$$np = n_i^2 e^{(F_N - F_P)/kT}$$

Assume

$$F_N - F_P = E_{F_n} - E_{F_p} = qV_A$$



Law of the junction

$$\boxed{np = n_i^2 e^{qV_A/kT}} \qquad -x_p \le x \le x_n$$

$$n(-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} \Rightarrow \boxed{\Delta n_p(-x_p) = \frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)}$$

Similarly, $\boxed{\Delta p_n(x_n) = \frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)}$

Ideal Diode

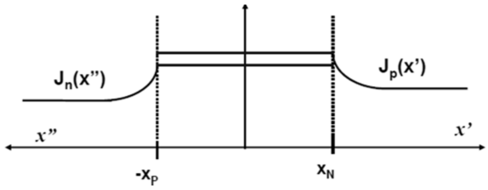
$\mathscr{E} \neq 0$ in the depletion region, minority carrier diff. eq. not applicable
From continuity equation,

$$0 = \frac{1}{q}\frac{dJ_N}{dx} + \cancel{\frac{\partial n}{\partial t}}|_{R-G}$$

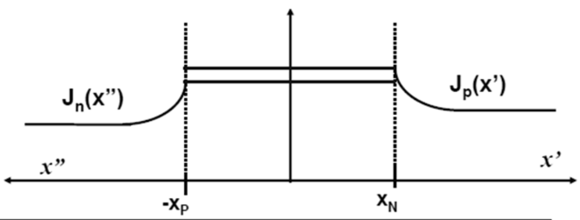
$$0 = -\frac{1}{q}\frac{dJ_P}{dx} + \cancel{\frac{\partial p}{\partial t}}|_{R-G}$$

Ideal Diode: assume no R-G in the depletion region
Current is a constant inside depletion region

$$J = J_N(-x_p) + J_P(x_n) = J_N(x'' = 0) + J_P(x' = 0)$$



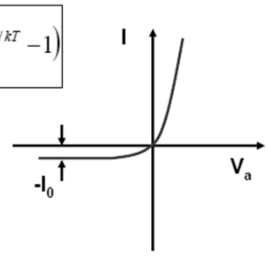
Ideal Diode, I-V



$$\boxed{J = J_n(x''=0) + J_p(x'=0) = q\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)\left(e^{qV_A/kT} - 1\right)}$$

$$\boxed{I = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)\left(e^{qV_A/kT} - 1\right)}$$

$$\boxed{I = I_0\left(e^{qV_A/kT} - 1\right)}$$



I_0 = Reverse saturation current

Note

- This review has been focusing on the physical concepts and understanding, but calculation is equally important!
- The exam is composed of 5 conceptual problems (35%) and 3 derivation problems (20%, 20%, 25%).