University of Michigan – Shanghai Jiao Tong University Joint Institute
Center of Optics and Optoelectronics

VE 320 – Summer 2012
Introduction to Semiconductor Device

Carrier Statistics

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Carrier Statistics

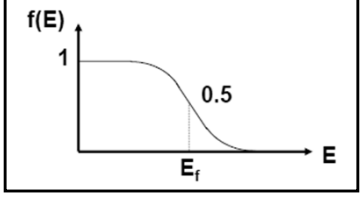
Density of States

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}$$

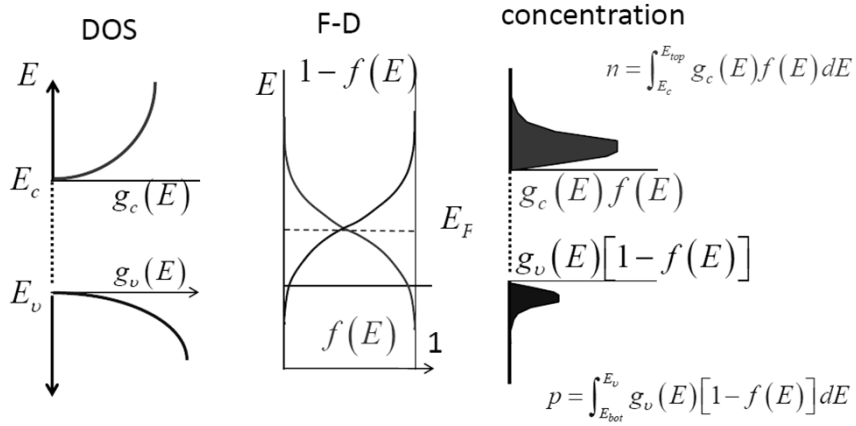
Fermi-Dirac distribution

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$



Now we are ready to calculate how many electrons are there !

Carrier Distribution



Electron Concentration in 3D Solid

$$n=\int_{E_c}^{E_{top}} g_c(E)f(E)dE$$

$$=\int_{E_c}^{E_{top}} 2\times\frac{m_n^*\sqrt{2m_n^*(E-E_C)}}{2\pi^2\hbar^3}\frac{1}{1+e^{\beta(E-E_F)}}dE$$

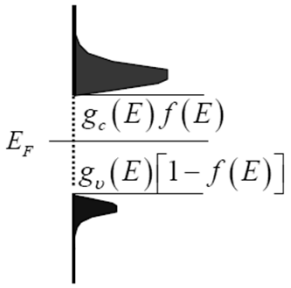
$$\simeq\int_{E_c}^{\infty}\frac{m_n^*\sqrt{2m_n^*(E-E_C)}}{\pi^2\hbar^3}\frac{1}{1+e^{\beta(E-E_c)}e^{\beta(E_c-E_F)}}dE$$

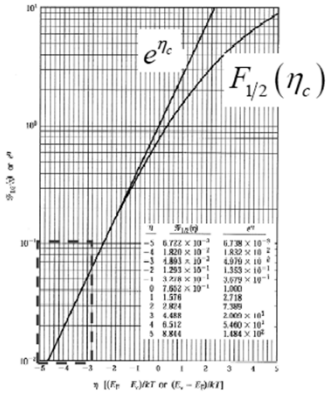
$$=N_C\frac{2}{\sqrt{\pi}}F_{1/2}(\eta_c)\qquad \eta_c\equiv\beta(E_F-E_C)$$

$$N_C\equiv 2\left(\frac{2\pi m_n^*\beta}{h^2}\right)^{3/2}\qquad F_{1/2}(\eta)=\int_0^\infty\frac{\sqrt{\xi}d\xi}{1+e^{\xi-\eta}}$$

Boltzmann vs. Fermi-Dirac Statistics

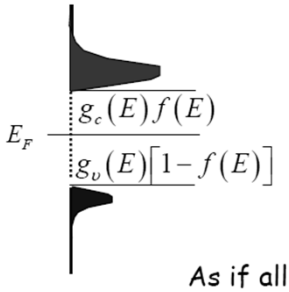
$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \to N_C e^{\eta_c} \quad if \quad -\eta_c \equiv \beta(E_C - E_F) > 3$$

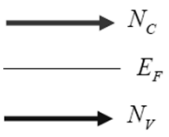




Effective Density of States

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \to N_C e^{-\beta(E_c - E_F)} \quad if \quad E_c - E_F > 3\beta$$





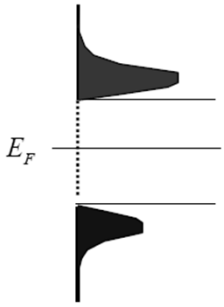
As if all states are at a single level E_C

Law of Mass-Action

$$n = N_C e^{-\beta(E_c - E_F)}$$

$$p = N_V e^{+\beta(E_v - E_F)}$$

$$n \times p = N_C N_V e^{-\beta(E_c - E_v)}$$
$$= N_C N_V e^{-\beta E_g}$$



Fermi-Level for Intrinsic Semiconductors

$$n = p = n_i$$

$$\boxed{n_i^2 = N_C N_V e^{-\beta E_g}}$$

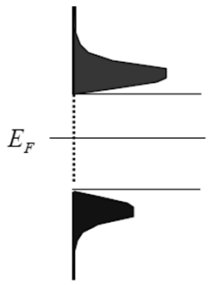
$$n_i = \sqrt{N_C N_V}\, e^{-\beta E_g/2}$$

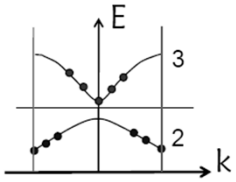
$$E_F \equiv E_i$$

$$n = p \Rightarrow N_C e^{-\beta(E_c - E_i)} = N_V e^{+\beta(E_v - E_i)}$$

$$E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

$$\boxed{np = n_i^2}$$

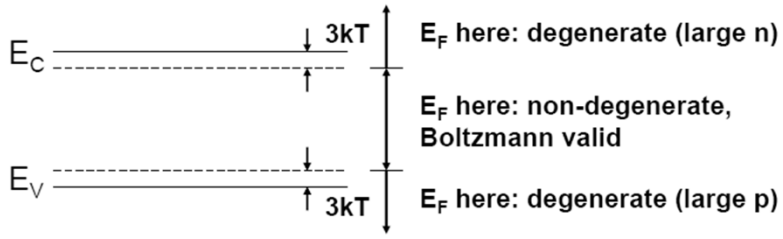




Degenerate and Nondegenerate

For $E_V + 3kT < E_f < E_C - 3kT$, Boltzmann approximation is accurate

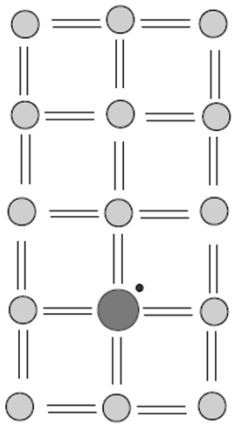
Semiconductor is said to be "non-degenerate"



Doping

- We can see that the number of carrier of intrinsic semiconductor is very small.
- Is there a way to change the carrier concentration?

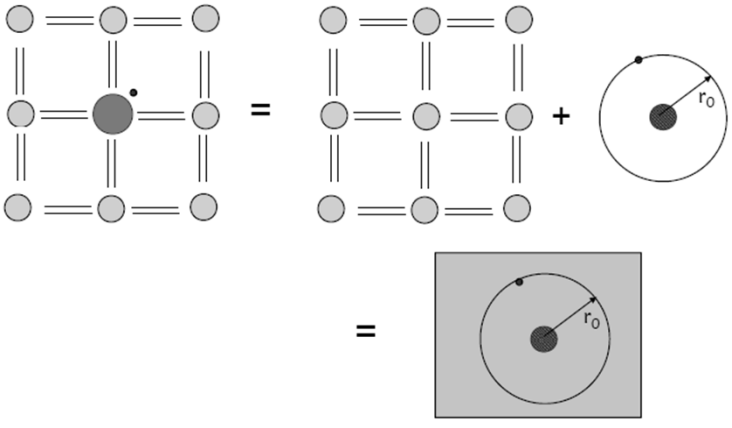
Doping



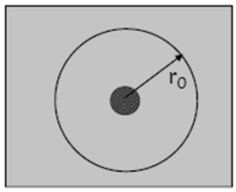
II	III	IV	V	VI
4 Be	5 B	6 C	7 N	8 O
12 Mg	13 **Al**	14 **Si**	15 **P**	16 S
30 **Zn**	31 **Ga**	32 Ge	33 **As**	34 **Se**
48 Cd	49 In	50 Sn	51 Sb	52 Te
80 Hg	81 Tl	82 Pb	83 Bi	84 Po

Even with donors, material is charge neutral

Donor Atom with Hydrogen Analogy



Donor Atom in Real and Energy Space

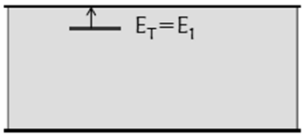


$$E_1 = -\frac{m^*_{host} q^4}{2\left(4\pi\varepsilon_0 K_{s,host}\hbar\right)^2}$$

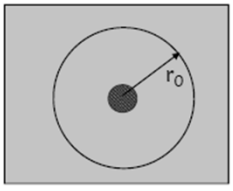
$$= -\frac{m_0 q^4}{2\left(4\pi\varepsilon_0\hbar\right)^2}\frac{m^*_{host}}{m_0}\frac{1}{K_{s,host}{}^2}$$

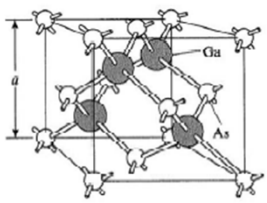
$$= -13.6\times\frac{m^*_{host}}{m_0}\frac{1}{K_{s,host}{}^2}$$

~10s meV



Assumption of Large Radius





$$r_{1,P} = \frac{4\pi\varepsilon_0 K_{s,host}\hbar^2}{m^*_{host} q^2}$$

$$= \frac{4\pi\varepsilon_0\hbar^2}{m_0 q^2}\frac{m_0 K_{s,host}}{m^*_{host}}$$

$$= r_{1,H}\frac{K_{s,host}}{m^*_{host}/m_0}$$

$$r_{1,P} = 0.53\ A\times\frac{12.9}{0.53} = 12.9\ A$$

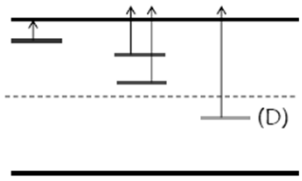
(see tables 1.6 and 4.1)

Characteristic of Donor Atoms

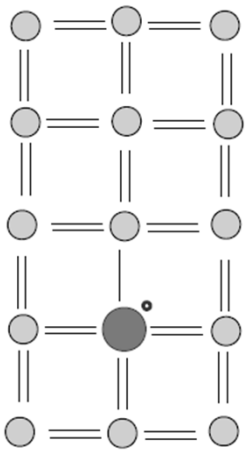
The number of donor atoms is much smaller compared to host atoms. Therefore, the electrons from one donor atom can go to the other donor atoms only via the conduction /valence bands of the host crystal.

Just like a Hydrogen atom, it is possible to have multiple localized level for a given atom (see the blue levels).

Good donors live close to the conduction band, so that they can offer electrons easily. However, if they are below the midgap, the donor levels are marked with (D) to differentiate them from acceptor atoms (which live close to the valence band).



Acceptor Atoms



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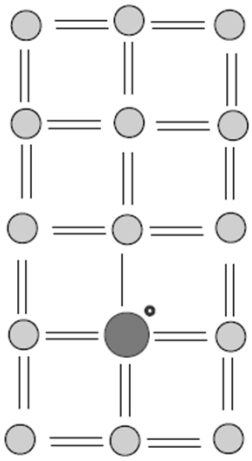
Even with acceptor, material is charge neutral

Characteristic of Acceptor Atoms



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Even with acceptor, material is charge neutral

Amphoteric Dopants

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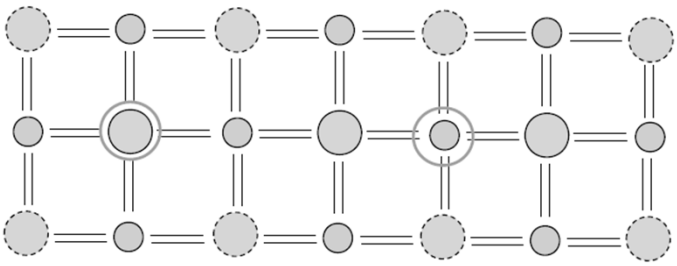
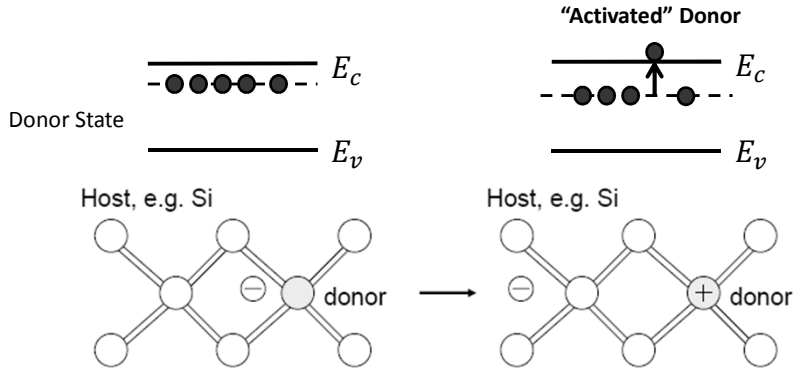
Donor-type acceptor-type



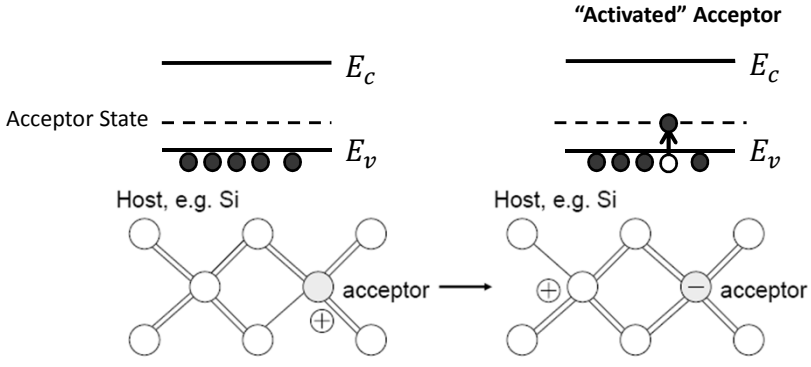
Donor Action

- A net concentration of donors provides a higher electron concentration than intrinsic material
- n-type semiconductor

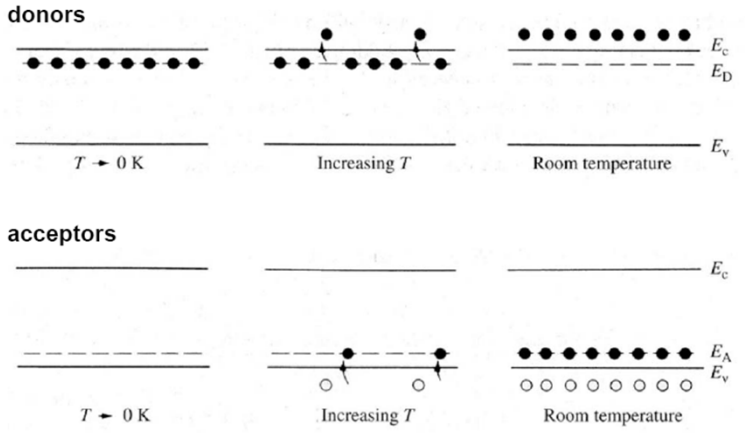


Acceptor Action

- A net concentration of acceptors provides a higher hole concentration than intrinsic material
- p-type semiconductor



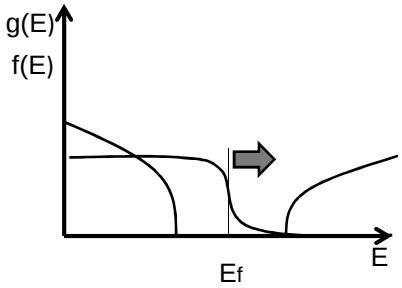
Temperature Activation of Dopants



For "good dopant" majority of dopants ionized at room temperature

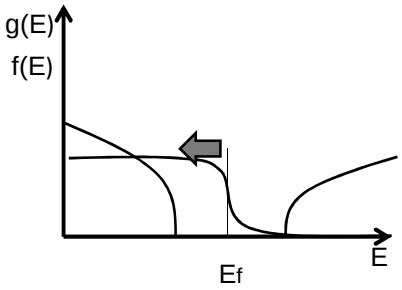
Fermi Level for Doped Semiconductor

- n-type semiconductors have excess electrons in the conduction band, therefore the probability of finding an electron in the conduction band increases, so the Fermi level moves to a higher energy.



Fermi Level for Doped Semiconductor

- p-type semiconductors have excess holes in the valence band, therefore the probability of finding an electron in the valence band decreases, so the Fermi level moves to a lower energy.



Occupation of Donor/Acceptor Levels

- The number of activated dopants are dependent on binding energy and temperature

# of Activated Donor/Acceptor

$$\frac{N_D^+}{N_D} = \frac{1}{1 + g_D e^{(E_f - E_D)/kT}} \qquad g_D = 2$$

# of Donor/Acceptor

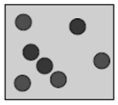
$$\frac{N_A^-}{N_A} = \frac{1}{1 + g_A e^{(E_A - E_f)/kT}} \qquad g_A = 4$$

Degeneracy factors

- E_D, E_A are Donor and Acceptor levels
- Spin up, spin down for donors
- Spin up, spin down, heavy hole, light hole for acceptors

Charge Neutral Relationship

A bulk material must be charge neutral over all …



$$\int \left[p - n + N_D^+ - N_A^- \right] dV = 0$$

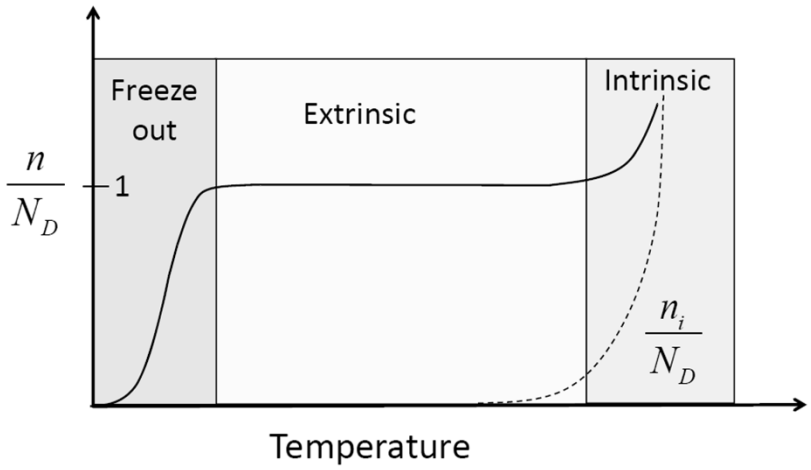
Further if the material is *spatially homogeneous*

$$p - n + N_D^+ - N_A^- = 0$$

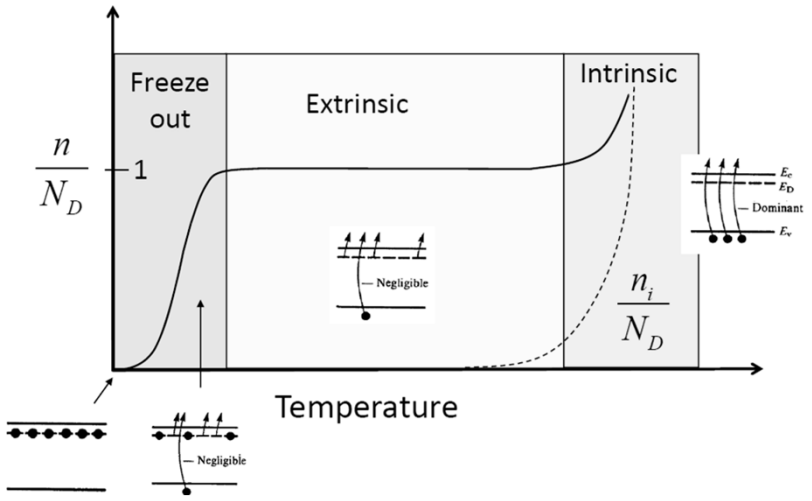
$$N_V e^{-(E_F - E_V)/k_B T} - N_c e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0$$

Note that with dopants, it can **no longer** be assumed that ***n=p***.

Temperature Activation of Dopants



Physical Interpretation



Charge Neutral Relationship

In the extrinsic region, one can assume that all donors and acceptors are ionized (activated). Therefore,

$$n - p + N_D - N_A = 0$$

Law of mass action $\quad np = n_i^2$

Carrier density

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$p = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2}$$

Common Case

In doped semiconductor where

$N_D - N_A \approx N_D >> n_i$

$$n \approx N_D \quad \text{Majority carriers}$$

$$p \approx n_i^2 / N_D \quad \text{Minority carriers}$$

or $N_A - N_D \approx N_A >> n_i$

$$p \approx N_A$$

$$n \approx n_i^2 / N_A$$

Carrier Concentration

Si: $n_i = 1.6 \times 10^{10}\ cm^{-3}$

GaAs: $n_i = 2 \times 10^{6}\ cm^{-3}$

Ge: $n_i = 2 \times 10^{13}\ cm^{-3}$

Doping level for silicon ~ $10^{13} \sim 10^{18}\ cm^{-3}$
(> 10^{18} for heavily doped silicon)

Atomic number density of silicon ~ $10^{22} cm^{-3}$

Fermi-Level for Doped Semiconductors

$$n = n_i e^{(E_F - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_F)/k_B T}$$

Summary