University of Michigan – Shanghai Jiao Tong University Joint Institute
Center of Optics and Optoelectronics

VE 320 – Summer 2012
Introduction to Semiconductor Device

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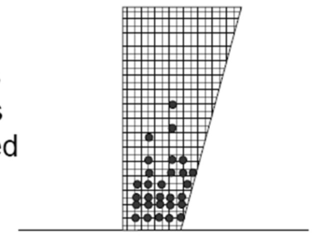
Carrier Density

Carrier number = Number of states x filling factor

Quantum Mechanics

Statistical Mechanics

Total number of occupants
= Number of apartments
X The fraction occupied



Statistical Laws

Need to define laws that the particles obey

Some common distribution laws:

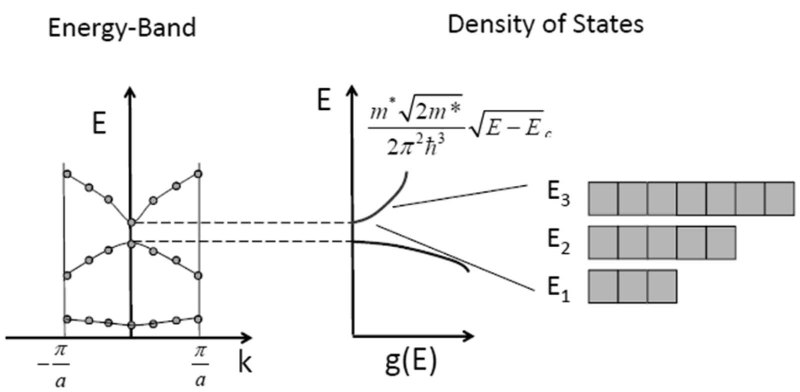
Maxwell Boltzmann:
particles distinguishable by number, no limit on number of particles in each energy state

Bose-Einstein:
particles are indistinguishable, no limit on number of particles in each energy state

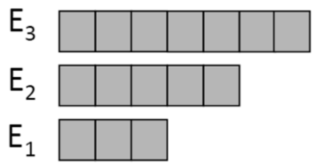
Fermi-Dirac:
particles are indistinguishable, only one particle in each energy state

Which would you expect to apply for electrons in a semiconductor?

E-k diagram and Electronic states

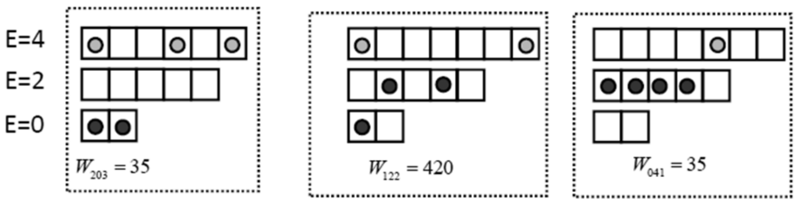


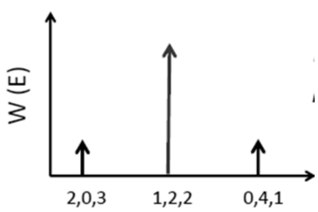
Rules for Filling up States



- Pauli Principle: Only one electron per state
- Total number of electrons is conserved $N_T = \sum_i N_i$
- Total energy of the system is conserved $E_T = \sum_i E_i N_i$

Occupation Statistics





Choose the most probable configuration.

Occupation Statistics

E=4

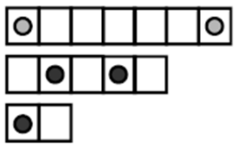
E=2

E=0

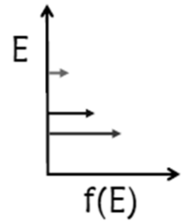
$$f_3^* = \frac{2}{7}$$

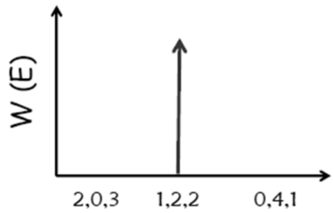
$$f_2^* = \frac{2}{5}$$

$$f_1^* = \frac{1}{2}$$

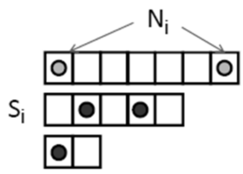


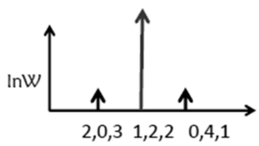
$W_{122} = 420$





For N States





$$W_c = \prod_i \frac{S_i!}{(S_i - N_i)!N_i!}$$

Recall. $W_{203} = \frac{2!}{1!2!} \bullet \frac{5!}{0!5!} \bullet \frac{7!}{3!4!}$

$$\ln(W) = \sum_i \left[\ln S_i! - \ln(S_i - N_i)! - \ln N_i!\right]$$

Stirling approx.

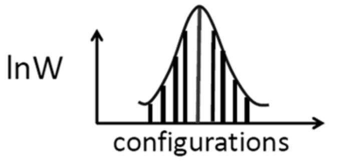
$$\simeq \sum_i \left[S_i \ln S_i - S_i - (S_i - N_i)\ln(S_i - N_i) + (S_i - N_i) - N_i \ln N_i + N_i\right]$$

$$= \sum_i \left[S_i \ln S_i - (S_i - N_i)\ln(S_i - N_i) - N_i \ln N_i\right]$$

Optimization with Lagrange-Multiplier

Choose the most probable configuration.

$$\delta \ln(W) = \sum_i \frac{\partial \ln W}{\partial N_i} dN_i$$



$$\delta \ln(W) = \sum_i \frac{\partial \ln W}{\partial N_i} dN_i$$

$$\simeq \sum_i \left[\ln\left(\frac{S_i}{N_i} - 1 \right) dN_i \right] - \alpha \sum_i dN_i - \beta \sum_i E_i dN_i$$

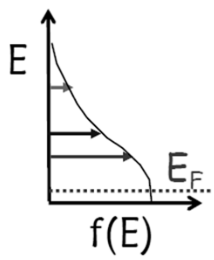
$$= \sum_i \left[\ln\left(\frac{S_i}{N_i} - 1 \right) - \alpha - \beta E_i \right] dN_i$$

$E_T = \sum_i E_i N_i$

$N_T = \sum_i N_i$

Final Steps

$$\left[\ln\left(\frac{S_i}{N_i} - 1 \right) - \alpha - \beta E_i \right] = 0$$



$$f(E) \equiv \frac{N_i}{S_i} = \frac{1}{1 + e^{\alpha + \beta E}}$$

f_{max}(E)=1

$$\text{At } E = E_F,\ f(E_F) \equiv \frac{1}{2} \quad \Rightarrow \alpha + \beta E_F = 0$$

$$f(E) = \frac{N_i}{S_i} = \frac{1}{1 + e^{\beta(E - E_F)}} = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

$$\text{At } E \to \infty,\ f_{Boltzman}(E) = A e^{-E/k_B T} \quad \Rightarrow \quad \beta = \frac{1}{k_B T}$$

Fermi-Dirac Distribution Function

$$\boxed{f(E) = \frac{N(E)}{g(E)} = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}}$$

$k = \text{Boltzmann Constant}$

$k = 8.617 \times 10^{-5} eV/K$

$kT = 0.026eV$ at $\text{T} = 300\text{K}$

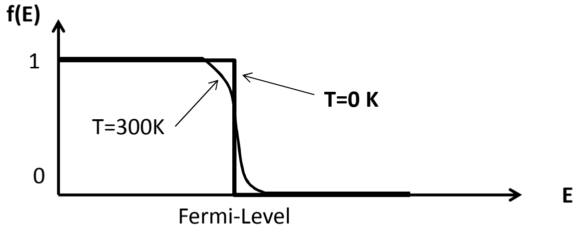
Can you see f(E) is always smaller than one?

BTW, from now on, since we will not talk about wave number any more, we will use k = k_B, which is the Boltzmann constant, to be consistent with the textbook.

Fermi-Dirac Distribution

$$\boxed{f(E) = \frac{N(E)}{g(E)} = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}}$$

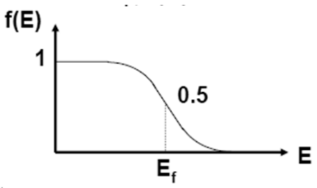
Assume arbitrary Fermi-level, plot f(E) at T=0 K and T= 300 K

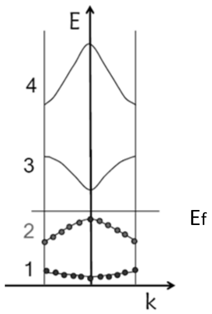


Apply the Fermi-Dirac distribution function to electrons in a semiconductor. If g(E) is the density of states, what does f(E) tell us?

Fermi Energy

At E_f, $f(E)$=1/2





Note that the Fermi function is the probability of occupation, regardless of whether or not a state exists!

For a semiconductor, if the number of electrons is the same as the number of holes, where is E_f located at?

Approximation to Fermi-Dirac Statistics

For $E-E_F \gg kT$

$$f(E) \approx \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)} = \exp\left(-\frac{E-E_F}{kT}\right)$$

You can assume this approximation is valid for $(E-E_f) > 3kT$

Summary

- The distribution of electrons follows Fermi-Dirac statistics
- Fermi level is critical for determining the number of carriers in semiconductors.