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Center of Optics and Optoelectronics

## VE 320 – Summer 2012 Introduction to Semiconductor Device

### PN Junction DC Response

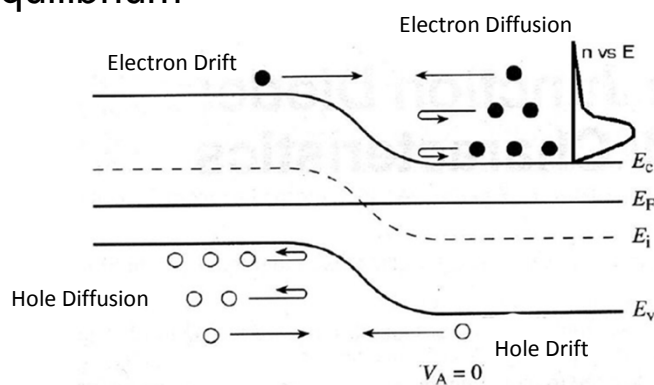
Instructor: Professor Hua Bao

**NANO ENERGY LAB**

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## Equilibrium



At equilibrium, the total current balances due to the sum of the individual components

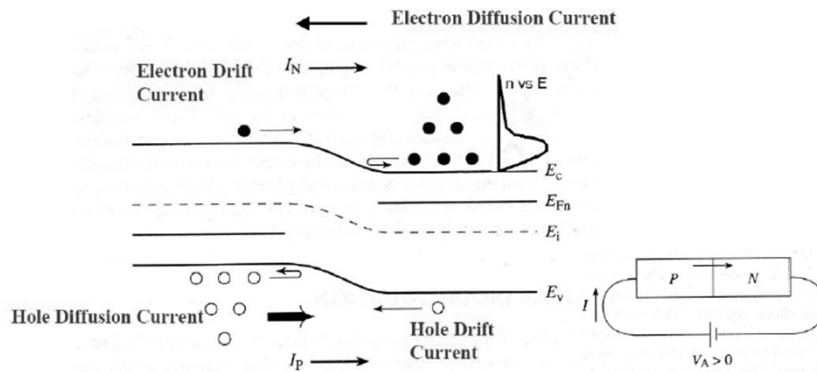


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## Forward Bias



Current flow is proportional to  $e^{(qV_A/kT)}$  due to the exponential increase of carriers in the majority carrier bands

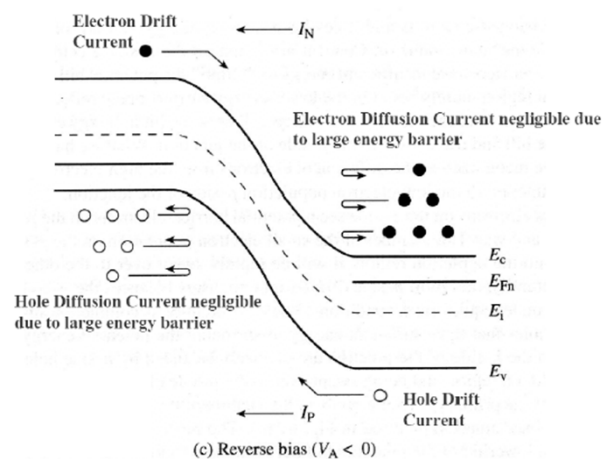


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## Reverse Bias



Reverse current caused by minority carriers being swept away by electric field, and independent of the size of  $V_A$



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## Qualitative Analysis

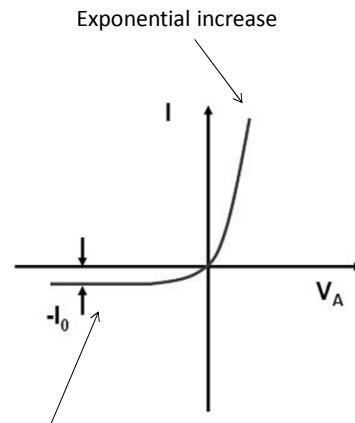
$$I = I^+ - I^-$$

$$I^+ \propto e^{qV_A/kT}$$

$$I^- = I_0 \text{ Independent of } V_A$$

$$I(V_A = 0) = 0$$

$$\rightarrow I = I_0(e^{qV_A/kT} - 1)$$



(very small) Constant  
saturation current



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## I-V Relation: Quantitative

Determine current flow assuming steady state, low-level injection

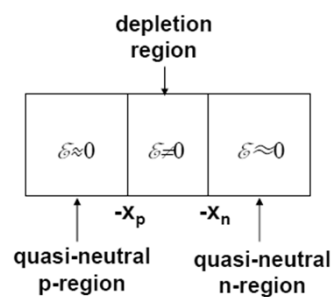
$$I = AJ \quad J = J_n + J_p$$

$$J_n = nq\mu_n\mathcal{E} + qD_n \frac{dn}{dx}$$

$$J_p = pq\mu_p\mathcal{E} - qD_p \frac{dp}{dx}$$

Need to determine carrier densities  
and electric field....

Solve continuity equations



Three regions of interest



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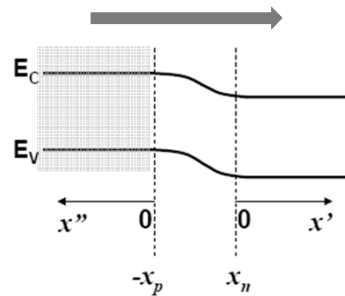
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## Quantitative Studies

Minority carrier diffusion equations apply in the “quasi-neutral” regions

In the quasi-neutral p-region

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$



• **Steady State:**

Currents are the same everywhere.

$$J = J_N(x) + J_P(x)$$

• **No Light**



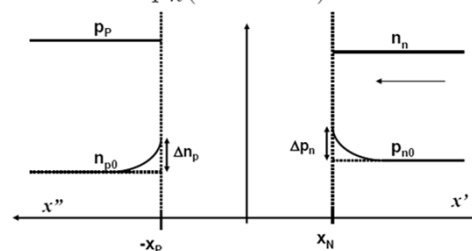
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## Minority Carrier Densities

Wide base diode  $\Delta n_p(x \rightarrow -\infty) = 0$   
 $\Delta p_n(x \rightarrow +\infty) = 0$



Solution to Minority Carrier Diffusion Equation

$$\Delta n_p(x) = \Delta n_p(x''=0)e^{-x''/L_N} \quad \Delta p_n(x) = \Delta p_n(x'=0)e^{-x'/L_P}$$

$$L_N = \sqrt{D_N \tau_N}$$

$$L_P = \sqrt{D_P \tau_P}$$



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## Minority Carrier Diffusion Current

In quasi-neutral P region

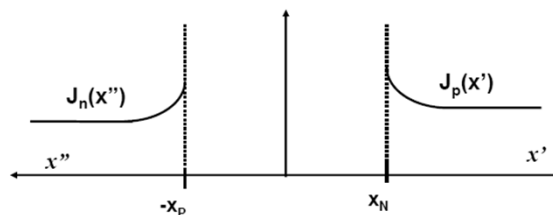
$$J_n = nq\mu_n \mathcal{E} + qD_n \frac{dn}{dx} \approx qD_n \frac{dn_p}{dx} = -qD_n \frac{d\Delta n_p}{dx''}$$

$$\Delta n_p(x) = \Delta n(x''=0)e^{-x''/L_n}$$

In quasi-neutral N region

$$J_p = pq\mu_p \mathcal{E} - qD_p \frac{dp}{dx} \approx -qD_p \frac{dp_n}{dx} = -qD_p \frac{d\Delta p_n}{dx'}$$

$$\Delta p_n(x) = \Delta p(x'=0)e^{-x'/L_p}$$



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## Ideal Diode

$\Delta n_p(-x_p)$ , and  $\Delta p_n(x_n)$  are still not known

By definition

$$np = n_i^2 e^{(F_N - F_P)/kT}$$

Assume

$$F_N - F_P = E_{F_n} - E_{F_p} = qV_A$$

Law of the junction

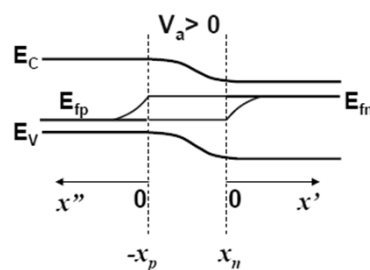
$$np = n_i^2 e^{qV_A/kT} \quad -x_p \leq x \leq x_n$$

$$n(-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

Similarly,

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1)$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$$



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## Minority Carrier Densities

Solution of Minority Carrier Diffusion Equation

$$\Delta n_p(x) = \Delta n(x''=0)e^{-x''/L_N} \quad \Delta p_n(x) = \Delta p(x'=0)e^{-x'/L_P}$$

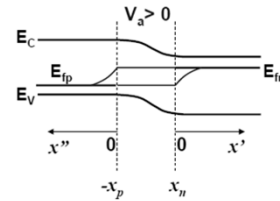
$$\Delta n_p(x'') = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1) e^{-x''/L_N} \quad \Delta p_n(x') = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1) e^{-x'/L_P}$$

Minority electron diffusion current on P side

$$J_n = -qD_N \frac{d\Delta n_p}{dx''} = q \frac{D_N}{L_N} \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1) e^{-x''/L_N}$$

Minority hole diffusion current on N side

$$J_p = -qD_P \frac{d\Delta p_n}{dx'} = q \frac{D_P}{L_P} \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1) e^{-x'/L_P}$$

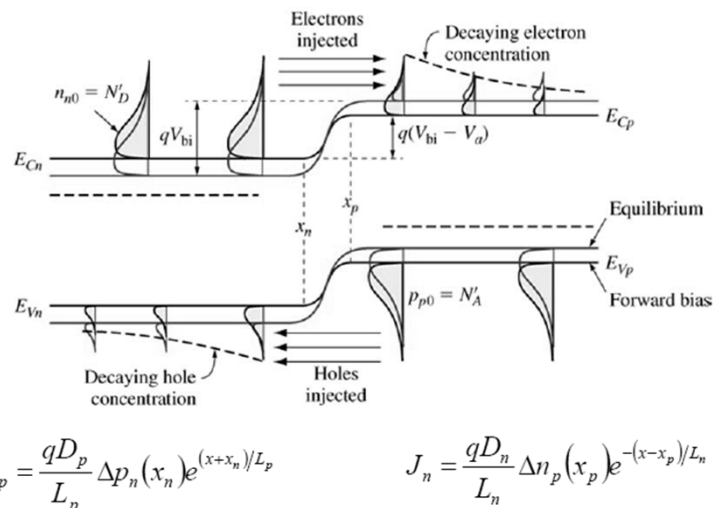


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## Diffusion Current



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## Ideal Diode

$\mathcal{E} \neq 0$  in the depletion region, minority carrier diff. eq. not applicable

From continuity equation,

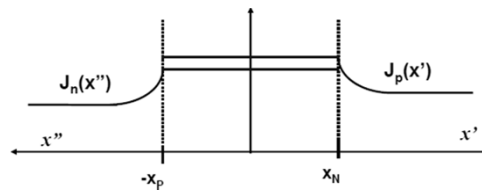
$$0 = \frac{1}{q} \frac{dJ_N}{dx} + \frac{\partial n}{\partial t} \Big|_{R-G}$$

$$0 = -\frac{1}{q} \frac{dJ_P}{dx} + \frac{\partial p}{\partial t} \Big|_{R-G}$$

Ideal Diode: assume no R-G in the depletion region

Current is a constant inside depletion region

$$J = J_N(-x_p) + J_P(x_n)$$

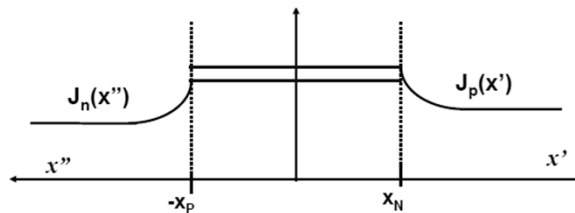


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## Ideal Diode, I-V

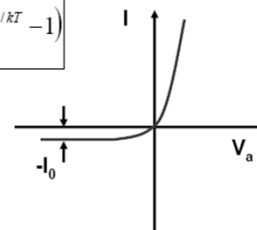


$$J = J_n(x''=0) + J_p(x'=0) = q \left( \frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right) (e^{qV_A/kT} - 1)$$

$$I = qA \left( \frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right) (e^{qV_A/kT} - 1)$$

$$I = I_0 (e^{qV_A/kT} - 1)$$

$I_0$  = Reverse saturation current

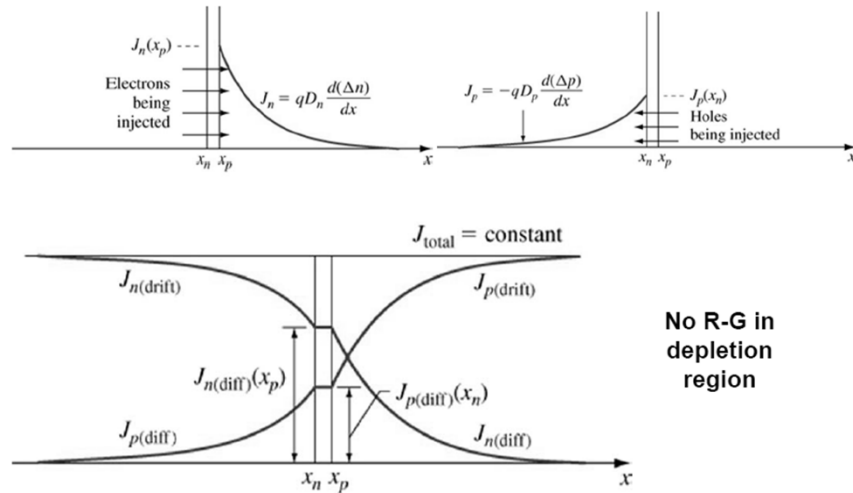


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## Total Current



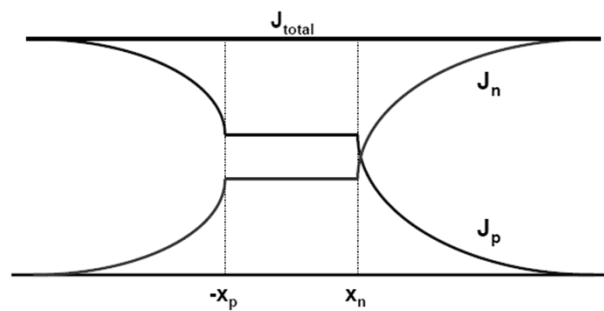
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## Electron and Hole Current

For all  $x$ ,  $J_{\text{total}} = J_n + J_p = \text{constant}$



- For every minority carrier recombination event, a majority carrier is injected from the contact
- $J_{\text{total}} > 0$  for  $V_a > 0$
- $J_{\text{total}} < 0$  for  $V_a < 0$



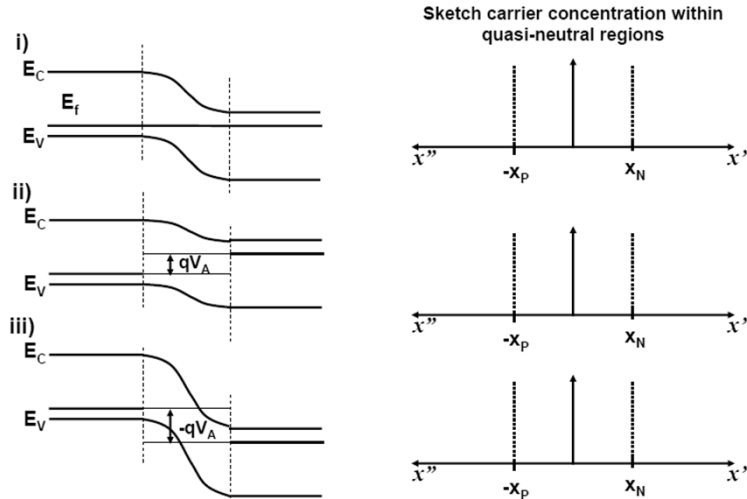
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## Carrier Concentration under Bias

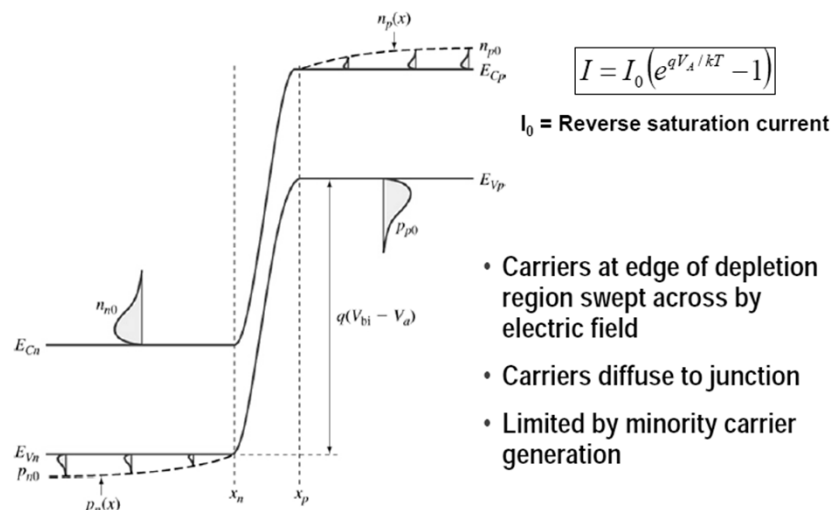


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## Diffusion Current –Reverse Bias



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## P-N Diode With Asymmetric Doping

$$I = qA \left( \frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right) (e^{qV_A/kT} - 1)$$

Typically, one side of the junction is doped much more heavily

$N_A = 10^{18} \text{ cm}^{-3}$	$N_D = 10^{15} \text{ cm}^{-3}$
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**p<sup>+</sup>-n junction**

$N_A = 10^{15} \text{ cm}^{-3}$	$N_D = 10^{18} \text{ cm}^{-3}$
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**n<sup>+</sup>-p junction**

The ideal diode equation may be simplified with asymmetric doping,  
rewrite expressions for p<sup>+</sup>-n and n<sup>+</sup>-p



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