3: Data-Based Generalization

John H Maindonald

April 3, 2018

Ideas and issues illustrated by the graphs in this vignette

A central concern of data analysis is to generalize from results from the one data set that is available for analysis to some wider relevance. Ideas of sample and population are crucial for such generalization. A further important notion is that of a *sampling distribution*,

Mechanisms for assessing predictive accuracy include the use of theory, simulation (which extends the use of theory into areas where the calculations may be intractable), the training/test approach, cross-validation, and bootstrap methods.

Note: Figures 3.15 and 3.16 show results from repeated sampling – simulation or bootstrap sampling. The versions of these figures that are shown in Section 2 are for a substantially reduced number of repeat samples, relative to the text *Statistically Informed Data Mining*.

1 R Functions for Creating Chapter 3 Figures

```
xlab="Weight (kg)", main="", freq=FALSE, xpd=TRUE)
title(main="B: Density histogram", adj=0, line=1.5, cex.main=1.05)
par(xpd=FALSE)
hist(x, xlim=c(2,3), xlab="Weight (kg)",
     main="", freq=FALSE)
axis(1)
lines(density(x), xlab="Weight (kg)", col="gray40")
title(main="C: Histogram, density is overlaid", adj=0, line=1.5,
      cex.main=1.05)
plot(density(x), xlim=c(2,3), xlab="Height (cm)",
     main="", sub="", bty="1", type="1")
av \leftarrow mean(x)
sdev \leftarrow sd(x)
xval <- pretty(c(2,3), n=40)</pre>
lines(xval, dnorm(xval, mean=av, sd=sdev), lty=2, col="gray40")
title(main="D: Density curve estimate", adj=0, line=1.5, cex.main=1.05)
par(opar)
par(mfrow=c(1,1))
```

```
fig3.3 <-
function (x=fCatBwt, plotit=TRUE){
    av <- mean(x); sdev <- sd(x); sampsize <- length(x)</pre>
    simmat <- cbind(x, matrix(rnorm(sampsize*5, mean=av, sd=sdev),</pre>
                                  ncol=5)
    simdf <- as.data.frame(simmat)</pre>
    names(simdf) <- c("Source", paste("normal", 1:5, sep=""))</pre>
    simdf <- stack(simdf)</pre>
    names(simdf) <- c("height", "Sample")</pre>
    denplotSimple <- densityplot(~height, groups=Sample, data=simdf,</pre>
                                   xlab="Body weight (kg)")
    denplotn <- update(denplotSimple, scales=list(tck=0.5),</pre>
                        main=list(expression(plain("A: Simulation (Densities)")),
                        cex.title=0.9, x=0.05, just="left"),
                        par.settings=simpleTheme(lty=1:6))
    bwpltBasic <- bwplot(Sample ~ height, data=simdf,</pre>
                          xlab="Body weight (kg)",
                          auto.key=list(columns=3))
    bwplotn <- update(bwpltBasic, scales=list(tck=0.5),</pre>
                       main=list(expression(plain("B: Simulation (Boxplots)")),
                                  cex.title=0.9, x=0.05, just="left"))
    if(plotit){
        print(denplotn, position=c(0,0,0.5,1))
        print(bwplotn, position=c(0.5,0,1,1),newpage=FALSE)
```

```
invisible(list(denplotn, bwplotn))
fig3.4 <-
function (x=fCatBwt, plotit=TRUE)
    sampsize <- length(x)</pre>
    bootmat <- cbind(x, matrix(0, ncol=5, nrow=sampsize))</pre>
    for(i in 2:6) bootmat[,i] <- sample(x, replace=TRUE)</pre>
    colnames(bootmat) <- c("Source", paste("normal", 1:5, sep=""))</pre>
    bootdf <- stack(as.data.frame(bootmat))</pre>
    names(bootdf) <- c("height", "Sample")</pre>
    denplotSimple <- densityplot(~ height, groups=Sample, data=bootdf,</pre>
                                   xlab="Body weight (kg)")
    legendA <- expression(plain("A: Bootstrap (Densities)"))</pre>
    denplot <- update(denplotSimple, scales=list(tck=0.5),</pre>
                       main=list(legendA, x=0.05, just="left"), cex.title=0.9,
                       par.settings=simpleTheme(lty=1:6))
    bwpltBasic <- bwplot(Sample ~ height, data=bootdf,</pre>
                          xlab="Body weight (kg)",
                          auto.key=list(columns=3))
    legendB <- expression(plain("B: Bootstrap (Boxplots)"))</pre>
    bwplot <- update(bwpltBasic, scales=list(tck=0.5),</pre>
                      main=list(legendB, x=0.05, just="left"), cex.title=0.9)
    if(plotit){
        print(denplot, position=c(0,0,0.5,1))
        print(bwplot, position=c(0.5,0,1,1),newpage=FALSE)
    invisible(list(denplot, bwplot))
fig3.5 <-
function ()
    opar \leftarrow par(mgp=c(2,.75,0), mfrow=c(1,2))
    curve(dnorm(x), from = -3, to = 3,
          ylab=expression("dnorm("*italic(x)*")"),
          xlab=expression("Normal deviate "*italic(x)))
    curve(pnorm(x), from = -3, to = 3,
          ylab=expression("pnorm("*italic(x)*")"),
          xlab=expression("Normal deviate "*italic(x)))
    par(opar)
```

```
function (){
    heights <- na.omit(subset(survey, Sex=="Female")$Height)
    plot(density(heights), bty="l", main="",
         cex.axis=1.15, cex.lab=1.15)
    av <- mean(heights); sdev <- sd(heights)</pre>
    abline(v=c(av-sdev, av, av+sdev), col="gray", lty=c(2,1,2))
    ## Show fitted normal curve
    xval <- pretty(heights, n=40)</pre>
    normal_den <- dnorm(xval, mean=av, sd=sdev)</pre>
    lines(xval, normal_den, col="gray40", lty=2)
    ytop \leftarrow par() usr[4] - 0.25 * par() cxy[2]
    text(c(av-sdev, av+sdev), ytop,
         labels=c("mean-SD", "mean+SD"), col="gray40", xpd=TRUE)
fig3.7 <-
function (wts=fCatBwt){
    opar <- par(pty="s")</pre>
    qqnorm(wts)
    par(opar)
fig3.8 <-
function (wts=fCatBwt)
    opar \leftarrow par(mfrow=c(1,2), mar=c(2.1, 3.6, 3.6, 2.6),
                mgp=c(2.25, 0.5,0))
    av <- numeric(1000)
    for (i in 1:1000)
        av[i] \leftarrow mean(rnorm(47, mean=2.36, sd=0.27))
    avdens <- density(av)</pre>
    xval \leftarrow pretty(c(2.36-3*0.27, 2.36+3*0.27), 50)
    den <- dnorm(xval, mean=2.36, sd=0.27)</pre>
    plot(xval, den, type="1", xlab="", xlim=c(1.5, 3.75),
         ylab="Density", ylim=c(0,max(avdens$y)),
         col="gray", lwd=2, lty=2)
    lines(avdens)
    mtext(side=3, line=0.75, "A: Simulation (from a normal distribution)",
          adj=0)
    legend("bottomright",
           legend=c("Source", "Sampling\ndistribution\nof mean"),
           col=c("gray", "black"), lty=c(2,1), lwd=c(2,1), bty="n",
```

fig3.6 <-

y.intersp=0.75, inset=c(0,0.2),

```
cex=0.8)
    av <- numeric(1000)</pre>
    for (i in 1:1000)
        av[i] <- mean(sample(wts, size=length(wts), replace=TRUE))</pre>
    avdens <- density(av)</pre>
    plot(density(wts), ylim=c(0, max(avdens$y)),
         xlab="", ylab="Density", xlim=c(1.5, 3.75),
         col="gray", lwd=2, lty=2, main="")
    lines(avdens)
    mtext(side=3, line=0.75,
          "B: Bootstrap samples (resample sample)", adj=0)
    legend("bottomright",
           legend=c("Source",
                       "Sampling\ndistribution\nof mean"),
           col=c("gray", "black"), lty=c(2,1), lwd=c(2,1), bty="n",
           y.intersp=0.75, inset=c(0,0.2),
           cex=0.8)
    par(opar)
    par(mfrow=c(1,1))
fig3.9 <-
function ()
    xleft <- 0:3; xrt <- 1:4
    ybot \leftarrow \text{rep}(0,4); ytop \leftarrow \text{rep}(1,4) - 0.05
    opar <- par(mar=rep(0.1,4))
    plot(c(0,5), c(-1,4), xlab="", ylab="", axes=F, type="n")
    for(i in 0:3){
        i1 <- i+1
        rect(xleft, ybot+i, xrt, ytop+i)
        xli <- xleft[i+1]; xri <- xrt[i+1];</pre>
        yboti <- (ybot+i)[i+1]; ytopi <- (ytop+i)[i+1]</pre>
        rect(xli, yboti, xri, ytopi, col="gray80")
        text(0.5*(xli+xri), 0.5*(yboti+ytopi), "TEST")
        text(0.5*(xleft[-i1]+xrt[-i1]), 0.5*(ybot[-i1]+ytop[-i1])+i, "Training")
        text(4+strwidth("TE"), i+0.475, paste("Fold", i1), adj=0)
fig3.10 <-
function (dset=cuckoos, plotit=TRUE)
```

```
parset1 <- lattice::simpleTheme(pch=1:6, alpha=0.8)</pre>
    plt1 <- lattice::xyplot(length ~ breadth, groups=species, data=dset,</pre>
                    par.settings=parset1, aspect=1,
                    scales=list(tck=0.5),
                    auto.key=list(columns=2, alpha=1),
                    main=grid::textGrob("A:", x=unit(.025, "npc"),
                    y = unit(.25, "npc"), just="left",
                    gp=gpar(cex=1))
    Species <- factor(c(rep("other", 5), "wren")[unclass(cuckoos$species)])</pre>
    parset2 <- lattice::simpleTheme(pch=c(0,6), alpha=0.8,</pre>
                            col=trellis.par.get()$superpose.symbol$col[c(7,6)])
    plt2 <- lattice::xyplot(length ~ breadth, groups=Species, data=dset,</pre>
                    par.settings=parset2,
                    aspect=1, ylab="", scales=list(tck=0.25),
                    auto.key=list(columns=1, alpha=1),
                    main=grid::textGrob("B:", x=unit(.05, "npc"),
                    y = unit(.25, "npc"), just="left",
                    gp=grid::gpar(cex=1))
    plt2 <- update(plt2,</pre>
                    par.settings=list(layout.heights=list(key.top=1.5)))
    if(plotit){
        print(plt1, position=c(0,0,0.515,1))
        print(plt2, position=c(0.485,0,1,1), newpage=FALSE)
    invisible(list(plt1, plt2))
fig3.11 <-
function (dset=cuckoos)
    parset <- list(dot.symbol=list(pch=1, alpha=0.6))</pre>
    dotwren <- dotplot(species %in% "wren" ~ length, data=dset,</pre>
                        scales=list(y=list(labels=c("Other", "Wren"))),
                        par.settings=parset, xlab="Length (mm)")
    dotwren
fig3.12 <-
function(dset=cuckoos)
    avdiff <- numeric(100)</pre>
```

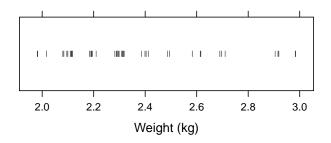
```
for(i in 1:100){
        avs <- with(dset, sapply(split(length, species %in% "wren"),</pre>
                                      function(x)mean(sample(x, replace=TRUE))))
        avdiff[i] <- avs[1] - avs[2] # FALSE (non-wren) minus TRUE (wren)</pre>
    xtxt <- paste("Means of bootstrap samples of length difference,\n",
                   "non-wren - wren (mm)")
    dotdiff <- dotplot(~ avdiff, xlab=xtxt,</pre>
                        par.settings=list(dot.symbol=list(pch=1, alpha=0.6)))
    dotdiff
fig3.13 <-
function (dset=mcats)
    xyplot(Hwt ~ Bwt, data=dset,
           type=c("p","r"))
fig3.14 <-
function(dset=mcats)
    mcats.lm <- lm(Hwt ~ Bwt, data=dset)</pre>
    res <- resid(mcats.lm)</pre>
    plot(density(res), main="")
    rug(res, col="gray")
fig3.15 <-
function(dset=mcats, nrepeats=100)
    bootmat <- bootreg(formula = Hwt ~ Bwt,</pre>
                        data = dset,
                        nboot = nrepeats)
    bootdf <- as.data.frame(bootmat)</pre>
    names(bootdf) <- c("Intercept", "Slope")</pre>
    colr <- adjustcolor(rep("black",3),</pre>
                         alpha.f=0.25)
    if(packageVersion('car') < '3.0.0'){</pre>
    scatterplot(Slope ~ Intercept, col=colr,
                 data=bootdf, boxplots="xy",
                 reg.line=NA, smooth=FALSE) } else
```

```
fig3.16 <-
function (dset=mcats, plotit=TRUE, nrepeats=100)
    bootmat <- bootreg(formula = Hwt ~ Bwt,</pre>
                        data = dset[-97, ],
                        nboot = nrepeats)
    bootdf0 <- as.data.frame(bootmat)</pre>
    names(bootdf0) <- c("Intercept", "Slope")</pre>
    gphA <- xyplot(Slope ~ Intercept, data=bootdf0, alpha=0.25,</pre>
                    main=paste("A:", nrepeats, "bootstrap samples"),
                    cex.title=1.1)
        simmat <- simreg(formula = Hwt ~ Bwt,</pre>
                           data=dset[-97, ], nsim=nrepeats)
    simdf <- as.data.frame(simmat)</pre>
    names(simdf) <- c("Intercept", "Slope")</pre>
    gphB <- xyplot(Slope ~ Intercept, data=simdf, alpha=0.25,</pre>
                    main=paste("B:", nrepeats, "simulations"),
                    cex.title=1.1)
    if(plotit){
        print(gphA, position=c(0,0,0.515,1))
        print(gphB, position=c(0.485,0,1,1), newpage=FALSE)
    invisible(list(gphA, gphB))
```

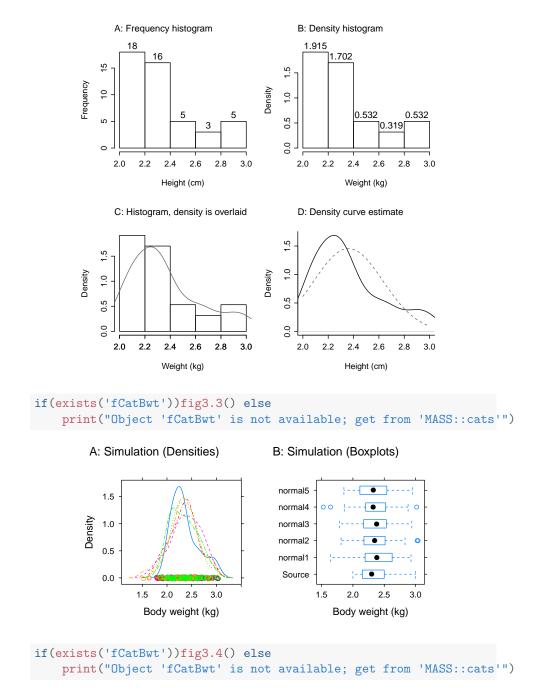
2 Show the Figures

```
msg <- "Data object 'cats', from 'MASS', may not be available"
if(!requireNamespace("MASS"))print(msg) else {
  mcats <- subset(MASS::cats, Sex=="M")
  fcats <- subset(MASS::cats, Sex=="F")
  fCatBwt <- na.omit(fcats[, "Bwt"])
}
if(!exists("cuckoos")){
      msg <- "Cannot find either 'cuckoos' or 'DAAG::cuckoos',"
if(requireNamespace("DAAG"))cuckoos <- DAAG::cuckoos else
      print(msg)
  }</pre>
```

```
if(exists('fCatBwt'))fig3.1() else
    print("Object 'fCatBwt' is not available; get from 'MASS::cats'")
```



```
if(exists('fCatBwt'))fig3.2() else
    print("Object 'fCatBwt' is not available; get from 'MASS::cats'")
```



A: Bootstrap (Densities)

2.0 2.0 1.5 0.5 0.0 2.0 2.5 3.0 Body weight (kg)

B: Bootstrap (Boxplots)

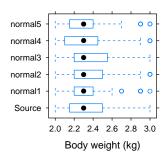
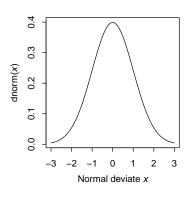


fig3.5()



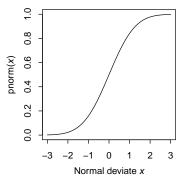
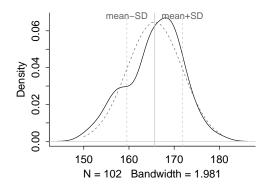
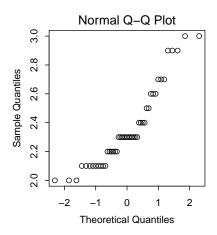


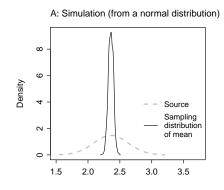
fig3.6()



```
if(exists("fCatBwt"))fig3.7() else
    print("Object 'fCatBwt' was not found; get from 'MASS::cats'")
```



```
if(exists("fCatBwt"))fig3.8() else
    print("Object 'fCatBwt' was not found; get from 'MASS::cats'")
```



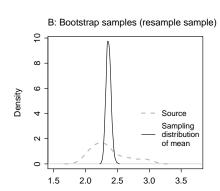
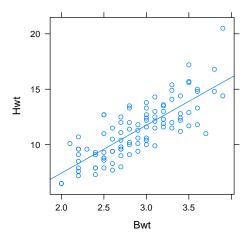


fig3.9()

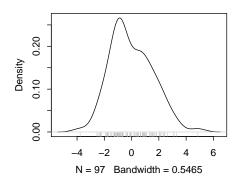
Training	Training	Training	TEST	Fold 4
Training	Training	TEST	Training	Fold 3
Training	TEST	Training	Training	Fold 2
TEST	Training	Training	Training	Fold 1

```
if(exists("cuckoos"))fig3.10() else
    print("Object 'cuckoos' was not found; get 'DAAG::cuckoos'")
                                          B:
    A:
     hedge.sparrow
meadow.pipit
                            robin
                                                       other \ \square
                            tree.pipit
                                                       wren
     pied.wagtail
                            wren
       25
                                            25
                                                                    24
       24
       23
                                            23
    length
                                            22
       22
       21
                                            21
       20
                                            20
                                               15.0 15.5 16.0 16.5 17.0 17.5
           15.0 15.5 16.0 16.5 17.0 17.5
                   breadth
                                                       breadth
if(exists("cuckoos"))fig3.11() else
    print("Object 'cuckoos' was not found; get 'DAAG::cuckoos'")
                  Wren
                  Other
                                    21
                                      22
                                                        25
                          20
                                            23
                                                  24
                                    Length (mm)
if(exists("cuckoos"))fig3.12() else
    print("Object 'cuckoos' was not found; get 'DAAG::cuckoos'")
                      1.0
                            1.2
                                  1.4
                                        1.6
                                              1.8
                                                   2.0
                                                         2.2
                    Means of bootstrap samples of length difference,
                              non-wren - wren (mm)
```

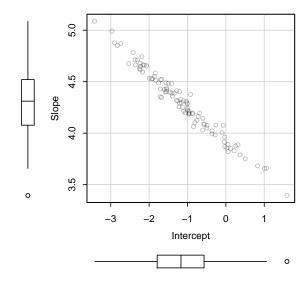
```
if(exists('mcats'))fig3.13() else
    print("Object 'mcats' was not found; subset from 'MASS::cats'")
```



```
if(exists('mcats'))fig3.14() else
    print("Object 'mcats' was not found; subset from 'MASS::cats'")
```



```
if(!require(car)){
    print("Figure 3.15 requires the 'car' package")
    return("The 'car' package needs to be installed.")
}
if(exists("mcats"))fig3.15(nrepeats=100) else
    print("Object 'mcats' was not found; subset from 'MASS::cats'")
```



if(exists('mcats'))fig3.16() else
 print("Object 'mcats' was not found; subset from 'MASS::cats'")

