

$$\forall x \exists \underline{z} (f(x) = c \wedge \neg (g(x, \underline{z}) = d))$$

$$\models \forall x (f(x) = c \wedge \neg (g(x, h(x)) = d))$$

$$\forall y \exists z \exists u (P(\underline{u}, y) \rightarrow Q(y, \underline{z}))$$

$$\forall y \exists u (P(u, y) \rightarrow Q(y, f(y)))$$

$$\forall y (P(g(y), y) \rightarrow Q(y, f(y)))$$

$$\exists x \forall u \forall y \exists z (P(x, u) \vee \neg (S(y) \rightarrow R(z)))$$

$$\forall u \forall y \exists z (P(c, u) \vee \neg (S(y) \rightarrow R(z)))$$

$$\forall u \forall y (P(c, u) \vee \neg (S(y) \rightarrow R(h(u, y))))$$

$$\forall z \forall x \exists u \forall v (Q(x, z) \vee R(x)) \rightarrow R(u) \vee \neg Q(v, u)$$

$$\forall z \forall x \forall v (Q(x, z) \vee R(x)) \rightarrow R(f(z, x)) \vee \neg Q(v, f(z, x))$$

$$\vdash \phi \rightarrow (\neg \phi \rightarrow \psi)$$

$$\begin{array}{c} \vdash \phi \rightarrow (\neg \phi \rightarrow \psi) \\ \hline \text{I.D. } \{ \phi \} \vdash \neg \phi \rightarrow \psi \quad \text{I.D. } \{ \phi, \neg \phi \} \vdash \psi \end{array}$$

$$1) \{ \phi, \neg \phi \} \vdash \phi$$

$$2) \{ \phi, \neg \phi \} \vdash \neg \phi$$

$$3) \{ \phi, \neg \phi \} \vdash \neg \phi \rightarrow (\neg \psi \rightarrow \neg \phi) \quad A1$$

$$4) \{ \phi, \neg \phi \} \vdash \neg \psi \rightarrow \neg \phi \quad \text{MP}(2,3)$$

$$5) \{ \phi, \neg \phi \} \vdash (\neg \psi \rightarrow \neg \phi) \rightarrow (\phi \rightarrow \psi)$$

$$6) \{ \phi, \neg \phi \} \vdash \phi \rightarrow \psi \quad \text{MP. 5}$$

$$7) \{ \phi, \neg \phi \} \vdash \psi \quad (\text{MP. 6.1})$$

$$8) \text{I.D.} \dots \text{I.D.}$$

$$\vdash \phi \rightarrow (\neg \phi \rightarrow \psi)$$