

$$\text{Prem: } \Pi \cup \{\neg\psi\} \vdash \neg(\psi \rightarrow \psi) \Rightarrow \psi$$

$$(1) \Pi \vdash \neg\psi \Rightarrow \neg(\psi \rightarrow \psi) \quad \text{T.D.}$$

$$(2) \Pi \vdash (\neg\psi \Rightarrow \neg(\psi \rightarrow \psi)) \rightarrow ((\psi \rightarrow \psi) \rightarrow \psi) \quad (A3)$$

$$(3) \Pi \vdash (\psi \rightarrow \psi) \rightarrow \psi$$

MP. (1), (2)

$$(4) \Pi \vdash \psi \rightarrow \psi$$

2.45

$$(5) \Pi \vdash \psi$$

$$\begin{aligned} & \text{Vrem} \quad \vdash (\varphi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi)) \text{ T.D.} \\ \Leftrightarrow & \{(\varphi \rightarrow \psi)\} \vdash (\psi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi) \Leftrightarrow \text{T.D.} \\ \Leftrightarrow & \underbrace{\{(\varphi \rightarrow \psi), (\psi \rightarrow \chi), \varphi\}}_{\text{TP}} \vdash \chi \text{ T.D.} \end{aligned}$$

$$\begin{array}{l|l} \begin{array}{l} (1) \text{TP} \vdash \varphi \rightarrow \psi \\ (2) \text{TP} \vdash \varphi \\ (3) \text{TP} \vdash \psi \end{array} & \begin{array}{l} \text{MP} \\ (1)(2) \end{array} \end{array} \quad \begin{array}{l} (4) \text{TP} \vdash \psi \rightarrow \chi \\ (5) \text{TP} \vdash \chi \end{array} \quad \text{MP (3)(4)}$$

Thm:  $\vdash \varphi \rightarrow \varphi$  • 2.45

$$(1) \quad \vdash \overline{\varphi} \rightarrow ((\varphi \rightarrow \overline{\varphi}) \rightarrow \overline{\varphi}) \rightarrow ((\varphi \rightarrow (\varphi \rightarrow \varphi)) \rightarrow (\varphi \rightarrow \varphi))$$

$$(2) \quad \vdash \varphi \rightarrow ((\varphi \rightarrow \varphi) \rightarrow \varphi) \quad (A1) \quad (A2)$$

$$(3) \quad \vdash (\varphi \rightarrow (\varphi \rightarrow \varphi)) \rightarrow (\varphi \rightarrow \varphi) \quad \text{MP. (1) (2)}$$

$$(4) \quad \vdash \varphi \rightarrow (\varphi \rightarrow \varphi) \quad A1$$

$$(5) \quad \vdash \varphi \rightarrow \varphi \quad \text{M.P. (3) (4)}$$