Vrem: $\Gamma \cup \{\neg\psi\} \vdash \neg(\varphi \to \varphi) \implies \psi$

(1) $\Gamma \vdash \neg\psi \to \neg(\varphi \to \varphi)$ T.D.

(2) $\Gamma \vdash (\neg\psi \to \neg(\varphi \to \varphi)) \to ((\varphi \to \varphi) \to \psi)$ (A3)

(3) $\Gamma \vdash (\varphi \to \varphi) \to \psi$ MP. (1),(2)

(4) $\Gamma \vdash \varphi \to \varphi$ 2.45

(5) $\Gamma \vdash \psi$

Vroum $\vdash (\varphi \to \psi) \to ((\psi \to \chi) \to (\varphi \to \chi))$ T.D.

$(\Leftarrow)$ $\{(\varphi \to \psi)\} \vdash (\psi \to \chi) \to (\varphi \to \chi) \Leftarrow$ T.D.

$(\Leftarrow)$ $\underbrace{\{(\varphi \to \psi), (\psi \to \chi), \varphi\}}_{\Gamma} \vdash \chi$ T.D.

(1) $\Gamma \vdash \varphi \to \psi$

(2) $\Gamma \vdash \varphi$

(3) $\Gamma \vdash \psi$

$\left. \begin{array}{l} \end{array} \right\}$ MP

(1) R)

(4) $\Gamma \vdash \psi \to \chi$

(5) $\Gamma \vdash \chi$

MP (3)(4)

⓶

Theorem: $\vdash \varphi \to \varphi$. $\boxed{2.45}$ $\qquad\qquad \varphi \to (\psi \to \chi) \to ((\varphi \to \psi) \to (\varphi \to \chi))$

(1) $\vdash \overset{1}{\varphi} \to \left[(\varphi \overset{2}{\to} \varphi) \to \overset{1}{\varphi} \right] \to \left[(\varphi \to (\varphi \to \varphi)) \to (\varphi \to \varphi) \right]$ (A2)

(2) $\vdash \varphi \to \left((\varphi \to \varphi) \to \varphi \right)$ (A1)

(3) $\vdash (\varphi \to (\varphi \to \varphi)) \to (\varphi \to \varphi)$ M.P. (1)(2)

(4) $\vdash \varphi \to (\varphi \to \varphi)$ A1

(5) $\vdash \varphi \to \varphi$ M.P. (3)(4)