The Beauty and Joy of Computing

Lecture #7
Algorithms II

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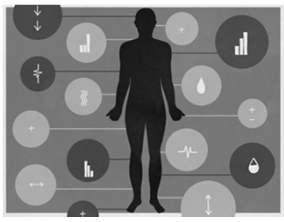
PAGING DOCTOR ELIZA

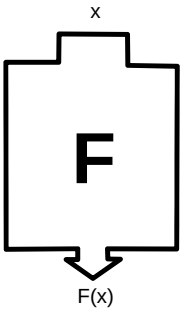


Apple enters the medical app field while Aetna pulls out. Fitbits are useful but only offer diagnostics. AI diagnosis needs more data for accuracy. There are definitely significant hurdles for CS in the medical field, but they are being tackled as we speak.

`http://www.nytimes.com/2014/09/21/sunday-review/high-tech-health-care-useful-to-a-point.html?_r=0`

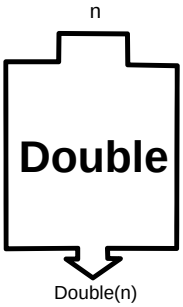
Functional Abstraction (review)

- A block, or function has inputs & outputs
  - Possibly no inputs
  - Possibly no outputs (if block is a command)
    - In this case, it would have a "side effect", i.e., what it does (e.g., move a robot)
- The contract describing what that block does is called a specification or spec



What is IN a spec? (review)

- Typically they all have
  - `NAME`
  - `INPUT(s)`
    - (and types, if appropriate)
    - Requirements
  - `OUTPUT`
    - Can write "none"
  - `(SIDE-EFFECTS)`
  - `EXAMPLE CALLS`
- Example
  - `NAME` : **Double**
  - `INPUT` : **n** (a number)
  - `OUTPUT`: **n + n**



What is NOT IN a spec?

- How!
  - That's the beauty of a functional abstraction; it doesn't say how it will do its job.
- Example: Double
  - Could be n * 2
  - Could be n + n
  - Could be n+1 (n times)
    - if n is a positive integer
- This gives great freedom to author!
  - <u>You</u> choose Algorithm(s)!

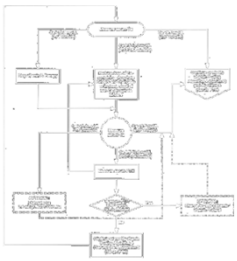
What do YOU think?

Which factor below is the most important in choosing the algorithm to use?

A. Simplest?
B. Easiest to implement?
C. Takes less time?
D. Uses up less space (memory)?
E. Gives a more precise answer?

Algorithm analysis : the basics

- An algorithm is correct if, for every input, it reports the correct output <u>and</u> doesn't run forever or cause an error.
  - Incorrect algorithms may run forever, or may crash, or may not return the correct answer.
    - They could still be useful!
    - Consider an approximation…
  - For now, we'll only consider <u>correct</u> algorithms

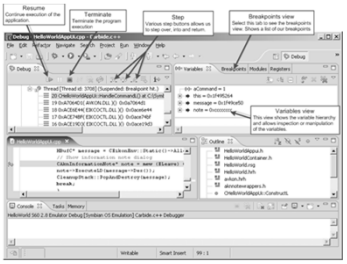


Algorithm for managing Vitamin D sterols based on serum calcium levels.

www.kidney.org/professionals/kdoqi/guidelines_bone/guide8b.htm

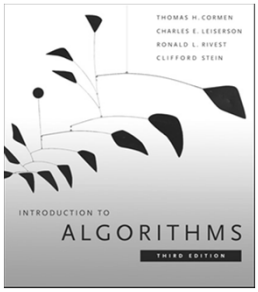
How do you know if "it" is correct?

- Mathematical proof for algorithms
- Empirical verification through testing of programs:
  - Unit Testing
  - Debugging
- Can get a mathematical proof for within a certain bound of the answer.



Reference text

- This book launched a generation of CS students into Algorithm Analysis
  - It's on everyone's shelf
  - It might be hard to parse at this point, but if you go on in CS, remember it & own it!
    - …but get the most recent year's

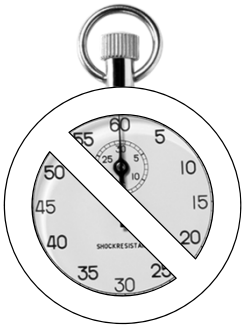


Algorithm analysis : running time

- One commonly used criterion in making a decision is **running time**
  - how long does the algorithm take to run and finish its task?
- How do we measure it?

Runtime analysis problem & solution

- Time w/stopwatch, but…
  - Different computers may have different runtimes. ☹
  - Same computer may have different runtime on the <u>same</u> input. ☹
  - Need to implement the algorithm first to run it. ☹
- *Solution*: Count the number of "steps" involved, not time!
  - Each operation = 1 step
  - *If we say "running time", we'll mean # of steps, not time!*



Runtime analysis : input size & efficiency

- Definition
  - Input size: the # of things in the input.
  - E.g., # of things in a list
  - Running time as a function of input size
  - Measures efficiency
- Important!
  - In CS10 <u>we won't care</u> about the efficiency of your solutions!
  - …in CS61B we will

CS10

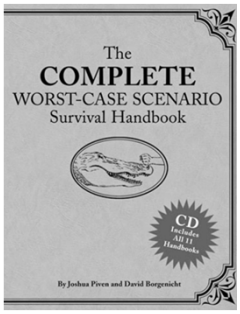
CS61A

CS61B

CS61C

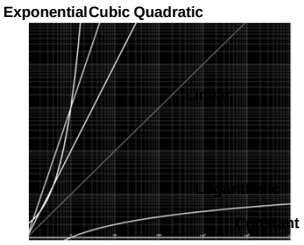
Runtime analysis : worst or avg case?

- Could use avg case
  - Average running time over a vast # of inputs
- Instead: use worst case
  - Consider running time as input grows
- Why?
  - Nice to know most time we'd <u>ever</u> spend
  - Worst case happens often
  - Avg is often ~ worst



Runtime analysis: Final abstraction

- Instead of an exact number of operations we'll use abstraction
  - Want order of growth, or dominant term
- In CS10 we'll consider
  - Constant
  - ~~Fractional Exponent~~
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential
- E.g. $10\ n^2 + 4 \log n + n$
  - …is quadratic



Graph of order of growth curves on log-log plot

Example: Finding a student (by ID)

- Input
  - Unsorted list of student IDs L
  - Particular student S
- Output
  - True if S is in L, else False
- Pseudocode Algorithm
  - Go through one by one, checking for match.
  - If match, true
  - If exhausted L and didn't find S, false

- Worst-case running time as function of the size of L?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Example: Finding a student (by ID)

- Input
  - Sorted list of students IDs L
  - Particular student S
- Output : same
- Pseudocode Algorithm
  - Start in middle
  - If match, report true
  - If ID is after the one you checked, throw away the top half of L and check middle again. Similar for if the name is before
  - If nobody left, report false

- Worst-case running time as function of the size of L?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Example: Finding a student (by ID)

- What if L were given to you in advance and you had infinite storage?
  - Could you do any better than logarithmic?
- Better yet, imagine instead of students you had n playing cards in a regular deck…
  - If you had a table could you presort your cards for single step lookup?

- Worst-case running time as function of the size of L?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

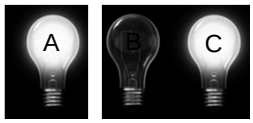
Example: Finding a shared birthday

- Input
  - Unsorted list L (of size n) of birthdays of team
- Output
  - True if any two people shared birthday, else False
- What's the worst-case running time?

- Worst-case running time as function of the size of L?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Example: Finding subsets

- Input:
  - n different light bulbs
- Output
  - All the subsets (i.e. on and off combinations of those bulbs)
- Worst-case running time? (as function of n)
- E.g., for 3 bulbs (a,b,c):
  - 1 empty: { }
  - 3 1-bulb: {a, b, c}
  - 3 2-bulb: {ab, bc, ac}
  - 1 3-bulb: {abc}



- Worst-case running time as function of the size of n?
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Limits

- We can prove mathematically that some algorithms **are never solvable!**
- We can (almost) prove mathematically that some algorithms **will never be efficient**!
  - Famous problem P = NP ?
  - Example: Travelling Salesman Problem
  - BUT: Can use heuristics for approximation

Summary

- When developing an algorithm, could optimize for
  - Simplest
  - Easiest to implement?
  - Most efficient
  - Uses up least resources
  - Gives most precision
  - …
- In CS10 we'll consider
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential

- There are empirical and formal methods to verify efficiency and correctness
- Some algorithms cannot be implemented efficiently