Variable Radii Connected Sensor Cover in Sensor Networks

Zongheng Zhou,† Samir Das, Himanshu Gupta
Department of Computer Science, SUNY, Stony Brook, NY 11794
Email: zzhou, samir, hgupta@cs.sunysb.edu

Abstract— **One of the useful approaches to exploit redundancy in a sensor network is to keep active only a small subset of sensors that are sufficient to cover the region required to be monitored. The set of active sensors should also form a connected communication graph, so that they can autonomously respond to application queries and/or tasks. Such a set of active sensors is known as a connected sensor cover, and the problem of selecting a minimum connected sensor cover has been well studied when the transmission radius and sensing radius of each sensor is fixed. In this article, we address the problem of selecting a minimum energy-cost connected sensor cover, when each sensor node can vary its sensing and transmission radius; larger sensing or transmission radius entails higher energy cost. For the above problem, we design various centralized and distributed algorithms, and compare their performance through extensive experiments. One of the designed centralized algorithms (called CGA) is shown to perform within an $O(\log n)$ factor of the optimal solution, where n is the size of the network. We have also designed a localized algorithm based on Voronoi diagrams which is empirically shown to perform very close to CGA, and due to its communication-efficiency results in significantly prolonging the network lifetime.**

I. Introduction

Wireless sensor networks are often deployed for passive gathering of sensor data in a geographical region. The "grand challenge" of sensor network design for data gathering activities is to maintain the fidelity of the gathered data while minimizing energy usage in the network. Energy is spent due to message transmissions by the radio interface, or due to the sensing activities by the signal processing electronics. Energy can be saved if these activities are used only to the extent absolutely needed, and no more.

Two important properties of a sensor network play critical roles in the design approach. They are *coverage* and *connectivity*. Loosely speaking, coverage describes how well sensors in the network can monitor a geographical region in question. This can include multiple parameters, such as whether every point in the region can be monitored by at least one sensor within a given confidence. The confidence typically depends on the physical distance of the point from the monitoring sensor, as distance weakens the signal and thus worsens the signal-to-noise ratio introducing measurement errors. In a simplified model, this confidence can be specified in terms of a *sensing range* [1]. Connectivity, on the other hand, simply describes the connectivity properties of the underlying network topology. It is often desirable that the network is connected. If the network is partitioned, the entire sensor network data cannot be gathered to a central decision-making node.

†Partially supported by NSF Grant ANI-0308631.

It is expected that in most deployment scenarios, it will be cost-effective to deploy the sensors randomly in a redundant fashion ([2], [3]). The sensor hardware is cheap, relative to the logistics or opportunity cost of deployment. Thus, it is useful to deploy the sensors redundantly, and employ sophisticated protocol support so that only a "minimally sufficient subset" of the sensors is actually active at a time – thus conserving energy and prolonging the sensor network lifetime. Also, in many scenarios the logistics for designed placement of sensor nodes at specific geographical locations will be very complex. Thus, in these scenarios, random deployment is the only feasible method. This means that the "minimally sufficient subset" cannot be pre-determined. The sensor nodes must be able to compute this on-line, by executing appropriate algorithms.

In this paper, our goal is to investigate such algorithms for energy efficient connectivity and coverage. We investigate the situation where both sensing and transmission range can be varied in the sensors. This uncovers an interesting design problem, where a minimally sufficient subset of sensors must be selected along with the assignment of sensing and transmission ranges for individual sensors, such that both coverage and connectivity are guaranteed with a *minimum total* energy cost. The assumption here is that the energy cost for an individual sensor increases with higher sensing range or transmission range. This is because with a larger sensing range, more energy is needed for appropriate filtering and signal processing methods to improve the signal-to-noise ratio in order to achieve the desired confidence level. Similarly, with a larger transmission range, transmission power is to be increased to reach larger distances. It is expected that with sophisticated sensors that can control their sensing and transmission ranges, the overall energy budget of the network can be reduced relative to the case where sensors have fixed sensing and transmission ranges. Note that a similar problem has been investigated in literature by varying transmission ranges of sensor nodes for minimum energy topology construction in wireless ad hoc networks ([4], [5], [6], [7]); however, this line of work does not involve any notion of sensing range.

The rest of the paper is organized as follows. In the next section, we review the related work on energy efficient connectivity and coverage problem. Then, in section III, we describe our sensor network model and give the formal definition of the variable radii connectivity and coverage problem. We start with presenting a fully localized algorithm based on Voronoi diagram and relative neighbor graph in Section IV. In Section V, we present centralized and distributed greedy algorithms, and in Section VI, we present another centralized

Steiner tree based algorithm. Section VII presents simulation results.

II. Related Work

Connectivity is a fundamental issue in wireless ad hoc environment, and many schemes have been addressed to conserve energy while maintaining connectivity in the network topology. One of the most related problem in the above context is the minimum connected dominating set problem [8]. The work in wireless network research community ([9], [10], [11], [12], [13], [14], [15]) has primarily focussed on developing energy-efficient distributed algorithms to construct a near-optimal connected dominating set. All the above works assume fixed transmission range for each sensor node. The works in [4], [5], [6], [7] a ddress the related NP-complete problem of constructing a minimum energy broadcast tree in a network, where every node can adjust its transmission power/range. None of the above described works involve any notion of sensing range or coverage.

Recently, there has been a lot of research done to address the coverage problem in sensor networks. In particular, the authors in [16] design a centralized heuristic to select mutually exclusive sensor covers that independently cover the network region. In [1], the authors investigate linear programming techniques to optimally place a set of sensors on a sensor field (three dimensional grid) for a complete coverage of the field. Meguerdichian et al. ([17], [18]) consider a slightly different definition of coverage and address the problem of finding maximal paths of lowest and highest observabilities in a sensor network.

Recently, researchers have also considered connectivity and coverage in an integrated platform. In particular, the authors in [19] consider an unreliable sensor network, and derive necessary and sufficient conditions for the coverage of the region and connectivity of the network with high probability. The PEAS protocol [2] considers a probing technique that maintains only a necessary set of sensors in working mode to ensure coverage and connectivity with high probability under certain assumptions. Wang et al. [3] present a localized heuristic in which they use the SPAN [20] protocol to maintain connectivity, and a seperate CCP protocol to maintain coverage. In our prior work [21], we designed a greedy approximation algorithm that delivers a connected sensor cover within a $O(\lg n)$ factor of the optimal solution. While prior works on connected sensor coverage have only considered a nodes with fixed radii, in this article, we consider the network model where each sensor has the ability to control its transmission and sensing power/radii.

III. Problem Formulation

In this section, we motivate and formulate the variable radii connected sensor cover problem addressed in this paper. We start with describing the sensor network model used in this paper.

A sensor network consists of a large number of sensors distributed randomly in a geographical region. Each sensor I has a unique ID, and is associated with a maximum sensing radius S^* and a maximum transmission radius T^*. We assume that the maximum radii associated are same for all the sensors in the network.[1] Each sensor I also chooses (or, is assigned) a sensing radius $S(I)$ ($< S^*$) and a transmission radius $T(I)$ ($< T^*$), such that it is capable of sensing up to a distance of $S(I)$ and can communicate directly with sensors that are within a distance of $T(I)$ units. The *assigned sensing region* $\theta(I)$ associated with a sensor I is a disk of radius $S(I)$ centered at the location of sensor I. Throughout this article, we use $d(x, y)$ is used to denote the euclidean distance between points x and y.

The variable radii connected sensor cover (VRCSC) problem in the above described sensor model can be informally stated as follows. Given a sensor network and a query region, select a subset of sensors with specified sensing and transmission radii, such that (a) each point in the query region can be sensed by at least one of the selected sensors, and (b) the selected sensors form a connected communication graph using their assigned transmission radii (considering only bidirectional link). Our goal is to minimize the total energy cost of the selected sensors, i.e., the sum of the sensing and communication energy costs of all the selected sensor nodes. Essentially, for a given query region in a sensor network, we wish to select a subset of sensors to be powered ON and assign them sensing and transmission radii, such that the given query region is covered and the selected set of sensors form a connected communication graph. The query region can also be thought of as a surveillance region that needs to be monitored by the sensor network.

Motivation for Variable Radii. Energy is a critical resource in sensor networks. One of the key characteristics in wireless communication is that the energy consumption increases with the transmission distance. Thus, it is often assumed that a wireless device can change its transmission range to save energy [5] [7] [4]. In conventional sensor design, the energy spent in sensing has an inverse relationship with the amount of signal energy received by the sensor. This is because, if the signal energy is weak, the signal to noise ratio needs to be suitably improved for reliable detection via appropriate signal processing methods.[2] Note also that the signal energy decays with distance of the sensor from the signal source according to an inverse power law. Thus, it is fair to model the energy spent in sensing as an increasing function of a power of the sensing radius. The same model is also used in [22].

We now formally define the variable radii connected sensor cover (VRCSC) problem. We start with a few definitions.

Definition 1: (Energy Cost) Consider a sensor I with an assigned sensing radius of $S(I)$ and a transmission radius of $T(I)$. We model the energy cost of I as $E(I) = f(S(I)) +$

[1]This assumption is needed only for the Voronoi based approach presented in Section IV.

[2]The actual relationship between the energy spent in sensing and signal energy incident on the sensor cannot be easily generalized, as it is dependent on the sensor technology and electronic circuitry for detection, but it is not important for our purposes.

$g(T(I)) + C$, where $f(x)$ and $g(x)$ are monotonically non-decreasing functions in x, and C is a constant that represents the idle-state energy cost. □

Definition 2: ((Full) Communication Graph) Given a set of sensors M in a sensor network, the *communication graph* of M is a graph with M as the set of vertices and an edge between any two sensors if they can directly communicate with each other using their *assigned* transmission radii. The *full-communication graph* of a set $\mathcal{I}$ of sensors is the communication graph of $\mathcal{I}$ when each node in $\mathcal{I}$ is assigned the maximum transmission radius T^*. □

Definition 3: (Communication Distance) A path of nodes/sensors between I_i and I_j in the communication graph is called a *communication path* between the sensors I_i and I_j. The *communication distance* between two sensors I_i and I_j is the weight of the minimum node-weighted path between I_i and I_j in the communication graph, where the weight at an intermediate sensor node I is the transmission energy cost $g(T(I))$ of the sensor node. □

Definition 4: (Variable Radii Connected Sensor Cover) Consider a sensor network. Let S^* and T^* be the maximum sensing and transmission radius respectively. Given a query region R_Q in the network, a set of sensors $M = \{I_1, I_2, \ldots, I_m\}$ in the sensor network, where each sensor I_j is *assigned* a sensing radius $S(I_j)$ $(< S^*)$ and a transmission radius $T(I_j)$ $(< T^*)$, is said to be a *variable radii connected sensor cover* for the query region R_Q if the following two conditions hold:

1) $R_Q \subseteq \theta(I_1) \cup \ \theta(I_2) \cup \ \ldots \theta(I_m)$, where $\theta(I_j)$ is the sensing region of I_j, i.e., a circular region of radius $S(I_j)$ centered around the sensor I_j, and
2) the communication graph of M is connected.

A set of sensors that satisfies only the first condition is called a *variable radii sensor cover*. □

The variable radii connected sensor coverage problem of computing a minimum energy-cost variable radii connected sensor cover is NP-hard as the less general problem of connected sensor cover with fixed radii is known to be NP-hard [21].

IV. Voronoi Based Algorithm

In this section, we design a distributed algorithm for the variable radii connected sensor cover problem based on the computational geometric concepts of Voronoi diagram and Relative-Neighbor Graph (RNG). The developed algorithm is a localized algorithm in the sense that each sensor makes decisions based only upon local neighborhood information. In this section, each sensor node is assumed to be either active (powered on) or inactive. Below, we recall definitions of Voronoi Diagrams and Relative-Neighbor Graphs.

Definition 5: (Voronoi Diagram/Cell/Neighbor) Given n nodes in a plane, the *voronoi diagram* is defined as the partitioning of the plane into n convex polygons such that each polygon contains exactly one of the n nodes and every point in a given polygon is closer to its central node than to any other node [23]. The *voronoi cell* of a node is the convex polygon in the voronoi diagram that contains the node. Two nodes whose voronoi cells share a common edge are called *voronoi neighbors*. □

Definition 6: (Relative Neighbor Graph (RNG)) Given n nodes in a 2D plane, the *relative neighbor graph* is the graph where an edge exists between any two nodes u, v, *iff* there is no node w such that $d(u, w) < d(u, v)$ and $d(v, w) < d(u, v)$. It is well-known that the relative neighbor graph contains the minimum spanning tree of the euclidean graph over the given n nodes [6]. □

Definition 7: (l-hop Active Neighborhood) The l-hop active neighborhood of an active node I, denoted as $N(I, l)$, is defined as the set of active nodes that are at most at a distance of l hops from I in the unweighted full-communication graph of the entire sensor network. □

In our proposed localized algorithm, each sensor node I builds its voronoi cell based upon locations of nodes in $N(I, l)$. A low l can result in construction of inaccurate voronoi cells, since each sensor node has only limited (l-hop) information. However, a low value of l does not affect the correctness of our proposed algorithm. The constant l is chosen carefully – larger l results in better performance, but higher communication cost. For ease of presentation, we will assume that l is a constant in the rest of the discussion.

Definition 8: (Local Voronoi Cell/Neighbor) A *local voronoi cell* $LV(I)$ of a node I is a set of points p such that p is in the given query region and $d(p, I) \leq d(p, J)$ for all $J \in \ N(I, l)$. Note that local voronoi cells of a set of nodes in a 2D plane may not be disjoint. For a node I, the *size* of its local voronoi cell $LV(I)$ is the maximum distance of a point in $LV(I)$ from I.

A node J is a *local voronoi neighbor* of I if J is a voronoi neighbor of I in the voronoi diagram over the set of nodes $N(I, l)$. Note that the local voronoi neighbor relationship is not symmetric, i.e., I may not be a local voronoi neighbor of J even if J is a local voronoi neighbor of I. We use $LN(I)$ to denote the set of local voronoi neighbors of I. □

The following method of assignment of radii to a set of active sensor nodes in a sensor network forms the core of our voronoi based algorithm.

V-R Assignment of Radii. Consider a set of active sensors A in a sensor network. Let the set of sensor nodes whose maximum sensing region intersects with the given query region be M. The *V-R assignment* of sensing and transmission radii is defined as follows. Each sensor node I in M is assigned a sensing radius equal to the size of its local voronoi cell or the maximum sensing radius, whichever is smaller. Each sensor node I in M is assigned a transmission radius equal to the maximum distance over all its neighboring nodes in the RNG graph of M. All active nodes that are not in M are assigned zero sensing and transmission radius. The following theorem shows that the V-R assignment ensures coverage and connectivity of the query region.

Theorem 1: Given a set of active sensors A and a query region in a sensor networ, such that the query region is covered by the union of the maximum sensing regions of nodes in A, the V-R assignment of radii ensures coverage of the query region.

Let the set of active sensor nodes whose maximum sensing region intersects with the query region be M. If the full-communication graph of M is connected, then the V-R assignment of transmission radii ensures connectivity of M.

Proof: It is easy to see that $(V(I) \cap R_Q) \subseteq LV(I)$, where $V(I)$ is the voronoi cell of I and R_Q is the query region. Consider a point p in the query region, and let I_p be the active sensor node that is closest to p. Now, $p \in V(I_p)$ and hence, $p \in LV(I_p)$. Since p is covered by the maximum sensing region of at least one active sensor node, it is covered by the maximum sensing region of I_p, and hence, the assigned sensing region of I_p covers p.

As RNG is a superset of the minimum spanning tree, the V-R assignment ensures connectivity of M. ■

Voronoi Based Algorithm Description. The V-R assignment of sensing and transmission radii is key in the design of our voronoi based algorithm. Informally, the voronoi based algorithm works as follows. We start with all sensors in the network as active nodes, and use the V-R assignment method to assign their sensing and transmission radius. At each stage, certain sensor nodes become inactive, and the assignment of sensing and transmission radii is redone for the remaining active nodes. A sensor node is chosen to become inactive only if the remaining active sensors are capable of covering the query region and maintain connectivity of their communication graph. We use an appropriately defined concept of "benefit" to choose the best sensor nodes to become inactive. The algorithm terminates when no more sensors can be made inactive. In the end, the set of active sensor nodes form the desired VRCSC solution. Formally, our proposed Voronoi Based Algorithm consists of the following steps.

1) Initially, each sensor node in the sensor network is active, and gathers locations of all the nodes in the l-hop active neighborhood.
2) Each active sensor node computes its local voronoi cell, and the neighbors in the RNG over active nodes. It uses the V-R assignment method to assign itself a sensing and a transmission radius.
3) Each node I computes its *sleeping benefit* (formally defined later), which is the decrease in the total energy cost of the "local" active sensors if I is inactivated.
4) A sensor node I is considered *removable*, if it satisfies the following two conditions.
   - For every pair of communication neighbors of I, there exists a communication path P in the full-communication graph of $N(I,l)$, such that all the intermediate nodes in P have a higher node-ID than that of I. This condition ensures connectivity of active nodes, if I is made inactive [15].
   - The region $(LV(I) \cap \theta(I))$ is covered by the union of the maximum sensing regions of the local voronoi neighbors of I. We show in Theorem 2 that the above condition ensures coverage of the query region, if I is made inactive.
5) If I is removable and has the most *sleeping benefit* among all its local voronoi neighbors, then I becomes inactive.
6) Go to Step 2.

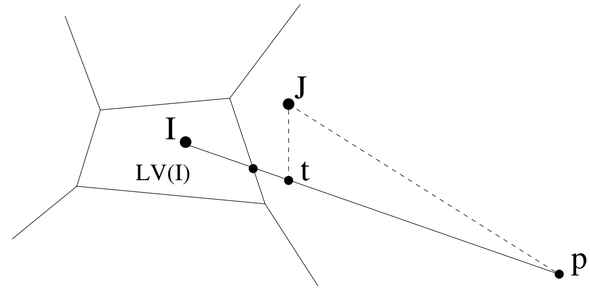


Fig. 1. Proof of Theorem 2

The above described algorithm can be easily implemented in a distributed setting, where the communication model is reliable. To ensure correctness in an unreliable communication model, we need to add certain tedious steps as discussed later. This completes the description of the algorithm.

Below, we show that the above described algorithm maintains coverage of the query region, if the query region was initially covered by the active sensors. We use $\theta^*(I)$ to represent the maximum sensing region (corresponding to the maximum sensing radius S^*) of I. Also, recall that $LN(I)$ is the set of local voronoi neighbors of I. We start with a lemma.

Lemma 1: Let I be an active sensor, and $\theta(I)$ be the sensing radius assigned by the V-R assignment method (step (2) of the Voronoi Based algorithm). If $LV(I) \cap \theta(I) \subseteq \bigcup_{j \in LN(I)} \theta^*(j)$, then $\theta^*(I) \subseteq \bigcup_{j \in LN(I)} \theta^*(j)$.

Proof: Consider an arbitrary point p in $\theta^*(I)$. We show that $p \in \bigcup_{j \in LN(I)} \theta^*(j)$. Let us consider two cases depending on whether $LV(I)$ contains p.

First, consider the case when $p \in LV(I)$. In V-R assignment of radii, either $LV(I) \subseteq \theta(I)$ or $\theta(I) = \theta^*(I)$. Thus, we have $p \in \theta(I)$. Hence, $p \in \bigcup_{j \in LN(I)} \theta^*(j)$.

Now, consider the case when $p \notin LV(I)$. As shown in Figure 1, there exists a point $t \notin LV(I)$ on the line segment $\overline{pI}$. Also, there is a sensor $J \in LN(I)$, such that $d(J,t) < d(I,t)$. Now,

$$\begin{aligned} d(J,p) &< d(t,p) + d(t,J) \\ &< d(t,p) + d(t,I) \\ &= d(p,I) \\ &< S^* \end{aligned}$$

Thus, $p \in \theta^*(J)$, and $p \in \bigcup_{j \in LN(I)} \theta^*(j)$. ■

Theorem 2: Given a set of active sensors A and a query region in a sensor network, such that the query region is covered by the union of the maximum sensing regions of nodes in A, the Voronoi Based algorithm ensures coverage of the query region.

Proof: We know by Theorem 1 that the initial V-R assignment ensures coverage of the query region. Below, we show that for any point p in the query region, there is an active sensor node K at any stage of the algorithm that cannot be inactivated.

Let $C(p)$ denote the set of active sensors that can cover a point p using their maximum sensing regions. Consider a point p in the query region such that $C(p) \neq \emptyset$. Let K be the sensor node with minimum sleeping benefit in $C(p)$. We show that the sensor node K will not be inactivated by the Voronoi Based algorithm. Let us assume the contrary that the sensor node K is inactivated, which means that $(LV(K) \cap \theta(K)) \subseteq \bigcup_{j \in LN(K)} \theta^*(j)$ and K's sleeping benefit is more than that of any sensor in $LN(K)$. From Lemma 1, we know that there exists a sensor $J \in LN(K)$ such that $p \in \theta^*(J)$. Thus, $J \in C(p)$ and J's sleeping benefit is less than that of K, which yields a contradiction. ■

Calculating Sleeping Benefit. The *sleeping benefit* $B(I)$ of an active node I is defined as the decrease in total energy cost of the set of active sensors in the networks due to inactivation of the node I. More precisely,

$$B(I) = E(I) - \sum_{J \in N(I)} (E_{new}(J) - E(J)),$$

where $E(X)$ is the current energy cost of a node X, $N(I)$ is the set of local neighbors (local voronoi neighbors union 1-hop communication neighbors) of I, and $E_{new}(X)$ is the new energy cost of a node X after inactivation of I. Each node I is aware of the current assignment of sensing and transmission radii (and hence, the energy cost) of all its local neighbors. Thus, to compute its sleeping benefit, a node I only needs to compute the increase in sensing and transmission radii of nodes in its local neighborhood.

Based on the V-R assignment, only the local voronoi neighbors of I need to increase their assigned sensing radius when I is inactivated. The local voronoi neighbors increase their sensing radii to cover the local voronoi cell $LV(I)$ of I, and the increase in sensing radius of a local voronoi neighbor can be computed using the polygon clipping method [24]. Note that only the nodes in $N(I, 1)$ may increase their transmission radius due to inactivation of I, and the increase in transmission energy cost of the nodes in $N(I, 1)$ can be easily computed by first constructing the induced subgraphs of RNG over $N(I, 1)$, with and without I.

Unreliable Communication Model. The above described algorithm needs to be augmented with certain handshake messages to ensure correctness in a communication model where message delays cannot be bounded. Below, we discuss the issues that arise in an unreliable communication model, and propose solutions to handle them.

The first problem in an unreliable communication model occurs if a node I doesn't have the updated benefit (which is sent in a message) of J, one of its local voronoi neighbors. In such a case, the second condition of removability could result in a cyclic condition in a distributed setting, and two *mutually* local voronoi neighbors I and J may both delete themselves and thus, possibly render the query region uncovered. To prevent such a scenario from happening we require the following. A sensor I that wishes to inactivate itself, sends an inquiry to each of its local voronoi neighbors; the node I enters sleeping mode only after it has received positive confirmation from all of its local voronoi neighbors. Inquiries are resent on failures, and a sensor node that sends a positive confirmation doesn't enter a sleeping state. We omit further minor details.

The second problem arises because a sensor node I may not be able to accurately compute its $N(I, l)$, the active l-hop neighborhood, because of message losses. In particular, a node may not know which neighboring nodes are active or inactive. We solve this problem by requiring each active sensor to send a periodic `hello` message to its l-hop neighbors. By default, a node I assumes that each node J in the l-hop neighborhood is inactive, unless it receives a `hello` message from J. This results in an *underestimation* of $N(I, l)$ due to possible message losses. Underestimation of $N(I, l)$ only results in overestimation of $LV(I)$, and hence, does not affect the claims in Theorem 1 and Theorem 2, and the correctness of the algorithm. The inaccuracy of neighborhood information doesn't cause any problems in maintenance of connectivity of the active nodes, as long as each node *initially* start with accurate information of one-hop communication neighbors and the active neighborhood nodes are eventually discovered.

V. Greedy Algorithm

In this section, we present a greedy algorithm for the variable radii connected sensor coverage problem. We present a centralized as well a distributed version of the algorithm. In contrast with the Voronoi-based approach, the centralized version of the greedy algorithm provably delivers a VRCSC whose total energy cost is at most $O(r \log n)$ times the optimal energy cost. Here, r is the link radius of the sensor network (defined later) and n is the total number of sensors in the entire network. The distributed version of the greedy algorithm empirically performs close to the centralized version, but incurs higher communication cost compared to the Voronoi approach due to the size of the messages. Moreover, for the greedy algorithm, we need to make an assumption that each sensor has only a finite number of choices for the sensing radii. In particular, we assume that each sensor I chooses from k sensing radii $S_1, S_2, \ldots, S_k = S^*$. The greedy algorithm presented here is a generalization of the greedy approximation algorithm presented in [21] for the fixed radii version of the problem. We start with describing the centralized version of the greedy algorithm.

Basic Idea. Informally, the proposed greedy algorithm works as follows. The algorithm maintains a set of selected sensors M along with their assigned transmission and sensing radii, and increases the covered region while keeping connectivity of M. At each stage, we either add to M a "path" of sensors or increase the sensing radius of a sensor in M, whichever gives the maximum "benefit." The algorithm terminates when the given query region is completely covered by the assigned sensing regions of the sensors in M. A more formal and complete description of the algorithm is given below. We first start with a few more definitions.

Definition 9: (Candidate Sensor; Candidate Path) Let M be the set of sensors already selected by the algorithm. A sensor c is called a *candidate sensor* if $c \notin M$ and there is a sensor m in M such that $d(c,m) < S^* + S(m)$. In other words, a sensor c is a candidate sensor if $c \notin M$ and its maximum sensing region (corresponding to the sensing radius S^*) intersects with the assigned sensing region ($\theta(m)$) of some sensor m in M.

A *candidate path* is a sequence/path of sensors $<p_0, p_1, \ldots, p_l>$ such that p_0 is a candidate sensor, $p_l \in M$, $p_i \notin M$ for $i < l$, and the sequence of sensors forms a communication path in the full-communication graph of the entire sensor network. Also, to ensure that the sequence of sensors P forms a communication path with minimum transmission energy cost, we make the following assignment of radii.

$$\begin{aligned}
T(p_0) &= d(p_0, p_1) \\
T(p_i) &= \mathrm{Max}(d(p_i, p_{i-1}), d(p_i, p_{i+1}) \quad \forall\ 0 < i < l \\
T(p_l) &= \mathrm{Max}(d(p_l, p_{l-1}), T(p_l)) \\
S(p_i) &= 0 \quad \text{for } 0 < i < l
\end{aligned}$$

In addition, the sensing radius of the candidate sensor p_0 is chosen to maximize the benefit of the candidate path (defined later). The sensing radius of p_l, which is in M, is kept unchanged. □

Definition 10: (Subelement; Valid Subelement) Recall that each sensor has a choice of k possible sensing regions (corresponding to the k different sensing radii). A *subelement* is a set of points. Two points belong to same subelement if and only if they are covered by the same set of possible sensing regions. If a subelement intersects with the given query region, then it is called a *valid subelement*. □

Definition 11: (Benefit of a Candidate Path) Benefit of a candidate path P with respect to M, an already selected set of sensors, is defined as the number of valid subelements newly (not covered by M) covered by P divided by the increase in energy cost of M due to addition of P. More formally, the benefit of a candidate path P with respect to a set of selected sensors M is:

$$\frac{V(M \cup P) - V(M)}{E(M \cup P) - E(M)},$$

where $V(\mathcal{I})$ is the number of valid subelements covered by a set of sensors $\mathcal{I}$, and $E(\mathcal{I})$ is the total energy cost of $\mathcal{I}$. □

Definition 12: (Optimal Incremental Benefit) Let M be the set of sensors already selected by the greedy algorithm, and m be a sensor node in M with an assigned sensing radius of $S(m)$. The *incremental benefit* of increasing m's sensing radius from $S(m)$ to $S'(m)$ is defined as the number of valid subelements newly (not covered by M) covered by the increased sensing region $\theta'(m)$ divided by the increase in energy cost of m. The sensing radius $S'(m)$ of m that results in the maximum incremental benefit is called the *optimal incremental radius* of m with respect to M, and the corresponding incremental benefit is called the *optimal incremental benefit* of m. □

Centralized Greedy Algorithm. We now give a formal and complete description of the Centralized Greedy Algorithm. Initially, M consists of an arbitrary sensor I whose minimum sensing region (S_1) intersects with the given query region. The sensor I's sensing radius is set to the minimum and its transmission radius is set to zero. At each subsequent stage, the algorithm finds the candidate path $\widehat{P}$ (after finding all the candidate sensors) that has the maximum benefit with respect to M. Also, for each sensor m in M, the algorithm computes its optimal incremental benefit (as defined above), and picks the sensor $\widehat{m}$ that has the highest optimal incremental benefit. If the optimal incremental benefit of $\widehat{m}$ is higher than the benefit of selected $\widehat{P}$, then $\widehat{m}$'s sensing radius is increased to its optimal incremental radius, otherwise the candidate path $\widehat{P}$ is added to M. That completes one stage of the algorithm. The above process is repeated until the given query region is completely covered by M.

Algorithm 1: Centralized Greedy Algorithm

Input: A sensor network and a query region R_Q.

Output: A set of connected sensor cover M. Each with assigned sensing and transmission radius.

BEGIN
  Let M denote the set of sensors selected.
  Let I be a node whose minimum sensing region intersects R_Q.
  $S(I)$ = Minimum sensing radius S_1;
  $T(I) = 0$;
  $M := I$;
  while (R_Q is not covered by M)
    Let SP be the set of candidate paths, and $\widehat{P} \in SP$ be the candidate path with maximum benefit;
    Let $\widehat{m} \in M$ be the sensor node with most optimal incremental benefit;
    BP = Benefit of $\widehat{P}$;
    Bm = Optimal incremental benefit of $\widehat{m}$;
    if (*BP* > *Bm*)
      $M = M \cup \widehat{P}$
    else
      Set $S(\widehat{m})$ to $\widehat{m}$'s optimum incremental radius.
    end if;
  end while;
  RETURN M**;**
END ◇

The above described Algorithm 1 can be implemented in $O(n^3)$ time, where n is the size of the network. The following theorem proves the near-optimality of the solution delivered by the algorithm. We omit the proof, as it is similar to the proof of the centralized approximation algorithm in [21].

Definition 13: (Link Radius) The *link radius* is defined as the maximum communication distance between any two sensors whose maximum sensing regions intersect. □

Theorem 3: Algorithm 1 returns a connected sensor cover whose energy cost is at most $O(r(1 + \log d))|OPT|$, where r is the link radius of the sensor network, d is the maximum number of subelements in any sensing region, and $|OPT|$ is the energy cost of an optimum solution. Since, $d = O((nk)^2)$ ([21]), the solution delivered by Algorithm 1 is within $O(r \log(nk))$ factor of the optimal solution. Recall that k is the total number of sensing radius choices available to a sensor node. □

A. Distributed Greedy Algorithm (DGA)

In this section, we briefly describe the distributed version of the Algorithm 1 proposed in the previous section. The distributed algorithm presented here is similar to the distributed approximation algorithm proposed in [21] for constructing a connected sensor cover. The Distributed Greedy Algorithm (DGA) works in stages, and at each stage, a *candidate path* is added to the already selected sensor set M, or the sensing range of a sensor in M is increased, until the whole query region is covered by M. Throughout the algorithm, the following variables are maintained:

- M, the set of sensors that have already been selected.
- SP, the set of candidate paths.
- $\widehat{P}$, the most recently added candidate path.
- $\hat{C}$, the candidate sensor associated with $\widehat{P}$.

Each stage of the the distributed algorithm consists of four phases as described below:

- Candidate Path Search (CPS). In this phase, the most recently added candidate sensor $\hat{C}$ broadcasts a CPS message within a range of $2r$ *communication distance*. In this broadcast phase, each sensor broadcast the CPS message with the maximum transmission range.
- Candidate Path Response(CPR). Any sensor that receives the CPS message checks whether it is a new candidate sensor (by checking whether its maximum sensing region intersects with any sensor in $\widehat{P}$). If so, it sends a CPR message (along with the associated candidate path formed by the routing path took by the CPS message) to $\hat{C}$, the originator of the CPS message.
- Selection of Best Candidate Path/Sensor. After gathering all CPR message, the sensor $\hat{C}$ calculates the benefit of each of the candidate paths and picks the candidate path $\widehat{P}_{new}$ (and the corresponding candidate sensor $\hat{C}_{new}$) that has the highest benefit. Moreover, it computes the optimal incremental benefit of each sensor in M, and picks the sensor $\widehat{m} \in M$ that has the maximum optimal incremental benefit. If the benefit of $\widehat{P}_{new}$ is greater than the optimal incremental benefit of $\widehat{m}$, then the sensor $\hat{C}$ unicasts all the required parameters to $\hat{C}_{new}$ after adding $\widehat{P}_{new}$ to M, and the $\widehat{P}_{new}$ and $\hat{C}_{new}$ now become the new (and current) $\widehat{P}$ and $\hat{C}$ respectively. If the optimal incremental benefit of $\widehat{m}$ is greater than the benefit of $\widehat{P}_{new}$, then the sensor $\hat{C}$ unicasts all the required parameters to $\widehat{m}$, which becomes the new (and current) $\widehat{P}$ and $\hat{C}$. Also, $\widehat{m}$'s sensing radius is increased to attain the optimal incremental benefit.
- Repeat. The new $\hat{C}$ broadcasts the CPS messages again and initiates a new stage. This continues, until a leading sensor $\hat{C}$ decides that the sensing region R_M successfully covers the whole query region R_Q.

We make similar optimization as in [21] to reduce the communication cost incurred by the distributed algorithm. In Section VII, we show that the solution returned by the above described Distributed Greedy Algorithm is very close to that returned by the Centralized Greedy Algorithm (Algorithm 1).

VI. Steiner Tree Based Algorithm

In this section, we present an alternate centralized algorithm to construct a variable radii connected sensor cover. We refer to this algorithm as the *Steiner Tree Based* algorithm, and it consists of two phases:

1) In the first phase, we construct a variable radii sensor cover (not necessarily connected).
2) In the second phase, we construct a Steiner tree to connect the sensor cover constructed in the first phase.

Each of the above phases can be solved near-optimally, i.e., within a factor of the optimal solution, as shown below.

Constructing a Variable Radii Sensor Cover (VRSC). The problem of constructing a variable radii sensor cover is similar to that of the well-known set cover problem, wherein the greedy algorithm delivers a near-optimal set cover. For the variable radii sensor cover problem, the greedy algorithm maintains a set of sensors M, and increments the set M by either increasing the sensing radius of a sensor already in M or by adding a new sensor along with an assigned sensing radius into M. In particular, during each stage of the greedy algorithm, we compute the optimal incremental benefit of each sensor in the network with respect to M (assuming that the sensors not in M have an assigned sensing radius of zero), and pick the sensor (for increasing the sensing radius or addition to M) that gives the the maximum optimal incremental benefit at that stage. This continues till the whole query region is covered. In this phase, the transmission radius assigned to the sensors is zero. Let M_1 be the variable radius sensor cover constructed by the above greedy algorithm. The following theorem follows easily from the approximation result of the greedy algorithm for weighted set cover problem.

Theorem 4: The above described greedy algorithm used to construct a variable radii sensor cover delivers a solution whose energy cost is within $O(\log(nk))$ factor of the optimal energy cost, where n is the total number of sensors in the network and k is the total number of sensing radii each sensor can choose from. □

Constructing a Steiner Tree. In the second phase, we construct a Steiner tree to connect the VRSC M_1 constructed in the first phase. We treat the problem as constructing an optimal Steiner tree problem over an edge-weighted sensor network graph G, where an edge exists between sensors x and y if $d(x, y) < T^*$, where $d(x, y)$ is the distance between x and

y, and the weight assigned to the edge is $g(d(x, y))^3$. We use the well known 2-approximation Steiner tree algorithm [25] to connect M_1 in the edge-weighted sensor network graph G. Let the Steiner tree thus constructed be τ. Each sensor node in τ is assigned a transmission radius equal to the maximum distance from all its neighbors in τ. We omit the proof of the following theorem, which follows from a similar result in [7] for minimum energy broadcast trees.

Theorem 5: The total transmission energy cost of the Steiner tree τ constructed by the above described second phase is at most 24 times the minimum transmission energy cost required to connect the set of sensors M_1, the VRSC returned by the first phase. □

VII. Performance Evaluation

We built a specific simulator for the distributed algorithms, and carried out extensive experiments to evaluate the performance of the proposed algorithms. The simulator randomly places sensors within a given region. The simulator does not model any link layer protocol or wireless channel characteristics. Thus, all messages in the simulator are transmitted in an error-free manner. While such a simulator models an idealized communication subsystem, it is sufficient for our purpose of comparing the performance of our proposed algorithms.

Cost Model. The sensing energy cost function depends on the specific sensor type and environment, but is usually of the form $S(I)^x$, where $S(I)$ is the assigned sensing radius and x is a constant [22]. Similarly, the transmission energy cost function is of the form $T(I)^y$, where $T(I)$ is the assigned transmission radius and y is a constant between 2 to 4 [7]. For our experiments, we chose $x = y = 4$. We assume that total energy cost incurred (sensing and transmission) in keeping a sensor node active for a unit time is:

$$E(I) = \alpha S(I)^4 + (1 - \alpha)T(I)^4 + C,$$

where α is a parameter that signifies relative weight of sensing and transmission energies. In our experiments, we use three different values of α viz. 0.1, 0.5, and 0.9 to simulate different sensor types. Essentially, when α is 0.1, the energy consumption due to sensing is relatively much less than the energy consumption due to transmission. We measure the performance of our algorithms for all these three energy cost models.

Network and Battery Parameter Values. We run our experiments with the following choice of parameter values. The maximum sensing radius S^* as well as the maximum transmission radius T^* for each sensor node is chosen to be 10. Each sensor can choose from 5 different sensing and transmission radius: 2, 4, 6, 8, or 10. We randomly distribute a certain number of sensor nodes in a query region of size 50×50. The total size n of sensor network is between 100 to 600, representing scarce to significantly dense sensor network density. In our experiments, we set each sensor node's battery power as 12,000,000 units, and the constant C in the energy cost function is set at 2,000 units. If the sensing and transmission radii of a sensor node are set to the maximum (10), the total energy cost incurred in keeping the node active for a unit time is 12,000 units. In a naive approach wherein all sensor nodes are kept active with maximum sensing and transmission radii, the sensor network will last for 1,000 time slots, for any value of α. During the construction phase (execution of an algorithm to construct a VRCSC), the energy cost incurred in transmitting a message is proportional to the size of the message. We assume that an active sensor can transmit 100 bytes of data in unit time; thus, the energy cost incurred in transmitting a message of size ℓ bytes during the construction phase is $(1 - \alpha)10^4\ell/100$.

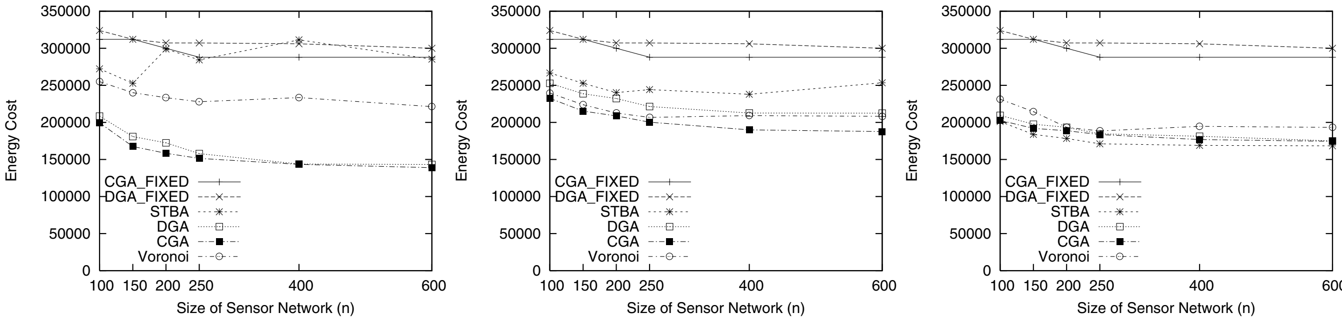
[3] Recall that, $g(t)$ is the transmission energy cost component of a sensor node with assigned transmission radius t.

Algorithms and Experiments. We compare the performance of the following algorithms in our experiments.

- Voronoi Based Algorithm (Voronoi) – The localized distributed algorithm described in section IV.
- Steiner Tree Based Algorithm (STBA) – the centralized algorithm described in Section VI.
- Centralized Greedy Algorithm (CGA) – the greedy approximation algorithm described in Algorithm 1.
- Distributed Greedy Algorithm (DGA) – the distributed version of Algorithm 1 described in Section V-A.
- Centralized Greedy Algorithm for Fixed Radii (CGA_FIXED) – the centralized greedy algorithm proposed in [21] for the fixed radius connected sensor cover. We choose the fixed sensing/transmission radius to be 10 (the maximum). The distributed version of the algorithm is denoted as DGA_FIXED.

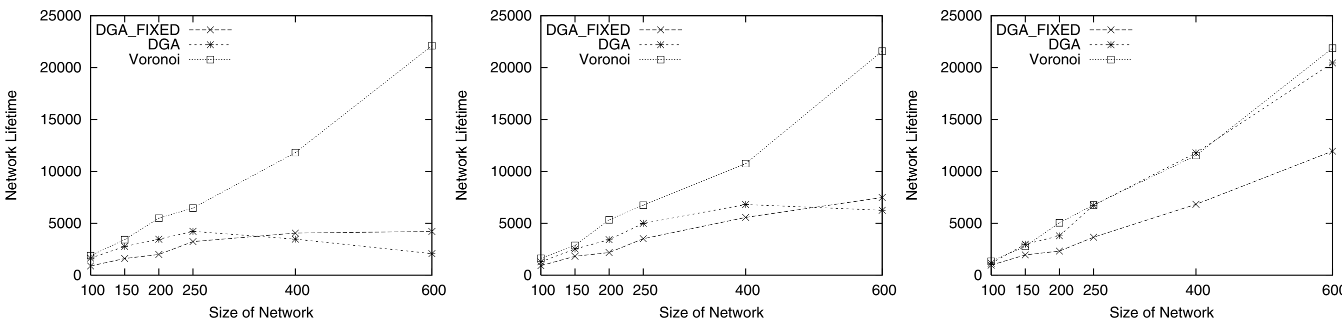
We have conducted two sets of experiments. The first set of experiments is to compare the performance of the various algorithms in terms of the total energy cost of the connected sensor cover delivered by the algorithm. The experiment results are presented in Figure 2. In the second set of experiments, we compare the performance of the various distributed algorithms (DGA, DGA_FIXED, Voronoi) in terms of their effectiveness in in prolonging the sensor network lifetime.

As shown in Figure 2, we can see that among all algorithms, the Centralized Greedy Algorithm (CGA) delivers the solution with least total energy cost. The energy cost of the solution delivered by CGA is almost half of the total energy cost of the solution delivered by CGA_FIXED, the best known approximation algorithm for the fixed radius connected sensor coverage problem. Also, we see that DGA performs very close to CGA. In general, the Voronoi algorithm also performs quite close to the CGA and DGA algorithm, except for the case when $\alpha = 0.1$, i.e., when transmission energy cost has a much higher weightage than the sensing energy cost – implying that the RNG approach of assignment transmission radius can be potentially improved further. It seems rather surprising that Steiner Tree Based Algorithm (STBA) performs quite bad for the cases when α is 0.1 or 0.5. The low performance of STBA is probably due to independent treatment of coverage and connectivity aspects, which leads to high transmission



(a) $\alpha = 0.1$ (b) $\alpha = 0.5$ (c) $\alpha = 0.9$

Fig. 2. Total energy cost of the solution returned by various algorithms.



(a) $\alpha = 0.1$ (b) $\alpha = 0.5$ (c) $\alpha = 0.9$

Fig. 3. Sensor network lifetime for various distributed algorithms.

energy cost, since in the first phase, sensor nodes are selected for coverage without taking into consideration the cost of connecting them.

Figure 3 shows that our approaches also result in significantly prolonging the lifetime of the sensor network. Due to the small size of messages in the Voronoi based approach compared to DGA, the Voronoi approach has a much lower transmission energy overhead during the construction phase. Hence, the Voronoi approach performs significantly better than the other distributed algorithms (DGA and DGA_FIXED) in terms of prolonging the network lifetime, except for the case when $\alpha = 0.9$. When α is 0.9, the transmission cost has very minimal weightage, and the performance of the algorithms is primarily determined by the sizes of the VRCSC returned. For dense networks, DGA performs slightly worse than DGA_FIXED due to much higher construction cost. Since DGA_FIXED uses maximum sensing radius, the number of stages of DGA_FIXED is much less than DGA, and at the end of each stage of these algorithms, a fairly large message containing the entire state information (proportional to the size of the network) is transmitted.

VIII. Conclusions

Given that the sensor networks are typically redundant, we presented an approach to conserve energy by exploiting redundancy in the network. In particular, we addressed the problem of constructing a connected sensor cover in a sensor network model wherein each sensor can control/adjust its sensing and transmission power/range. For the above problem we proposed various centralized approximation and communication-efficient distributed algorithms. Through extensive experiments, we demonstrated the usefulness of our approaches in prolonging the network lifetime. In particular, our proposed Voronoi based localized algorithm performs very well.

References

[1] K. Charkrabarty, S. Iyengar, H. Qi, and E. Cho, "Grid coverage for surveillance and target location in distributed sensor networks," *IEEE Transaction on Computers*, no. 12, 2002.

[2] F. Ye, G. Zhong, S. Lu, and L. Zhang, "Peas: A robust energy conserving protocol for long-lived sensor networks," in *Proceedings of the International Conference on Distributed Computing Systems*, 2003.

[3] X. Wang, G. Xing, Y. Zhang, C. Lu, R. Pless, and C. Gill, "Integrated coverage and connectivity configuration in wireless sensor networks," in *Proceedings of the ACM Conference on Embedded Networked Sensor Systems (SenSys)*, 2003.

[4] J. E. Wieselther, G. D. Nguyen, and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks," in *Proceedings of the IEEE INFOCOM*, 2000.

[5] M. Cagalj, J. Hubaux, and C. Enz, "Minimum-energy broadcast in all-wireless networks: Np-completeness and distributed issues," in *Proceedings of the International Conference on Mobile Computing and Networking (MobiCom)*, 2002.

[6] J. Cartigny, D. Simplot, and I. Stojmenovic, "Localized minimum-energy broadcasting in ad hoc networks," in *Proceedings of the IEEE INFOCOM*, 2003.

[7] P. Wan, G. Calinescu, X. Li, and O. Frieder, "Minimum-energy broadcast routing in static ad hoc wireless networks," in *Proceedings of the IEEE INFOCOM*, 2001.

[8] S. Guha and S. Khuller, "Approximation algorithms for connected dominating sets," *Algorithmica*, vol. 20, no. 4, 1998.

[9] P. Wan, K. Alzoubi, and O. Frieder, "Distributed construction of connected dominating set in wireless ad hoc networks," in *Proceedings of the IEEE INFOCOM*, 2002.

[10] B. Das, R. Sivakumar, and V. Bhargavan, "Routing in ad hoc networks using a spine," in *Proceedings of the International Conference on Computer Communications and Networks (IC3N)*, 1997.

[11] A. Laouiti, A. Qayyum, and L. Viennot, "Multipoint relaying: An efficient technique for flooding in mobile wireless networks," in *Proceedings of the Hawaii International Conference on System Sciences*, 2002.

[12] J. Wu and H. Li, "A dominating-set-based routing scheme in ad hoc wireless networks," *Telecommunication Systems Journal*, vol. 3, 2001.

[13] K. M. Alzoubi, P.-J. Wan, and O. Frieder, "Message-optimal connected dominating sets in mobile ad hoc networks," in *Proceedings of the International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2002.

[14] Y. Chen and A. Liestman, "Approximating minimum size weakly-connected dominating sets for clustering mobile ad hoc networks," in *Proceedings of the International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2002.

[15] J. Wu and F. Dai, "Broadcasting in ad hoc networks based on self-pruning," in *Proceedings of the IEEE INFOCOM*, 2003.

[16] S. Slijepcevic and M. Potkonjak, "Power efficient organization of wireless sensor networks," in *Proceedings of the International Conference on Communications (ICC)*, 2001.

[17] S. Meguerdichian, F. Koushanfar, G. Qu, and M. Potkonjak, "Exposure in wireless ad-hoc sensor networks," in *Proceedings of the International Conference on Mobile Computing and Networking (MobiCom)*, 2001.

[18] S. Meguerdichian, F. Koushanfar, M. Potkonjak, and M. Srivastava, "Coverage problems in wireless ad-hoc sensor networks," in *Proceedings of the IEEE INFOCOM*, 2001.

[19] S. Shakkottai, R. Srikant, and N. Shroff, "Unreliable sensor grids: Coverage, connectivity and diameter," in *Proceedings of the IEEE INFOCOM*, 2003.

[20] B. Chen, K. Jamieson, H. Balakrishnan, and R. Morris, "SPAN: An energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks," in *Proceedings of the International Conference on Mobile Computing and Networking (MobiCom)*, 2001.

[21] H. Gupta, S. Das, and Q. Gu, "Self-organization of sensor networks for efficient query execution," in *Proceedings of the International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2003.

[22] S. Pattem, S. Poduri, and B. Krishnamachari, "Energy-quality tradeoffs for target tracking in wireless sensor networks," in *Proceedings of the International Workshop on Information Processing in Sensor Networks (IPSN)*, 2003.

[23] Http://mathworld.wolfram.com/VoronoiDiagram.html.

[24] J. Foley, A. V. Dam, S. Feiner, and J. Hughes, *Computer Graphics, Principles and Practice*. Addison Wesley, 1990.

[25] P. Berman and V. Ramaiyer, "Improved approximations for the steiner tree problem," *Journal of Algorithms*, no. 3, 1994.