Gradual Typing from a Categorical Perspective

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References

A The Complete Spec of Grady

termvar, x			
index, k			
t	$::=$		term
	$\mid$	x	variable
	$\mid$	triv	unit
	$\mid$	$\lambda x : T.t$	λ-abstraction
	$\mid$	$t_1\ t_2$	function application
	$\mid$	(t_1, t_2)	pair constructor
	$\mid$	$\mathsf{fst}\ t$	first projection
	$\mid$	$\mathsf{snd}\ t$	second projection
	$\mid$	$\mathsf{succ}\ t$	successor function
	$\mid$	0	zero
	$\mid$	(t) S	

T	$::=$		type
	$\mid$	1	unit type
	$\mid$	$\mathbb{N}$	natural number type
	$\mid$	$?$	untyped universe
	$\mid$	$T_1 \to T_2$	function type
	$\mid$	$T_1 \times T_2$	cartesian product type
	$\mid$	(T) S	

Γ	$::=$		typing context
	$\mid$	$\cdot$	empty context
	$\mid$	$\Gamma, x : T$	cons

$\boxed{T_1 \sim_U T_2}$ T_1 can can be converted into T_2

$$\frac{}{T \sim_U T}\ \text{CV\_REFL}$$

$$\frac{T_1 \sim_U T_2 \quad T_2 \sim_U T_3}{T_1 \sim_U T_3} \quad \text{CV\_TRANS}$$

$$\frac{}{(? \to ?) \sim_U ?} \quad \text{CV\_INJ}$$

$$\frac{}{? \sim_U (? \to ?)} \quad \text{CV\_SURJ}$$

$$\frac{T_1 \sim_U T_1'}{(T_1 \times T_2) \sim_U (T_1' \times T_2)} \quad \text{CV\_PAIR1}$$

$$\frac{T_2 \sim_U T_2'}{(T_1 \times T_2) \sim_U (T_1 \times T_2')} \quad \text{CV\_PAIR2}$$

$$\frac{T_1 \sim_U T_1'}{(T_1 \to T_2) \sim_U (T_1' \to T_2)} \quad \text{CV\_FUN1}$$

$$\frac{T_2 \sim_U T_2'}{(T_1 \to T_2) \sim_U (T_1 \to T_2')} \quad \text{CV\_FUN2}$$

$\boxed{T_1 \sim T_2}$ T_1 is consistent with T_2

$$\frac{}{T \sim T} \quad \text{CS\_REFL}$$

$$\frac{}{? \sim T} \quad \text{CS\_UL}$$

$$\frac{}{T \sim ?} \quad \text{CS\_UR}$$

$$\frac{T_1 \sim_U T_2}{T_1 \sim T_2} \quad \text{CS\_CONV}$$

$$\frac{T_1 \sim T_1' \quad T_2 \sim T_2'}{(T_1 \times T_2) \sim (T_1' \times T_2')} \quad \text{CS\_PAIR}$$

$$\frac{T_1 \sim T_1' \quad T_2 \sim T_2'}{(T_1 \to T_2) \sim (T_1' \to T_2')} \quad \text{CS\_ARROW}$$

$\boxed{\Gamma \vdash t : T}$ t has type T in context Γ

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} : 1} \quad \text{UNIT}$$

$$\frac{}{\Gamma \vdash 0 : \mathbb{N}} \quad \text{ZERO}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{succ}\, t : \mathbb{N}} \quad \text{SUCC}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2} \quad \text{PAIR}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \mathsf{fst}\, t : T_1} \quad \text{FST}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \mathsf{snd}\, t : T_2} \quad \text{SND}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x : T_1 . t : T_1 \to T_2} \quad \text{ABS}$$

$$\frac{\Gamma \vdash t : T_1 \quad T_1 \sim_U T_2}{\Gamma \vdash t : T_2} \quad \text{U}$$

$$\frac{\Gamma \vdash t_1 : T_1 \to T_2 \quad \Gamma \vdash t_2 : T_3 \quad T_3 \sim T_1}{\Gamma \vdash t_1\, t_2 : T_2} \quad \text{APP}$$

$\boxed{t_1 \rightsquigarrow t_2}$ t_1 reduces to t_2

$$\frac{}{(\lambda x : T . t_2)\, t_1 \rightsquigarrow [t_1/x]t_2} \quad \text{RD\_BETA}$$

$$\frac{}{(\lambda x : T . t\, x) \rightsquigarrow t} \quad \text{RD\_ETA}$$

$$\frac{}{\mathsf{fst}\, (t_1, t_2) \rightsquigarrow t_1} \quad \text{RD\_PROJ1}$$

$$\frac{}{\mathsf{snd}\, (t_1, t_2) \rightsquigarrow t_2} \quad \text{RD\_PROJ2}$$

$$\frac{}{(\mathsf{fst}\, t, \mathsf{snd}\, t) \rightsquigarrow t} \quad \text{RD\_ETAP}$$