$termvar, x, y, z, f$
$typevar, X, Y, Z$
$index, i, j, k$

$t,\ c,\ v,\ s,\ n$ ::=
| x
| triv
| box
| unbox
| $\Lambda(X <: A).t$
| $[A]t$
| $\lambda(x : A).t$
| $t_1\ t_2$
| (t_1, t_2)
| fst t
| snd t
| succ t
| 0
| case t of $t_3 \rightarrow t_1, t_4 \rightarrow t_2$
| $[]$
| $t :: t'$
| (t) S

K ::=
| $\star$

$A,\ B,\ C,\ D,\ E,\ S,\ U$ ::=
| $\top$
| $\mathbb{S}$
| X
| List A
| $\forall(X <: A).B$
| Unit
| Nat
| ?
| $A_1 \rightarrow A_2$
| $A_1 \times A_2$
| (A) S

Γ ::=
| $\cdot$
| $\Gamma, X <: A$
| $\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \top : \star} \quad \text{K\_TOP}$$

$$\frac{}{\Gamma \vdash \mathbb{S} : \star} \quad \text{K\_SL}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star} \quad \text{K\_UNIT}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star} \quad \text{K\_NAT}$$

$$\frac{}{\Gamma \vdash \,? : \star} \quad \text{K\_UNITYPE}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\, A : \star} \quad \text{K\_LIST}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \quad \text{K\_ARROW}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \text{K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A).B : \star} \quad \text{K\_FORALL}$$

$\boxed{\Gamma \, \mathsf{Ok}}$

$$\frac{}{\cdot \, \mathsf{Ok}} \quad \text{OK\_EMPTY}$$

$$\frac{\Gamma \, \mathsf{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \, \mathsf{Ok}} \quad \text{OK\_TYPEVAR}$$

$$\frac{\Gamma \, \mathsf{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \, \mathsf{Ok}} \quad \text{OK\_VAR}$$

$\boxed{\Gamma \vdash A \sim B}$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \sim A} \quad \text{C\_REFL}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \sim \,?} \quad \text{C\_BOX}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \,? \sim A} \quad \text{C\_UNBOX}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\mathsf{List}\, A) \sim (\mathsf{List}\, B)} \quad \text{C\_LIST}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \to B_1) \sim (A_2 \to B_2)} \quad \text{C\_ARROW}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A).B_1) \sim (\forall (X <: A).B_2)} \quad \text{C\_FORALL}$$

$\boxed{A \sqsubseteq B}$

$$\frac{}{A \sqsubseteq \,?} \quad \text{P\_U}$$

$$\frac{}{A \sqsubseteq A} \quad \text{P\_REFL}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad \text{P\_ARROW}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P\_PROD}$$

$$\frac{A \sqsubseteq B}{(\text{List } A) \sqsubseteq (\text{List } B)} \quad \text{P\_LIST}$$

$$\frac{A_1 \sqsubseteq A_2 \quad B_1 \sqsubseteq B_2}{(\forall(X <: A_1).B_1) \sqsubseteq (\forall(X <: A_2).B_2)} \quad \text{P\_FORALL}$$

$\boxed{t \sqsubseteq t'}$

$$\frac{}{t \sqsubseteq t} \quad \text{TP\_REFL}$$

$$\frac{t_1 \sqsubseteq t_2}{(\text{succ } t_1) \sqsubseteq (\text{succ } t_2)} \quad \text{TP\_SUCC}$$

$$\frac{t_1 \sqsubseteq t_4 \quad t_2 \sqsubseteq t_5 \quad t_3 \sqsubseteq t_6}{(\text{case } t_1 \text{ of } 0 \rightarrow t_2, (\text{succ } x) \rightarrow t_3) \sqsubseteq (\text{case } t_4 \text{ of } 0 \rightarrow t_5, (\text{succ } x) \rightarrow t_6)} \quad \text{TP\_NATE}$$

$$\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1, t_2) \sqsubseteq (t_3, t_4)} \quad \text{TP\_PAIR}$$

$$\frac{t_1 \sqsubseteq t_2}{(\text{fst } t_1) \sqsubseteq (\text{fst } t_2)} \quad \text{TP\_FST}$$

$$\frac{t_1 \sqsubseteq t_2}{(\text{snd } t_1) \sqsubseteq (\text{snd } t_2)} \quad \text{TP\_SND}$$

$$\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP\_CONS}$$

$$\frac{t_1 \sqsubseteq t_4 \quad t_2 \sqsubseteq t_5 \quad t_3 \sqsubseteq t_6}{(\text{case } t_1 \text{ of } [] \rightarrow t_2, (x :: y) \rightarrow t_3) \sqsubseteq (\text{case } t_4 \text{ of } 0 \rightarrow t_5, (x :: y) \rightarrow t_6)} \quad \text{TP\_LISTE}$$

$$\frac{t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{(\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP\_FUN}$$

$$\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1 \, t_2) \sqsubseteq (t_3 \, t_4)} \quad \text{TP\_APP}$$

$$\frac{t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{(\Lambda(X <: A_1).t_1) \sqsubseteq (\Lambda(X <: A_2).t_2)} \quad \text{TP\_TFUN}$$

$$\frac{t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{[A]t_1 \sqsubseteq [B]t_2} \quad \text{TP\_TAPP}$$

$\boxed{\Gamma \vdash A \lesssim B}$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim A} \quad \text{S\_REFL}$$

$$\frac{X <: A' \in \Gamma \quad \Gamma \vdash A' \sim A}{\Gamma \vdash X \lesssim A} \quad \text{S\_VAR}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim \top} \quad \text{S\_TOP}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim ?} \quad \text{S\_BOX}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash\ ?\lesssim A} \quad \text{S\_Unbox}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{Nat} \lesssim \mathbb{S}} \quad \text{S\_NatSL}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{Unit} \lesssim \mathbb{S}} \quad \text{S\_UnitSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash \mathsf{List}\ A \lesssim \mathbb{S}} \quad \text{S\_ListSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \to B \lesssim \mathbb{S}} \quad \text{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \times B \lesssim \mathbb{S}} \quad \text{S\_ProdSL}$$

$$\frac{\Gamma \vdash A \lesssim B}{\Gamma \vdash (\mathsf{List}\ A) \lesssim (\mathsf{List}\ B)} \quad \text{S\_List}$$

$$\frac{\Gamma \vdash A_1 \lesssim A_2 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \times B_1) \lesssim (A_2 \times B_2)} \quad \text{S\_Prod}$$

$$\frac{\Gamma \vdash A_2 \lesssim A_1 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \to B_1) \lesssim (A_2 \to B_2)} \quad \text{S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 \lesssim B_2}{\Gamma \vdash (\forall (X <: A).B_1) \lesssim (\forall (X <: A).B_2)} \quad \text{S\_Forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A\ \in\ \Gamma \quad \Gamma\ \mathsf{Ok}}{\Gamma \vdash x : A} \quad \text{T\_varP}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{box} : \forall (X <: \mathbb{S}).(X \to\ ?)} \quad \text{T\_Box}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{unbox} : \forall (X <: \mathbb{S}).(? \to X)} \quad \text{T\_Unbox}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}} \quad \text{T\_unitP}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}} \quad \text{T\_zeroP}$$

$$\frac{\Gamma \vdash t : A \quad \mathsf{nat}(A) = \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\ t : \mathsf{Nat}} \quad \text{T\_succ}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : C \quad \mathsf{nat}(C) = \mathsf{Nat} \quad \Gamma \vdash A_1 \sim A \\ \Gamma \vdash t_1 : A_1 \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A_2 \quad \Gamma \vdash A_2 \sim A\end{array}}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 : A} \quad \text{T\_ncase}$$

$$\frac{\Gamma\ \mathsf{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall (X <:\ ?).\mathsf{List}\ X} \quad \text{T\_empty}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2 \quad \mathsf{list}(A_2) = \mathsf{List}\ A_3 \quad \Gamma \vdash A_1 \sim A_3}{\Gamma \vdash t_1 :: t_2 : \mathsf{List}\ A_3} \quad \text{T\_cons}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \quad \text{T\_PAIR}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \to B} \quad \text{T\_LAM}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B} \quad \text{T\_LAM}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash [A]t : [A/X]C} \quad \text{T\_TYPEAPP}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash t : B} \quad \text{T\_SUB}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : C \quad \mathsf{list}(C) = \mathsf{List}\, A \\ \Gamma \vdash t_1 : B_1 \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 : B_2 \quad \Gamma \vdash B_1 \sim B \quad \Gamma \vdash B_2 \sim B\end{array}}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 : B} \quad \text{T\_LCASE}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 : C \quad \mathsf{fun}(C) = A_1 \to B_1 \\ \Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_2 \sim A_1\end{array}}{\Gamma \vdash t_1\, t_2 : B_2} \quad \text{T\_APP}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\ t : A_1} \quad \text{T\_FST}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\ t : A_2} \quad \text{T\_SND}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ Cast insertion

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CI\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \Rightarrow \mathsf{box} : S \to ?} \quad \text{CI\_BOX}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \Rightarrow \mathsf{unbox} : ? \to S} \quad \text{CI\_UNBOX}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \mathsf{Nat}} \quad \text{CI\_ZERO}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \Rightarrow \mathsf{triv} : \mathsf{Unit}} \quad \text{CI\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\,(\mathsf{unbox}_{\mathsf{Nat}}\, t_2) : \mathsf{Nat}} \quad \text{CI\_SUCCU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\, t_2 : \mathsf{Nat}} \quad \text{CI\_SUCC}$$

$$\frac{\begin{array}{c}\Gamma \vdash t \Rightarrow t' : ? \\ \Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A\end{array}}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2) \Rightarrow (\mathsf{case}\,(\mathsf{unbox}_{\mathsf{Nat}}\, t')\ \mathsf{of}\ 0 \to t_1', (\mathsf{succ}\ x) \to t_2') : A} \quad \text{CI\_NCASEU}$$

$$\frac{\begin{array}{c}\Gamma \vdash t \Rightarrow t' : \mathsf{Nat} \\ \Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A\end{array}}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2) \Rightarrow (\mathsf{case}\ t'\ \mathsf{of}\ 0 \to t_1', (\mathsf{succ}\ x) \to t_2') : A} \quad \text{CI\_NCASE}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \quad \text{CI\_PAIR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, (\mathsf{split}_{(?\times?)}\, t_2) : ?} \quad \text{CI\_FST}\textsc{U}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, t_2 : A_1} \quad \text{CI\_FST}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, (\mathsf{split}_{(?\times?)}\, t_2) : ?} \quad \text{CI\_SND}\textsc{U}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, t_2 : B} \quad \text{CI\_SND}$$

$$\frac{}{\Gamma \vdash [] \Rightarrow [] : \forall(X <: ?).\mathsf{List}\, X} \quad \text{CI\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_2' : \mathsf{List}\, A_2 \quad \Gamma \vdash A_1 \lesssim A_2 \quad \mathsf{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c\, t_1') :: t_2') : \mathsf{List}\, A} \quad \text{CI\_CONS}$$

$$\frac{\begin{array}{c}\Gamma \vdash t \Rightarrow t' : ? \\ \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : ?, y : \mathsf{List}\, ? \vdash t_2 \Rightarrow t_2' : B\end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, (\mathsf{split}_{(\mathsf{List}\, ?)}\, t')\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B} \quad \text{CI\_LCASE}\textsc{U}$$

$$\frac{\begin{array}{c}\Gamma \vdash t \Rightarrow t : \mathsf{List}\, A \\ \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 \Rightarrow t_2' : B\end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B} \quad \text{CI\_LCASE}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \to A_2} \quad \text{CI\_LAM}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 \Rightarrow t_1' : ? \\ \Gamma \vdash t_2 \Rightarrow t_2' : A_2 \quad \mathsf{caster}(A_2, ?) = c\end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow (\mathsf{split}_{(?\to?)}\, t_1')\, (c\, t_2') : ?} \quad \text{CI\_APP}\textsc{U}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_2 \Rightarrow t_2' : A_2 \\ \Gamma \vdash t_1 \Rightarrow t_1' : A_1 \to B \quad \Gamma \vdash A_2 \lesssim A_1 \quad \mathsf{caster}(A_2, A_1) = c\end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow t_1'\, (c\, t_2') : B} \quad \text{CI\_APP}$$

$$\frac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall(X <: A).B} \quad \text{CI\_LAM}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall(X <: B).C \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A]t_2) : [A/X]C} \quad \text{CI\_TYPEAPP}$$

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Definition rules:          93 good     0 bad
Definition rule clauses: 181 good      0 bad
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