$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ v,\ s,\ n$	$::=$		
	$\mid$	x	
	$\mid$	triv	
	$\mid$	squash_S	
	$\mid$	split_S	
	$\mid$	box_A	
	$\mid$	unbox_A	
	$\mid$	$\Lambda X <: A.t$	
	$\mid$	$[A]t$	
	$\mid$	$\lambda x : A.t$	
	$\mid$	$t_1\ t_2$	
	$\mid$	(t_1, t_2)	
	$\mid$	$\mathsf{fst}\ t$	
	$\mid$	$\mathsf{snd}\ t$	
	$\mid$	$\mathsf{succ}\ t$	
	$\mid$	0	
	$\mid$	$\mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2$	
	$\mid$	(t)	S
K	$::=$		
	$\mid$	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U$	$::=$		
	$\mid$	X	
	$\mid$	$\forall X <: A.B$	
	$\mid$	$\top$	
	$\mid$	$\mathbb{S}$	
	$\mid$	Unit	
	$\mid$	Nat	
	$\mid$	$?$	
	$\mid$	$A_1 \to A_2$	
	$\mid$	$A_1 \times A_2$	
	$\mid$	(A)	S
Γ	$::=$		
	$\mid$	$.$	
	$\mid$	$\Gamma, X <: A$	
	$\mid$	$\Gamma, x : A$	

$\boxed{\Gamma \vdash A : \star}$

$$\frac{X <: A \in \Gamma}{\Gamma \vdash X : \star}\ \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star}\ \text{K\_UNIT}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star}\ \text{K\_NAT}$$

$$\frac{}{\Gamma \vdash ? : \star}\ \text{K\_UNITYPE}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \text{ K\_ARROW}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \text{ K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall X <: A.B : \star} \text{ K\_FORALL}$$

$\boxed{\Gamma \,\mathrm{Ok}}$

$$\frac{}{\cdot \,\mathrm{Ok}} \text{ OK\_EMPTY}$$

$$\frac{\Gamma \,\mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \,\mathrm{Ok}} \text{ OK\_TYPEVAR}$$

$$\frac{\Gamma \,\mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \,\mathrm{Ok}} \text{ OK\_VAR}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash A <: A} \text{ S\_REFL}$$

$$\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \text{ S\_TRANS}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash A <: \top} \text{ S\_TOP}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}} \text{ S\_NAT}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}} \text{ S\_UNIT}$$

$$\frac{X <: A \in \Gamma \quad \Gamma \,\mathrm{Ok}}{\Gamma \vdash X <: A} \text{ S\_VAR}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}} \text{ S\_ARROWSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}} \text{ S\_PRODSL}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \text{ S\_PROD}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2} \text{ S\_ARROW}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall X <: A.B_1 <: \forall X <: A.B_2} \text{ S\_FORALL}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma \,\mathrm{Ok}}{\Gamma \vdash x : A} \text{ VARP}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{box}_A : A \to ?} \text{ BOX}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{unbox}_A : ? \to A}\ \textsc{Unbox}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{squash}_U : U \to ?}\ \textsc{squash}\mathrm{P}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{split}_U : ? \to U}\ \textsc{split}\mathrm{P}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{unit}\mathrm{P}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{zero}\mathrm{P}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, t_1 \mid\mid x.t_2 : A}\ \textsc{case}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2}\ \textsc{pair}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1}\ \textsc{fst}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2}\ \textsc{snd}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A.t : A \to B}\ \textsc{lam}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B}\ \textsc{app}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda X <: A.t : \forall X <: A.B}\ \textsc{Lam}$$

$$\frac{\Gamma \vdash t : \forall X <: B.C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C}\ \textsc{typeApp}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B}\ \textsc{Sub}$$

$\boxed{t_1 \rightsquigarrow t_2}$

$$\frac{\cdot \vdash t : A \quad \cdot \vdash A <: B}{\mathsf{unbox}_A\, (\mathsf{box}_B\, t) \rightsquigarrow t}\ \textsc{rd\_retract}\mathrm{T}$$

$$\frac{\cdot \vdash t : U}{\mathsf{split}_U\, (\mathsf{squash}_U\, t) \rightsquigarrow t}\ \textsc{rd\_retract}\mathrm{U}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t'}\ \textsc{rd\_succ}$$

$$\frac{}{\mathsf{case}\, 0\, \mathsf{of}\, t_1 \mid\mid x.t_2 \rightsquigarrow t_1}\ \textsc{rd\_case}0$$

$$\frac{}{\mathsf{case}\, (\mathsf{succ}\, t)\, \mathsf{of}\, t_1 \mid\mid x.t_2 \rightsquigarrow [t/x]t_2}\ \textsc{rd\_case}\textsc{Succ}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\ t'\ \mathsf{of}\ t_1 \mid\mid x.t_2}\ \text{RD\_CASE1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\ t\ \mathsf{of}\ t_1' \mid\mid x.t_2}\ \text{RD\_CASE2}$$

$$\frac{t_2 \rightsquigarrow t_2'}{\mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2'}\ \text{RD\_CASE3}$$

$$\frac{}{(\lambda x : A_1.t_2)\ t_1 \rightsquigarrow [t_1/x]t_2}\ \text{RD\_BETA}$$

$$\frac{x \notin \mathsf{FV}(t)}{\lambda x : A_1.t\ x \rightsquigarrow t}\ \text{RD\_ETA}$$

$$\frac{}{\mathsf{fst}\ (t_1, t_2) \rightsquigarrow t_1}\ \text{RD\_PROJ1}$$

$$\frac{}{\mathsf{snd}\ (t_1, t_2) \rightsquigarrow t_2}\ \text{RD\_PROJ2}$$

$$\frac{}{(\mathsf{fst}\ t, \mathsf{snd}\ t) \rightsquigarrow t}\ \text{RD\_ETAP}$$

$$\frac{t \rightsquigarrow t'}{\lambda x : A.t \rightsquigarrow \lambda x : A.t'}\ \text{RD\_LAM}$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1\ t_2 \rightsquigarrow t_1'\ t_2}\ \text{RD\_APP1}$$

$$\frac{t_2 \rightsquigarrow t_2'}{t_1\ t_2 \rightsquigarrow t_1\ t_2'}\ \text{RD\_APP2}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{fst}\ t \rightsquigarrow \mathsf{fst}\ t'}\ \text{RD\_FST}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{snd}\ t \rightsquigarrow \mathsf{snd}\ t'}\ \text{RD\_SND}$$

$$\frac{t_1 \rightsquigarrow t_1'}{(t_1, t_2) \rightsquigarrow (t_1', t_2)}\ \text{RD\_PAIR1}$$

$$\frac{t_2 \rightsquigarrow t_2'}{(t_1, t_2) \rightsquigarrow (t_1, t_2')}\ \text{RD\_PAIR2}$$

$$\frac{}{[A](\Lambda X <: B.t) \rightsquigarrow [A/X]t}\ \text{RD\_TYPEBETA}$$

$$\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2}\ \text{RD\_TYPEAPP}$$

$$\frac{t_1 \rightsquigarrow t_2}{\Lambda X <: A.t_1 \rightsquigarrow \Lambda X <: A.t_2}\ \text{RD\_LAM}$$

```
Definition rules:          61 good    0 bad
Definition rule clauses: 112 good    0 bad
```