

$termvar, x, y, z, f$   
 $typevar, X, Y, Z$   
 $index, i, j, k$   
 $t, c, v, s, n$

$::=$   
 $|$   $x$   
 $|$  **triv**  
 $|$  **box**  
 $|$  **unbox**  
 $|$   $\Lambda(X <: A).t$   
 $|$   $[A]t$   
 $|$   $\lambda(x : A).t$   
 $|$   $t_1 t_2$   
 $|$   $(t_1, t_2)$   
 $|$  **fst**  $t$   
 $|$  **snd**  $t$   
 $|$  **succ**  $t$   
 $|$   $0$   
 $|$  **case**  $t$  **of**  $t_3 \rightarrow t_1, t_4 \rightarrow t_2$   
 $|$   $\square$   
 $|$   $t :: t'$   
 $|$   $(t)$  S

$K$   $::=$   
 $|$   $\star$

$A, B, C, D, E, S, U$   $::=$   
 $|$   $X$   
 $|$  **List**  $A$   
 $|$   $\forall(X <: A).B$   
 $|$   $\top$   
 $|$  **SL**  
 $|$  **Unit**  
 $|$  **Nat**  
 $|$   $?$   
 $|$   $A_1 \rightarrow A_2$   
 $|$   $A_1 \times A_2$   
 $|$   $(A)$  S

$\Gamma$   $::=$   
 $|$   $\cdot$   
 $|$   $\Gamma, X <: A$   
 $|$   $\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\begin{array}{c}
\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K\_VAR} \\
\\
\frac{}{\Gamma \vdash \text{Unit} : \star} \quad \text{K\_UNIT} \\
\\
\frac{}{\Gamma \vdash \text{Nat} : \star} \quad \text{K\_NAT}
\end{array}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash ? : \star} \text{K\_UNITYPE} \\
\frac{\Gamma \vdash A : \star}{\Gamma \vdash \text{List } A : \star} \text{K\_LIST} \\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \rightarrow B : \star} \text{K\_ARROW} \\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \text{K\_PROD} \\
\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A). B : \star} \text{K\_FORALL}
\end{array}$$

$\boxed{\Gamma \text{ Ok}}$

$$\begin{array}{c}
\frac{}{\cdot \text{Ok}} \text{OK\_EMPTY} \\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \text{Ok}} \text{OK\_TYPEVAR} \\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \text{Ok}} \text{OK\_VAR}
\end{array}$$

$\boxed{\Gamma \vdash A <: B}$

$$\begin{array}{c}
\frac{\Gamma \text{ Ok}}{\Gamma \vdash A <: A} \text{S\_REFL} \\
\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \text{S\_TRANS} \\
\frac{X <: A \in \Gamma \quad \Gamma \text{ Ok}}{\Gamma \vdash X <: A} \text{S\_VAR} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash A <: \top} \text{S\_TOP} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Nat} <: \text{SL}} \text{S\_NATS} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Unit} <: \text{SL}} \text{S\_UNITS} \\
\frac{\Gamma \vdash A <: \text{SL}}{\Gamma \vdash \text{List } A <: \text{SL}} \text{S\_LISTS} \\
\frac{\Gamma \vdash A <: \text{SL} \quad \Gamma \vdash B <: \text{SL}}{\Gamma \vdash (A \rightarrow B) <: \text{SL}} \text{S\_ARROWS} \\
\frac{\Gamma \vdash A <: \text{SL} \quad \Gamma \vdash B <: \text{SL}}{\Gamma \vdash (A \times B) <: \text{SL}} \text{S\_PRODS} \\
\frac{\Gamma \vdash A <: B}{\Gamma \vdash (\text{List } A) <: (\text{List } B)} \text{S\_LIST} \\
\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \times B_1) <: (A_2 \times B_2)} \text{S\_PROD} \\
\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \rightarrow B_1) <: (A_2 \rightarrow B_2)} \text{S\_ARROW}
\end{array}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash (\forall(X <: A).B_1) <: (\forall(X <: A).B_2)} \text{S\_FORALL}$$

$$\boxed{\Gamma \vdash A \sim B}$$

$$\frac{}{\Gamma \vdash A \sim A} \text{C\_REFL}$$

$$\frac{}{\Gamma \vdash A \sim ?} \text{C\_BOX}$$

$$\frac{}{\Gamma \vdash ? \sim A} \text{C\_UNBOX}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\text{List } A) \sim (\text{List } B)} \text{C\_LIST}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \rightarrow B_1) \sim (A_2 \rightarrow B_2)} \text{C\_ARROW}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \text{C\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall(X <: A).B_1) \sim (\forall(X <: A).B_2)} \text{C\_FORALL}$$

$$\boxed{\Gamma \vdash t : A}$$

$$\frac{x : A \in \Gamma \quad \Gamma \text{Ok}}{\Gamma \vdash x : A} \text{T\_VARP}$$

$$\frac{}{\Gamma \vdash \text{box} : \forall(X <: \text{SL}).(X \rightarrow ?)} \text{T\_BOX}$$

$$\frac{}{\Gamma \vdash \text{unbox} : \forall(X <: \text{SL}).(? \rightarrow X)} \text{T\_UNBOX}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash \text{triv} : \text{Unit}} \text{T\_UNITP}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash 0 : \text{Nat}} \text{T\_ZEROP}$$

$$\frac{\Gamma \vdash t : A \quad \text{nat}(A) = \text{Nat}}{\Gamma \vdash \text{succ } t : \text{Nat}} \text{T\_SUCC}$$

$$\frac{\Gamma \vdash t : C \quad \text{nat}(C) = \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \text{T\_NCASE}$$

$$\frac{\Gamma \text{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: \top).\text{List } X} \text{T\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : \text{List } A_2 \quad \Gamma \vdash A_1 \sim A_2}{\Gamma \vdash t_1 :: t_2 : \text{List } A_2} \text{T\_CONS}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \text{T\_PAIR}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \rightarrow B} \text{T\_LAM}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B} \text{T\_LAM}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C} \quad \text{T\_TYPEAPP}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B} \quad \text{T\_SUB}$$

$$\frac{\Gamma \vdash t : C \quad \text{list}(C) = \text{List } A \quad \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \text{List } A \vdash t_2 : B}{\Gamma \vdash \text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \quad \text{T\_LCASE}$$

$$\frac{\Gamma \vdash t_1 : C \quad \text{fun}(C) = A_1 \rightarrow B_1 \quad \Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_2 \sim A_1}{\Gamma \vdash t_1 t_2 : B_2} \quad \text{T\_APP}$$

$$\frac{\Gamma \vdash t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash \text{fst } t : A_1} \quad \text{T\_FST}$$

$$\frac{\Gamma \vdash t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash \text{snd } t : A_2} \quad \text{T\_SND}$$

$$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A} \quad \text{Cast insertion}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CL\_VAR}$$

$$\overline{\Gamma \vdash \text{box} \Rightarrow \text{box} : \forall(X <: \text{SL}).(X \rightarrow ?)} \quad \text{CL\_BOX}$$

$$\overline{\Gamma \vdash \text{unbox} \Rightarrow \text{unbox} : \forall(X <: \text{SL}).(? \rightarrow X)} \quad \text{CL\_UNBOX}$$

$$\overline{\Gamma \vdash 0 \Rightarrow 0 : \text{Nat}} \quad \text{CL\_ZERO}$$

$$\overline{\Gamma \vdash \text{triv} \Rightarrow \text{triv} : \text{Unit}} \quad \text{CL\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ}(\text{unbox}_{\text{Nat}} t_2) : \text{Nat}} \quad \text{CL\_SUCCU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \text{Nat}}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ } t_2 : \text{Nat}} \quad \text{CL\_SUCC}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash t_1 \Rightarrow t'_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 \Rightarrow t'_2 : A}{\Gamma \vdash (\text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2) \Rightarrow (\text{case } (\text{unbox}_{\text{Nat}} t') \text{ of } 0 \rightarrow t'_1, (\text{succ } x) \rightarrow t'_2) : A} \quad \text{CL\_NCASEU}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : \text{Nat} \quad \Gamma \vdash t_1 \Rightarrow t'_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 \Rightarrow t'_2 : A}{\Gamma \vdash (\text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2) \Rightarrow (\text{case } t' \text{ of } 0 \rightarrow t'_1, (\text{succ } x) \rightarrow t'_2) : A} \quad \text{CL\_NCASE}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \quad \text{CL\_PAIR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst}(\text{split}_{(? \times ?)} t_2) : ?} \quad \text{CL\_FSTU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst } t_2 : A_1} \quad \text{CL\_FST}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd}(\text{split}_{(? \times ?)} t_2) : ?} \quad \text{CL\_SNDU}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd } t_2 : B} \quad \text{CL\_SND} \\
\\
\frac{}{\Gamma \vdash [] \Rightarrow [] : \forall(X <: \top).\text{List } X} \quad \text{CL\_EMPTY} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t'_2 : \text{List } A_2 \quad \Gamma \vdash A_1 \sim A_2 \quad \text{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c \ t'_1) :: t'_2) : \text{List } A} \quad \text{CL\_CONS} \\
\\
\frac{\Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash t_1 \Rightarrow t'_1 : B \quad \Gamma, x : ?, y : \text{List } ? \vdash t_2 \Rightarrow t'_2 : B}{\Gamma \vdash (\text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2) \Rightarrow (\text{case } (\text{split}_{(\text{List } ?)} t') \text{ of } [] \rightarrow t'_1, (x :: y) \rightarrow t'_2) : B} \quad \text{CL\_LCASEU} \\
\\
\frac{\Gamma \vdash t \Rightarrow t' : \text{List } A \quad \Gamma \vdash t_1 \Rightarrow t'_1 : B \quad \Gamma, x : A, y : \text{List } A \vdash t_2 \Rightarrow t'_2 : B}{\Gamma \vdash (\text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2) \Rightarrow (\text{case } t' \text{ of } [] \rightarrow t'_1, (x :: y) \rightarrow t'_2) : B} \quad \text{CL\_LCASE} \\
\\
\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \rightarrow A_2} \quad \text{CL\_LAM} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t'_1 : ? \quad \Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \quad \text{caster}(A_2, ?) = c}{\Gamma \vdash t_1 \ t_2 \Rightarrow (\text{split}_{(? \rightarrow ?)} t'_1) (c \ t'_2) : ?} \quad \text{CL\_APPU} \\
\\
\frac{\Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \quad \Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \rightarrow B \quad \Gamma \vdash A_2 \sim A_1 \quad \text{caster}(A_2, A_1) = c}{\Gamma \vdash t_1 \ t_2 \Rightarrow t'_1 (c \ t'_2) : B} \quad \text{CL\_APP} \\
\\
\frac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall(X <: A).B} \quad \text{CL\_LAM} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A]t_2) : [A/X]C} \quad \text{CL\_TYPEAPP}
\end{array}$$

Definition rules: 72 good 0 bad

Definition rule clauses: 139 good 0 bad