

$termvar, x, y, z, f$   
 $typevar, X, Y, Z$   
 $index, i, j, k$   
 $t, c, v, s, n$

$::=$   
 $|$   $x$   
 $|$  **triv**  
 $|$  **box**  
 $|$  **unbox**  
 $|$   $\Lambda(X <: A).t$   
 $|$   $[A]t$   
 $|$   $\lambda(x : A).t$   
 $|$   $t_1 t_2$   
 $|$   $(t_1, t_2)$   
 $|$  **fst**  $t$   
 $|$  **snd**  $t$   
 $|$  **succ**  $t$   
 $|$   $0$   
 $|$  **case**  $t$  **of**  $t_3 \rightarrow t_1, t_4 \rightarrow t_2$   
 $|$   $\square$   
 $|$   $t :: t'$   
 $|$   $(t)$  S

$K$

$::=$   
 $|$   $\star$

$A, B, C, D, E, S, U$

$::=$   
 $|$   $\top$   
 $|$   $\mathbb{S}$   
 $|$   $X$   
 $|$  **List**  $A$   
 $|$   $\forall(X <: A).B$   
 $|$  **Unit**  
 $|$  **Nat**  
 $|$   $?$   
 $|$   $A_1 \rightarrow A_2$   
 $|$   $A_1 \times A_2$   
 $|$   $(A)$  S

$\Gamma$

$::=$   
 $|$   $\cdot$   
 $|$   $\Gamma, X <: A$   
 $|$   $\Gamma, x : A$

$\boxed{A \sim B}$

$\frac{}{A \sim A}$  C\_REFL  
 $\frac{}{A \sim ?}$  C\_BOX  
 $\frac{}{? \sim A}$  C\_UNBOX

$$\begin{array}{c}
\frac{A \sim B}{(\text{List } A) \sim (\text{List } B)} \quad \text{C\_LIST} \\
\frac{A_2 \sim A_1 \quad B_1 \sim B_2}{(A_1 \rightarrow B_1) \sim (A_2 \rightarrow B_2)} \quad \text{C\_ARROW} \\
\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{(A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C\_PROD} \\
\frac{B_1 \sim B_2}{(\forall(X <: A).B_1) \sim (\forall(X <: A).B_2)} \quad \text{C\_FORALL}
\end{array}$$

$$\boxed{A \sqsubseteq B}$$

$$\begin{array}{c}
\frac{}{A \sqsubseteq ?} \quad \text{P\_U} \\
\frac{}{A \sqsubseteq A} \quad \text{P\_REFL} \\
\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \rightarrow B) \sqsubseteq (C \rightarrow D)} \quad \text{P\_ARROW} \\
\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P\_PROD} \\
\frac{A \sqsubseteq B}{(\text{List } A) \sqsubseteq (\text{List } B)} \quad \text{P\_LIST} \\
\frac{B_1 \sqsubseteq B_2}{(\forall(X <: A).B_1) \sqsubseteq (\forall(X <: A).B_2)} \quad \text{P\_FORALL}
\end{array}$$

$$\boxed{t \sqsubseteq t'}$$

$$\begin{array}{c}
\frac{}{t \sqsubseteq t} \quad \text{TP\_REFL} \\
\frac{t_1 \sqsubseteq t_2}{(\text{succ } t_1) \sqsubseteq (\text{succ } t_2)} \quad \text{TP\_SUCC} \\
\frac{t_1 \sqsubseteq t_4 \quad t_2 \sqsubseteq t_5 \quad t_3 \sqsubseteq t_6}{(\text{case } t_1 \text{ of } 0 \rightarrow t_2, (\text{succ } x) \rightarrow t_3) \sqsubseteq (\text{case } t_4 \text{ of } 0 \rightarrow t_5, (\text{succ } x) \rightarrow t_6)} \quad \text{TP\_NATE} \\
\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1, t_2) \sqsubseteq (t_3, t_4)} \quad \text{TP\_PAIR} \\
\frac{t_1 \sqsubseteq t_2}{(\text{fst } t_1) \sqsubseteq (\text{fst } t_2)} \quad \text{TP\_FST} \\
\frac{t_1 \sqsubseteq t_2}{(\text{snd } t_1) \sqsubseteq (\text{snd } t_2)} \quad \text{TP\_SND} \\
\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP\_CONS} \\
\frac{t_1 \sqsubseteq t_4 \quad t_2 \sqsubseteq t_5 \quad t_3 \sqsubseteq t_6}{(\text{case } t_1 \text{ of } [] \rightarrow t_2, (x :: y) \rightarrow t_3) \sqsubseteq (\text{case } t_4 \text{ of } 0 \rightarrow t_5, (x :: y) \rightarrow t_6)} \quad \text{TP\_LISTE} \\
\frac{t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{(\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP\_FUN}
\end{array}$$

$$\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1 \ t_2) \sqsubseteq (t_3 \ t_4)} \quad \text{TP\_APP}$$

$$\frac{t_1 \sqsubseteq t_2}{(\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \quad \text{TP\_TFUN}$$

$$\frac{t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{[A]t_1 \sqsubseteq [B]t_2} \quad \text{TP\_TAPP}$$

$$\boxed{\Gamma \vdash_{\text{SG}} t : A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\text{SG}} x : A} \quad \text{T\_VARP}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : A}{\Gamma \vdash_{\text{SG}} \text{box}_A t : ?} \quad \text{T\_BOX}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : A}{\Gamma \vdash_{\text{SG}} \text{unbox}_A t : A} \quad \text{T\_UNBOX}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : S}{\Gamma \vdash_{\text{SG}} \text{squash}_S t : ?} \quad \text{T\_SQUASH}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : ?}{\Gamma \vdash_{\text{SG}} \text{split}_S t : S} \quad \text{T\_SPLIT}$$

$$\frac{}{\Gamma \vdash_{\text{SG}} \text{triv} : \text{Unit}} \quad \text{T\_UNITP}$$

$$\frac{}{\Gamma \vdash_{\text{SG}} 0 : \text{Nat}} \quad \text{T\_ZEROP}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : A \quad \text{nat}(A) = \text{Nat}}{\Gamma \vdash_{\text{SG}} \text{succ } t : \text{Nat}} \quad \text{T\_SUCC}$$

$$\frac{\begin{array}{l} \Gamma \vdash_{\text{SG}} t : C \quad \text{nat}(C) = \text{Nat} \quad A_1 \sim A \\ \Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma, x : \text{Nat} \vdash_{\text{SG}} t_2 : A_2 \quad A_2 \sim A \end{array}}{\Gamma \vdash_{\text{SG}} \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{T\_NCASE}$$

$$\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2}{\Gamma \vdash_{\text{SG}} (t_1, t_2) : A_1 \times A_2} \quad \text{T\_PAIR}$$

$$\frac{\Gamma, x : A \vdash_{\text{SG}} t : B}{\Gamma \vdash_{\text{SG}} \lambda(x : A).t : A \rightarrow B} \quad \text{T\_LAM}$$

$$\frac{\begin{array}{l} \Gamma \vdash_{\text{SG}} t_1 : C \\ \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{fun}(C) = A_1 \rightarrow B_1 \quad A_2 \sim A_1 \end{array}}{\Gamma \vdash_{\text{SG}} t_1 \ t_2 : B_1} \quad \text{T\_APP}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash_{\text{SG}} \text{fst } t : A_1} \quad \text{T\_FST}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash_{\text{SG}} \text{snd } t : A_2} \quad \text{T\_SND}$$

$$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CL\_VAR}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \text{Nat}} \quad \text{CL\_ZERO}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash \text{triv} \Rightarrow \text{triv} : \text{Unit}} \text{CI\_TRIV} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ } (\text{unbox}_{\text{Nat}} t_2) : \text{Nat}} \text{CI\_SUCCU} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \text{Nat}}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ } t_2 : \text{Nat}} \text{CI\_SUCC} \\
\\
\frac{
\begin{array}{l}
A_1 \sim A \quad A_2 \sim A \quad \text{caster}(A_2, A) = c_2 \quad \text{caster}(A_1, A) = c_1 \\
\Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma, x : \text{Nat} \vdash t_2 \Rightarrow t'_2 : A_2 \\
t'' = (\text{unbox}_{\text{Nat}} t') \quad t''_1 = (c_1 t'_1) \quad t''_2 = (c_2 t'_2)
\end{array}
}{\Gamma \vdash (\text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2) \Rightarrow (\text{case } t'' \text{ of } 0 \rightarrow t''_1, (\text{succ } x) \rightarrow t''_2) : A} \text{CI\_NCASEU} \\
\\
\frac{
\begin{array}{l}
\text{caster}(A_2, A) = c_2 \quad \text{caster}(A_1, A) = c_1 \\
t''_1 = (c_1 t'_1) \quad t''_2 = (c_2 t'_2) \\
\Gamma \vdash t \Rightarrow t' : \text{Nat} \quad A_1 \sim A \\
\Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma, x : \text{Nat} \vdash t_2 \Rightarrow t'_2 : A_2 \quad A_2 \sim A
\end{array}
}{\Gamma \vdash (\text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2) \Rightarrow (\text{case } t' \text{ of } 0 \rightarrow t'_1, (\text{succ } x) \rightarrow t'_2) : A} \text{CI\_NCASE} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \text{CI\_PAIR} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst } (\text{unbox}_{(?) \times ?} t_2) : ?} \text{CI\_FSTU} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst } t_2 : A_1} \text{CI\_FST} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd } (\text{unbox}_{(?) \times ?} t_2) : ?} \text{CI\_SNDU} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd } t_2 : B} \text{CI\_SND} \\
\\
\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \rightarrow A_2} \text{CI\_LAM} \\
\\
\frac{
\begin{array}{l}
\Gamma \vdash t_1 \Rightarrow t'_1 : ? \\
\Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \quad \text{caster}(A_2, ?) = c
\end{array}
}{\Gamma \vdash t_1 t_2 \Rightarrow (\text{unbox}_{(? \rightarrow ?)} t'_1) (c t'_2) : ?} \text{CI\_APPU} \\
\\
\frac{
\begin{array}{l}
\Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \\
\Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \rightarrow B \quad A_2 \sim A_1 \quad \text{caster}(A_2, A_1) = c
\end{array}
}{\Gamma \vdash t_1 t_2 \Rightarrow t'_1 (c t'_2) : B} \text{CI\_APP}
\end{array}$$

Definition rules: 54 good 0 bad

Definition rule clauses: 107 good 0 bad