

$termvar, x, y, z, f, r, ys$
 $typevar, X, Y, Z$
 $index, i, j, k$
 t, c, s

$::=$
 $|$ x
 $|$ $triv$
 $|$ box
 $|$ $unbox$
 $|$ $error_A$
 $|$ $error$
 $|$ $\Lambda(X <: A).t$
 $|$ $[A]t$
 $|$ $\lambda(x : A).t$
 $|$ $t_1 t_2$
 $|$ (t_1, t_2)
 $|$ $fst t$
 $|$ $snd t$
 $|$ $succ t$
 $|$ 0
 $|$ $case t : A \text{ of } t_3 \rightarrow t_1, t_4 \rightarrow t_2$
 $|$ \square
 $|$ $t :: t'$
 $|$ (t) S
 $|$ $squash$
 $|$ $split$

n, m

$::=$
 $|$ 0
 $|$ $succ n$

v

$::=$
 $|$ $triv$
 $|$ \square
 $|$ $unbox_A$
 $|$ $\Lambda(X <: A).t$
 $|$ $\lambda(x : A).t$
 $|$ n
 $|$ $case t : A \text{ of } t_3 \rightarrow t_1, t_4 \rightarrow t_2$

\mathcal{E}

$::=$
 $|$ \square
 $|$ $v \mathcal{E}$
 $|$ $succ \mathcal{E}$
 $|$ $fst \mathcal{E}$
 $|$ $snd \mathcal{E}$
 $|$ (\mathcal{E}, t)
 $|$ (t, \mathcal{E})
 $|$ $case \mathcal{E} : A \text{ of } t_3 \rightarrow t_1, t_4 \rightarrow t_2$

$A, B, C, D, E, S, U, K, T$

$::=$

	X	
	\top	
	$\text{List } A$	
	$\forall(X <: A).B$	
	\mathbb{S}	
	Unit	
	Nat	
	$?$	
	$A_1 \rightarrow A_2$	
	$A_1 \times A_2$	
	(A)	S

Γ	$::=$
	\cdot
	$\Gamma, X <: A$
	$\Gamma, x : A$
	$x : A$

$$\boxed{\Gamma \vdash A <: B}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash A <: A} \quad \text{S_REFL} \\
\frac{}{\Gamma \vdash A <: \top} \quad \text{S_TOP} \\
\frac{X <: A \in \Gamma}{\Gamma \vdash X <: A} \quad \text{S_VAR} \\
\frac{}{\Gamma \vdash \top <: \mathbb{S}} \quad \text{S_TOPSL} \\
\frac{}{\Gamma \vdash \text{Nat} <: \mathbb{S}} \quad \text{S_NATSL} \\
\frac{}{\Gamma \vdash \text{Unit} <: \mathbb{S}} \quad \text{S_UNITSL} \\
\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \text{List } A <: \mathbb{S}} \quad \text{S_LISTSL} \\
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \rightarrow B <: \mathbb{S}} \quad \text{S_ARROWSL} \\
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}} \quad \text{S_PROD SL} \\
\frac{\Gamma \vdash A <: B}{\Gamma \vdash \text{List } A <: \text{List } B} \quad \text{S_LIST} \\
\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \quad \text{S_PROD} \\
\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \rightarrow B_1 <: A_2 \rightarrow B_2} \quad \text{S_ARROW} \\
\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2} \quad \text{S_FORALL}
\end{array}$$

$$\boxed{\Gamma_1 \sqsubseteq \Gamma_2}$$

$$\frac{}{\overline{\Gamma \sqsubseteq \Gamma}} \text{CTXP_REFL}$$

$$\frac{\Gamma_1 \sqsubseteq \Gamma_2 \quad A \sqsubseteq A' \quad \Gamma_3 \sqsubseteq \Gamma_4}{\Gamma_1, x : A, \Gamma_3 \sqsubseteq \Gamma_2, x : A', \Gamma_4} \text{CTXP_EXT}$$

$$\boxed{A \sqsubseteq B}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{A \sqsubseteq ?} \text{P_U}$$

$$\frac{}{\overline{A \sqsubseteq A}} \text{P_REFL}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \rightarrow B) \sqsubseteq (C \rightarrow D)} \text{P_ARROW}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \text{P_PROD}$$

$$\frac{A \sqsubseteq B}{(\text{List } A) \sqsubseteq (\text{List } B)} \text{P_LIST}$$

$$\frac{B_1 \sqsubseteq B_2}{(\forall (X <: A). B_1) \sqsubseteq (\forall (X <: A). B_2)} \text{P_FORALL}$$

$$\boxed{\Gamma \vdash t \sqsubseteq t'}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \sqsubseteq x} \text{TP_VAR}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \text{split}_{S_1} \sqsubseteq \text{split}_{S_2}} \text{TP_SPLIT}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \text{squash}_{S_1} \sqsubseteq \text{squash}_{S_2}} \text{TP_SQUASH}$$

$$\frac{}{\overline{\Gamma \vdash \text{box} \sqsubseteq \text{box}}} \text{TP_BOX}$$

$$\frac{}{\overline{\Gamma \vdash \text{unbox} \sqsubseteq \text{unbox}}} \text{TP_UNBOX}$$

$$\frac{}{\overline{\Gamma \vdash 0 \sqsubseteq 0}} \text{TP_NAT}$$

$$\frac{}{\overline{\Gamma \vdash \text{triv} \sqsubseteq \text{triv}}} \text{TP_TRIV}$$

$$\frac{}{\overline{\Gamma \vdash [] \sqsubseteq []}} \text{TP_EMPTY}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\text{succ } t_1) \sqsubseteq (\text{succ } t_2)} \text{TP_SUCC}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : \text{Nat} \vdash t_3 \sqsubseteq t_6}{\Gamma \vdash (\text{case } t_1 : \text{Nat of } 0 \rightarrow t_2, (\text{succ } x) \rightarrow t_3) \sqsubseteq (\text{case } t_4 : \text{Nat of } 0 \rightarrow t_5, (\text{succ } x) \rightarrow t_6)} \text{TP_NATE}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1, t_2) \sqsubseteq (t_3, t_4)} \text{TP_PAIR}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\text{fst } t_1) \sqsubseteq (\text{fst } t_2)} \text{TP_FST}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\text{snd } t_1) \sqsubseteq (\text{snd } t_2)} \quad \text{TP_SND} \\
\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP_CONS} \\
\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : A_2, y : \text{List } A_2 \vdash t_3 \sqsubseteq t_6 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\text{case } t_1 : \text{List } A_1 \text{ of } [] \rightarrow t_2, (x :: y) \rightarrow t_3) \sqsubseteq (\text{case } t_4 : \text{List } A_2 \text{ of } 0 \rightarrow t_5, (x :: y) \rightarrow t_6)} \quad \text{TP_LISTE} \\
\frac{\Gamma, x : A_2 \vdash t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP_FUN} \\
\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 t_2) \sqsubseteq (t_3 t_4)} \quad \text{TP_APP} \\
\frac{\Gamma \vdash_{\text{CG}} t : ?}{\Gamma \vdash (\text{unbox}_A t) \sqsubseteq t} \quad \text{TP_UNBOXING} \\
\frac{\Gamma \vdash_{\text{CG}} t : A}{\Gamma \vdash t \sqsubseteq (\text{box}_A t)} \quad \text{TP_BOXING} \\
\frac{\Gamma \vdash_{\text{CG}} t : ?}{\Gamma \vdash (\text{split}_S t) \sqsubseteq t} \quad \text{TP_SPLITING} \\
\frac{\Gamma \vdash_{\text{CG}} t : S}{\Gamma \vdash t \sqsubseteq (\text{squash}_S t)} \quad \text{TP_SQUASHING} \\
\frac{\Gamma, X <: A \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \quad \text{TP_TFUN} \\
\frac{\Gamma \vdash t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{\Gamma \vdash [A]t_1 \sqsubseteq [B]t_2} \quad \text{TP_TAPP} \\
\frac{\Gamma \vdash_{\text{CG}} t : B \quad A \sqsubseteq B}{\Gamma \vdash \text{error}_A \sqsubseteq t} \quad \text{TP_ERROR}
\end{array}$$

$$\boxed{\Gamma \vdash_{\text{CG}} t : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma}{\Gamma \vdash_{\text{CG}} x : A} \quad \text{T_VARP} \\
\frac{x : A \in \Gamma}{\Gamma \vdash_{\text{CG}} x : A} \quad \text{T_VAR} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{box}_A : A \rightarrow ?} \quad \text{T_BOX} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{unbox}_A : ? \rightarrow A} \quad \text{T_UNBOX} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{box} : \forall(X <: \mathbb{S}).(X \rightarrow ?)} \quad \text{T_BoxP} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{unbox} : \forall(X <: \mathbb{S}).(? \rightarrow X)} \quad \text{T_UNBOXP} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{squash}_S : S \rightarrow ?} \quad \text{T_SQUASH} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{split}_S : ? \rightarrow S} \quad \text{T_SPLIT} \\
\frac{}{\Gamma \vdash_{\text{CG}} \text{triv} : \text{Unit}} \quad \text{T_UNITP}
\end{array}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash_{\text{CG}} 0 : \text{Nat}} \quad \text{T_ZEROP} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : \text{Nat}}{\Gamma \vdash_{\text{CG}} \text{succ } t : \text{Nat}} \quad \text{T_SUCC} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : \text{Nat} \quad \Gamma \vdash_{\text{CG}} t_1 : A \quad \Gamma, x : \text{Nat} \vdash_{\text{CG}} t_2 : A}{\Gamma \vdash_{\text{CG}} \text{case } t : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{T_NCASE} \\
\\
\frac{}{\Gamma \vdash_{\text{CG}} [] : \forall (X <: ?). \text{List } X} \quad \text{T_EMPTY} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t_1 : A \quad \Gamma \vdash_{\text{CG}} t_2 : \text{List } A}{\Gamma \vdash_{\text{CG}} t_1 :: t_2 : \text{List } A} \quad \text{T_CONS} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : \text{List } A \quad \Gamma \vdash_{\text{CG}} t_1 : B \quad \Gamma, x : A, y : \text{List } A \vdash_{\text{CG}} t_2 : B}{\Gamma \vdash_{\text{CG}} \text{case } t : \text{List } A \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \quad \text{T_LCASE} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t_1 : A_1 \quad \Gamma \vdash_{\text{CG}} t_2 : A_2}{\Gamma \vdash_{\text{CG}} (t_1, t_2) : A_1 \times A_2} \quad \text{T_PAIR} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\text{CG}} \text{fst } t : A_1} \quad \text{T_FST} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\text{CG}} \text{snd } t : A_2} \quad \text{T_SND} \\
\\
\frac{\Gamma, x : A \vdash_{\text{CG}} t : B}{\Gamma \vdash_{\text{CG}} \lambda(x : A). t : A \rightarrow B} \quad \text{T_LAM} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t_1 : A \rightarrow B \quad \Gamma \vdash_{\text{CG}} t_2 : A}{\Gamma \vdash_{\text{CG}} t_1 t_2 : B} \quad \text{T_APP} \\
\\
\frac{\Gamma, X <: A \vdash_{\text{CG}} t : B}{\Gamma \vdash_{\text{CG}} \Lambda(X <: A). t : \forall (X <: A). B} \quad \text{T_LAM} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : \forall (X <: B). C \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\text{CG}} [A]t : [A/X]C} \quad \text{T_TYPEAPP} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\text{CG}} t : B} \quad \text{T_SUB} \\
\\
\frac{}{\Gamma \vdash_{\text{CG}} \text{error}_A : A} \quad \text{T_ERROR}
\end{array}$$

$\boxed{t_1 \rightsquigarrow t_2}$ call by name

$$\begin{array}{c}
\frac{}{\text{unbox}_A (\text{box}_A t) \rightsquigarrow t} \quad \text{RD_RETRACT} \\
\\
\frac{A \neq B}{\text{unbox}_A (\text{box}_B t) \rightsquigarrow \text{error}_A} \quad \text{RD_RETRACTE} \\
\\
\frac{}{\text{split}_S (\text{squash}_S t) \rightsquigarrow t} \quad \text{RD_RETRACTU} \\
\\
\frac{x : B \vdash_{\text{CG}} \mathcal{E}[x] : A}{\mathcal{E}[\text{error}_B] \rightsquigarrow \text{error}_A} \quad \text{RD_ERROR} \\
\\
\frac{}{\text{case } 0 : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow t_1} \quad \text{RD_NCASE0}
\end{array}$$

$$\begin{array}{c}
\frac{}{\text{case } (\text{succ } t) : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow [t/x]t_2} \text{RD_NCASESUCC} \\
\\
\frac{}{\text{case } [] : \text{List } A \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow t_1} \text{RD_LCASEEMPTY} \\
\\
\frac{}{\text{case } (t_1 :: t_2) : \text{List } A \text{ of } [] \rightarrow t_3, (x :: y) \rightarrow t_4 \rightsquigarrow [t_1/x][t_2/y]t_4} \text{RD_LCASECONS} \\
\\
\frac{}{(\lambda(x : A_1).t_2) t_1 \rightsquigarrow [t_1/x]t_2} \text{RD_BETA} \\
\\
\frac{}{\text{fst}(t_1, t_2) \rightsquigarrow t_1} \text{RD_PROJ1} \\
\\
\frac{}{\text{snd}(t_1, t_2) \rightsquigarrow t_2} \text{RD_PROJ2} \\
\\
\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t} \text{RD_TYPEBETA} \\
\\
\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2} \text{RD_TYPEAPP} \\
\\
\frac{t_1 \rightsquigarrow t_2}{\mathcal{E}[t_1] \rightsquigarrow \mathcal{E}[t_2]} \text{RD_CONG}
\end{array}$$

Definition rules: 83 good 0 bad
 Definition rule clauses: 136 good 0 bad