$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$$
\begin{array}{lcll}
t,\ c,\ v,\ s,\ n & ::= & & \\
 & | & x & \\
 & | & \mathsf{triv} & \\
 & | & \mathsf{squash}_S & \\
 & | & \mathsf{split}_S & \\
 & | & \mathsf{box}_A & \\
 & | & \mathsf{unbox}_A & \\
 & | & \Lambda X <: A.t & \\
 & | & [A]t & \\
 & | & \lambda x : A.t & \\
 & | & t_1\ t_2 & \\
 & | & (t_1, t_2) & \\
 & | & \mathsf{fst}\ t & \\
 & | & \mathsf{snd}\ t & \\
 & | & \mathsf{succ}\ t & \\
 & | & 0 & \\
 & | & \mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2 & \\
 & | & (t) & \mathsf{S} \\
 & & & \\
K & ::= & & \\
 & | & \star & \\
 & & & \\
A,\ B,\ C,\ D,\ E,\ S,\ U & ::= & & \\
 & | & X & \\
 & | & \forall X <: A.B & \\
 & | & \top & \\
 & | & \mathbb{S} & \\
 & | & \mathsf{Unit} & \\
 & | & \mathsf{Nat} & \\
 & | & ? & \\
 & | & A_1 \to A_2 & \\
 & | & A_1 \times A_2 & \\
 & | & (A) & \mathsf{S} \\
 & & & \\
\Gamma & ::= & & \\
 & | & \cdot & \\
 & | & \Gamma, X <: A & \\
 & | & \Gamma, x : A &
\end{array}
$$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{X <: A \in \Gamma}{\Gamma \vdash X : \star}\ \textsc{K\_var}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star}\ \textsc{K\_unit}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star}\ \textsc{K\_nat}$$

$$\frac{}{\Gamma \vdash ? : \star}\ \textsc{K\_unitype}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \text{ K\_ARROW}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \text{ K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall X <: A.B : \star} \text{ K\_FORALL}$$

$\boxed{\Gamma \,\mathrm{Ok}}$

$$\frac{}{\cdot \,\mathrm{Ok}} \text{ OK\_EMPTY}$$

$$\frac{\Gamma \,\mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \,\mathrm{Ok}} \text{ OK\_TYPEVAR}$$

$$\frac{\Gamma \,\mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \,\mathrm{Ok}} \text{ OK\_VAR}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash A <: \top} \text{ S\_TOP}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}} \text{ S\_NAT}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}} \text{ S\_UNIT}$$

$$\frac{X <: A \in \Gamma \quad \Gamma \,\mathrm{Ok}}{\Gamma \vdash X <: A} \text{ S\_VAR}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}} \text{ S\_ARROWSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}} \text{ S\_PRODSL}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \text{ S\_PROD}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2} \text{ S\_ARROW}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall X <: A.B_1 <: \forall X <: A.B_2} \text{ S\_FORALL}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma \,\mathrm{Ok}}{\Gamma \vdash x : A} \text{ VARP}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{box}_A : A \to ?} \text{ BOX}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{unbox}_A : ? \to A} \text{ UNBOX}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{squash}_U : U \to ?} \text{ SQUASHP}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{split}_U : ? \to U}\ \textsc{splitP}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{unitP}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{zeroP}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat}\\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, t_1 \mid\mid x.t_2 : A}\ \textsc{case}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2}\ \textsc{pair}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1}\ \textsc{fst}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2}\ \textsc{snd}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A.t : A \to B}\ \textsc{lam}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B}\ \textsc{app}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda X <: A.t : \forall X <: A.B}\ \textsc{Lam}$$

$$\frac{\Gamma \vdash t : \forall X <: B.C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C}\ \textsc{typeApp}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B}\ \textsc{Sub}$$

$\boxed{t_1 \rightsquigarrow t_2}$

$$\frac{\cdot \vdash t : B \quad \cdot \vdash A <: B}{\mathsf{unbox}_A\, (\mathsf{box}_B\, t) \rightsquigarrow t}\ \textsc{rd\_retractT}$$

$$\frac{\cdot \vdash t : U}{\mathsf{split}_U\, (\mathsf{squash}_U\, t) \rightsquigarrow t}\ \textsc{rd\_retractU}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t'}\ \textsc{rd\_succ}$$

$$\frac{}{\mathsf{case}\, 0\, \mathsf{of}\, t_1 \mid\mid x.t_2 \rightsquigarrow t_1}\ \textsc{rd\_case0}$$

$$\frac{}{\mathsf{case}\, (\mathsf{succ}\, t)\, \mathsf{of}\, t_1 \mid\mid x.t_2 \rightsquigarrow [t/x]t_2}\ \textsc{rd\_caseSucc}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\, t\, \mathsf{of}\, t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\, t'\, \mathsf{of}\, t_1 \mid\mid x.t_2}\ \textsc{rd\_case1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\, t\, \mathsf{of}\, t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\, t\, \mathsf{of}\, t_1' \mid\mid x.t_2}\ \textsc{rd\_case2}$$

$$\frac{t_2 \leadsto t_2'}{\mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2 \leadsto \mathsf{case}\ t\ \mathsf{of}\ t_1 \mid\mid x.t_2'} \quad \textsc{rd\_case3}$$

$$\frac{}{(\lambda x : A_1.t_2)\ t_1 \leadsto [t_1/x]t_2} \quad \textsc{rd\_beta}$$

$$\frac{x \notin \mathsf{FV}(t)}{\lambda x : A_1.t\ x \leadsto t} \quad \textsc{rd\_eta}$$

$$\frac{}{\mathsf{fst}\ (t_1, t_2) \leadsto t_1} \quad \textsc{rd\_proj1}$$

$$\frac{}{\mathsf{snd}\ (t_1, t_2) \leadsto t_2} \quad \textsc{rd\_proj2}$$

$$\frac{}{(\mathsf{fst}\ t, \mathsf{snd}\ t) \leadsto t} \quad \textsc{rd\_etaP}$$

$$\frac{t \leadsto t'}{\lambda x : A.t \leadsto \lambda x : A.t'} \quad \textsc{rd\_lam}$$

$$\frac{t_1 \leadsto t_1'}{t_1\ t_2 \leadsto t_1'\ t_2} \quad \textsc{rd\_app1}$$

$$\frac{t_2 \leadsto t_2'}{t_1\ t_2 \leadsto t_1\ t_2'} \quad \textsc{rd\_app2}$$

$$\frac{t \leadsto t'}{\mathsf{fst}\ t \leadsto \mathsf{fst}\ t'} \quad \textsc{rd\_fst}$$

$$\frac{t \leadsto t'}{\mathsf{snd}\ t \leadsto \mathsf{snd}\ t'} \quad \textsc{rd\_snd}$$

$$\frac{t_1 \leadsto t_1'}{(t_1, t_2) \leadsto (t_1', t_2)} \quad \textsc{rd\_pair1}$$

$$\frac{t_2 \leadsto t_2'}{(t_1, t_2) \leadsto (t_1, t_2')} \quad \textsc{rd\_pair2}$$

$$\frac{}{[A](\Lambda X <: B.t) \leadsto [A/X]t} \quad \textsc{rd\_typeBeta}$$

$$\frac{t_1 \leadsto t_2}{[A]t_1 \leadsto [A]t_2} \quad \textsc{rd\_typeApp}$$

$$\frac{t_1 \leadsto t_2}{\Lambda X <: A.t_1 \leadsto \Lambda X <: A.t_2} \quad \textsc{rd\_Lam}$$

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Definition rules:         59 good    0 bad
Definition rule clauses: 108 good    0 bad
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