

$termvar, x, y, z, f$
 $typevar, X, Y, Z$
 $index, i, j, k$
 t, c, s

$::=$
 $|$ x
 $|$ $triv$
 $|$ $squash_U$
 $|$ $split_U$
 $|$ $Squash_S$
 $|$ $Split_S$
 $|$ box_C
 $|$ $unbox_C$
 $|$ $\lambda x : A. t$
 $|$ $t_1 t_2$
 $|$ (t_1, t_2)
 $|$ $fst\ t$
 $|$ $snd\ t$
 $|$ $succ\ t$
 $|$ 0
 $|$ $case\ t\ of\ 0 \rightarrow t_1, (succ\ x) \rightarrow t_2$
 $|$ (t) S

n
 $::=$
 $|$ 0
 $|$ $succ\ n$

v
 $::=$
 $|$ $triv$
 $|$ $\lambda x : A. t$
 $|$ n
 $|$ $split_U$
 $|$ $squash_U$
 $|$ box_C
 $|$ $unbox_C$

T
 $::=$
 $|$ $Unit$
 $|$ Nat

$A, B, C, D, E, R, X, Y, U, S$
 $::=$
 $|$ $Unit$
 $|$ Nat
 $|$ $?$
 $|$ $A_1 \rightarrow A_2$
 $|$ $A_1 \times A_2$
 $|$ (A) S

Γ
 $::=$
 $|$ \cdot
 $|$ $\Gamma, x : A$

$$\boxed{\Gamma \vdash t : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma}{\Gamma \vdash x : A} \text{ VAR} \\
\\
\frac{}{\Gamma \vdash \text{box}_T : T \rightarrow ?} \text{ BOX} \\
\\
\frac{}{\Gamma \vdash \text{unbox}_T : ? \rightarrow T} \text{ UNBOX} \\
\\
\frac{}{\Gamma \vdash \text{Box}_A : A \rightarrow ?} \text{ BOXG} \\
\\
\frac{}{\Gamma \vdash \text{Unbox}_A : ? \rightarrow A} \text{ UNBOXG} \\
\\
\frac{}{\Gamma \vdash \text{squash}_U : U \rightarrow ?} \text{ SQUASH} \\
\\
\frac{}{\Gamma \vdash \text{split}_U : ? \rightarrow U} \text{ SPLIT} \\
\\
\frac{}{\Gamma \vdash \text{Split}_S : S \rightarrow ?} \text{ SPLITG} \\
\\
\frac{}{\Gamma \vdash \text{Squash}_S : ? \rightarrow S} \text{ SQUASHG} \\
\\
\frac{}{\Gamma \vdash \text{triv} : \text{Unit}} \text{ UNIT} \\
\\
\frac{}{\Gamma \vdash 0 : \text{Nat}} \text{ ZERO} \\
\\
\frac{\Gamma \vdash t : \text{Nat}}{\Gamma \vdash \text{succ } t : \text{Nat}} \text{ SUCC} \\
\\
\frac{\Gamma \vdash t : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \text{ CASE} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \text{ PAIR} \\
\\
\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \text{fst } t : A_1} \text{ FST} \\
\\
\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \text{snd } t : A_2} \text{ SND} \\
\\
\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A. t : A \rightarrow B} \text{ LAM} \\
\\
\frac{\Gamma \vdash t_1 : A \rightarrow B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1 t_2 : B} \text{ APP}
\end{array}$$

$$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma}{\Gamma \vdash x \rightsquigarrow x : A} \text{ RD_VAR} \\
\\
\frac{\Gamma \vdash t : T}{\Gamma \vdash \text{unbox}_T (\text{box}_T t) \rightsquigarrow t : T} \text{ RD_RETRACT}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : T}{\Gamma \vdash \text{unbox}_T t_1 \rightsquigarrow \text{unbox}_T t_2 : T} \quad \text{RD_UNBOX} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash \text{Unbox}_A (\text{Box}_A t) \rightsquigarrow t : A} \quad \text{RD_RETRACTG} \\
\\
\frac{\Gamma \vdash t : S}{\Gamma \vdash \text{Split}_S (\text{Squash}_S t) \rightsquigarrow t : S} \quad \text{RD_RETRACTSG} \\
\\
\frac{\Gamma \vdash t : U}{\Gamma \vdash \text{split}_U (\text{squash}_U t) \rightsquigarrow t : U} \quad \text{RD_RETRACTU} \\
\\
\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : U}{\Gamma \vdash \text{split}_U t_1 \rightsquigarrow \text{split}_U t_2 : U} \quad \text{RD_SPLIT} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : \text{Nat}}{\Gamma \vdash \text{succ } t \rightsquigarrow \text{succ } t' : \text{Nat}} \quad \text{RD_SUCC} \\
\\
\frac{\Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } 0 \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow t_1 : A} \quad \text{RD_CASE0} \\
\\
\frac{\Gamma \vdash t : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } (\text{succ } t) \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow [t/x]t_2 : A} \quad \text{RD_CASESUCC} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow \text{case } t' \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{RD_CASE} \\
\\
\frac{\Gamma, x : A_1 \vdash t_1 : A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash (\lambda x : A_1. t_1) t_2 \rightsquigarrow [t_2/x]t_2 : A_2} \quad \text{RD_BETA} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \text{fst } (t_1, t_2) \rightsquigarrow t_1 : A_1} \quad \text{RD_PROJ1} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \text{snd } (t_1, t_2) \rightsquigarrow t_2 : A_2} \quad \text{RD_PROJ2} \\
\\
\frac{\Gamma \vdash t_1 \rightsquigarrow t'_1 : A_1 \rightarrow A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1 t_2 \rightsquigarrow t'_1 t_2 : A_2} \quad \text{RD_APP1} \\
\\
\frac{\Gamma \vdash v : A_1 \rightarrow A_2 \quad \Gamma \vdash t \rightsquigarrow t' : A_1}{\Gamma \vdash v t \rightsquigarrow v t' : A_2} \quad \text{RD_APP2} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \text{fst } t \rightsquigarrow \text{fst } t' : A_1} \quad \text{RD_FST} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \text{snd } t \rightsquigarrow \text{snd } t' : A_2} \quad \text{RD_SND} \\
\\
\frac{\Gamma \vdash t_1 \rightsquigarrow t'_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t'_1, t_2) : A_1 \times A_2} \quad \text{RD_PAIR1} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t'_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t'_2) : A_1 \times A_2} \quad \text{RD_PAIR2}
\end{array}$$

Definition rules: 38 good 0 bad
 Definition rule clauses: 69 good 0 bad