$termvar$, x, y, z, f
$typevar$, X, Y, Z
$index$, i, j, k

$t,\ c,\ v,\ s,\ n$	::=		term
	$\vert$	x	variable
	$\vert$	triv	unit
	$\vert$	squash_S	injection of the retract
	$\vert$	split_S	surjection of the retract
	$\vert$	box_C	generalize to the untyped universe
	$\vert$	unbox_C	specialize the untyped universe to a specific type
	$\vert$	$\Lambda X.t$	
	$\vert$	$[A]t$	
	$\vert$	$\langle A\rangle t$	type cast
	$\vert$	$\lambda x : A.t$	λ-abstraction
	$\vert$	$t_1\ t_2$	function application
	$\vert$	(t_1, t_2)	pair constructor
	$\vert$	$\mathsf{fst}\ t$	first projection
	$\vert$	$\mathsf{snd}\ t$	second projection
	$\vert$	$\mathsf{succ}\ t$	successor function
	$\vert$	0	zero
	$\vert$	$\mathsf{case}\ t\ \mathsf{of}\ t_1 \vert\vert x.t_2$	
	$\vert$	(t) S	
h	::=		head-normal forms
	$\vert$	triv	
	$\vert$	split_S	
	$\vert$	squash_S	
	$\vert$	box_C	
	$\vert$	unbox_C	
	$\vert$	$\lambda x : A.t$	
	$\vert$	(t_1, t_2)	
	$\vert$	$\mathsf{fst}\ t$	
	$\vert$	$\mathsf{snd}\ t$	
	$\vert$	$\mathsf{succ}\ t$	
	$\vert$	0	
T	::=		terminating types
	$\vert$	Unit	unit type
	$\vert$	Nat	natural number type
	$\vert$	$T_1 \to T_2$	function type
	$\vert$	$T_1 \times T_2$	cartesian product type
	$\vert$	(T) S	
K	::=		
	$\vert$	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U$	::=		type
	$\vert$	X	
	$\vert$	$\forall X.A$	
	$\vert$	Unit	unit type

	$\mid$ 1	
	$\mid$ Nat	natural number type
	$\mid$ $?$	untyped universe
	$\mid$ $A_1 \to A_2$	function type
	$\mid$ $A_1 \times A_2$	cartesian product type
	$\mid$ (A) S	

Γ	$::=$	typing context
	$\mid$ $\cdot$	empty context
	$\mid$ $\Gamma, X : \star$	
	$\mid$ $\Gamma, x : A$	cons

vd	$::=$
	$\mid$ $\vdash$
	$\mid$ $\nvdash$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{X : \star \in \Gamma}{\Gamma \vdash X : \star}\ \textsc{K\_var}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star}\ \textsc{K\_unit}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star}\ \textsc{K\_nat}$$

$$\frac{}{\Gamma \vdash ? : \star}\ \textsc{K\_unitype}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star}\ \textsc{K\_arrow}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star}\ \textsc{K\_prod}$$

$$\frac{\Gamma, X : \star \vdash A : \star}{\Gamma \vdash \forall X.A : \star}\ \textsc{K\_forall}$$

$\boxed{\Gamma\,\mathrm{Ok}}$

$$\frac{}{\cdot\,\mathrm{Ok}}\ \textsc{Ok\_empty}$$

$$\frac{\Gamma\,\mathrm{Ok}}{(\Gamma, X : \star)\,\mathrm{Ok}}\ \textsc{Ok\_typeVar}$$

$$\frac{\Gamma\,\mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\,\mathrm{Ok}}\ \textsc{Ok\_var}$$

$\boxed{A \sim B}$ A is consistent with B

$$\frac{}{A \sim A}\ \textsc{refl}$$

$$\frac{}{A \sim ?}\ \textsc{box}$$

$$\frac{}{? \sim A}\ \textsc{unbox}$$

$$\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{A_1 \to B_1 \sim A_2 \to B_2}\ \textsc{arrow}$$

$$\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{A_1 \times B_1 \sim A_2 \times B_2}\ \textsc{prod}$$

$\boxed{\Gamma v d t : A}$ t has type A in context Γ

$$\frac{x : A \in \Gamma}{\Gamma \vdash x : A}\ \textsc{var}$$

$$\frac{x : A \in \Gamma \quad \Gamma\,\mathrm{Ok}}{\Gamma \vdash x : A}\ \textsc{var}\mathrm{P}$$

$$\frac{}{\Gamma \vdash \mathsf{box}_C : C \to ?}\ \textsc{Box}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash \mathsf{box}_C : C \to ?}\ \textsc{Box}\mathrm{P}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox}_C : ? \to C}\ \textsc{Unbox}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash \mathsf{unbox}_C : ? \to C}\ \textsc{Unbox}\mathrm{P}$$

$$\frac{}{\Gamma \vdash \mathsf{Box}_A : A \to ?}\ \textsc{Box}\mathrm{G}$$

$$\frac{}{\Gamma \vdash \mathsf{Unbox}_A : ? \to A}\ \textsc{Unbox}\mathrm{G}$$

$$\frac{}{\Gamma \vdash \mathsf{squash}_U : U \to ?}\ \textsc{squash}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash \mathsf{squash}_U : U \to ?}\ \textsc{squash}\mathrm{P}$$

$$\frac{}{\Gamma \vdash \mathsf{split}_U : ? \to U}\ \textsc{split}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash \mathsf{split}_U : ? \to U}\ \textsc{split}\mathrm{P}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{unit}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{unit}\mathrm{P}$$

$$\frac{}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{zero}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{zero}\mathrm{P}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, t_1 \mid\mid x.t_2 : A}\ \textsc{case}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2}\ \textsc{pair}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1}\ \textsc{fst}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2} \quad \textsc{snd}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A_1.t : A \to B} \quad \textsc{lam}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B} \quad \textsc{app}$$

$$\frac{\Gamma, X : \star \vdash t : A}{\Gamma \vdash \Lambda X.t : \forall X.A} \quad \textsc{Lam}$$

$$\frac{\Gamma \vdash t : \forall X.B \quad \Gamma \vdash A : \star}{\Gamma \vdash [A]t : [A/X]B} \quad \textsc{typeApp}$$

$$\frac{\Gamma \vdash t : ?}{\Gamma \vdash \mathsf{succ}\, t : ?} \quad \textsc{succU}$$

$$\frac{\Gamma \vdash t : ?}{\Gamma \vdash \mathsf{fst}\, t : ?} \quad \textsc{fstU}$$

$$\frac{\Gamma \vdash t : ?}{\Gamma \vdash \mathsf{snd}\, t : ?} \quad \textsc{sndU}$$

$$\frac{\Gamma \vdash t_1 : ? \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : ?} \quad \textsc{appU}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 : A_1 \to B \\ \Gamma \vdash t_2 : A_2 \quad A_1 \sim A_2\end{array}}{\Gamma \vdash t_1\, t_2 : B} \quad \textsc{appC}$$

$$\frac{\Gamma \vdash t : A_1 \times B \quad A_1 \sim A_2}{\Gamma \vdash \mathsf{fst}\, t : A_2} \quad \textsc{fstC}$$

$$\frac{\Gamma \vdash t : A \times B_1 \quad B_1 \sim B_2}{\Gamma \vdash \mathsf{snd}\, t : B_2} \quad \textsc{sndC}$$

$$\frac{\Gamma \vdash t : A \quad A \sim B}{\Gamma \vdash \langle B \rangle t : B} \quad \textsc{cast}$$

$\boxed{\Gamma \vdash t_1 \leadsto t_2 : A}$ t_1 reduces to t_2 with type A in context Γ

$$\frac{\Gamma \vdash s : A}{\Gamma \vdash s \leadsto s : A} \quad \textsc{rd\_values}$$

$$\frac{\Gamma \vdash t : C}{\Gamma \vdash \mathsf{unbox}_C\, (\mathsf{box}_C\, t) \leadsto t : C} \quad \textsc{rd\_retracT}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathsf{Unbox}_A\, (\mathsf{Box}_A\, t) \leadsto t : A} \quad \textsc{rd\_retracTG}$$

$$\frac{\Gamma \vdash t : U}{\Gamma \vdash \mathsf{split}_U\, (\mathsf{squash}_U\, t) \leadsto t : U} \quad \textsc{rd\_retractU}$$

$$\frac{\Gamma \vdash t \leadsto t' : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t \leadsto \mathsf{succ}\, t' : \mathsf{Nat}} \quad \textsc{rd\_succ}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A}{\Gamma \vdash \mathsf{case}\, 0\, \mathsf{of}\, t_1 \mid\mid x.t_2 \leadsto t_1 : A} \quad \textsc{rd\_case0}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat}\\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\,(\mathsf{succ}\,t)\,\mathsf{of}\,t_1 \mid\mid x.t_2 \rightsquigarrow [t/x]t_2 : A}\ \textsc{rd\_caseSucc}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \rightsquigarrow t' : \mathsf{Nat}\\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\,t\,\mathsf{of}\,t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\,t'\,\mathsf{of}\,t_1 \mid\mid x.t_2 : A}\ \textsc{rd\_case1}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat}\\ \Gamma \vdash t_1 \rightsquigarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\,t\,\mathsf{of}\,t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\,t\,\mathsf{of}\,t_1' \mid\mid x.t_2 : A}\ \textsc{rd\_case2}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat}\\ \Gamma \vdash t : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \rightsquigarrow t_2' : A\end{array}}{\Gamma \vdash \mathsf{case}\,t\,\mathsf{of}\,t_1 \mid\mid x.t_2 \rightsquigarrow \mathsf{case}\,t\,\mathsf{of}\,t_1 \mid\mid x.t_2' : A}\ \textsc{rd\_case3}$$

$$\frac{\Gamma, x : A_1 \vdash t_2 : A_2 \quad \Gamma \vdash t_1 : A_1}{\Gamma \vdash (\lambda x : A_1.t_2)\,t_1 \rightsquigarrow [t_1/x]t_2 : A_2}\ \textsc{rd\_beta}$$

$$\frac{\Gamma \vdash t : A_1 \to A_2 \quad x \notin \mathsf{FV}(t)}{\Gamma \vdash \lambda x : A_1.t\,x \rightsquigarrow t : A_1 \to A_2}\ \textsc{rd\_eta}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{fst}\,(t_1, t_2) \rightsquigarrow t_1 : A_1}\ \textsc{rd\_proj1}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{snd}\,(t_1, t_2) \rightsquigarrow t_2 : A_2}\ \textsc{rd\_proj2}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash (\mathsf{fst}\,t, \mathsf{snd}\,t) \rightsquigarrow t : A_1 \times A_2}\ \textsc{rd\_etaP}$$

$$\frac{\Gamma, x : A_1 \vdash t \rightsquigarrow t' : A_2}{\Gamma \vdash \lambda x : A_1.t \rightsquigarrow \lambda x : A_1.t' : A_1 \to A_2}\ \textsc{rd\_lam}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \to A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1\,t_2 \rightsquigarrow t_1'\,t_2 : A_2}\ \textsc{rd\_app1}$$

$$\frac{\Gamma \vdash t_1 : A_1 \to A_2 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_1}{\Gamma \vdash t_1\,t_2 \rightsquigarrow t_1\,t_2' : A_2}\ \textsc{rd\_app2}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\,t \rightsquigarrow \mathsf{fst}\,t' : A_1}\ \textsc{rd\_fst}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\,t \rightsquigarrow \mathsf{snd}\,t' : A_2}\ \textsc{rd\_snd}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1', t_2) : A_1 \times A_2}\ \textsc{rd\_pair1}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t_2') : A_1 \times A_2}\ \textsc{rd\_pair2}$$

$$\frac{\Gamma, X : \star \vdash t : B}{\Gamma \vdash [A](\Lambda X.t) \rightsquigarrow [A/X]t : [A/X]B}\ \textsc{rd\_typeBeta}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : \forall X.B}{\Gamma \vdash [A]t_1 \rightsquigarrow [A]t_2 : [A/X]B}\ \textsc{rd\_typeApp}$$

$$\frac{\Gamma, X : \star \vdash t_1 \rightsquigarrow t_2 : A}{\Gamma \vdash \Lambda X.t_1 \rightsquigarrow \Lambda X.t_2 : \forall X.A}\ \textsc{rd\_Lam}$$

$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$ Reduction for annotated Siek16

$$\frac{\Gamma \vdash v : A}{\Gamma \vdash v \rightsquigarrow v : A} \quad \text{RDA\_VALUES}$$

$$\frac{\Gamma \vdash \mathsf{drop\text{-}cast}\ v : C}{\Gamma \vdash \langle C \rangle v \rightsquigarrow \mathsf{drop\text{-}cast}\ v : C} \quad \text{RDA\_CASTA}$$

$$\frac{\Gamma \vdash t : ?}{\Gamma \vdash \langle \mathsf{Nat} \rangle (\mathsf{succ}\ t) \rightsquigarrow \mathsf{succ}\ \langle \mathsf{Nat} \rangle t : \mathsf{Nat}} \quad \text{RDA\_CASTNAT}$$

$$\frac{\Gamma \vdash t : A_1 \to B_1 \quad (A_1 \to B_1) \sim (A_2 \to B_2)}{\Gamma \vdash \langle A_2 \to B_2 \rangle t \rightsquigarrow \lambda y : A_2. \langle B_2 \rangle (t\ \langle A_1 \rangle y) : A_2 \to B_2} \quad \text{RDA\_CASTARROW}$$

$$\frac{\Gamma \vdash t : A_1 \times B_1 \quad (A_1 \times B_1) \sim (A_2 \times B_2)}{\Gamma \vdash \langle A_2 \times B_2 \rangle t \rightsquigarrow (\langle A_2 \rangle (\mathsf{fst}\ t), \langle B_2 \rangle (\mathsf{snd}\ t)) : A_2 \times B_2} \quad \text{RDA\_CASTPAIR}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : A \quad A \sim B}{\Gamma \vdash \langle B \rangle t_1 \rightsquigarrow \langle B \rangle t_2 : B} \quad \text{RDA\_CAST}$$

$$\frac{\Gamma, x : A_1 \vdash t_2 : A_2 \quad \Gamma \vdash t_1 : A_1}{\Gamma \vdash (\lambda x : A_1.t_2)\ t_1 \rightsquigarrow [t_1/x] t_2 : A_2} \quad \text{RDA\_BETA}$$

$$\frac{\Gamma \vdash t : A_1 \to A_2 \quad x \notin \mathsf{FV}(t)}{\Gamma \vdash \lambda x : A_1.t\ x \rightsquigarrow t : A_1 \to A_2} \quad \text{RDA\_ETA}$$

$$\frac{\Gamma, x : A_1 \vdash t \rightsquigarrow t' : A_2}{\Gamma \vdash \lambda x : A_1.t \rightsquigarrow \lambda x : A_1.t' : A_1 \to A_2} \quad \text{RDA\_LAM}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \to A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1\ t_2 \rightsquigarrow t_1'\ t_2 : A_2} \quad \text{RDA\_APP1}$$

$$\frac{\Gamma \vdash t_1 : A_1 \to A_2 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_1}{\Gamma \vdash t_1\ t_2 \rightsquigarrow t_1\ t_2' : A_2} \quad \text{RDA\_APP2}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\ t \rightsquigarrow \mathsf{fst}\ t' : A_1} \quad \text{RDA\_FST}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\ t \rightsquigarrow \mathsf{snd}\ t' : A_2} \quad \text{RDA\_SND}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash (\mathsf{fst}\ t, \mathsf{snd}\ t) \rightsquigarrow t : A_1 \times A_2} \quad \text{RDA\_ETAP}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1', t_2) : A_1 \times A_2} \quad \text{RDA\_PAIR1}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t_2') : A_1 \times A_2} \quad \text{RDA\_PAIR2}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ Cast insertion from Siek16

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CI\_VAR}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : A} \quad \text{CI\_ZERO}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \Rightarrow \mathsf{triv} : \mathsf{Unit}} \quad \text{CI\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t_1 \Rightarrow \mathsf{succ}\, t_2 : \mathsf{Nat}} \quad \textsc{ci\_succ}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \quad \textsc{ci\_pair}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times B \quad A_1 \sim A_2 \quad A_1 \neq A_2}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, \langle A_2 \times B \rangle t_2 : A_2} \quad \textsc{ci\_fst}1$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, t_2 : A} \quad \textsc{ci\_fst}2$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B_1 \quad B_1 \sim B_2 \quad B_1 \neq B_2}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, \langle A \times B_2 \rangle t_2 : B_2} \quad \textsc{ci\_snd}1$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, t_2 : B} \quad \textsc{ci\_snd}2$$

$$\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda x : A_1 . t_1 \Rightarrow \lambda x : A_1 . t_2 : A_1 \to A_2} \quad \textsc{ci\_lam}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : ? \quad \Gamma \vdash t_2 \Rightarrow t_4 : A}{\Gamma \vdash t_1\, t_2 \Rightarrow (\langle A \to ? \rangle t_3)\, t_4 : ?} \quad \textsc{ci\_app}1$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \to B \\ \Gamma \vdash t_2 \Rightarrow t_4 : A_2 \quad A_1 \sim A_2 \quad A_1 \neq A_2\end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow t_1\, \langle A_1 \rangle t : B} \quad \textsc{ci\_app}2$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \to A_2 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_1}{\Gamma \vdash t_1\, t_2 \Rightarrow t_3\, t_4 : A_2} \quad \textsc{ci\_app}3$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{succ}\, t_1 \Rightarrow \langle ? \rangle \mathsf{succ}\, \langle \mathsf{Nat} \rangle t_2 : ?} \quad \textsc{ci\_succ}U$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, \langle ? \times ? \rangle t_2 : ?} \quad \textsc{ci\_fst}U$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, \langle ? \times ? \rangle t_2 : ?} \quad \textsc{ci\_snd}U$$

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Definition rules:        105 good    0 bad
Definition rule clauses: 200 good    0 bad
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