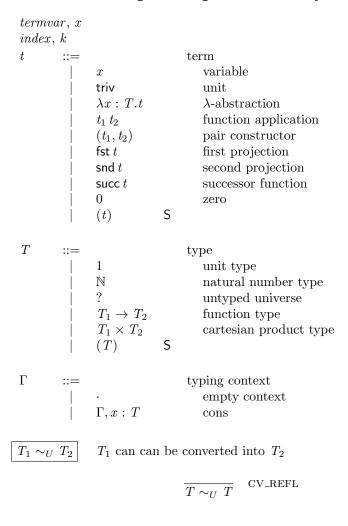
Gradual Typing from a Categorical Perspective

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References

A The Complete Spec of Grady



$$\begin{array}{c|c} T_1 \sim_U T_2 & T_2 \sim_U T_3 \\ \hline T_1 \sim_U T_3 & \text{CV_TRANS} \\ \hline (? \rightarrow ?) \sim_U ? & \text{CV_INJ} \\ \hline ? \sim_U (? \rightarrow ?) & \text{CV_SURJ} \\ \hline T_1 \sim_U T_1' \\ \hline (T_1 \times T_2) \sim_U (T_1' \times T_2) & \text{CV_PAIR1} \\ \hline \frac{T_2 \sim_U T_2'}{(T_1 \times T_2) \sim_U (T_1 \times T_2')} & \text{CV_PAIR2} \\ \hline \frac{T_1 \sim_U T_1'}{(T_1 \rightarrow T_2) \sim_U (T_1' \rightarrow T_2)} & \text{CV_FUN1} \\ \hline \frac{T_2 \sim_U T_2'}{(T_1 \rightarrow T_2) \sim_U (T_1 \rightarrow T_2')} & \text{CV_FUN2} \\ \hline \end{array}$$

 $T_1 \sim T_2$ T_1 is consistent with T_2

$$\begin{array}{ccc} \overline{T \sim T} & \text{CS_REFL} \\ \hline \hline ? \sim T & \text{CS_UL} \\ \hline \overline{T \sim ?} & \text{CS_UR} \\ \hline \hline \frac{T_1 \sim_U T_2}{T_1 \sim T_2} & \text{CS_CONV} \\ \hline \frac{T_1 \sim T_1' & T_2 \sim T_2'}{(T_1 \times T_2) \sim (T_1' \times T_2')} & \text{CS_PAIR} \\ \hline \frac{T_1 \sim T_1' & T_2 \sim T_2'}{(T_1 \rightarrow T_2) \sim (T_1' \rightarrow T_2')} & \text{CS_ARROW} \end{array}$$

 $\Gamma \vdash t : T$ t has type T in context Γ

$$\begin{array}{ccc} x:T\in\Gamma\\ \hline \Gamma\vdash x:T & \text{VAR} \\ \hline \hline \Gamma\vdash \text{triv}:1 & \text{UNIT} \\ \hline \hline \Gamma\vdash \text{triv}:1 & \text{SERO} \\ \hline \hline \Gamma\vdash t:\mathbb{N}\\ \hline \hline \Gamma\vdash \text{succ}\ t:\mathbb{N} & \text{SUCC} \\ \hline \hline \Gamma\vdash t_1:T_1 & \Gamma\vdash t_2:T_2\\ \hline \hline \Gamma\vdash (t_1,t_2):T_1\times T_2 & \text{PAIR} \\ \hline \end{array}$$

$$\begin{array}{ccc} \frac{\Gamma \vdash t: T_1 \times T_2}{\Gamma \vdash \mathsf{fst} \ t: T_1} & \mathsf{FST} \\ \\ \frac{\Gamma \vdash t: T_1 \times T_2}{\Gamma \vdash \mathsf{snd} \ t: T_2} & \mathsf{SND} \\ \\ \frac{\Gamma, x: T_1 \vdash t: T_2}{\Gamma \vdash \lambda x: T_1.t: T_1 \to T_2} & \mathsf{ABS} \\ \\ \frac{\Gamma \vdash t: T_1 \quad T_1 \sim_U T_2}{\Gamma \vdash t: T_2} & \mathsf{U} \\ \\ \frac{\Gamma \vdash t_1: T_1 \to T_2 \quad \Gamma \vdash t_2: T_3 \quad T_3 \sim T_1}{\Gamma \vdash t_1 \ t_2: T_2} & \mathsf{APP} \end{array}$$

 $t_1 \rightsquigarrow t_2$ t_1 reduces to t_2