

termvar, x, y, z, f

typevar, X, Y, Z

index, i, j, k

t, c, v, s, n

$::=$
 $| \quad x$
 $| \quad \text{triv}$
 $| \quad t : \text{ctag}$
 $| \quad \lambda x : A. t$
 $| \quad t_1 t_2$
 $| \quad (t_1, t_2)$
 $| \quad \text{fst } t$
 $| \quad \text{snd } t$
 $| \quad \text{succ } t$
 $| \quad 0$
 $| \quad (t) \quad \quad \quad \text{S}$

T

$::=$
 $| \quad \text{Unit}$
 $| \quad \text{Nat}$

R

$::=$
 $| \quad \text{Unit}$
 $| \quad \text{Nat}$
 $| \quad ? \rightarrow ?$

ctag

$::=$
 $| \quad A$
 $| \quad \text{ctag} \Rightarrow \text{ctag}'$
 $| \quad \text{ctag}$

A, B, C, D, E, S, U

$::=$
 $| \quad \text{Unit}$
 $| \quad \text{Nat}$
 $| \quad ?$
 $| \quad A_1 \rightarrow A_2$
 $| \quad A_1 \times A_2$
 $| \quad (A) \quad \quad \quad \text{S}$

Γ

$::=$
 $| \quad \cdot$
 $| \quad \Gamma, x : A$

vd

$::=$
 $| \quad \vdash$
 $| \quad \not\vdash$

$\boxed{A \sim B}$

$\overline{A \sim A} \quad \text{REFL}$

$\overline{A \sim ?} \quad \text{BOX}$

$$\boxed{\Gamma \vdash_S t : A}$$

$$\begin{array}{c}
\frac{}{? \sim A} \text{ UNBOX} \\
\\
\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{A_1 \rightarrow B_1 \sim A_2 \rightarrow B_2} \text{ ARROW} \\
\\
\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{A_1 \times B_1 \sim A_2 \times B_2} \text{ PROD} \\
\\
\frac{x : A \in \Gamma}{\Gamma \vdash_S x : A} \text{ S_VAR} \\
\\
\frac{}{\Gamma \vdash_S \text{triv} : \text{Unit}} \text{ S_UNIT} \\
\\
\frac{}{\Gamma \vdash_S 0 : \text{Nat}} \text{ S_ZERO} \\
\\
\frac{\Gamma \vdash_S t : A \quad \text{nat}(A) = \text{Nat}}{\Gamma \vdash_S \text{succ } t : \text{Nat}} \text{ S_SUCC} \\
\\
\frac{\Gamma \vdash_S t_1 : A_1 \quad \Gamma \vdash_S t_2 : A_2}{\Gamma \vdash_S (t_1, t_2) : A_1 \times A_2} \text{ S_PAIR} \\
\\
\frac{\Gamma \vdash_S t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash_S \text{fst } t : A_1} \text{ S_FST} \\
\\
\frac{\Gamma \vdash_S t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash_S \text{snd } t : A_2} \text{ S_SND} \\
\\
\frac{\Gamma, x : A \vdash_S t : B}{\Gamma \vdash_S \lambda x : A_1. t : A \rightarrow B} \text{ S_LAM} \\
\\
\frac{\Gamma \vdash_S t_1 : C \quad \text{fun}(C) = A_1 \rightarrow B_1 \quad \Gamma \vdash_S t_2 : A_2 \quad A_2 \sim A_1}{\Gamma \vdash_S t_1 t_2 : B_1} \text{ S_APP}
\end{array}$$

$$\boxed{\Gamma \vdash_C t : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma}{\Gamma \vdash_C x : A} \text{ C_VAR} \\
\\
\frac{}{\Gamma \vdash_C \text{triv} : \text{Unit}} \text{ C_UNIT} \\
\\
\frac{}{\Gamma \vdash_C 0 : \text{Nat}} \text{ C_ZERO} \\
\\
\frac{\Gamma \vdash_C t : \text{Nat}}{\Gamma \vdash_C \text{succ } t : \text{Nat}} \text{ C_SUCC} \\
\\
\frac{\Gamma \vdash_C t_1 : A_1 \quad \Gamma \vdash_C t_2 : A_2}{\Gamma \vdash_C (t_1, t_2) : A_1 \times A_2} \text{ C_PAIR} \\
\\
\frac{\Gamma \vdash_C t : A_1 \times A_2}{\Gamma \vdash_C \text{fst } t : A_1} \text{ C_FST} \\
\\
\frac{\Gamma \vdash_C t : A_1 \times A_2}{\Gamma \vdash_C \text{snd } t : A_2} \text{ C_SND} \\
\\
\frac{\Gamma, x : A \vdash_C t : B}{\Gamma \vdash_C \lambda x : A_1. t : A \rightarrow B} \text{ C_LAM}
\end{array}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} t_1 : A \rightarrow B \quad \Gamma \vdash_{\mathbf{C}} t_2 : A}{\Gamma \vdash_{\mathbf{C}} t_1 t_2 : B} \quad \mathbf{C_APP}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} t : A \quad A \sim B}{\Gamma \vdash_{\mathbf{C}} (t : A \Rightarrow B) : B} \quad \mathbf{C_CAST}$$

$$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : A}{\Gamma \vdash v \rightsquigarrow v : A} \quad \mathbf{rDA_VALUES}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : T}{\Gamma \vdash v : T \Rightarrow T \rightsquigarrow v : T} \quad \mathbf{rDA_CASTId}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : ?}{\Gamma \vdash v : ? \Rightarrow ? \rightsquigarrow v : ?} \quad \mathbf{rDA_CASTU}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : R}{\Gamma \vdash v : R \Rightarrow ? \rightsquigarrow v : R} \quad \mathbf{rDA_SUCCEED}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v_1 : A_1 \rightarrow B_1 \quad \Gamma \vdash_{\mathbf{C}} v_2 : A_2}{\Gamma \vdash (v_1 : (A_1 \rightarrow B_1) \Rightarrow (A_2 \rightarrow B_2)) v_2 \rightsquigarrow v_1 (v_2 : A_2 \Rightarrow A_1) : B_1 \Rightarrow B_2 : B} \quad \mathbf{rDA_CASTArrow}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : A \quad A \sim T \quad T \neq R \quad T \neq ?}{\Gamma \vdash v : A \Rightarrow ? \rightsquigarrow v : A \Rightarrow T \Rightarrow ? : ?} \quad \mathbf{rDA_CASTGROUND}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : ? \quad A \sim T \quad T \neq R \quad T \neq ?}{\Gamma \vdash v : ? \Rightarrow A \rightsquigarrow v : ? \Rightarrow T \Rightarrow A : A} \quad \mathbf{rDA_CASTEXPAND}$$

$$\frac{\Gamma, x : A_1 \vdash_{\mathbf{C}} t : A_2 \quad \Gamma \vdash_{\mathbf{C}} v : A_1}{\Gamma \vdash (\lambda x : A_1. t) v \rightsquigarrow [v/x]t : A_2} \quad \mathbf{rDA_BETA}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t'_1 : A_1 \rightarrow A_2 \quad \Gamma \vdash_{\mathbf{C}} t_2 : A_1}{\Gamma \vdash t_1 t_2 \rightsquigarrow t'_1 t_2 : A_2} \quad \mathbf{rDA_APP1}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} v : A_1 \rightarrow A_2 \quad \Gamma \vdash t \rightsquigarrow t' : A_1}{\Gamma \vdash v t \rightsquigarrow v t' : A_2} \quad \mathbf{rDA_APP2}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \text{fst } t \rightsquigarrow \text{fst } t' : A_1} \quad \mathbf{rDA_FST}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \text{snd } t \rightsquigarrow \text{snd } t' : A_2} \quad \mathbf{rDA_SND}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t'_1 : A_1 \quad \Gamma \vdash_{\mathbf{C}} t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t'_1, t_2) : A_1 \times A_2} \quad \mathbf{rDA_PAIR1}$$

$$\frac{\Gamma \vdash_{\mathbf{C}} t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t'_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t'_2) : A_1 \times A_2} \quad \mathbf{rDA_PAIR2}$$

$$\boxed{\Gamma \vdash t_1 \boxRightarrow t_2 : A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \boxRightarrow x : A} \quad \mathbf{CL_VAR}$$

$$\frac{}{\Gamma \vdash 0 \boxRightarrow 0 : A} \quad \mathbf{CL_ZERO}$$

$$\frac{}{\Gamma \vdash \text{triv} \boxRightarrow \text{triv} : \text{Unit}} \quad \mathbf{CL_TRIV}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 \sqsupseteq t_2 : ?}{\Gamma \vdash \text{succ } t_1 \sqsupseteq \text{succ } (t_2 : ? \Rightarrow \text{Nat}) : \text{Nat}} \quad \text{CI_SUCCONE} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_2 : \text{Nat}}{\Gamma \vdash \text{succ } t_1 \sqsupseteq \text{succ } t_2 : \text{Nat}} \quad \text{CI_SUCCTWO} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_3 : A_1 \quad \Gamma \vdash t_2 \sqsupseteq t_4 : A_2}{\Gamma \vdash (t_1, t_2) \sqsupseteq (t_3, t_4) : A_1 \times A_2} \quad \text{CI_PAIR} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_2 : ?}{\Gamma \vdash \text{fst } t_1 \sqsupseteq \text{fst } (t_2 : ? \Rightarrow (? \times ?)) : ?} \quad \text{CI_FSTONE} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_2 : A \times B}{\Gamma \vdash \text{fst } t_1 \sqsupseteq \text{fst } t_2 : A} \quad \text{CI_FSTTWO} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_2 : ?}{\Gamma \vdash \text{snd } t_1 \sqsupseteq \text{snd } (t_2 : ? \Rightarrow (? \times ?)) : ?} \quad \text{CI_SNDONE} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_2 : A \times B}{\Gamma \vdash \text{snd } t_1 \sqsupseteq \text{snd } t_2 : B} \quad \text{CI_SNDTWO} \\
\\
\frac{\Gamma, x : A_1 \vdash t_1 \sqsupseteq t_2 : A_2}{\Gamma \vdash \lambda x : A_1. t_1 \sqsupseteq \lambda x : A_1. t_2 : A_1 \rightarrow A_2} \quad \text{CI_LAM} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_3 : ? \quad \Gamma \vdash t_2 \sqsupseteq t_4 : A}{\Gamma \vdash t_1 t_2 \sqsupseteq (t_3 : ? \Rightarrow (A \rightarrow ?)) t_4 : ?} \quad \text{CI_APP1} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_3 : A_1 \rightarrow B \quad \Gamma \vdash t_2 \sqsupseteq t_4 : A_2 \quad A_1 \sim A_2 \quad A_1 \neq A_2}{\Gamma \vdash t_1 t_2 \sqsupseteq t_3 (t_4 : A_2 \Rightarrow A_1) : B} \quad \text{CI_APP2} \\
\\
\frac{\Gamma \vdash t_1 \sqsupseteq t_3 : A_1 \rightarrow A_2 \quad \Gamma \vdash t_2 \sqsupseteq t_4 : A_1}{\Gamma \vdash t_1 t_2 \sqsupseteq t_3 t_4 : A_2} \quad \text{CI_APP3}
\end{array}$$

Definition rules: 52 good 0 bad

Definition rule clauses: 97 good 0 bad