$termvar, x, y, z, f$
$typevar, X, Y, Z$
$index, i, j, k$

t, c, v, s, n	$::=$		term
	$\mid$	x	variable
	$\mid$	triv	unit
	$\mid$	$t : ctag$	type cast
	$\mid$	$\lambda x : A.t$	λ-abstraction
	$\mid$	$t_1\ t_2$	function application
	$\mid$	(t_1, t_2)	pair constructor
	$\mid$	$\mathsf{fst}\ t$	first projection
	$\mid$	$\mathsf{snd}\ t$	second projection
	$\mid$	$\mathsf{succ}\ t$	successor function
	$\mid$	0	zero
	$\mid$	(t) S	

T	$::=$		terminating types
	$\mid$	Unit	unit type
	$\mid$	Nat	natural number type

R	$::=$		terminating types
	$\mid$	Unit	unit type
	$\mid$	Nat	natural number type
	$\mid$	$? \to ?$	

$ctag$	$::=$	
	$\mid$	$\{A\}$
	$\mid$	$ctag \Rightarrow ctag'$
	$\mid$	$ctag$

A, B, C, D, E, S, U	$::=$		type
	$\mid$	Unit	unit type
	$\mid$	Nat	natural number type
	$\mid$	$?$	untyped universe
	$\mid$	$A_1 \to A_2$	function type
	$\mid$	$A_1 \times A_2$	cartesian product type
	$\mid$	(A) S	

Γ	$::=$		typing context
	$\mid$	$\cdot$	empty context
	$\mid$	$\Gamma, x : A$	cons

vd	$::=$	
	$\mid$	$\vdash$
	$\mid$	$\not\vdash$

$\boxed{A \sim B}$ A is consistent with B

$$\frac{}{A \sim A}\ \text{REFL}$$

$$\frac{}{A \sim ?}\ \text{BOX}$$

$$\frac{}{? \sim A}\ \textsc{unbox}$$

$$\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{A_1 \to B_1 \sim A_2 \to B_2}\ \textsc{arrow}$$

$$\frac{A_1 \sim A_2 \quad B_1 \sim B_2}{A_1 \times B_1 \sim A_2 \times B_2}\ \textsc{prod}$$

$\boxed{\Gamma \vdash_{\mathsf{S}} t : A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{S}} x : A}\ \textsc{S\_var}$$

$$\frac{}{\Gamma \vdash_{\mathsf{S}} \mathsf{triv} : \mathsf{Unit}}\ \textsc{S\_unit}$$

$$\frac{}{\Gamma \vdash_{\mathsf{S}} 0 : \mathsf{Nat}}\ \textsc{S\_zero}$$

$$\frac{\Gamma \vdash_{\mathsf{S}} t : A \quad \mathsf{nat}(A) = \mathsf{Nat}}{\Gamma \vdash_{\mathsf{S}} \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{S\_succ}$$

$$\frac{\Gamma \vdash_{\mathsf{S}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{S}} t_2 : A_2}{\Gamma \vdash_{\mathsf{S}} (t_1, t_2) : A_1 \times A_2}\ \textsc{S\_pair}$$

$$\frac{\Gamma \vdash_{\mathsf{S}} t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash_{\mathsf{S}} \mathsf{fst}\, t : A_1}\ \textsc{S\_fst}$$

$$\frac{\Gamma \vdash_{\mathsf{S}} t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash_{\mathsf{S}} \mathsf{snd}\, t : A_2}\ \textsc{S\_snd}$$

$$\frac{\Gamma, x : A \vdash_{\mathsf{S}} t : B}{\Gamma \vdash_{\mathsf{S}} \lambda x : A_1.t : A \to B}\ \textsc{S\_lam}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\mathsf{S}} t_1 : C \quad \mathsf{fun}(C) = A_1 \to B_1 \\ \Gamma \vdash_{\mathsf{S}} t_2 : A_2 \quad A_2 \sim A_1\end{array}}{\Gamma \vdash_{\mathsf{S}} t_1\ t_2 : B_1}\ \textsc{S\_app}$$

$\boxed{\Gamma \vdash_{\mathsf{C}} t : A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{C}} x : A}\ \textsc{C\_var}$$

$$\frac{}{\Gamma \vdash_{\mathsf{C}} \mathsf{triv} : \mathsf{Unit}}\ \textsc{C\_unit}$$

$$\frac{}{\Gamma \vdash_{\mathsf{C}} 0 : \mathsf{Nat}}\ \textsc{C\_zero}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t : \mathsf{Nat}}{\Gamma \vdash_{\mathsf{C}} \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{C\_succ}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{C}} t_2 : A_2}{\Gamma \vdash_{\mathsf{C}} (t_1, t_2) : A_1 \times A_2}\ \textsc{C\_pair}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{C}} \mathsf{fst}\, t : A_1}\ \textsc{C\_fst}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{C}} \mathsf{snd}\, t : A_2}\ \textsc{C\_snd}$$

$$\frac{\Gamma, x : A \vdash_{\mathsf{C}} t : B}{\Gamma \vdash_{\mathsf{C}} \lambda x : A_1.t : A \to B}\ \textsc{C\_lam}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t_1 : A \to B \quad \Gamma \vdash_{\mathsf{C}} t_2 : A}{\Gamma \vdash_{\mathsf{C}} t_1\, t_2 : B} \quad \text{C\_APP}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t : A \quad A \sim B}{\Gamma \vdash_{\mathsf{C}} (t : \{A\} \Rightarrow \{B\}) : B} \quad \text{C\_CAST}$$

$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2; A}$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : A}{\Gamma \vdash v \rightsquigarrow v; A} \quad \text{RDA\_VALUES}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : T}{\Gamma \vdash v : \{T\} \Rightarrow \{T\} \rightsquigarrow v; T} \quad \text{RDA\_CASTID}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : ?}{\Gamma \vdash v : \{?\} \Rightarrow \{?\} \rightsquigarrow v; ?} \quad \text{RDA\_CASTU}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : R}{\Gamma \vdash v : \{R\} \Rightarrow \{?\} \Rightarrow \{R\} \rightsquigarrow v; R} \quad \text{RDA\_SUCCEED}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v_1 : A_1 \to B_1 \quad \Gamma \vdash_{\mathsf{C}} v_2 : A_2}{\Gamma \vdash (v_1 : \{A_1 \to B_1\} \Rightarrow \{A_2 \to B_2\})\, v_2 \rightsquigarrow v_1\, (v_2 : \{A_2\} \Rightarrow \{A_1\}) : \{B_1\} \Rightarrow \{B_2\}; B} \quad \text{RDA\_CASTARROW}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : A \quad A \sim T \quad T \neq R \quad T \neq ?}{\Gamma \vdash v : \{A\} \Rightarrow \{?\} \rightsquigarrow v : \{A\} \Rightarrow \{T\} \Rightarrow \{?\}; ?} \quad \text{RDA\_CASTGROUND}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : ? \quad A \sim T \quad T \neq R \quad T \neq ?}{\Gamma \vdash v : \{?\} \Rightarrow \{A\} \rightsquigarrow v : \{?\} \Rightarrow \{T\} \Rightarrow \{A\}; A} \quad \text{RDA\_CASTEXPAND}$$

$$\frac{\Gamma, x : A_1 \vdash_{\mathsf{C}} t : A_2 \quad \Gamma \vdash_{\mathsf{C}} v : A_1}{\Gamma \vdash (\lambda x : A_1.t)\, v \rightsquigarrow [v/x]t; A_2} \quad \text{RDA\_BETA}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1'; A_1 \to A_2 \quad \Gamma \vdash_{\mathsf{C}} t_2 : A_1}{\Gamma \vdash t_1\, t_2 \rightsquigarrow t_1'\, t_2; A_2} \quad \text{RDA\_APP1}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} v : A_1 \to A_2 \quad \Gamma \vdash t \rightsquigarrow t'; A_1}{\Gamma \vdash v\, t \rightsquigarrow v\, t'; A_2} \quad \text{RDA\_APP2}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t'; A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t \rightsquigarrow \mathsf{fst}\, t'; A_1} \quad \text{RDA\_FST}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t'; A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t \rightsquigarrow \mathsf{snd}\, t'; A_2} \quad \text{RDA\_SND}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1'; A_1 \quad \Gamma \vdash_{\mathsf{C}} t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1', t_2); A_1 \times A_2} \quad \text{RDA\_PAIR1}$$

$$\frac{\Gamma \vdash_{\mathsf{C}} t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t_2'; A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t_2'); A_1 \times A_2} \quad \text{RDA\_PAIR2}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ Cast insertion from Siek16

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Definition rules:        38 good    0 bad
Definition rule clauses: 70 good    0 bad
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