Gradual Typing from a Categorical Perspective

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June 2016

References

A The Complete Spec of Grady

termvar, x, z			
index, k			
t	::=		term
	|	x	variable
	|	triv	unit
	|	squash	injection of the retract
	|	split	surjection of the retract
	|	box_T	generalize to the untyped universe
	|	unbox	specialize the untyped universe to a specific type
	|	$\lambda x : A.t$	λ-abstraction
	|	$t_1\ t_2$	function application
	|	(t_1, t_2)	pair constructor
	|	$\mathsf{fst}\ t$	first projection
	|	$\mathsf{snd}\ t$	second projection
	|	$\mathsf{succ}\ t$	successor function
	|	0	zero
	|	(t) S	
T	::=		terminating types
	|	1	unit type
	|	$\mathbb{N}$	natural number type
	|	$T_1 \to T_2$	function type
	|	$T_1 \times T_2$	cartesian product type
	|	(T) S	
A	::=		type
	|	T	terminating type
	|	$?$	untyped universe
	|	$A_1 \to A_2$	function type

$$
\begin{array}{llll}
 & | & A_1 \times A_2 & \text{cartesian product type} \\
 & | & (A) \quad \mathsf{S} & \\
 & & & \\
\Gamma & ::= & & \text{typing context} \\
 & | & \cdot & \text{empty context} \\
 & | & \Gamma, x : A & \text{cons}
\end{array}
$$

$\boxed{\Gamma \vdash t : A}$ t has type A in context Γ

$$\frac{x : A \in \Gamma}{\Gamma \vdash x : A} \quad \text{VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{box}_T : T \to ?} \quad \text{BOX}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} : ? \to T} \quad \text{UNBOX}$$

$$\frac{}{\Gamma \vdash \mathsf{squash} : (? \to ?) \to ?} \quad \text{INJ}$$

$$\frac{}{\Gamma \vdash \mathsf{split} : ? \to (? \to ?)} \quad \text{SURJ}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} : 1} \quad \text{UNIT}$$

$$\frac{}{\Gamma \vdash 0 : \mathbb{N}} \quad \text{ZERO}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{succ}\, t : \mathbb{N}} \quad \text{SUCC}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \quad \text{PAIR}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1} \quad \text{FST}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2} \quad \text{SND}$$

$$\frac{\Gamma, x : A_1 \vdash t : A_2}{\Gamma \vdash \lambda x : A_1 . t : A_1 \to A_2} \quad \text{ABS}$$

$$\frac{\Gamma \vdash t_1 : A_1 \to A_2 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash t_1\, t_2 : A_2} \quad \text{APP}$$

$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$ t_1 is reduces to t_2

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash \mathsf{unbox}\,(\mathsf{box}_T\, t) \rightsquigarrow t : T} \quad \text{RD\_RETRACT}\mathrm{T}$$

$$\frac{\Gamma \vdash t : ? \to ?}{\Gamma \vdash \mathsf{split}\,(\mathsf{squash}\, t) \rightsquigarrow t : ? \to ?} \quad \text{RD\_RETRACT}\mathrm{U}$$

$$\frac{\Gamma \vdash t : A_1 \to A_2 \quad x \notin \mathsf{FV}(t)}{\Gamma \vdash \lambda x : A_1.t\, x \rightsquigarrow t : A_1 \to A_2}\ \textsc{rd\_eta}$$

$$\frac{\Gamma, x : A_1 \vdash t_2 : A_2 \quad \Gamma \vdash t_1 : A_1}{\Gamma \vdash (\lambda x : A_1.t_2)\, t_1 \rightsquigarrow [t_1/x]t_2 : A_2}\ \textsc{rd\_beta}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{fst}\,(t_1, t_2) \rightsquigarrow t_1 : A_1}\ \textsc{rd\_proj}1$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{snd}\,(t_1, t_2) \rightsquigarrow t_2 : A_2}\ \textsc{rd\_proj}2$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash (\mathsf{fst}\, t, \mathsf{snd}\, t) \rightsquigarrow t : A_1 \times A_2}\ \textsc{rd\_eta}\mathrm{P}$$

$$\frac{\Gamma, x : A_1 \vdash t \rightsquigarrow t' : A_2}{\Gamma \vdash \lambda x : A_1.t \rightsquigarrow \lambda x : A_1.t' : A_1 \to A_2}\ \textsc{rd\_lam}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \to A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1\, t_2 \rightsquigarrow t_1'\, t_2 : A_2}\ \textsc{rd\_app}1$$

$$\frac{\Gamma \vdash t_1 : A_1 \to A_2 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_1}{\Gamma \vdash t_1\, t_2 \rightsquigarrow t_1\, t_2' : A_2}\ \textsc{rd\_app}2$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t \rightsquigarrow \mathsf{fst}\, t' : A_1}\ \textsc{rd\_fst}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t \rightsquigarrow \mathsf{snd}\, t' : A_2}\ \textsc{rd\_snd}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1', t_2) : A_1 \times A_2}\ \textsc{rd\_pair}1$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t_2') : A_1 \times A_2}\ \textsc{rd\_pair}2$$