$termvar$, x, y, z, f
$typevar$, X, Y, Z
$blamelabels$, ℓ
$index$, i, j, k

$$
\begin{array}{llll}
t,\ c,\ s & ::= & & \\
 & | & x & \\
 & | & \mathsf{triv} & \\
 & | & \mathsf{squash}^{\ell}_{U} & \\
 & | & \mathsf{split}^{\ell}_{U} & \\
 & | & \mathsf{box}^{\ell} & \\
 & | & \mathsf{unbox}^{\ell} & \\
 & | & \mathsf{blame}_{A}\,\ell & \\
 & | & \Lambda(X <: A).t & \\
 & | & [A]t & \\
 & | & \lambda(x : A).t & \\
 & | & t_1\; t_2 & \\
 & | & (t_1, t_2) & \\
 & | & \mathsf{fst}\, t & \\
 & | & \mathsf{snd}\, t & \\
 & | & \mathsf{succ}\, t & \\
 & | & 0 & \\
 & | & \mathsf{case}\, t\, \mathsf{of}\, t_3 \to t_1, t_4 \to t_2 & \\
 & | & [] & \\
 & | & t :: t' & \\
 & | & (t) & \mathsf{S} \\
 & & & \\
n,\ m & ::= & & \\
 & | & 0 & \\
 & | & \mathsf{succ}\, n & \\
 & & & \\
v & ::= & & \\
 & | & \mathsf{triv} & \\
 & | & \mathsf{squash}^{\ell}_{S} & \\
 & | & \mathsf{split}^{\ell}_{S} & \\
 & | & \mathsf{box}^{\ell}_{A} & \\
 & | & \mathsf{unbox}^{\ell}_{A} & \\
 & | & \Lambda(X <: A).t & \\
 & | & \lambda(x : A).t & \\
 & | & n & \\
 & & & \\
K & ::= & & \\
 & | & \star & \\
 & & & \\
A,\ B,\ C,\ D,\ E,\ S,\ U & ::= & & \\
 & | & X & \\
 & | & \mathsf{List}\, A & \\
 & | & \forall(X <: A).B & \\
 & | & \top & \\
 & | & \mathbb{S} & \\
 & | & \mathsf{Unit} & \\
 & | & \mathsf{Nat} &
\end{array}
$$

$$
\begin{array}{lll}
 & | & ? \\
 & | & A_1 \to A_2 \\
 & | & A_1 \times A_2 \\
 & | & (A) \qquad \mathsf{S}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma & ::= & \\
 & | & . \\
 & | & \Gamma, X <: A \\
 & | & \Gamma, x : A
\end{array}
$$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \textsc{K\_var}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star} \quad \textsc{K\_unit}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star} \quad \textsc{K\_nat}$$

$$\frac{}{\Gamma \vdash ? : \star} \quad \textsc{K\_unitype}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\, A : \star} \quad \textsc{K\_list}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \quad \textsc{K\_arrow}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \textsc{K\_prod}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A).B : \star} \quad \textsc{K\_forall}$$

$\boxed{\Gamma\, \mathrm{Ok}}$

$$\frac{}{\cdot\, \mathrm{Ok}} \quad \textsc{Ok\_empty}$$

$$\frac{\Gamma\, \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A)\, \mathrm{Ok}} \quad \textsc{Ok\_typeVar}$$

$$\frac{\Gamma\, \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\, \mathrm{Ok}} \quad \textsc{Ok\_var}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash A <: A} \quad \textsc{S\_Refl}$$

$$\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \quad \textsc{S\_Trans}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash A <: \top} \quad \textsc{S\_Top}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}} \quad \textsc{S\_Nat}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}} \quad \textsc{S\_Unit}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{List}\, A <: \mathbb{S}}\ \textsc{S\_ListSL}$$

$$\frac{X <: A \in \Gamma \quad \Gamma\, \mathrm{Ok}}{\Gamma \vdash X <: A}\ \textsc{S\_Var}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash \mathsf{List}\, A <: \mathsf{List}\, B}\ \textsc{S\_List}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}}\ \textsc{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}}\ \textsc{S\_ProdSL}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2}\ \textsc{S\_Prod}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2}\ \textsc{S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2}\ \textsc{S\_Forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma\, \mathrm{Ok}}{\Gamma \vdash x : A}\ \textsc{varP}$$

$$\frac{}{\Gamma \vdash \mathsf{box}^{\ell} : \forall(X <: \mathbb{S}).(X \to ?)}\ \textsc{Box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox}^{\ell} : \forall(X <: \mathbb{S}).(? \to X)}\ \textsc{Unbox}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{squash}^{\ell}_{U} : U \to ?}\ \textsc{squashP}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{split}^{\ell}_{U} : ? \to U}\ \textsc{splitP}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{unitP}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{zeroP}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A}\ \textsc{ncase}$$

$$\frac{\Gamma\, \mathrm{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: \top).\mathsf{List}\, X}\ \textsc{empty}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma \vdash t_2 : \mathsf{List}\, A}{\Gamma \vdash t_1 :: t_2 : \mathsf{List}\, A}\ \textsc{cons}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{List}\, A \\ \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 : B\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 : B}\ \textsc{lcase}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \quad \textsc{pair}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1} \quad \textsc{fst}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2} \quad \textsc{snd}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \to B} \quad \textsc{lam}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B} \quad \textsc{app}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B} \quad \textsc{Lam}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C} \quad \textsc{typeApp}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B} \quad \textsc{Sub}$$

$$\frac{}{\Gamma \vdash \mathsf{blame}_A\, \ell : A} \quad \textsc{blame}$$

$\boxed{t_1 \rightsquigarrow t_2}$ call by name

$$\frac{}{\mathsf{unbox}^{\ell_1}_A\, (\mathsf{box}^{\ell_2}_A\, t) \rightsquigarrow t} \quad \textsc{rd\_retracT}$$

$$\frac{A \neq B}{\mathsf{unbox}^{\ell_1}_A\, (\mathsf{box}^{\ell_2}_B\, t) \rightsquigarrow \mathsf{blame}_A\, \ell_1} \quad \textsc{rd\_retracTB}$$

$$\frac{}{\mathsf{split}^{\ell_1}_U\, (\mathsf{squash}^{\ell_2}_U\, t) \rightsquigarrow t} \quad \textsc{rd\_retractU}$$

$$\frac{U_1 \neq U_2}{\mathsf{split}^{\ell_1}_{U_1}\, (\mathsf{squash}^{\ell_1}_{U_2}\, t) \rightsquigarrow \mathsf{blame}_{U_1}\, \ell_1} \quad \textsc{rd\_retractUB}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t'} \quad \textsc{rd\_succ}$$

$$\frac{}{\mathsf{case}\, 0\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow t_1} \quad \textsc{rd\_ncase0}$$

$$\frac{}{\mathsf{case}\, (\mathsf{succ}\, t)\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow [t/x]t_2} \quad \textsc{rd\_ncaseSucc}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow \mathsf{case}\, t'\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2} \quad \textsc{rd\_ncase1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow \mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2} \quad \textsc{rd\_ncase2}$$

$$\frac{t_2 \rightsquigarrow t_2'}{\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow \mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2'} \quad \textsc{rd\_ncase3}$$

$$\frac{}{\mathsf{case}\, []\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 \rightsquigarrow t_1} \quad \textsc{rd\_lcaseEmpty}$$

$$\frac{}{\mathsf{case}\, (t_1 :: t_2)\, \mathsf{of}\, [] \to t_3, (x :: y) \to t_4 \rightsquigarrow [t_1/x][t_2/y]t_4} \quad \textsc{rd\_lcaseCons}$$

$$\frac{t_1 \leadsto t_1'}{t_1 :: t_2 \leadsto t_1' :: t_2} \quad \textsc{rd\_head}$$

$$\frac{t_2 \leadsto t_2'}{t_1 :: t_2 \leadsto t_1 :: t_2'} \quad \textsc{rd\_tail}$$

$$\frac{t \leadsto t'}{\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \leadsto \mathsf{case}\ t'\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2} \quad \textsc{rd\_lcase}1$$

$$\frac{t_1 \leadsto t_1'}{\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \leadsto \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1', (x :: y) \to t_2} \quad \textsc{rd\_lcase}2$$

$$\frac{t_2 \leadsto t_2'}{\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \leadsto \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2'} \quad \textsc{rd\_lcase}3$$

$$\frac{}{(\lambda(x : A_1).t_2)\, t_1 \leadsto [t_1/x]t_2} \quad \textsc{rd\_beta}$$

$$\frac{x \notin \mathsf{FV}(t)}{\lambda(x : A_1).t\, x \leadsto t} \quad \textsc{rd\_eta}$$

$$\frac{}{\mathsf{fst}\ (t_1, t_2) \leadsto t_1} \quad \textsc{rd\_proj}1$$

$$\frac{}{\mathsf{snd}\ (t_1, t_2) \leadsto t_2} \quad \textsc{rd\_proj}2$$

$$\frac{}{(\mathsf{fst}\ t, \mathsf{snd}\ t) \leadsto t} \quad \textsc{rd\_eta}\mathrm{P}$$

$$\frac{t_1 \leadsto t_1'}{t_1\ t_2 \leadsto t_1'\ t_2} \quad \textsc{rd\_app}1$$

$$\frac{t_2 \leadsto t_2'}{t_1\ t_2 \leadsto t_1\ t_2'} \quad \textsc{rd\_app}2$$

$$\frac{t \leadsto t'}{\mathsf{fst}\ t \leadsto \mathsf{fst}\ t'} \quad \textsc{rd\_fst}$$

$$\frac{t \leadsto t'}{\mathsf{snd}\ t \leadsto \mathsf{snd}\ t'} \quad \textsc{rd\_snd}$$

$$\frac{t_1 \leadsto t_1'}{(t_1, t_2) \leadsto (t_1', t_2)} \quad \textsc{rd\_pair}1$$

$$\frac{t_2 \leadsto t_2'}{(t_1, t_2) \leadsto (t_1, t_2')} \quad \textsc{rd\_pair}2$$

$$\frac{}{[A](\Lambda(X <: B).t) \leadsto [A/X]t} \quad \textsc{rd\_type}\mathrm{Beta}$$

$$\frac{t_1 \leadsto t_2}{[A]t_1 \leadsto [A]t_2} \quad \textsc{rd\_type}\mathrm{App}$$

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Definition rules:        75 good    0 bad
Definition rule clauses: 134 good    0 bad
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