

$termvar, x, y, z, f$   
 $typevar, X, Y, Z$   
 $index, i, j, k$   
 $t, c, s$

```

::=
|  x
|  triv
|  squashU
|  splitU
|  boxC
|  unboxC
|  λx : A. t
|  t1 t2
|  (t1, t2)
|  fst t
|  snd t
|  succ t
|  0
|  case t of 0 → t1, (succ x) → t2
|  (t)
S

```

$n$

```

::=
|  0
|  succ n

```

$v$

```

::=
|  triv
|  λx : A. t
|  n
|  splitU
|  squashU
|  boxC
|  unboxC

```

$U$

```

::=
|  ? → ?
|  ? × ?

```

$T$

```

::=
|  Unit
|  Nat

```

$A, B, C, D, E, R$

```

::=
|  Unit
|  Nat
|  ?
|  A1 → A2
|  A1 × A2
|  (A)
S

```

$\Gamma$

```

::=

```

$$\begin{array}{c} | \\ | \end{array} \cdot \\ \Gamma, x : A$$

$$\boxed{\Gamma \vdash t : A}$$

$$\begin{array}{c} \frac{x : A \in \Gamma}{\Gamma \vdash x : A} \text{ VAR} \\ \\ \overline{\Gamma \vdash \text{box}_T : T \rightarrow ?} \text{ BOX} \\ \\ \overline{\Gamma \vdash \text{unbox}_T : ? \rightarrow T} \text{ UNBOX} \\ \\ \overline{\Gamma \vdash \text{Box}_A : A \rightarrow ?} \text{ BOXG} \\ \\ \overline{\Gamma \vdash \text{Unbox}_A : ? \rightarrow A} \text{ UNBOXG} \\ \\ \overline{\Gamma \vdash \text{squash}_U : U \rightarrow ?} \text{ SQUASH} \\ \\ \overline{\Gamma \vdash \text{split}_U : ? \rightarrow U} \text{ SPLIT} \\ \\ \overline{\Gamma \vdash \text{triv} : \text{Unit}} \text{ UNIT} \\ \\ \overline{\Gamma \vdash 0 : \text{Nat}} \text{ ZERO} \\ \\ \frac{\Gamma \vdash t : \text{Nat}}{\Gamma \vdash \text{succ } t : \text{Nat}} \text{ SUCC} \\ \\ \frac{\Gamma \vdash t : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \text{ CASE} \\ \\ \frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \text{ PAIR} \\ \\ \frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \text{fst } t : A_1} \text{ FST} \\ \\ \frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \text{snd } t : A_2} \text{ SND} \\ \\ \frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A. t : A \rightarrow B} \text{ LAM} \\ \\ \frac{\Gamma \vdash t_1 : A \rightarrow B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1 t_2 : B} \text{ APP} \end{array}$$

$$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$$

$$\begin{array}{c} \frac{\Gamma \vdash s : A}{\Gamma \vdash s \rightsquigarrow s : A} \text{ RD\_VALUES} \\ \\ \overline{\Gamma \vdash t : T} \text{ RD\_RETRACT} \\ \Gamma \vdash \text{unbox}_T (\text{box}_T t) \rightsquigarrow t : T \\ \\ \overline{\Gamma \vdash t : A} \text{ RD\_RETRACTG} \\ \Gamma \vdash \text{Unbox}_A (\text{Box}_A t) \rightsquigarrow t : A \end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t : U}{\Gamma \vdash \text{split}_U (\text{squash}_U t) \rightsquigarrow t : U} \quad \text{RD\_RETRACTU} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : \text{Nat}}{\Gamma \vdash \text{succ } t \rightsquigarrow \text{succ } t' : \text{Nat}} \quad \text{RD\_SUCC} \\
\\
\frac{\Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } 0 \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow t_1 : A} \quad \text{RD\_CASE0} \\
\\
\frac{\Gamma \vdash t : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } (\text{succ } t) \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow [t/x]t_2 : A} \quad \text{RD\_CASESUCC} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow \text{case } t' \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{RD\_CASE} \\
\\
\frac{\Gamma, x : A_1 \vdash t_1 : A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash (\lambda x : A_1. t_1) v \rightsquigarrow [t_2/x]t_2 : A_2} \quad \text{RD\_BETA} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \text{fst } (t_1, t_2) \rightsquigarrow t_1 : A_1} \quad \text{RD\_PROJ1} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \text{snd } (t_1, t_2) \rightsquigarrow t_2 : A_2} \quad \text{RD\_PROJ2} \\
\\
\frac{\Gamma \vdash t_1 \rightsquigarrow t'_1 : A_1 \rightarrow A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1 t_2 \rightsquigarrow t'_1 t_2 : A_2} \quad \text{RD\_APP1} \\
\\
\frac{\Gamma \vdash v : A_1 \rightarrow A_2 \quad \Gamma \vdash t \rightsquigarrow t' : A_1}{\Gamma \vdash v t \rightsquigarrow v t' : A_2} \quad \text{RD\_APP2} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \text{fst } t \rightsquigarrow \text{fst } t' : A_1} \quad \text{RD\_FST} \\
\\
\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \text{snd } t \rightsquigarrow \text{snd } t' : A_2} \quad \text{RD\_SND} \\
\\
\frac{\Gamma \vdash t_1 \rightsquigarrow t'_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t'_1, t_2) : A_1 \times A_2} \quad \text{RD\_PAIR1} \\
\\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t'_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t'_2) : A_1 \times A_2} \quad \text{RD\_PAIR2}
\end{array}$$

Definition rules: 33 good 0 bad

Definition rule clauses: 61 good 0 bad