Gradual Typing from a Categorical Perspective

Harley Eades III

June 2016

References

A The Complete Spec of Grady

termvar, x
index, k

t	::=			term
	|	x		variable
	|	triv		unit
	|	squash		injection of the retract
	|	split		surjection of the retract
	|	gen		generalize to the untyped universe
	|	$\mathsf{gen}\, t\langle T\rangle$		generalize to the untyped universe
	|	spec		specialize the untyped universe to a specific type
	|	$\lambda x : T.t$		λ-abstraction
	|	$t_1\, t_2$		function application
	|	(t_1, t_2)		pair constructor
	|	$\mathsf{fst}\, t$		first projection
	|	$\mathsf{snd}\, t$		second projection
	|	$\mathsf{succ}\, t$		successor function
	|	0		zero
	|	(t)	S	

T	::=			type
	|	1		unit type
	|	$\mathbb{N}$		natural number type
	|	$?$		untyped universe
	|	$T_1 \to T_2$		function type
	|	$T_1 \times T_2$		cartesian product type
	|	(T)	S	

Γ	::=			typing context
	|	$\cdot$		empty context

$| \quad \Gamma, x : T \qquad \text{cons}$

$\boxed{T_1 \sim T_2}$ T_1 is consistent with T_2

$$\frac{}{T \sim T} \quad \text{CS\_REFL}$$

$$\frac{}{? \sim T} \quad \text{CS\_UL}$$

$$\frac{}{T \sim ?} \quad \text{CS\_UR}$$

$$\frac{T_1 \sim T_1' \quad T_2 \sim T_2'}{(T_1 \times T_2) \sim (T_1' \times T_2')} \quad \text{CS\_PAIR}$$

$$\frac{T_1 \sim T_1' \quad T_2 \sim T_2'}{(T_1 \to T_2) \sim (T_1' \to T_2')} \quad \text{CS\_ARROW}$$

$\boxed{\Gamma \vdash t : T}$ t has type T in context Γ

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{gen} : T \to ?} \quad \text{GEN}$$

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash \mathsf{gen}\, t\langle T \rangle : ?} \quad \text{GEN2}$$

$$\frac{}{\Gamma \vdash \mathsf{spec} : ? \to T} \quad \text{SPEC}$$

$$\frac{\Gamma \vdash \mathsf{gen}\, t\langle T \rangle : ?}{\Gamma \vdash \mathsf{spec}\, t : T} \quad \text{SPEC2}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} : 1} \quad \text{UNIT}$$

$$\frac{}{\Gamma \vdash 0 : \mathbb{N}} \quad \text{ZERO}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{succ}\, t : \mathbb{N}} \quad \text{SUCC}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2} \quad \text{PAIR}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \mathsf{fst}\, t : T_1} \quad \text{FST}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \mathsf{snd}\, t : T_2} \quad \text{SND}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x : T_1 . t : T_1 \to T_2} \quad \text{ABS}$$

$$\frac{\Gamma \vdash t_1 : T_1 \to T_2 \quad \Gamma \vdash t_2 : T_3}{\Gamma \vdash t_1\, t_2 : T_2} \quad \textsc{app}$$

$$\frac{}{\Gamma \vdash \mathsf{squash} : (? \to ?) \to ?} \quad \textsc{inj}$$

$$\frac{}{\Gamma \vdash \mathsf{split} : ? \to (? \to ?)} \quad \textsc{surj}$$

$\boxed{t_1 \leadsto t_2}$ t_1 reduces to t_2

$$\frac{}{(\lambda x : T.t_2)\, t_1 \leadsto [t_1/x]t_2} \quad \textsc{rd\_beta}$$

$$\frac{}{\mathsf{spec}\,(\mathsf{gen}\; t) \leadsto t} \quad \textsc{rd\_retract}$$

$$\frac{}{(\lambda x : T.t\; x) \leadsto t} \quad \textsc{rd\_eta}$$

$$\frac{}{\mathsf{fst}\,(t_1, t_2) \leadsto t_1} \quad \textsc{rd\_proj}1$$

$$\frac{}{\mathsf{snd}\,(t_1, t_2) \leadsto t_2} \quad \textsc{rd\_proj}2$$

$$\frac{}{(\mathsf{fst}\; t, \mathsf{snd}\; t) \leadsto t} \quad \textsc{rd\_eta}\mathrm{P}$$

$$\frac{}{\mathsf{split}\,(\mathsf{squash}\; t) \leadsto t} \quad \textsc{rd\_retract}\mathrm{S}$$