Proving error properties in the Kleisli category

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1 Definitions

Let $\mathcal{C}$ be a cartesian closed category with all finite coproducts. Then $\mathcal{C}$ is a monoidal category with associator

$$\alpha_{A,B,C} : A \times (B \times C) \longrightarrow (A \times B) \times C = \langle\langle \pi_1, \pi_2; \pi_1\rangle, \pi_2; \pi_2\rangle$$

and unitors

$$\begin{array}{rcl} r_A : 1 \times A \longrightarrow A & = & \pi_2 \\ l_A : A \times 1 \longrightarrow A & = & \pi_1 \end{array}$$

2 Strong monad diagrams

$$\begin{tikzcd}
1 \times (A+1) \arrow[r, "r_A"] \arrow[dr, "t_{1,A}"'] & TA \arrow[d, "r_A + \mathsf{id}_1"] \\
 & (1 \times A) + 1
\end{tikzcd}$$

$$\begin{tikzcd}
(A \times B) \times (C+1) \arrow[rr, "\mathsf{st}_{A \times B, C}"] \arrow[d, "\alpha A{,}B{,}C{+}1"'] & & ((A \times B) \times C) + 1 \arrow[d, "\alpha_{A,B,C} + \mathsf{id}_1"'] \\
A \times (B \times (C+1)) \arrow[r, "\mathsf{id}_A \times \mathsf{st}_{B,C}"] & A \times ((B \times C) + 1) \arrow[r, "\mathsf{st}_{A, B \times C}"] & (A \times (B \times C)) + 1
\end{tikzcd}$$

$$
\begin{tikzcd}
A \times B \arrow[d, "\mathsf{id}_A \times \eta_B"'] \arrow[dr, "\eta_{A\times B}"] & & \\
A \times (B+1) \arrow[r, "\mathsf{st}_{A,B}"] & (A \times B) + 1 & \\
A \times ((B+1)+1) \arrow[u, "\mathsf{id}_A \times \mu_B"'] \arrow[r, "\mathsf{st}_{A,B+1}"] & (A \times (B+1)) + 1 \arrow[r, "\mathsf{st}_{A,B} + \mathsf{id}_1"] & ((A \times B)+1)+1 \arrow[ul, "\mu_{A\times B}"']
\end{tikzcd}
$$

3 Expanded versions

$$
\begin{tikzcd}
1 \times (A+1) \arrow[r, "\pi_2"] \arrow[dr, "\mathsf{st}_{1,A}"'] & A+1 \arrow[d, "\langle \mathsf{triv}_A, \mathsf{id}_A \rangle + \mathsf{id}_1"] \\
& (1 \times A) + 1
\end{tikzcd}
$$