## Proving error properties in the Kleisli category

#### Richard Blair

#### 1 Definitions

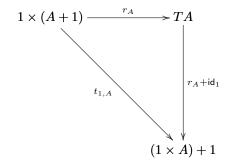
Let  $\mathcal C$  be a cartesian closed category with all finite coproducts. Then  $\mathcal C$  is a monoidal category with associator

$$\alpha_{A,B,C}: A \times (B \times C) \longrightarrow (A \times B) \times C = \langle \langle \pi_1, \pi_2; \pi_1 \rangle, \pi_2; \pi_2 \rangle$$

and unitors

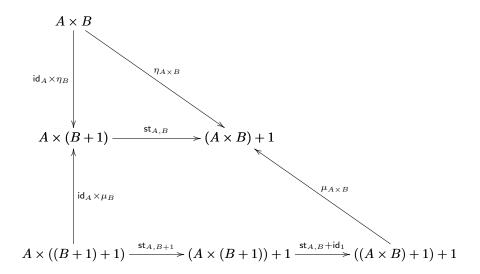
$$r_A: 1 \times A \longrightarrow A = \pi_2$$
  
 $l_A: A \times 1 \longrightarrow A = \pi_1$ 

### 2 Strong monad diagrams



$$(A \times B) \times (C+1) \xrightarrow{\operatorname{st}_{A \times B,C}} ((A \times B) \times C) + 1$$

$$\alpha_{A,B,C+1} \downarrow \qquad \qquad \alpha_{A,B,C+id_1} \downarrow \qquad \qquad \alpha_{A,B,C+id_1} \downarrow \qquad \qquad A \times (B \times (C+1)) \xrightarrow{\operatorname{id}_{A} \times \operatorname{st}_{B,C}} A \times ((B \times C) + 1) \xrightarrow{\operatorname{st}_{A,B \times C}} (A \times (B \times C)) + 1$$



# 3 Expanded versions

