

$termvar, x, y, z, f, r, ys$
 $typevar, X, Y, Z$
 $index, i, j, k$
 t, c, s

$::=$
 $|$ x
 $|$ $triv$
 $|$ box
 $|$ $unbox$
 $|$ $error_A$
 $|$ $error$
 $|$ $\Lambda(X <: A).t$
 $|$ $[A]t$
 $|$ $\lambda(x : A).t$
 $|$ $t_1 t_2$
 $|$ (t_1, t_2)
 $|$ $fst t$
 $|$ $snd t$
 $|$ $succ t$
 $|$ 0
 $|$ $case t : A \text{ of } t_3 \rightarrow t_1, t_4 \rightarrow t_2$
 $|$ \square
 $|$ $t :: t'$
 $|$ (t) S
 $|$ $squash$
 $|$ $split$

n, m

$::=$
 $|$ 0
 $|$ $succ n$

v

$::=$
 $|$ $triv$
 $|$ \square
 $|$ $squash_S$
 $|$ $split_S$
 $|$ box_A
 $|$ $unbox_A$
 $|$ $\Lambda(X <: A).t$
 $|$ $\lambda(x : A).t$
 $|$ n
 $|$ $case t : A \text{ of } t_3 \rightarrow t_1, t_4 \rightarrow t_2$

Kd

$::=$
 $|$ \star

A, B, C, D, E, S, U, K

$::=$
 $|$ X
 $|$ \top
 $|$ $List A$
 $|$ $\forall(X <: A).B$

	\mathbb{S}	
	Unit	
	Nat	
	$?$	
	$A_1 \rightarrow A_2$	
	$A_1 \times A_2$	
	(A)	\mathbb{S}

Γ	$::=$
	\cdot
	$\Gamma, X <: A$
	$\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\begin{array}{c}
\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K_VAR} \\
\\
\frac{}{\Gamma \vdash \mathbf{Unit} : \star} \quad \text{K_UNIT} \\
\\
\frac{}{\Gamma \vdash \mathbf{Nat} : \star} \quad \text{K_NAT} \\
\\
\frac{}{\Gamma \vdash ? : \star} \quad \text{K_UNITYPE} \\
\\
\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathbf{List} A : \star} \quad \text{K_LIST} \\
\\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \rightarrow B : \star} \quad \text{K_ARROW} \\
\\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \text{K_PROD} \\
\\
\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall(X <: A).B : \star} \quad \text{K_FORALL}
\end{array}$$

$\boxed{\Gamma \text{ Ok}}$

$$\begin{array}{c}
\frac{}{\cdot \text{Ok}} \quad \text{OK_EMPTY} \\
\\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \text{ Ok}} \quad \text{OK_TYPEVAR} \\
\\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \text{ Ok}} \quad \text{OK_VAR}
\end{array}$$

$\boxed{\Gamma \vdash A <: B}$

$$\begin{array}{c}
\frac{\Gamma \vdash A : \star}{\Gamma \vdash A <: A} \quad \text{S_REFL} \\
\\
\frac{\Gamma \vdash A : \star}{\Gamma \vdash A <: \top} \quad \text{S_TOP} \\
\\
\frac{X <: A \in \Gamma \quad \Gamma \text{ Ok}}{\Gamma \vdash X <: A} \quad \text{S_VAR}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \text{Ok}}{\Gamma \vdash \top <: \mathbb{S}} \quad \text{S_TOPSL} \\
\\
\frac{\Gamma \text{Ok}}{\Gamma \vdash \text{Nat} <: \mathbb{S}} \quad \text{S_NATSL} \\
\\
\frac{\Gamma \text{Ok}}{\Gamma \vdash \text{Unit} <: \mathbb{S}} \quad \text{S_UNITSL} \\
\\
\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \text{List } A <: \mathbb{S}} \quad \text{S_LISTSL} \\
\\
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \rightarrow B <: \mathbb{S}} \quad \text{S_ARROWSL} \\
\\
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}} \quad \text{S_PRODSL} \\
\\
\frac{\Gamma \vdash A <: B}{\Gamma \vdash \text{List } A <: \text{List } B} \quad \text{S_LIST} \\
\\
\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \quad \text{S_PROD} \\
\\
\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \rightarrow B_1 <: A_2 \rightarrow B_2} \quad \text{S_ARROW} \\
\\
\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2} \quad \text{S_FORALL}
\end{array}$$

$$\boxed{\Gamma_1 \sqsubseteq \Gamma_2}$$

$$\begin{array}{c}
\overline{\Gamma \sqsubseteq \Gamma} \quad \text{CTXP_REFL} \\
\\
\frac{\Gamma_1 \sqsubseteq \Gamma_2 \quad A \sqsubseteq A' \quad \Gamma_3 \sqsubseteq \Gamma_4}{\Gamma_1, x : A, \Gamma_3 \sqsubseteq \Gamma_2, x : A', \Gamma_4} \quad \text{CTXP_EXT}
\end{array}$$

$$\boxed{A \sqsubseteq B}$$

$$\begin{array}{c}
\frac{\Gamma \vdash A <: \mathbb{S}}{A \sqsubseteq ?} \quad \text{P_U} \\
\\
\overline{A \sqsubseteq A} \quad \text{P_REFL} \\
\\
\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \rightarrow B) \sqsubseteq (C \rightarrow D)} \quad \text{P_ARROW} \\
\\
\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P_PROD} \\
\\
\frac{A \sqsubseteq B}{(\text{List } A) \sqsubseteq (\text{List } B)} \quad \text{P_LIST} \\
\\
\frac{B_1 \sqsubseteq B_2}{(\forall(X <: A).B_1) \sqsubseteq (\forall(X <: A).B_2)} \quad \text{P_FORALL}
\end{array}$$

$$\boxed{\Gamma \vdash t \sqsubseteq t'}$$

$$\frac{x : A \in \Gamma \quad \Gamma \text{Ok}}{\Gamma \vdash x \sqsubseteq x} \quad \text{TP_VAR}$$

$$\begin{array}{c}
\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \text{split}_{S_1} \sqsubseteq \text{split}_{S_2}} \quad \text{TP_SPLIT} \\
\\
\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \text{squash}_{S_1} \sqsubseteq \text{squash}_{S_2}} \quad \text{TP_SQUASH} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{box} \sqsubseteq \text{box}} \quad \text{TP_BOX} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{unbox} \sqsubseteq \text{unbox}} \quad \text{TP_UNBOX} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash 0 \sqsubseteq 0} \quad \text{TP_NAT} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{triv} \sqsubseteq \text{triv}} \quad \text{TP_TRIV} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash [] \sqsubseteq []} \quad \text{TP_EMPTY} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\text{succ } t_1) \sqsubseteq (\text{succ } t_2)} \quad \text{TP_SUCC} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : \text{Nat} \vdash t_3 \sqsubseteq t_6}{\Gamma \vdash (\text{case } t_1 : \text{Nat of } 0 \rightarrow t_2, (\text{succ } x) \rightarrow t_3) \sqsubseteq (\text{case } t_4 : \text{Nat of } 0 \rightarrow t_5, (\text{succ } x) \rightarrow t_6)} \quad \text{TP_NATE} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1, t_2) \sqsubseteq (t_3, t_4)} \quad \text{TP_PAIR} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\text{fst } t_1) \sqsubseteq (\text{fst } t_2)} \quad \text{TP_FST} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\text{snd } t_1) \sqsubseteq (\text{snd } t_2)} \quad \text{TP_SND} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP_CONS} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : A_2, y : \text{List } A_2 \vdash t_3 \sqsubseteq t_6 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\text{case } t_1 : \text{List } A_1 \text{ of } [] \rightarrow t_2, (x :: y) \rightarrow t_3) \sqsubseteq (\text{case } t_4 : \text{List } A_2 \text{ of } 0 \rightarrow t_5, (x :: y) \rightarrow t_6)} \quad \text{TP_LISTE} \\
\\
\frac{\Gamma, x : A_2 \vdash t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP_FUN} \\
\\
\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 \ t_2) \sqsubseteq (t_3 \ t_4)} \quad \text{TP_APP} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : ?}{\Gamma \vdash (\text{unbox}_A t) \sqsubseteq t} \quad \text{TP_UNBOXING} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : A}{\Gamma \vdash t \sqsubseteq (\text{box}_A t)} \quad \text{TP_BOXING} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : ?}{\Gamma \vdash (\text{split}_S t) \sqsubseteq t} \quad \text{TP_SPLITING} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : S}{\Gamma \vdash t \sqsubseteq (\text{squash}_S t)} \quad \text{TP_SQUASHING}
\end{array}$$

$$\frac{\Gamma, X <: A \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \quad \text{TP_TFUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{\Gamma \vdash [A]t_1 \sqsubseteq [B]t_2} \quad \text{TP_TAPP}$$

$$\frac{\Gamma \vdash_{\text{CG}} t : B \quad A \sqsubseteq B}{\Gamma \vdash \text{error}_A \sqsubseteq t} \quad \text{TP_ERROR}$$

$$\boxed{\Gamma \vdash_{\text{CG}} t : A}$$

$$\frac{x : A \in \Gamma \quad \Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} x : A} \quad \text{T_VARP}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} \text{box} : \forall(X <: \mathbb{S}).(X \rightarrow ?)} \quad \text{T_BOX}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} \text{unbox} : \forall(X <: \mathbb{S}).(? \rightarrow X)} \quad \text{T_UNBOX}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} \text{squash}_S : S \rightarrow ?} \quad \text{T_SQUASH}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} \text{split}_S : ? \rightarrow S} \quad \text{T_SPLIT}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} \text{triv} : \text{Unit}} \quad \text{T_UNITP}$$

$$\frac{\Gamma \text{Ok}}{\Gamma \vdash_{\text{CG}} 0 : \text{Nat}} \quad \text{T_ZEROP}$$

$$\frac{\Gamma \vdash_{\text{CG}} t : \text{Nat}}{\Gamma \vdash_{\text{CG}} \text{succ } t : \text{Nat}} \quad \text{T_SUCC}$$

$$\frac{\Gamma \vdash_{\text{CG}} t : \text{Nat} \quad \Gamma \vdash_{\text{CG}} t_1 : A \quad \Gamma, x : \text{Nat} \vdash_{\text{CG}} t_2 : A}{\Gamma \vdash_{\text{CG}} \text{case } t : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{T_NCASE}$$

$$\frac{\Gamma \text{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash_{\text{CG}} [] : \forall(X <: ?).\text{List } X} \quad \text{T_EMPTY}$$

$$\frac{\Gamma \vdash_{\text{CG}} t_1 : A \quad \Gamma \vdash_{\text{CG}} t_2 : \text{List } A}{\Gamma \vdash_{\text{CG}} t_1 :: t_2 : \text{List } A} \quad \text{T_CONS}$$

$$\frac{\Gamma \vdash_{\text{CG}} t : \text{List } A \quad \Gamma \vdash_{\text{CG}} t_1 : B \quad \Gamma, x : A, y : \text{List } A \vdash_{\text{CG}} t_2 : B}{\Gamma \vdash_{\text{CG}} \text{case } t : \text{List } A \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \quad \text{T_LCASE}$$

$$\frac{\Gamma \vdash_{\text{CG}} t_1 : A_1 \quad \Gamma \vdash_{\text{CG}} t_2 : A_2}{\Gamma \vdash_{\text{CG}} (t_1, t_2) : A_1 \times A_2} \quad \text{T_PAIR}$$

$$\frac{\Gamma \vdash_{\text{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\text{CG}} \text{fst } t : A_1} \quad \text{T_FST}$$

$$\frac{\Gamma \vdash_{\text{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\text{CG}} \text{snd } t : A_2} \quad \text{T_SND}$$

$$\frac{\Gamma, x : A \vdash_{\text{CG}} t : B}{\Gamma \vdash_{\text{CG}} \lambda(x : A).t : A \rightarrow B} \quad \text{T_LAM}$$

$$\begin{array}{c}
\frac{\Gamma \vdash_{\text{CG}} t_1 : A \rightarrow B \quad \Gamma \vdash_{\text{CG}} t_2 : A}{\Gamma \vdash_{\text{CG}} t_1 t_2 : B} \quad \text{T_APP} \\
\\
\frac{\Gamma, X <: A \vdash_{\text{CG}} t : B}{\Gamma \vdash_{\text{CG}} \Lambda(X <: A).t : \forall(X <: A).B} \quad \text{T_LAM} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\text{CG}} [A]t : [A/X]C} \quad \text{T_TYPEAPP} \\
\\
\frac{\Gamma \vdash_{\text{CG}} t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\text{CG}} t : B} \quad \text{T_SUB} \\
\\
\frac{}{\Gamma \vdash_{\text{CG}} \text{error}_A : A} \quad \text{T_ERROR}
\end{array}$$

$t_1 \rightsquigarrow t_2$ call by name

$$\begin{array}{c}
\frac{}{\text{unbox}_A(\text{box}_A t) \rightsquigarrow t} \quad \text{RD_RETRACT} \\
\\
\frac{A \neq B}{\text{unbox}_A(\text{box}_B t) \rightsquigarrow \text{error}_A} \quad \text{RD_RETRACTE} \\
\\
\frac{}{\text{split}_S(\text{squash}_S t) \rightsquigarrow t} \quad \text{RD_RETRACTU} \\
\\
\frac{S_1 \neq S_2}{\text{split}_{S_1}(\text{squash}_{S_2} t) \rightsquigarrow \text{error}_{S_1}} \quad \text{RD_RETRACTUE} \\
\\
\frac{t \rightsquigarrow t'}{\text{succ } t \rightsquigarrow \text{succ } t'} \quad \text{RD_SUCC} \\
\\
\frac{}{\text{case } 0 : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow t_1} \quad \text{RD_NCASE0} \\
\\
\frac{}{\text{case } (\text{succ } t) : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow [t/x]t_2} \quad \text{RD_NCASESUCC} \\
\\
\frac{t \rightsquigarrow t'}{\text{case } t : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow \text{case } t' : \text{Nat of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2} \quad \text{RD_NCASE} \\
\\
\frac{}{\text{case } [] : \text{List } A \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow t_1} \quad \text{RD_LCASEEMPTY} \\
\\
\frac{}{\text{case } (t_1 :: t_2) : \text{List } A \text{ of } [] \rightarrow t_3, (x :: y) \rightarrow t_4 \rightsquigarrow [t_1/x][t_2/y]t_4} \quad \text{RD_LCASECONS} \\
\\
\frac{t_1 \rightsquigarrow t'_1}{t_1 :: t_2 \rightsquigarrow t'_1 :: t_2} \quad \text{RD_HEAD} \\
\\
\frac{t_2 \rightsquigarrow t'_2}{t_1 :: t_2 \rightsquigarrow t_1 :: t'_2} \quad \text{RD_TAIL} \\
\\
\frac{t \rightsquigarrow t'}{\text{case } t : \text{List } A \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow \text{case } t' : \text{List } A \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2} \quad \text{RD_LCASE} \\
\\
\frac{}{(\lambda(x : A_1).t_2) t_1 \rightsquigarrow [t_1/x]t_2} \quad \text{RD_BETA} \\
\\
\frac{}{\text{fst}(t_1, t_2) \rightsquigarrow t_1} \quad \text{RD_PROJ1} \\
\\
\frac{}{\text{snd}(t_1, t_2) \rightsquigarrow t_2} \quad \text{RD_PROJ2}
\end{array}$$

$$\begin{array}{c}
\frac{t_1 \rightsquigarrow t'_1}{t_1 t_2 \rightsquigarrow t'_1 t_2} \quad \text{RD_APP} \\
\\
\frac{t_2 \rightsquigarrow t'_2}{v t_2 \rightsquigarrow v t'_2} \quad \text{RD_APP2} \\
\\
\frac{t \rightsquigarrow t'}{\text{split}_S t \rightsquigarrow \text{split}_S t'} \quad \text{RD_SPLIT} \\
\\
\frac{t \rightsquigarrow t'}{\text{unbox}_A t \rightsquigarrow \text{unbox}_A t'} \quad \text{RD_UNBOX} \\
\\
\frac{t \rightsquigarrow t'}{\text{fst } t \rightsquigarrow \text{fst } t'} \quad \text{RD_FST} \\
\\
\frac{t \rightsquigarrow t'}{\text{snd } t \rightsquigarrow \text{snd } t'} \quad \text{RD_SND} \\
\\
\frac{t_1 \rightsquigarrow t'_1}{(t_1, t_2) \rightsquigarrow (t'_1, t_2)} \quad \text{RD_PAIR1} \\
\\
\frac{t_2 \rightsquigarrow t'_2}{(t_1, t_2) \rightsquigarrow (t_1, t'_2)} \quad \text{RD_PAIR2} \\
\\
\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t} \quad \text{RD_TYPEBETA} \\
\\
\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2} \quad \text{RD_TYPEAPP}
\end{array}$$

Definition rules: 103 good 0 bad
 Definition rule clauses: 191 good 0 bad