$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ s$ ::=
| x
| triv
| squash_U
| split_U
| Squash_S
| Split_S
| box_C
| unbox_C
| $\lambda x : A.t$
| $t_1\ t_2$
| (t_1, t_2)
| $\mathsf{fst}\ t$
| $\mathsf{snd}\ t$
| $\mathsf{succ}\ t$
| 0
| $\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2$
| $\mathsf{error}\ B$
| (t) S

n ::=
| 0
| $\mathsf{succ}\ n$

v ::=
| triv
| $\lambda x : A.t$
| n
| split_U
| squash_U
| box_C
| unbox_C

$\mathcal{C}$ ::=
| $\Box$
| $\lambda x : A.\mathcal{C}$
| $\mathcal{C}\ t_2$
| $t_1\ \mathcal{C}$
| $(\mathcal{C}, t_2)$
| $(t_1, \mathcal{C})$
| $\mathsf{fst}\ \mathcal{C}$
| $\mathsf{snd}\ \mathcal{C}$
| $\mathsf{succ}\ \mathcal{C}$
| $\mathsf{case}\ \mathcal{C}\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2$
| $\mathsf{case}\ t\ \mathsf{of}\ 0 \to \mathcal{C}, (\mathsf{succ}\ x) \to t_2$
| $\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to \mathcal{C}$

$$
\begin{array}{lll}
T & ::= & \\
 & | & \mathsf{Unit} \\
 & | & \mathsf{Nat}
\end{array}
$$

$$
\begin{array}{llll}
A,\ B,\ C,\ D,\ E,\ R,\ X,\ Y,\ U,\ S & ::= & & \\
 & | & \mathsf{Unit} & \\
 & | & \mathsf{Nat} & \\
 & | & ? & \\
 & | & A_1 \to A_2 & \\
 & | & A_1 \times A_2 & \\
 & | & (A) & \mathsf{S}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma & ::= & \\
 & | & \cdot \\
 & | & \Gamma, x : A
\end{array}
$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x : A}\ \textsc{var}$$

$$\frac{}{\Gamma \vdash \mathsf{box}_T : T \to ?}\ \textsc{Box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox}_T : ? \to T}\ \textsc{Unbox}$$

$$\frac{}{\Gamma \vdash \mathsf{Box}_A : A \to ?}\ \textsc{BoxG}$$

$$\frac{}{\Gamma \vdash \mathsf{Unbox}_A : ? \to A}\ \textsc{UnboxG}$$

$$\frac{}{\Gamma \vdash \mathsf{squash}_U : U \to ?}\ \textsc{squash}$$

$$\frac{}{\Gamma \vdash \mathsf{split}_U : ? \to U}\ \textsc{split}$$

$$\frac{}{\Gamma \vdash \mathsf{Split}_S : S \to ?}\ \textsc{SplitG}$$

$$\frac{}{\Gamma \vdash \mathsf{Squash}_S : ? \to S}\ \textsc{SquashG}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{unit}$$

$$\frac{}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{zero}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}}\ \textsc{succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A}\ \textsc{case}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2}\ \textsc{pair}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1}\ \textsc{fst}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2}\ \textsc{snd}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A_1 . t : A \to B}\ \textsc{lam}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B}\ \textsc{app}$$

$$\frac{}{\Gamma \vdash \mathsf{error}\, B : B}\ \textsc{error}$$

$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \rightsquigarrow x : A}\ \textsc{rd\_var}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathsf{unbox}_A\, (\mathsf{box}_A\, t) \rightsquigarrow t : A}\ \textsc{rd\_retrac}\mathrm{T}$$

$$\frac{\Gamma \vdash t : A \quad A \neq B}{\Gamma \vdash \mathsf{unbox}_B\, (\mathsf{box}_A\, t) \rightsquigarrow \mathsf{error}\, B : B}\ \textsc{rd\_retract}\mathrm{E}$$

$$\frac{\Gamma, x : B \vdash \mathcal{C}[x] : A}{\Gamma \vdash \mathcal{C}[\mathsf{error}\, B] \rightsquigarrow \mathsf{error}\, A : A}\ \textsc{rd\_error}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : T}{\Gamma \vdash \mathsf{unbox}_T\, t_1 \rightsquigarrow \mathsf{unbox}_T\, t_2 : T}\ \textsc{rd\_unbox}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathsf{Unbox}_A\, (\mathsf{Box}_A\, t) \rightsquigarrow t : A}\ \textsc{rd\_retrac}\mathrm{TG}$$

$$\frac{\Gamma \vdash t : S}{\Gamma \vdash \mathsf{Split}_S\, (\mathsf{Squash}_S\, t) \rightsquigarrow t : S}\ \textsc{rd\_retrac}\mathrm{TSG}$$

$$\frac{\Gamma \vdash t : U}{\Gamma \vdash \mathsf{split}_U\, (\mathsf{squash}_U\, t) \rightsquigarrow t : U}\ \textsc{rd\_retract}\mathrm{U}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : U}{\Gamma \vdash \mathsf{split}_U\, t_1 \rightsquigarrow \mathsf{split}_U\, t_2 : U}\ \textsc{rd\_split}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t' : \mathsf{Nat}}\ \textsc{rd\_succ}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A}{\Gamma \vdash \mathsf{case}\, 0\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow t_1 : A}\ \textsc{rd\_case}0$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, (\mathsf{succ}\, t)\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow [t/x]t_2 : A}\ \textsc{rd\_case}\mathrm{S}\textsc{ucc}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \rightsquigarrow t' : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow \mathsf{case}\, t'\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A}\ \textsc{rd\_case}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 : A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash (\lambda x : A_1 . t_1)\, t_2 \rightsquigarrow [t_2/x]t_2 : A_2}\ \textsc{rd\_beta}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{fst}\, (t_1, t_2) \rightsquigarrow t_1 : A_1}\ \textsc{rd\_proj}1$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{snd}\,(t_1, t_2) \leadsto t_2 : A_2} \quad \text{RD\_PROJ2}$$

$$\frac{\Gamma \vdash t_1 \leadsto t_1' : A_1 \to A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1\, t_2 \leadsto t_1'\, t_2 : A_2} \quad \text{RD\_APP1}$$

$$\frac{\Gamma \vdash v : A_1 \to A_2 \quad \Gamma \vdash t \leadsto t' : A_1}{\Gamma \vdash v\, t \leadsto v\, t' : A_2} \quad \text{RD\_APP2}$$

$$\frac{\Gamma \vdash t \leadsto t' : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t \leadsto \mathsf{fst}\, t' : A_1} \quad \text{RD\_FST}$$

$$\frac{\Gamma \vdash t \leadsto t' : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t \leadsto \mathsf{snd}\, t' : A_2} \quad \text{RD\_SND}$$

$$\frac{\Gamma \vdash t_1 \leadsto t_1' : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \leadsto (t_1', t_2) : A_1 \times A_2} \quad \text{RD\_PAIR1}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \leadsto t_2' : A_2}{\Gamma \vdash (t_1, t_2) \leadsto (t_1, t_2') : A_1 \times A_2} \quad \text{RD\_PAIR2}$$

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Definition rules:        41 good    0 bad
Definition rule clauses: 74 good    0 bad
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