$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ v,\ s,\ n$	$::=$		
	$\mid$	x	
	$\mid$	triv	
	$\mid$	box	
	$\mid$	unbox	
	$\mid$	$\Lambda(X <: A).t$	
	$\mid$	$[A]t$	
	$\mid$	$\lambda(x : A).t$	
	$\mid$	$t_1\ t_2$	
	$\mid$	(t_1, t_2)	
	$\mid$	$\mathsf{fst}\ t$	
	$\mid$	$\mathsf{snd}\ t$	
	$\mid$	$\mathsf{succ}\ t$	
	$\mid$	0	
	$\mid$	$\mathsf{case}\ t\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
	$\mid$	$[]$	
	$\mid$	$t :: t'$	
	$\mid$	(t)	S
K	$::=$		
	$\mid$	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U$	$::=$		
	$\mid$	X	
	$\mid$	$\mathsf{List}\ A$	
	$\mid$	$\forall(X <: A).B$	
	$\mid$	$\top$	
	$\mid$	SL	
	$\mid$	Unit	
	$\mid$	Nat	
	$\mid$	$?$	
	$\mid$	$A_1 \to A_2$	
	$\mid$	$A_1 \times A_2$	
	$\mid$	(A)	S
Γ	$::=$		
	$\mid$	$.$	
	$\mid$	$\Gamma, X <: A$	
	$\mid$	$\Gamma, x : A$	

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star}\ \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star}\ \text{K\_UNIT}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star}\ \text{K\_NAT}$$

$$\frac{}{\Gamma \vdash ? : \star} \quad \text{K\_UNITYPE}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\ A : \star} \quad \text{K\_LIST}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \quad \text{K\_ARROW}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \text{K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A).B : \star} \quad \text{K\_FORALL}$$

$\boxed{\Gamma\ \mathrm{Ok}}$

$$\frac{}{\cdot\ \mathrm{Ok}} \quad \text{OK\_EMPTY}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A)\ \mathrm{Ok}} \quad \text{OK\_TYPEVAR}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\ \mathrm{Ok}} \quad \text{OK\_VAR}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash A <: A} \quad \text{S\_REFL}$$

$$\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \quad \text{S\_TRANS}$$

$$\frac{X <: A \in \Gamma \quad \Gamma\ \mathrm{Ok}}{\Gamma \vdash X <: A} \quad \text{S\_VAR}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash A <: \top} \quad \text{S\_TOP}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathsf{SL}} \quad \text{S\_NATS}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathsf{SL}} \quad \text{S\_UNITS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL}}{\Gamma \vdash \mathsf{List}\ A <: \mathsf{SL}} \quad \text{S\_LISTS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL} \quad \Gamma \vdash B <: \mathsf{SL}}{\Gamma \vdash (A \to B) <: \mathsf{SL}} \quad \text{S\_ARROWS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL} \quad \Gamma \vdash B <: \mathsf{SL}}{\Gamma \vdash (A \times B) <: \mathsf{SL}} \quad \text{S\_PRODS}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash (\mathsf{List}\ A) <: (\mathsf{List}\ B)} \quad \text{S\_LIST}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \times B_1) <: (A_2 \times B_2)} \quad \text{S\_PROD}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \to B_1) <: (A_2 \to B_2)} \quad \text{S\_ARROW}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash (\forall(X <: A).B_1) <: (\forall(X <: A).B_2)} \quad \text{S\_Forall}$$

$\boxed{\Gamma \vdash A \sim B}$

$$\frac{}{\Gamma \vdash A \sim A} \quad \text{C\_Refl}$$

$$\frac{}{\Gamma \vdash A \sim ?} \quad \text{C\_Box}$$

$$\frac{}{\Gamma \vdash ? \sim A} \quad \text{C\_Unbox}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\mathsf{List}\, A) \sim (\mathsf{List}\, B)} \quad \text{C\_List}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \to B_1) \sim (A_2 \to B_2)} \quad \text{C\_Arrow}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C\_Prod}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall(X <: A).B_1) \sim (\forall(X <: A).B_2)} \quad \text{C\_Forall}$$

$\boxed{A \sqsubseteq B}$

$$\frac{}{A \sqsubseteq ?} \quad \text{P\_U}$$

$$\frac{}{A \sqsubseteq A} \quad \text{P\_refl}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad \text{P\_arrow}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P\_prod}$$

$$\frac{A \sqsubseteq B}{(\mathsf{List}\, A) \sqsubseteq (\mathsf{List}\, B)} \quad \text{P\_list}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(\forall(X <: A).B) \sqsubseteq (\forall(X <: C).D)} \quad \text{P\_forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma\,\mathrm{Ok}}{\Gamma \vdash x : A} \quad \text{T\_varP}$$

$$\frac{}{\Gamma \vdash \mathsf{box} : \forall(X <: \mathsf{SL}).(X \to ?)} \quad \text{T\_Box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} : \forall(X <: \mathsf{SL}).(? \to X)} \quad \text{T\_Unbox}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}} \quad \text{T\_unitP}$$

$$\frac{\Gamma\,\mathrm{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}} \quad \text{T\_zeroP}$$

$$\frac{\Gamma \vdash t : A \quad \mathsf{nat}(A) = \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}} \quad \text{T\_succ}$$

$$\frac{\Gamma \vdash t : C \quad \mathsf{nat}(C) = \mathsf{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 : A}\ \text{T\_NCASE}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: \top).\mathsf{List}\ X}\ \text{T\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : \mathsf{List}\ A_2 \quad \Gamma \vdash A_1 \sim A_2}{\Gamma \vdash t_1 :: t_2 : \mathsf{List}\ A_2}\ \text{T\_CONS}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2}\ \text{T\_PAIR}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \to B}\ \text{T\_LAM}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B}\ \text{T\_LAM}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C}\ \text{T\_TYPEAPP}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B}\ \text{T\_SUB}$$

$$\frac{\Gamma \vdash t : C \quad \mathsf{list}(C) = \mathsf{List}\ A \quad \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\ A \vdash t_2 : B}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 : B}\ \text{T\_LCASE}$$

$$\frac{\Gamma \vdash t_1 : C \quad \mathsf{fun}(C) = A_1 \to B_1 \quad \Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_2 \sim A_1}{\Gamma \vdash t_1\ t_2 : B_2}\ \text{T\_APP}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\ t : A_1}\ \text{T\_FST}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\ t : A_2}\ \text{T\_SND}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ Cast insertion

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A}\ \text{CI\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \Rightarrow \mathsf{box} : \forall(X <: \mathsf{SL}).(X \to ?)}\ \text{CI\_BOX}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \Rightarrow \mathsf{unbox} : \forall(X <: \mathsf{SL}).(? \to X)}\ \text{CI\_UNBOX}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \mathsf{Nat}}\ \text{CI\_ZERO}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \Rightarrow \mathsf{triv} : \mathsf{Unit}}\ \text{CI\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\ (\mathsf{unbox}_{\mathsf{Nat}}\ t_2) : \mathsf{Nat}}\ \text{CI\_SUCCU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\ t_2 : \mathsf{Nat}}\ \text{CI\_SUCC}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2) \Rightarrow (\mathsf{case}\, (\mathsf{unbox}_{\mathsf{Nat}}\, t')\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2') : A}\ \textsc{ci\_ncaseU}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : \mathsf{Nat} \quad \Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2') : A}\ \textsc{ci\_ncase}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2}\ \textsc{ci\_pair}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, (\mathsf{split}_{(? \times ?)}\, t_2) : ?}\ \textsc{ci\_fstU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, t_2 : A_1}\ \textsc{ci\_fst}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, (\mathsf{split}_{(? \times ?)}\, t_2) : ?}\ \textsc{ci\_sndU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, t_2 : B}\ \textsc{ci\_snd}$$

$$\frac{}{\Gamma \vdash [] \Rightarrow [] : \forall (X <: \top).\mathsf{List}\, X}\ \textsc{ci\_Empty}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_2' : \mathsf{List}\, A_2 \quad \Gamma \vdash A_1 \sim A_2 \quad \mathsf{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c\, t_1') :: t_2') : \mathsf{List}\, A}\ \textsc{ci\_cons}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : ?, y : \mathsf{List}\, ? \vdash t_2 \Rightarrow t_2' : B}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, (\mathsf{split}_{(\mathsf{List}\, ?)}\, t')\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B}\ \textsc{ci\_lcaseU}$$

$$\frac{\Gamma \vdash t \Rightarrow t : \mathsf{List}\, A \quad \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 \Rightarrow t_2' : B}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B}\ \textsc{ci\_lcase}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \to A_2}\ \textsc{ci\_lam}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_1' : ? \quad \Gamma \vdash t_2 \Rightarrow t_2' : A_2 \quad \mathsf{caster}(A_2, ?) = c}{\Gamma \vdash t_1\, t_2 \Rightarrow (\mathsf{split}_{(? \to ?)}\, t_1')\, (c\, t_2') : ?}\ \textsc{ci\_appU}$$

$$\frac{\Gamma \vdash t_2 \Rightarrow t_2' : A_2 \quad \Gamma \vdash t_1 \Rightarrow t_1' : A_1 \to B \quad \Gamma \vdash A_2 \sim A_1 \quad \mathsf{caster}(A_2, A_1) = c}{\Gamma \vdash t_1\, t_2 \Rightarrow t_1'\, (c\, t_2') : B}\ \textsc{ci\_app}$$

$$\frac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall (X <: A).B}\ \textsc{ci\_Lam}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall (X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A]t_2) : [A/X]C}\ \textsc{ci\_typeApp}$$

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Definition rules:         78 good    0 bad
Definition rule clauses: 149 good    0 bad
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