

$termvar, x, y, z, f$
 $typevar, X, Y, Z$
 $index, i, j, k$
 t, c, v, s, n

$::=$
 $|$ x
 $|$ $triv$
 $|$ $squash_S$
 $|$ $split_S$
 $|$ box
 $|$ box_A
 $|$ $unbox_A$
 $|$ $unbox$
 $|$ $\Lambda(X <: A).t$
 $|$ $[A]t$
 $|$ $\lambda(x : A).t$
 $|$ $t_1 t_2$
 $|$ (t_1, t_2)
 $|$ $fst t$
 $|$ $snd t$
 $|$ $succ t$
 $|$ 0
 $|$ $ncase t \text{ of } 0 \rightarrow t_1, t' \rightarrow t_2$
 $|$ \square
 $|$ $t :: t'$
 $|$ $lcase t \text{ of } \square \rightarrow t_1, t' \rightarrow t_2$
 $|$ (t) S

K

$::=$
 $|$ \star

A, B, C, D, E, S, U

$::=$
 $|$ X
 $|$ $List A$
 $|$ $\forall(X <: A).B$
 $|$ \top
 $|$ \mathbb{S}
 $|$ $Unit$
 $|$ Nat
 $|$ $?$
 $|$ $A_1 \rightarrow A_2$
 $|$ $A_1 \times A_2$
 $|$ (A) S

Γ

$::=$
 $|$ \cdot
 $|$ $\Gamma, X <: A$
 $|$ $\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\begin{array}{c}
\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K_VAR} \\
\\
\frac{}{\Gamma \vdash \text{Unit} : \star} \quad \text{K_UNIT} \\
\\
\frac{}{\Gamma \vdash \text{Nat} : \star} \quad \text{K_NAT} \\
\\
\frac{}{\Gamma \vdash ? : \star} \quad \text{K_UNITYPE} \\
\\
\frac{\Gamma \vdash A : \star}{\Gamma \vdash \text{List } A : \star} \quad \text{K_LIST} \\
\\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \rightarrow B : \star} \quad \text{K_ARROW} \\
\\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \text{K_PROD} \\
\\
\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall(X <: A).B : \star} \quad \text{K_FORALL}
\end{array}$$

$\boxed{\Gamma \text{ Ok}}$

$$\begin{array}{c}
\frac{}{\cdot \text{Ok}} \quad \text{OK_EMPTY} \\
\\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \text{ Ok}} \quad \text{OK_TYPEVAR} \\
\\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \text{ Ok}} \quad \text{OK_VAR}
\end{array}$$

$\boxed{\Gamma \vdash A <: B}$

$$\begin{array}{c}
\frac{\Gamma \text{ Ok}}{\Gamma \vdash A <: A} \quad \text{S_REFL} \\
\\
\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \quad \text{S_TRANS} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash A <: \top} \quad \text{S_TOP} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Nat} <: \mathbb{S}} \quad \text{S_NAT} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Unit} <: \mathbb{S}} \quad \text{S_UNIT} \\
\\
\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \text{List } A <: \mathbb{S}} \quad \text{S_LISTSL} \\
\\
\frac{X <: A \in \Gamma \quad \Gamma \text{ Ok}}{\Gamma \vdash X <: A} \quad \text{S_VAR} \\
\\
\frac{\Gamma \vdash A <: B}{\Gamma \vdash \text{List } A <: \text{List } B} \quad \text{S_LIST} \\
\\
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \rightarrow B <: \mathbb{S}} \quad \text{S_ARROWSL}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}} \quad \text{S_PROD SL} \\
\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \quad \text{S_PROD} \\
\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \rightarrow B_1 <: A_2 \rightarrow B_2} \quad \text{S_ARROW} \\
\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2} \quad \text{S_FORALL}
\end{array}$$

$$\boxed{\Gamma \vdash t : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma \quad \Gamma \text{ Ok}}{\Gamma \vdash x : A} \quad \text{VARP} \\
\frac{}{\Gamma \vdash \text{box} : \forall(X <: \mathbb{S}).(X \rightarrow ?)} \quad \text{BOX} \\
\frac{}{\Gamma \vdash \text{unbox} : \forall(X <: \mathbb{S}).(? \rightarrow X)} \quad \text{UNBOX} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{squash}_U : U \rightarrow ?} \quad \text{SQUASHP} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{split}_U : ? \rightarrow U} \quad \text{SPLITP} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{triv} : \text{Unit}} \quad \text{UNITP} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash 0 : \text{Nat}} \quad \text{ZEROP} \\
\frac{\Gamma \vdash t : \text{Nat}}{\Gamma \vdash \text{succ } t : \text{Nat}} \quad \text{SUCC} \\
\frac{\Gamma \vdash t : \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{ncase } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{NCASE} \\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: \top).\text{List } X} \quad \text{EMPTY} \\
\frac{\Gamma \vdash t_1 : A \quad \Gamma \vdash t_2 : \text{List } A}{\Gamma \vdash t_1 :: t_2 : \text{List } A} \quad \text{CONS} \\
\frac{\Gamma \vdash t : \text{List } A \quad \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \text{List } A \vdash t_2 : B}{\Gamma \vdash \text{lcase } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \quad \text{LCASE} \\
\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \quad \text{PAIR} \\
\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \text{fst } t : A_1} \quad \text{FST} \\
\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \text{snd } t : A_2} \quad \text{SND}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \rightarrow B} \text{LAM} \\
\frac{\Gamma \vdash t_1 : A \rightarrow B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1 t_2 : B} \text{APP} \\
\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B} \text{LAM} \\
\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C} \text{TYPEAPP} \\
\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B} \text{SUB}
\end{array}$$

$$\boxed{t_1 \rightsquigarrow t_2}$$

$$\begin{array}{c}
\frac{\cdot \vdash t : A}{\text{unbox}_A(\text{box}_B t) \rightsquigarrow t} \text{RD_RETRACT} \\
\frac{\cdot \vdash t : U}{\text{split}_U(\text{squash}_U t) \rightsquigarrow t} \text{RD_RETRACTU} \\
\frac{t \rightsquigarrow t'}{\text{succ } t \rightsquigarrow \text{succ } t'} \text{RD_SUCC} \\
\frac{}{\text{ncase } 0 \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow t_1} \text{RD_NCASE0} \\
\frac{}{\text{ncase } (\text{succ } t) \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow [t/x]t_2} \text{RD_NCASESUCC} \\
\frac{t \rightsquigarrow t'}{\text{ncase } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow \text{ncase } t' \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2} \text{RD_NCASE1} \\
\frac{t_1 \rightsquigarrow t'_1}{\text{ncase } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow \text{ncase } t \text{ of } 0 \rightarrow t'_1, (\text{succ } x) \rightarrow t_2} \text{RD_NCASE2} \\
\frac{t_2 \rightsquigarrow t'_2}{\text{ncase } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 \rightsquigarrow \text{ncase } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t'_2} \text{RD_NCASE3} \\
\frac{}{\text{lcase } [] \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow t_1} \text{RD_LCASEEMPTY} \\
\frac{}{\text{lcase } (t_1 :: t_2) \text{ of } [] \rightarrow t_3, (x :: y) \rightarrow t_4 \rightsquigarrow [t_1/x][t_2/y]t_4} \text{RD_LCASECONS} \\
\frac{t_1 \rightsquigarrow t'_1}{t_1 :: t_2 \rightsquigarrow t'_1 :: t_2} \text{RD_HEAD} \\
\frac{t_2 \rightsquigarrow t'_2}{t_1 :: t_2 \rightsquigarrow t_1 :: t'_2} \text{RD_TAIL} \\
\frac{t \rightsquigarrow t'}{\text{lcase } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow \text{lcase } t' \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2} \text{RD_LCASE1} \\
\frac{t_1 \rightsquigarrow t'_1}{\text{lcase } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow \text{lcase } t \text{ of } [] \rightarrow t'_1, (x :: y) \rightarrow t_2} \text{RD_LCASE2} \\
\frac{t_2 \rightsquigarrow t'_2}{\text{lcase } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow \text{lcase } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t'_2} \text{RD_LCASE3}
\end{array}$$

$$\begin{array}{c}
\frac{}{(\lambda(x : A_1).t_2) \, t_1 \rightsquigarrow [t_1/x]t_2} \quad \text{RD_BETA} \\
\\
\frac{x \notin \text{FV}(t)}{\lambda(x : A_1).t \, x \rightsquigarrow t} \quad \text{RD_ETA} \\
\\
\frac{}{\text{fst}(t_1, t_2) \rightsquigarrow t_1} \quad \text{RD_PROJ1} \\
\\
\frac{}{\text{snd}(t_1, t_2) \rightsquigarrow t_2} \quad \text{RD_PROJ2} \\
\\
\frac{}{(\text{fst } t, \text{snd } t) \rightsquigarrow t} \quad \text{RD_ETAP} \\
\\
\frac{t \rightsquigarrow t'}{\lambda(x : A).t \rightsquigarrow \lambda(x : A).t'} \quad \text{RD_LAM} \\
\\
\frac{t_1 \rightsquigarrow t'_1}{t_1 \, t_2 \rightsquigarrow t'_1 \, t_2} \quad \text{RD_APP1} \\
\\
\frac{t_2 \rightsquigarrow t'_2}{t_1 \, t_2 \rightsquigarrow t_1 \, t'_2} \quad \text{RD_APP2} \\
\\
\frac{t \rightsquigarrow t'}{\text{fst } t \rightsquigarrow \text{fst } t'} \quad \text{RD_FST} \\
\\
\frac{t \rightsquigarrow t'}{\text{snd } t \rightsquigarrow \text{snd } t'} \quad \text{RD_SND} \\
\\
\frac{t_1 \rightsquigarrow t'_1}{(t_1, t_2) \rightsquigarrow (t'_1, t_2)} \quad \text{RD_PAIR1} \\
\\
\frac{t_2 \rightsquigarrow t'_2}{(t_1, t_2) \rightsquigarrow (t_1, t'_2)} \quad \text{RD_PAIR2} \\
\\
\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t} \quad \text{RD_TYPEBETA} \\
\\
\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2} \quad \text{RD_TYPEAPP} \\
\\
\frac{t_1 \rightsquigarrow t_2}{\Lambda(X <: A).t_1 \rightsquigarrow \Lambda(X <: A).t_2} \quad \text{RD_LAM}
\end{array}$$

Definition rules: 74 good 0 bad
 Definition rule clauses: 135 good 0 bad