$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ v,\ s,\ n$	::=		
	|	x	
	|	triv	
	|	squash_S	
	|	split_S	
	|	box	
	|	box_A	
	|	unbox_A	
	|	unbox	
	|	$\Lambda(X <: A).t$	
	|	$[A]t$	
	|	$\lambda(x : A).t$	
	|	$t_1\ t_2$	
	|	(t_1, t_2)	
	|	fst t	
	|	snd t	
	|	succ t	
	|	0	
	|	case t of $t_3 \to t_1, t_4 \to t_2$	
	|	$[]$	
	|	$t :: t'$	
	|	(t)	S
K	::=		
	|	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U$	::=		
	|	X	
	|	List A	
	|	$\forall(X <: A).B$	
	|	$\top$	
	|	$\mathbb{S}$	
	|	Unit	
	|	Nat	
	|	$?$	
	|	$A_1 \to A_2$	
	|	$A_1 \times A_2$	
	|	(A)	S
Γ	::=		
	|	$\cdot$	
	|	$\Gamma, X <: A$	
	|	$\Gamma, x : A$	

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star}\ \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star}\ \textsc{K\_unit}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star}\ \textsc{K\_nat}$$

$$\frac{}{\Gamma \vdash ? : \star}\ \textsc{K\_unitype}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\ A : \star}\ \textsc{K\_list}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star}\ \textsc{K\_arrow}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star}\ \textsc{K\_prod}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A).B : \star}\ \textsc{K\_forall}$$

$\boxed{\Gamma\ \mathrm{Ok}}$

$$\frac{}{\cdot\ \mathrm{Ok}}\ \textsc{Ok\_empty}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A)\ \mathrm{Ok}}\ \textsc{Ok\_typeVar}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\ \mathrm{Ok}}\ \textsc{Ok\_var}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash A <: A}\ \textsc{S\_Refl}$$

$$\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C}\ \textsc{S\_Trans}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash A <: \top}\ \textsc{S\_Top}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}}\ \textsc{S\_Nat}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}}\ \textsc{S\_Unit}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{List}\ A <: \mathbb{S}}\ \textsc{S\_ListSL}$$

$$\frac{X <: A \in \Gamma \quad \Gamma\ \mathrm{Ok}}{\Gamma \vdash X <: A}\ \textsc{S\_Var}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash \mathsf{List}\ A <: \mathsf{List}\ B}\ \textsc{S\_List}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}}\ \textsc{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}}\ \textsc{S\_ProdSL}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \text{ S\_Prod}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2} \text{ S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2} \text{ S\_Forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma \,\mathrm{Ok}}{\Gamma \vdash x : A} \text{ varP}$$

$$\frac{}{\Gamma \vdash \mathsf{box} : \forall(X <: \mathbb{S}).(X \to ?)} \text{ Box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} : \forall(X <: \mathbb{S}).(? \to X)} \text{ Unbox}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{squash}_U : U \to ?} \text{ squashP}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{split}_U : ? \to U} \text{ splitP}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}} \text{ unitP}$$

$$\frac{\Gamma \,\mathrm{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}} \text{ zeroP}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t : \mathsf{Nat}} \text{ succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A} \text{ ncase}$$

$$\frac{\Gamma \,\mathrm{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: \top).\mathsf{List}\, X} \text{ empty}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma \vdash t_2 : \mathsf{List}\, A}{\Gamma \vdash t_1 :: t_2 : \mathsf{List}\, A} \text{ cons}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{List}\, A \\ \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 : B\end{array}}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 : B} \text{ lcase}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \text{ pair}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1} \text{ fst}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2} \text{ snd}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \to B} \text{ lam}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B}\ \textsc{app}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B}\ \textsc{Lam}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C}\ \textsc{typeApp}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B}\ \textsc{Sub}$$

$\boxed{t_1 \rightsquigarrow t_2}$

$$\frac{\cdot \vdash t : A}{\mathsf{unbox}_A\,(\mathsf{box}_B\ t) \rightsquigarrow t}\ \textsc{rd\_retracT}$$

$$\frac{\cdot \vdash t : U}{\mathsf{split}_U\,(\mathsf{squash}_U\ t) \rightsquigarrow t}\ \textsc{rd\_retractU}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{succ}\ t \rightsquigarrow \mathsf{succ}\ t'}\ \textsc{rd\_succ}$$

$$\frac{}{\mathsf{case}\ 0\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 \rightsquigarrow t_1}\ \textsc{rd\_ncase0}$$

$$\frac{}{\mathsf{case}\ (\mathsf{succ}\ t)\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 \rightsquigarrow [t/x]t_2}\ \textsc{rd\_ncaseSucc}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 \rightsquigarrow \mathsf{case}\ t'\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2}\ \textsc{rd\_ncase1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 \rightsquigarrow \mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1', (\mathsf{succ}\ x) \to t_2}\ \textsc{rd\_ncase2}$$

$$\frac{t_2 \rightsquigarrow t_2'}{\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 \rightsquigarrow \mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2'}\ \textsc{rd\_ncase3}$$

$$\frac{}{\mathsf{case}\ []\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow t_1}\ \textsc{rd\_lcaseEmpty}$$

$$\frac{}{\mathsf{case}\ (t_1 :: t_2)\ \mathsf{of}\ [] \to t_3, (x :: y) \to t_4 \rightsquigarrow [t_1/x][t_2/y]t_4}\ \textsc{rd\_lcaseCons}$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1 :: t_2 \rightsquigarrow t_1' :: t_2}\ \textsc{rd\_head}$$

$$\frac{t_2 \rightsquigarrow t_2'}{t_1 :: t_2 \rightsquigarrow t_1 :: t_2'}\ \textsc{rd\_tail}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow \mathsf{case}\ t'\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2}\ \textsc{rd\_lcase1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1', (x :: y) \to t_2}\ \textsc{rd\_lcase2}$$

$$\frac{t_2 \rightsquigarrow t_2'}{\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2'}\ \textsc{rd\_lcase3}$$

$$\frac{}{(\lambda(x : A_1).t_2)\ t_1 \rightsquigarrow [t_1/x]t_2}\ \textsc{rd\_beta}$$

$$\frac{x \notin \mathsf{FV}(t)}{\lambda(x : A_1).t\ x \rightsquigarrow t}\ \textsc{rd\_eta}$$

$$\frac{}{\mathsf{fst}\,(t_1, t_2) \rightsquigarrow t_1}\ \textsc{rd\_proj1}$$

$$\frac{}{\mathsf{snd}\,(t_1, t_2) \rightsquigarrow t_2}\ \textsc{rd\_proj2}$$

$$\frac{}{(\mathsf{fst}\,t, \mathsf{snd}\,t) \rightsquigarrow t}\ \textsc{rd\_etaP}$$

$$\frac{t \rightsquigarrow t'}{\lambda(x : A).t \rightsquigarrow \lambda(x : A).t'}\ \textsc{rd\_lam}$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1\ t_2 \rightsquigarrow t_1'\ t_2}\ \textsc{rd\_app1}$$

$$\frac{t_2 \rightsquigarrow t_2'}{t_1\ t_2 \rightsquigarrow t_1\ t_2'}\ \textsc{rd\_app2}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{fst}\ t \rightsquigarrow \mathsf{fst}\ t'}\ \textsc{rd\_fst}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{snd}\ t \rightsquigarrow \mathsf{snd}\ t'}\ \textsc{rd\_snd}$$

$$\frac{t_1 \rightsquigarrow t_1'}{(t_1, t_2) \rightsquigarrow (t_1', t_2)}\ \textsc{rd\_pair1}$$

$$\frac{t_2 \rightsquigarrow t_2'}{(t_1, t_2) \rightsquigarrow (t_1, t_2')}\ \textsc{rd\_pair2}$$

$$\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t}\ \textsc{rd\_typeBeta}$$

$$\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2}\ \textsc{rd\_typeApp}$$

$$\frac{t_1 \rightsquigarrow t_2}{\Lambda(X <: A).t_1 \rightsquigarrow \Lambda(X <: A).t_2}\ \textsc{rd\_Lam}$$

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Definition rules:         74 good    0 bad
Definition rule clauses: 135 good    0 bad
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