$termvar,\ x,\ y,\ z,\ f,\ r,\ ys$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ s$	$::=$	
	$\mid\quad x$	
	$\mid\quad \mathsf{triv}$	
	$\mid\quad \mathsf{box}$	
	$\mid\quad \mathsf{unbox}$	
	$\mid\quad \mathsf{error}_A$	
	$\mid\quad \mathsf{error}$	
	$\mid\quad \Lambda(X <: A).t$	
	$\mid\quad [A]t$	
	$\mid\quad \lambda(x : A).t$	
	$\mid\quad t_1\ t_2$	
	$\mid\quad (t_1, t_2)$	
	$\mid\quad \mathsf{fst}\ t$	
	$\mid\quad \mathsf{snd}\ t$	
	$\mid\quad \mathsf{succ}\ t$	
	$\mid\quad 0$	
	$\mid\quad \mathsf{case}\ t : A\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
	$\mid\quad []$	
	$\mid\quad t :: t'$	
	$\mid\quad (t)$	S
	$\mid\quad \mathsf{squash}$	
	$\mid\quad \mathsf{split}$	
$n,\ m$	$::=$	
	$\mid\quad 0$	
	$\mid\quad \mathsf{succ}\ n$	
v	$::=$	
	$\mid\quad \mathsf{triv}$	
	$\mid\quad []$	
	$\mid\quad \mathsf{unbox}_A$	
	$\mid\quad \Lambda(X <: A).t$	
	$\mid\quad \lambda(x : A).t$	
	$\mid\quad n$	
	$\mid\quad \mathsf{case}\ t : A\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
$\mathcal{E}$	$::=$	
	$\mid\quad \square$	
	$\mid\quad v\ \mathcal{E}$	
	$\mid\quad \mathsf{succ}\ \mathcal{E}$	
	$\mid\quad \mathsf{fst}\ \mathcal{E}$	
	$\mid\quad \mathsf{snd}\ \mathcal{E}$	
	$\mid\quad (\mathcal{E}, t)$	
	$\mid\quad (t, \mathcal{E})$	
$A,\ B,\ C,\ D,\ E,\ S,\ U,\ K,\ T$	$::=$	
	$\mid\quad X$	

$$
\begin{array}{lll}
| & \top & \\
| & \mathsf{List}\ A & \\
| & \forall (X <: A).B & \\
| & \mathbb{S} & \\
| & \mathsf{Unit} & \\
| & \mathsf{Nat} & \\
| & ? & \\
| & A_1 \to A_2 & \\
| & A_1 \times A_2 & \\
| & (A) & \mathsf{S}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma & ::= & \\
& | & . \\
& | & \Gamma, X <: A \\
& | & \Gamma, x : A \\
& | & x : A
\end{array}
$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{}{\Gamma \vdash A <: A}\ \text{S\_Refl}$$

$$\frac{}{\Gamma \vdash A <: \top}\ \text{S\_Top}$$

$$\frac{X <: A \in \Gamma}{\Gamma \vdash X <: A}\ \text{S\_Var}$$

$$\frac{}{\Gamma \vdash \top <: \mathbb{S}}\ \text{S\_TopSL}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}}\ \text{S\_NatSL}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}}\ \text{S\_UnitSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{List}\ A <: \mathbb{S}}\ \text{S\_ListSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}}\ \text{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}}\ \text{S\_ProdSL}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash \mathsf{List}\ A <: \mathsf{List}\ B}\ \text{S\_List}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2}\ \text{S\_Prod}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2}\ \text{S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall (X <: A).B_1 <: \forall (X <: A).B_2}\ \text{S\_Forall}$$

$\boxed{\Gamma_1 \sqsubseteq \Gamma_2}$

$$\frac{}{\Gamma \sqsubseteq \Gamma}\ \text{CtxP\_refl}$$

$$\frac{\Gamma_1 \sqsubseteq \Gamma_2 \quad A \sqsubseteq A' \quad \Gamma_3 \sqsubseteq \Gamma_4}{\Gamma_1, x : A, \Gamma_3 \sqsubseteq \Gamma_2, x : A', \Gamma_4} \quad \textsc{CtxP\_ext}$$

$\boxed{A \sqsubseteq B}$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{A \sqsubseteq ?} \quad \textsc{P\_U}$$

$$\frac{}{A \sqsubseteq A} \quad \textsc{P\_refl}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad \textsc{P\_arrow}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \textsc{P\_prod}$$

$$\frac{A \sqsubseteq B}{(\mathsf{List}\, A) \sqsubseteq (\mathsf{List}\, B)} \quad \textsc{P\_list}$$

$$\frac{B_1 \sqsubseteq B_2}{(\forall (X <: A).B_1) \sqsubseteq (\forall (X <: A).B_2)} \quad \textsc{P\_forall}$$

$\boxed{\Gamma \vdash t \sqsubseteq t'}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \sqsubseteq x} \quad \textsc{TP\_var}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \mathsf{split}_{S_1} \sqsubseteq \mathsf{split}_{S_2}} \quad \textsc{TP\_split}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \mathsf{squash}_{S_1} \sqsubseteq \mathsf{squash}_{S_2}} \quad \textsc{TP\_squash}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \sqsubseteq \mathsf{box}} \quad \textsc{TP\_box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \sqsubseteq \mathsf{unbox}} \quad \textsc{TP\_unbox}$$

$$\frac{}{\Gamma \vdash 0 \sqsubseteq 0} \quad \textsc{TP\_nat}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \sqsubseteq \mathsf{triv}} \quad \textsc{TP\_triv}$$

$$\frac{}{\Gamma \vdash [] \sqsubseteq []} \quad \textsc{TP\_empty}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{succ}\, t_1) \sqsubseteq (\mathsf{succ}\, t_2)} \quad \textsc{TP\_succ}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : \mathsf{Nat} \vdash t_3 \sqsubseteq t_6}{\Gamma \vdash (\mathsf{case}\, t_1 : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_2, (\mathsf{succ}\, x) \to t_3) \sqsubseteq (\mathsf{case}\, t_4 : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_5, (\mathsf{succ}\, x) \to t_6)} \quad \textsc{TP\_Nate}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1, t_2) \sqsubseteq (t_3, t_4)} \quad \textsc{TP\_pair}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{fst}\, t_1) \sqsubseteq (\mathsf{fst}\, t_2)} \quad \textsc{TP\_fst}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{snd}\, t_1) \sqsubseteq (\mathsf{snd}\, t_2)} \quad \textsc{TP\_snd}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP\_CONS}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : A_2, y : \mathsf{List}\, A_2 \vdash t_3 \sqsubseteq t_6 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\mathsf{case}\, t_1 : \mathsf{List}\, A_1\, \mathsf{of}\, [] \to t_2, (x :: y) \to t_3) \sqsubseteq (\mathsf{case}\, t_4 : \mathsf{List}\, A_2\, \mathsf{of}\, 0 \to t_5, (x :: y) \to t_6)} \quad \text{TP\_LISTE}$$

$$\frac{\Gamma, x : A_2 \vdash t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP\_FUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1\, t_2) \sqsubseteq (t_3\, t_4)} \quad \text{TP\_APP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : ?}{\Gamma \vdash (\mathsf{unbox}_A\, t) \sqsubseteq t} \quad \text{TP\_UNBOXING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A}{\Gamma \vdash t \sqsubseteq (\mathsf{box}_A\, t)} \quad \text{TP\_BOXING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : ?}{\Gamma \vdash (\mathsf{split}_S\, t) \sqsubseteq t} \quad \text{TP\_SPLITING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : S}{\Gamma \vdash t \sqsubseteq (\mathsf{squash}_S\, t)} \quad \text{TP\_SQUASHING}$$

$$\frac{\Gamma, X <: A \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \quad \text{TP\_TFUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{\Gamma \vdash [A]t_1 \sqsubseteq [B]t_2} \quad \text{TP\_TAPP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : B \quad A \sqsubseteq B}{\Gamma \vdash \mathsf{error}_A \sqsubseteq t} \quad \text{TP\_ERROR}$$

$\boxed{\Gamma \vdash_{\mathsf{CG}} t : A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{CG}} x : A} \quad \text{T\_VARP}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{CG}} x : A} \quad \text{T\_VAR}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box}_A : A \to ?} \quad \text{T\_BOX}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox}_A : ? \to A} \quad \text{T\_UNBOX}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box} : \forall(X <: \mathbb{S}).(X \to ?)} \quad \text{T\_BOXP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox} : \forall(X <: \mathbb{S}).(? \to X)} \quad \text{T\_UNBOXP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{squash}_S : S \to ?} \quad \text{T\_SQUASH}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{split}_S : ? \to S} \quad \text{T\_SPLIT}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{triv} : \mathsf{Unit}} \quad \text{T\_UNITP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} 0 : \mathsf{Nat}} \quad \text{T\_ZEROP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{succ}\, t : \mathsf{Nat}} \quad \mathrm{T\_SUCC}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat} \\ \Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash_{\mathsf{CG}} t_2 : A\end{array}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A} \quad \mathrm{T\_NCASE}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} [] : \forall (X <: ?).\mathsf{List}\, X} \quad \mathrm{T\_EMPTY}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma \vdash_{\mathsf{CG}} t_2 : \mathsf{List}\, A}{\Gamma \vdash_{\mathsf{CG}} t_1 :: t_2 : \mathsf{List}\, A} \quad \mathrm{T\_CONS}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\mathsf{CG}} t : \mathsf{List}\, A \\ \Gamma \vdash_{\mathsf{CG}} t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash_{\mathsf{CG}} t_2 : B\end{array}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t : \mathsf{List}\, A\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 : B} \quad \mathrm{T\_LCASE}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A_2}{\Gamma \vdash_{\mathsf{CG}} (t_1, t_2) : A_1 \times A_2} \quad \mathrm{T\_PAIR}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{fst}\, t : A_1} \quad \mathrm{T\_FST}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{snd}\, t : A_2} \quad \mathrm{T\_SND}$$

$$\frac{\Gamma, x : A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \lambda (x : A).t : A \to B} \quad \mathrm{T\_LAM}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \to B \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A}{\Gamma \vdash_{\mathsf{CG}} t_1\, t_2 : B} \quad \mathrm{T\_APP}$$

$$\frac{\Gamma, X <: A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \Lambda (X <: A).t : \forall (X <: A).B} \quad \mathrm{T\_LAM}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \forall (X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} [A]t : [A/X]C} \quad \mathrm{T\_TYPEAPP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} t : B} \quad \mathrm{T\_SUB}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{error}_A : A} \quad \mathrm{T\_ERROR}$$

$\boxed{t_1 \rightsquigarrow t_2}$ call by name

$$\frac{}{\mathsf{unbox}_A\, (\mathsf{box}_A\, t) \rightsquigarrow t} \quad \mathrm{RD\_RETRACT T}$$

$$\frac{A \neq B}{\mathsf{unbox}_A\, (\mathsf{box}_B\, t) \rightsquigarrow \mathsf{error}_A} \quad \mathrm{RD\_RETRACTE}$$

$$\frac{}{\mathsf{split}_S\, (\mathsf{squash}_S\, t) \rightsquigarrow t} \quad \mathrm{RD\_RETRACTU}$$

$$\frac{x : B \vdash_{\mathsf{CG}} \mathcal{E}[x] : A}{\mathcal{E}[\mathsf{error}_B] \rightsquigarrow \mathsf{error}_A} \quad \mathrm{RD\_ERROR}$$

$$\frac{}{\mathsf{case}\, 0 : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow t_1} \quad \mathrm{RD\_NCASE0}$$

$$\frac{}{\mathsf{case}\, (\mathsf{succ}\, t) : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow [t/x]t_2} \quad \mathrm{RD\_NCASESUCC}$$

$$\frac{}{\mathsf{case}\,[]\colon \mathsf{List}\,A\,\mathsf{of}\,[] \to t_1, (x :: y) \to t_2 \rightsquigarrow t_1}\ \textsc{rd\_lcaseEmpty}$$

$$\frac{}{\mathsf{case}\,(t_1 :: t_2)\colon \mathsf{List}\,A\,\mathsf{of}\,[] \to t_3, (x :: y) \to t_4 \rightsquigarrow [t_1/x][t_2/y]t_4}\ \textsc{rd\_lcaseCons}$$

$$\frac{}{(\lambda(x : A_1).t_2)\,t_1 \rightsquigarrow [t_1/x]t_2}\ \textsc{rd\_beta}$$

$$\frac{}{\mathsf{fst}\,(t_1, t_2) \rightsquigarrow t_1}\ \textsc{rd\_proj1}$$

$$\frac{}{\mathsf{snd}\,(t_1, t_2) \rightsquigarrow t_2}\ \textsc{rd\_proj2}$$

$$\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t}\ \textsc{rd\_typeBeta}$$

$$\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2}\ \textsc{rd\_typeApp}$$

$$\frac{t_1 \rightsquigarrow t_2}{\mathcal{E}[t_1] \rightsquigarrow \mathcal{E}[t_2]}\ \textsc{rd\_Cong}$$

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Definition rules:        83 good    0 bad
Definition rule clauses: 136 good    0 bad
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