

$termvar, x, y, z, f$
 $typevar, X, Y, Z$
 $index, i, j, k$
 t, c, v, s, n

$::=$
 $|$ x
 $|$ **triv**
 $|$ **box**
 $|$ **unbox**
 $|$ $\Lambda(X <: A).t$
 $|$ $[A]t$
 $|$ $\lambda(x : A).t$
 $|$ $t_1 t_2$
 $|$ (t_1, t_2)
 $|$ **fst** t
 $|$ **snd** t
 $|$ **succ** t
 $|$ 0
 $|$ **case** t **of** $t_3 \rightarrow t_1, t_4 \rightarrow t_2$
 $|$ \square
 $|$ $t :: t'$
 $|$ (t) S

K $::=$
 $|$ \star

A, B, C, D, E, S, U $::=$
 $|$ \top
 $|$ \mathbb{S}
 $|$ X
 $|$ **List** A
 $|$ $\forall(X <: A).B$
 $|$ **Unit**
 $|$ **Nat**
 $|$ $?$
 $|$ $A_1 \rightarrow A_2$
 $|$ $A_1 \times A_2$
 $|$ (A) S

Γ $::=$
 $|$ \cdot
 $|$ $\Gamma, X <: A$
 $|$ $\Gamma, x : A$

$\boxed{\Gamma \vdash A \sim B}$

$\frac{}{\Gamma \vdash A \sim A}$ C-REFL
 $\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash A \sim ?}$ C-BoxP
 $\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash ? \sim A}$ C-UNBOXP

$$\begin{array}{c}
\frac{}{\Gamma \vdash A \sim ?} \text{ C_BOX} \\
\frac{}{\Gamma \vdash ? \sim A} \text{ C_UNBOX} \\
\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\text{List } A) \sim (\text{List } B)} \text{ C_LIST} \\
\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \rightarrow B_1) \sim (A_2 \rightarrow B_2)} \text{ C_ARROW} \\
\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \text{ C_PROD} \\
\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A). B_1) \sim (\forall (X <: A). B_2)} \text{ C_FORALL}
\end{array}$$

$$\boxed{A \sqsubseteq B}$$

$$\begin{array}{c}
\frac{\Gamma \vdash A \lesssim \mathbb{S}}{A \sqsubseteq ?} \text{ P_UP} \\
\frac{}{A \sqsubseteq ?} \text{ P_U} \\
\frac{}{A \sqsubseteq A} \text{ P_REFL} \\
\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \rightarrow B) \sqsubseteq (C \rightarrow D)} \text{ P_ARROW} \\
\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \text{ P_PROD} \\
\frac{A \sqsubseteq B}{(\text{List } A) \sqsubseteq (\text{List } B)} \text{ P_LIST} \\
\frac{B_1 \sqsubseteq B_2}{(\forall (X <: A). B_1) \sqsubseteq (\forall (X <: A). B_2)} \text{ P_FORALL}
\end{array}$$

$$\boxed{t \sqsubseteq t'}$$

$$\begin{array}{c}
\frac{}{t \sqsubseteq t} \text{ TP_REFL} \\
\frac{t_1 \sqsubseteq t_2}{(\text{succ } t_1) \sqsubseteq (\text{succ } t_2)} \text{ TP_SUCC} \\
\frac{t_1 \sqsubseteq t_4 \quad t_2 \sqsubseteq t_5 \quad t_3 \sqsubseteq t_6}{(\text{case } t_1 \text{ of } 0 \rightarrow t_2, (\text{succ } x) \rightarrow t_3) \sqsubseteq (\text{case } t_4 \text{ of } 0 \rightarrow t_5, (\text{succ } x) \rightarrow t_6)} \text{ TP_NATE} \\
\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1, t_2) \sqsubseteq (t_3, t_4)} \text{ TP_PAIR} \\
\frac{t_1 \sqsubseteq t_2}{(\text{fst } t_1) \sqsubseteq (\text{fst } t_2)} \text{ TP_FST} \\
\frac{t_1 \sqsubseteq t_2}{(\text{snd } t_1) \sqsubseteq (\text{snd } t_2)} \text{ TP_SND} \\
\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \text{ TP_CONS}
\end{array}$$

$$\begin{array}{c}
\frac{t_1 \sqsubseteq t_4 \quad t_2 \sqsubseteq t_5 \quad t_3 \sqsubseteq t_6}{(\text{case } t_1 \text{ of } [] \rightarrow t_2, (x :: y) \rightarrow t_3) \sqsubseteq (\text{case } t_4 \text{ of } 0 \rightarrow t_5, (x :: y) \rightarrow t_6)} \quad \text{TP_LISTE} \\
\\
\frac{t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{(\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP_FUN} \\
\\
\frac{t_1 \sqsubseteq t_3 \quad t_2 \sqsubseteq t_4}{(t_1 \ t_2) \sqsubseteq (t_3 \ t_4)} \quad \text{TP_APP} \\
\\
\frac{t_1 \sqsubseteq t_2}{(\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \quad \text{TP_TFUN} \\
\\
\frac{t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{[A]t_1 \sqsubseteq [B]t_2} \quad \text{TP_TAPP}
\end{array}$$

$$\boxed{\Gamma \vdash A \lesssim B}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash A \lesssim A} \quad \text{S_REFL} \\
\\
\frac{X <: A' \in \Gamma \quad \Gamma \vdash A' \sim A}{\Gamma \vdash X \lesssim A} \quad \text{S_VAR} \\
\\
\frac{}{\Gamma \vdash A \lesssim \top} \quad \text{S_TOP} \\
\\
\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash A \lesssim ?} \quad \text{S_BOX} \\
\\
\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash ? \lesssim A} \quad \text{S_UNBOX} \\
\\
\frac{}{\Gamma \vdash \top \lesssim \mathbb{S}} \quad \text{S_TOPSL} \\
\\
\frac{}{\Gamma \vdash ? \lesssim \mathbb{S}} \quad \text{S_USL} \\
\\
\frac{}{\Gamma \vdash \text{Nat} \lesssim \mathbb{S}} \quad \text{S_NATSL} \\
\\
\frac{}{\Gamma \vdash \text{Unit} \lesssim \mathbb{S}} \quad \text{S_UNITSL} \\
\\
\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash \text{List } A \lesssim \mathbb{S}} \quad \text{S_LISTSL} \\
\\
\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \rightarrow B \lesssim \mathbb{S}} \quad \text{S_ARROWSL} \\
\\
\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \times B \lesssim \mathbb{S}} \quad \text{S_PROD SL} \\
\\
\frac{\Gamma \vdash A \lesssim B}{\Gamma \vdash (\text{List } A) \lesssim (\text{List } B)} \quad \text{S_LIST} \\
\\
\frac{\Gamma \vdash A_1 \lesssim A_2 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \times B_1) \lesssim (A_2 \times B_2)} \quad \text{S_PROD} \\
\\
\frac{\Gamma \vdash A_2 \lesssim A_1 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \rightarrow B_1) \lesssim (A_2 \rightarrow B_2)} \quad \text{S_ARROW} \\
\\
\frac{\Gamma, X <: A \vdash B_1 \lesssim B_2}{\Gamma \vdash (\forall(X <: A).B_1) \lesssim (\forall(X <: A).B_2)} \quad \text{S_FORALL}
\end{array}$$

$$\boxed{\Gamma \vdash_{\text{SG}} t : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma}{\Gamma \vdash_{\text{SG}} x : A} \quad \text{T_VARP} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : A}{\Gamma \vdash_{\text{SG}} \text{box}_A t : ?} \quad \text{T_BOX} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : A}{\Gamma \vdash_{\text{SG}} \text{unbox}_A t : A} \quad \text{T_UNBOX} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : S}{\Gamma \vdash_{\text{SG}} \text{squash}_S t : ?} \quad \text{T_SQUASH} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : ?}{\Gamma \vdash_{\text{SG}} \text{split}_S t : S} \quad \text{T_SPLIT} \\
\\
\frac{}{\Gamma \vdash_{\text{SG}} \text{triv} : \text{Unit}} \quad \text{T_UNITP} \\
\\
\frac{}{\Gamma \vdash_{\text{SG}} 0 : \text{Nat}} \quad \text{T_ZEROP} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : A \quad \text{nat}(A) = \text{Nat}}{\Gamma \vdash_{\text{SG}} \text{succ } t : \text{Nat}} \quad \text{T_SUCC} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : C \quad \text{nat}(C) = \text{Nat} \quad \Gamma \vdash A_1 \sim A \quad \Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma, x : \text{Nat} \vdash_{\text{SG}} t_2 : A_2 \quad \Gamma \vdash A_2 \sim A}{\Gamma \vdash_{\text{SG}} \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \quad \text{T_NCASE} \\
\\
\frac{}{\Gamma \vdash_{\text{SG}} [] : \forall (X <: \top). \text{List } X} \quad \text{T_EMPTY} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{list}(A_2) = \text{List } A_3 \quad \Gamma \vdash A_1 \sim A_3}{\Gamma \vdash_{\text{SG}} t_1 :: t_2 : \text{List } A_3} \quad \text{T_CONS} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2}{\Gamma \vdash_{\text{SG}} (t_1, t_2) : A_1 \times A_2} \quad \text{T_PAIR} \\
\\
\frac{\Gamma, x : A \vdash_{\text{SG}} t : B}{\Gamma \vdash_{\text{SG}} \lambda(x : A). t : A \rightarrow B} \quad \text{T_LAM} \\
\\
\frac{\Gamma, X <: A \vdash_{\text{SG}} t : B}{\Gamma \vdash_{\text{SG}} \Lambda(X <: A). t : \forall (X <: A). B} \quad \text{T_LAM} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : \forall (X <: B). C \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash_{\text{SG}} [A]t : [A/X]C} \quad \text{T_TYPEAPP} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : A \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash_{\text{SG}} t : B} \quad \text{T_SUB} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : C \quad \text{list}(C) = \text{List } A \quad \Gamma \vdash_{\text{SG}} t_1 : B_1 \quad \Gamma, x : A, y : \text{List } A \vdash_{\text{SG}} t_2 : B_2 \quad \Gamma \vdash B_1 \sim B \quad \Gamma \vdash B_2 \sim B}{\Gamma \vdash_{\text{SG}} \text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \quad \text{T_LCASE} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t_1 : C \quad \Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{fun}(C) = A_1 \rightarrow B_1}{\Gamma \vdash_{\text{SG}} t_1 t_2 : B_1} \quad \text{T_APP} \\
\\
\frac{\Gamma \vdash_{\text{SG}} t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash_{\text{SG}} \text{fst } t : A_1} \quad \text{T_FST}
\end{array}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash_{\text{SG}} \text{snd } t : A_2} \quad \text{T_SND}$$

$$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CI_VAR}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \text{Nat}} \quad \text{CI_ZERO}$$

$$\frac{}{\Gamma \vdash \text{triv} \Rightarrow \text{triv} : \text{Unit}} \quad \text{CI_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ } (\text{unbox}_{\text{Nat}} t_2) : \text{Nat}} \quad \text{CI_SUCCU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \text{Nat}}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ } t_2 : \text{Nat}} \quad \text{CI_SUCC}$$

$$\frac{\begin{array}{l} \Gamma \vdash A_1 \sim A \quad \Gamma \vdash A_2 \sim A \quad \text{caster}(A_2, A) = c_2 \quad \text{caster}(A_1, A) = c_1 \\ \Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma, x : \text{Nat} \vdash t_2 \Rightarrow t'_2 : A_2 \\ t'' = (\text{unbox}_{\text{Nat}} t') \quad t'_1 = (c_1 t'_1) \quad t'_2 = (c_2 t'_2) \end{array}}{\Gamma \vdash (\text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2) \Rightarrow (\text{case } t'' \text{ of } 0 \rightarrow t'_1, (\text{succ } x) \rightarrow t'_2) : A} \quad \text{CI_NCASEU}$$

$$\frac{\begin{array}{l} \Gamma \vdash t \Rightarrow t' : \text{Nat} \quad \Gamma \vdash A_1 \sim A \\ \Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma, x : \text{Nat} \vdash t_2 \Rightarrow t'_2 : A_2 \quad \Gamma \vdash A_2 \sim A \end{array}}{\Gamma \vdash (\text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2) \Rightarrow (\text{case } t' \text{ of } 0 \rightarrow t'_1, (\text{succ } x) \rightarrow t'_2) : A} \quad \text{CI_NCASE}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \quad \text{CI_PAIR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst } (\text{unbox}_{(?) \times ?} t_2) : ?} \quad \text{CI_FSTU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst } t_2 : A_1} \quad \text{CI_FST}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd } (\text{unbox}_{(?) \times ?} t_2) : ?} \quad \text{CI_SNDU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd } t_2 : B} \quad \text{CI_SND}$$

$$\frac{}{\Gamma \vdash [] \Rightarrow [] : \forall(X <: \top). \text{List } X} \quad \text{CI_EMPTY}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t'_2 : ?}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((\text{box}_{A_1} t'_1) :: (\text{unbox}_{(\text{List } ?)} t'_2)) : \text{List } ?} \quad \text{CI_CONS U}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t'_2 : \text{List } A_2 \quad \Gamma \vdash A_1 \lesssim A_2 \quad \text{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c t'_1) :: t'_2) : \text{List } A_2} \quad \text{CI_CONS}$$

$$\frac{\begin{array}{l} \Gamma \vdash t \Rightarrow t' : ? \quad \text{caster}(B_1, B) = c_1 \quad \text{caster}(B_2, B) = c_2 \\ \Gamma \vdash t_1 \Rightarrow t'_1 : B_1 \quad \Gamma, x : ?, y : \text{List } ? \vdash t_2 \Rightarrow t'_2 : B_2 \quad \Gamma \vdash B_1 \sim B \quad \Gamma \vdash B_2 \sim B \end{array}}{\Gamma \vdash (\text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2) \Rightarrow (\text{case } (\text{unbox}_{(\text{List } ?)} t') \text{ of } [] \rightarrow (c_1 t'_1), (x :: y) \rightarrow (c_2 t'_2)) : B} \quad \text{CI_LCASEU}$$

$$\frac{\begin{array}{l} \Gamma \vdash t \Rightarrow t : \text{List } A \quad \text{caster}(B_1, B) = c_1 \quad \text{caster}(B_2, B) = c_2 \\ \Gamma \vdash t_1 \Rightarrow t'_1 : B_1 \quad \Gamma, x : A, y : \text{List } A \vdash t_2 \Rightarrow t'_2 : B_2 \quad \Gamma \vdash B_1 \sim B \quad \Gamma \vdash B_2 \sim B \end{array}}{\Gamma \vdash (\text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2) \Rightarrow (\text{case } t' \text{ of } [] \rightarrow (c_1 t'_1), (x :: y) \rightarrow (c_2 t'_2)) : B} \quad \text{CI_LCASE}$$

$$\begin{array}{c}
\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \rightarrow A_2} \quad \text{CI_LAM} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t'_1 : ? \quad \Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \quad \text{caster}(A_2, ?) = c}{\Gamma \vdash t_1 t_2 \Rightarrow (\text{unbox}_{(? \rightarrow ?)} t'_1) (c t'_2) : ?} \quad \text{CI_APPU} \\
\\
\frac{\Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \quad \Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \rightarrow B \quad \Gamma \vdash A_2 \sim A_1 \quad \text{caster}(A_2, A_1) = c}{\Gamma \vdash t_1 t_2 \Rightarrow t'_1 (c t'_2) : B} \quad \text{CI_APP} \\
\\
\frac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall(X <: A).B} \quad \text{CI_LAM} \\
\\
\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall(X <: B).C \quad \Gamma \vdash A \sim A' \quad \Gamma \vdash A' <: B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A']t_2) : [A'/X]C} \quad \text{CI_TYPEAPP}
\end{array}$$

Definition rules: 86 good 0 bad
 Definition rule clauses: 164 good 0 bad