

$termvar, x, y, z, f$
 $typevar, X, Y, Z$
 $index, i, j, k$
 t, c, v, s, n

$::=$
 $|$ x
 $|$ **triv**
 $|$ **box**
 $|$ **unbox**
 $|$ $\Lambda(X <: A).t$
 $|$ $[A]t$
 $|$ $\lambda(x : A).t$
 $|$ $t_1 t_2$
 $|$ (t_1, t_2)
 $|$ **fst** t
 $|$ **snd** t
 $|$ **succ** t
 $|$ 0
 $|$ **case** t **of** $t_3 \rightarrow t_1, t_4 \rightarrow t_2$
 $|$ \square
 $|$ $t :: t'$
 $|$ (t) S

K $::=$
 $|$ \star

A, B, C, D, E, S, U $::=$
 $|$ X
 $|$ **List** A
 $|$ $\forall(X <: A).B$
 $|$ \top
 $|$ \mathbb{C}
 $|$ \mathbb{S}
 $|$ **Unit**
 $|$ **Nat**
 $|$ $?$
 $|$ $A_1 \rightarrow A_2$
 $|$ $A_1 \times A_2$
 $|$ (A) S

Γ $::=$
 $|$ \cdot
 $|$ $\Gamma, X <: A$
 $|$ $\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K_VAR}$$

$$\frac{}{\Gamma \vdash \mathbf{Unit} : \star} \quad \text{K_UNIT}$$

$$\frac{}{\Gamma \vdash \mathbf{Nat} : \star} \quad \text{K_NAT}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash ? : \star} \text{K_UNITYTYPE} \\
\frac{\Gamma \vdash A : \star}{\Gamma \vdash \text{List } A : \star} \text{K_LIST} \\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \rightarrow B : \star} \text{K_ARROW} \\
\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \text{K_PROD} \\
\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A). B : \star} \text{K_FORALL}
\end{array}$$

$\boxed{\Gamma \text{ Ok}}$

$$\begin{array}{c}
\frac{}{\cdot \text{Ok}} \text{OK_EMPTY} \\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A) \text{ Ok}} \text{OK_TYPEVAR} \\
\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \text{ Ok}} \text{OK_VAR}
\end{array}$$

$\boxed{\Gamma \vdash A <: B}$

$$\begin{array}{c}
\frac{\Gamma \text{ Ok}}{\Gamma \vdash A <: A} \text{S_REFL} \\
\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \text{S_TRANS} \\
\frac{X <: A \in \Gamma \quad \Gamma \text{ Ok}}{\Gamma \vdash X <: A} \text{S_VAR} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash A <: \top} \text{S_TOP} \\
\frac{\Gamma \vdash A <: \mathbb{C}}{\Gamma \vdash \text{List } A <: \mathbb{C}} \text{S_LISTC} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash ? <: \mathbb{C}} \text{S_U} \\
\frac{\Gamma \vdash A <: \mathbb{C} \quad \Gamma \vdash B <: \mathbb{C}}{\Gamma \vdash (A \rightarrow B) <: \mathbb{C}} \text{S_ARROWC} \\
\frac{\Gamma \vdash A <: \mathbb{C} \quad \Gamma \vdash B <: \mathbb{C}}{\Gamma \vdash (A \times B) <: \mathbb{C}} \text{S_PRODC} \\
\frac{}{\Gamma \vdash \mathbb{S} <: \mathbb{C}} \text{S_SL} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Nat} <: \mathbb{S}} \text{S_NATS} \\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Unit} <: \mathbb{S}} \text{S_UNITS} \\
\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \text{List } A <: \mathbb{S}} \text{S_LISTS}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash (A \rightarrow B) <: \mathbb{S}} \quad \text{S_ARROWS} \\
\\
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash (A \times B) <: \mathbb{S}} \quad \text{S_PRODS} \\
\\
\frac{\Gamma \vdash A <: B}{\Gamma \vdash (\text{List } A) <: (\text{List } B)} \quad \text{S_LIST} \\
\\
\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \times B_1) <: (A_2 \times B_2)} \quad \text{S_PROD} \\
\\
\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \rightarrow B_1) <: (A_2 \rightarrow B_2)} \quad \text{S_ARROW} \\
\\
\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash (\forall (X <: A). B_1) <: (\forall (X <: A). B_2)} \quad \text{S_FORALL}
\end{array}$$

$$\boxed{\Gamma \vdash A \sim B}$$

$$\begin{array}{c}
\frac{}{\Gamma \vdash A \sim A} \quad \text{C_REFL} \\
\\
\frac{\Gamma \vdash A <: \mathbb{C}}{\Gamma \vdash A \sim ?} \quad \text{C_BOX} \\
\\
\frac{\Gamma \vdash A <: \mathbb{C}}{\Gamma \vdash ? \sim A} \quad \text{C_UNBOX} \\
\\
\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\text{List } A) \sim (\text{List } B)} \quad \text{C_LIST} \\
\\
\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \rightarrow B_1) \sim (A_2 \rightarrow B_2)} \quad \text{C_ARROW} \\
\\
\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C_PROD} \\
\\
\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A). B_1) \sim (\forall (X <: A). B_2)} \quad \text{C_FORALL}
\end{array}$$

$$\boxed{\Gamma \vdash t : A}$$

$$\begin{array}{c}
\frac{x : A \in \Gamma \quad \Gamma \text{ Ok}}{\Gamma \vdash x : A} \quad \text{VARP} \\
\\
\frac{}{\Gamma \vdash \text{box} : \forall (X <: \mathbb{S}). (X \rightarrow ?)} \quad \text{BOX} \\
\\
\frac{}{\Gamma \vdash \text{unbox} : \forall (X <: \mathbb{S}). (? \rightarrow X)} \quad \text{UNBOX} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{triv} : \text{Unit}} \quad \text{UNITP} \\
\\
\frac{\Gamma \text{ Ok}}{\Gamma \vdash 0 : \text{Nat}} \quad \text{ZEROP} \\
\\
\frac{\Gamma \vdash t : \text{Nat}}{\Gamma \vdash \text{succ } t : \text{Nat}} \quad \text{SUCC}
\end{array}$$

$$\frac{\Gamma \vdash t : C \quad \text{nat}(C) = \text{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A}{\Gamma \vdash \text{case } t \text{ of } 0 \rightarrow t_1, (\text{succ } x) \rightarrow t_2 : A} \text{ NCASE}$$

$$\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall (X <: \top). \text{List } X} \text{ EMPTY}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma \vdash t_2 : \text{List } A}{\Gamma \vdash t_1 :: t_2 : \text{List } A} \text{ CONS}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \text{ PAIR}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A). t : A \rightarrow B} \text{ LAM}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A). t : \forall (X <: A). B} \text{ LAM}$$

$$\frac{\Gamma \vdash t : \forall (X <: B). C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C} \text{ TYPEAPP}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B} \text{ SUB}$$

$$\frac{\Gamma \vdash t : C \quad \text{list}(C) = \text{List } A \quad \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \text{List } A \vdash t_2 : B}{\Gamma \vdash \text{case } t \text{ of } [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \text{ LCASE}$$

$$\frac{\Gamma \vdash t_1 : C \quad \text{fun}(C) = A_1 \rightarrow B \quad \Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_1 \sim A_2}{\Gamma \vdash t_1 t_2 : B} \text{ APP}$$

$$\frac{\Gamma \vdash t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash \text{fst } t : A_1} \text{ FST}$$

$$\frac{\Gamma \vdash t : B \quad \text{prod}(B) = A_1 \times A_2}{\Gamma \vdash \text{snd } t : A_2} \text{ SND}$$

$$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$$

Cast insertion

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \text{ CL_VAR}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : A} \text{ CL_ZERO}$$

$$\frac{}{\Gamma \vdash \text{triv} \Rightarrow \text{triv} : \text{Unit}} \text{ CL_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \text{Nat}}{\Gamma \vdash \text{succ } t_1 \Rightarrow \text{succ } t_2 : \text{Nat}} \text{ CL_SUCC}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \text{ CL_PAIR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \text{fst } t_1 \Rightarrow \text{fst } t_2 : A} \text{ CL_FST2}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \text{snd } t_1 \Rightarrow \text{snd } t_2 : B} \text{ CL_SND2}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1). t_1 \Rightarrow \lambda(x : A_1). t_2 : A_1 \rightarrow A_2} \text{ CL_LAM}$$

Definition rules: 62 good 0 bad
Definition rule clauses: 117 good 0 bad