# Gradual Typing from a Categorical Perspective

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## References

## A The Complete Spec of Grady

| | | | | |
|---|---|---|---|---|
| *termvar*, $x$ | | | | |
| *index*, $k$ | | | | |
| $t$ | ::= | | | term |
| | \| | $x$ | | variable |
| | \| | $\mathsf{triv}$ | | unit |
| | \| | $\lambda x : T.t$ | | $\lambda$-abstraction |
| | \| | $t_1\, t_2$ | | function application |
| | \| | $(t_1, t_2)$ | | pair constructor |
| | \| | $\mathsf{fst}\, t$ | | first projection |
| | \| | $\mathsf{snd}\, t$ | | second projection |
| | \| | $\mathsf{succ}\, t$ | | successor function |
| | \| | $0$ | | zero |
| | \| | $(t)$ | S | |
| | \| | $[t_1/x]t_2$ | M | |
| | \| | $t$ | M | |
| $T$ | ::= | | | type |
| | \| | $1$ | | unit type |
| | \| | $\mathbb{N}$ | | natural number type |
| | \| | $?$ | | untyped universe |
| | \| | $T_1 \to T_2$ | | function type |
| | \| | $T_1 \times T_2$ | | cartesian product type |
| | \| | $(T)$ | S | |
| $\Gamma$ | ::= | | | typing context |
| | \| | $\cdot$ | | empty context |
| | \| | $\Gamma, x : T$ | | cons |
| | \| | $\Gamma, \Gamma'$ | M | append |

| | | | |
|---|---|---|---|
| *terminals* | ::= | | |
| | \| | $\rightarrow$ | |
| | \| | $\vdash$ | |
| | \| | $1$ | |
| | \| | $\mathbb{N}$ | |
| | \| | $\mathsf{succ}$ | |
| | \| | $?$ | |
| | \| | $\cdot$ | |
| | \| | $\sim$ | |
| | \| | $\sim_U$ | |
| | \| | $\in$ | |
| | \| | $\rightsquigarrow$ | |
| | \| | $\mathsf{triv}$ | |
| *formula* | ::= | | |
| | \| | *judgement* | |
| | \| | $x : T \in \Gamma$ | |
| | \| | $formula_1 \quad formula_2$ | |
| | \| | **not** *formula* | |
| | \| | *formula* $\quad\quad \mathsf{S}$ | |
| *ConvType* | ::= | | |
| | \| | $T_1 \sim_U T_2$ | $T_1$ can can be converted into $T_2$ |
| | \| | $T_1 \sim T_2$ | $T_1$ is consistent with $T_2$ |
| *Typing* | ::= | | |
| | \| | $\Gamma \vdash t : T$ | $t$ has type $T$ in context $\Gamma$ |
| *Reduction* | ::= | | |
| | \| | $t_1 \rightsquigarrow t_2$ | $t_1$ reduces to $t_2$ |
| *judgement* | ::= | | |
| | \| | *ConvType* | |
| | \| | *Typing* | |
| | \| | *Reduction* | |
| *user_syntax* | ::= | | |
| | \| | *termvar* | |
| | \| | *index* | |
| | \| | $t$ | |

| $T$
| $\Gamma$
| *terminals*
| *formula*

$\boxed{T_1 \sim_U T_2}$ $T_1$ can can be converted into $T_2$

$$\frac{}{T \sim_U T} \quad \text{CV\_REFL}$$

$$\frac{T_1 \sim_U T_2 \quad T_2 \sim_U T_3}{T_1 \sim_U T_3} \quad \text{CV\_TRANS}$$

$$\frac{}{(? \to ?) \sim_U ?} \quad \text{CV\_INJ}$$

$$\frac{}{? \sim_U (? \to ?)} \quad \text{CV\_SURJ}$$

$$\frac{T_1 \sim_U T_1'}{(T_1 \times T_2) \sim_U (T_1' \times T_2)} \quad \text{CV\_PAIR1}$$

$$\frac{T_2 \sim_U T_2'}{(T_1 \times T_2) \sim_U (T_1 \times T_2')} \quad \text{CV\_PAIR2}$$

$$\frac{T_1 \sim_U T_1'}{(T_1 \to T_2) \sim_U (T_1' \to T_2)} \quad \text{CV\_FUN1}$$

$$\frac{T_2 \sim_U T_2'}{(T_1 \to T_2) \sim_U (T_1 \to T_2')} \quad \text{CV\_FUN2}$$

$\boxed{T_1 \sim T_2}$ $T_1$ is consistent with $T_2$

$$\frac{}{T \sim T} \quad \text{CS\_REFL}$$

$$\frac{}{? \sim T} \quad \text{CS\_UL}$$

$$\frac{}{T \sim ?} \quad \text{CS\_UR}$$

$$\frac{T_1 \sim_U T_2}{T_1 \sim T_2} \quad \text{CS\_CONV}$$

$$\frac{T_1 \sim T_1' \quad T_2 \sim T_2'}{(T_1 \times T_2) \sim (T_1' \times T_2')} \quad \text{CS\_PAIR}$$

$$\frac{T_1 \sim T_1' \quad T_2 \sim T_2'}{(T_1 \to T_2) \sim (T_1' \to T_2')} \quad \text{CS\_ARROW}$$

$\boxed{\Gamma \vdash t : T}$ $t$ has type $T$ in context $\Gamma$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} : 1} \quad \text{UNIT}$$

$$\frac{}{\Gamma \vdash 0 : \mathbb{N}} \quad \text{ZERO}$$

$$\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \mathsf{succ}\, t : \mathbb{N}} \quad \text{SUCC}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2} \quad \text{PAIR}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \mathsf{fst}\, t : T_1} \quad \text{FST}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \mathsf{snd}\, t : T_2} \quad \text{SND}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x : T_1 . t : T_1 \to T_2} \quad \text{ABS}$$

$$\frac{\Gamma \vdash t : T_1 \quad T_1 \sim_U T_2}{\Gamma \vdash t : T_2} \quad \text{U}$$

$$\frac{\Gamma \vdash t_1 : T_1 \to T_2 \quad \Gamma \vdash t_2 : T_3 \quad T_3 \sim T_1}{\Gamma \vdash t_1\, t_2 : T_2} \quad \text{APP}$$

$\boxed{t_1 \leadsto t_2}$ $t_1$ reduces to $t_2$

$$\frac{}{(\lambda x : T . t_2)\, t_1 \leadsto [t_1/x] t_2} \quad \text{RD\_BETA}$$

$$\frac{}{(\lambda x : T . t\, x) \leadsto t} \quad \text{RD\_ETA}$$

$$\frac{}{\mathsf{fst}\, (t_1, t_2) \leadsto t_1} \quad \text{RD\_PROJ1}$$

$$\frac{}{\mathsf{snd}\, (t_1, t_2) \leadsto t_2} \quad \text{RD\_PROJ2}$$

$$\frac{}{(\mathsf{fst}\, t, \mathsf{snd}\, t) \leadsto t} \quad \text{RD\_ETAP}$$