$termvar$, x, y, z, f
$typevar$, X, Y, Z
$index$, i, j, k

$t,\ c,\ v,\ s,\ n$	::=		
	$\vert$	x	
	$\vert$	triv	
	$\vert$	box	
	$\vert$	unbox	
	$\vert$	$\Lambda(X <: A).t$	
	$\vert$	$[A]t$	
	$\vert$	$\lambda(x : A).t$	
	$\vert$	$t_1\ t_2$	
	$\vert$	(t_1, t_2)	
	$\vert$	$\mathsf{fst}\ t$	
	$\vert$	$\mathsf{snd}\ t$	
	$\vert$	$\mathsf{succ}\ t$	
	$\vert$	0	
	$\vert$	$\mathsf{case}\ t\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
	$\vert$	$[]$	
	$\vert$	$t :: t'$	
	$\vert$	(t)	S
K	::=		
	$\vert$	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U$	::=		
	$\vert$	$\top$	
	$\vert$	$\mathbb{S}$	
	$\vert$	X	
	$\vert$	$\mathsf{List}\ A$	
	$\vert$	$\forall(X <: A).B$	
	$\vert$	Unit	
	$\vert$	Nat	
	$\vert$	$?$	
	$\vert$	$A_1 \to A_2$	
	$\vert$	$A_1 \times A_2$	
	$\vert$	(A)	S
Γ	::=		
	$\vert$	$\cdot$	
	$\vert$	$\Gamma, X <: A$	
	$\vert$	$\Gamma, x : A$	

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star}\quad \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \top : \star}\quad \text{K\_TOP}$$

$$\frac{}{\Gamma \vdash \mathbb{S} : \star}\quad \text{K\_SL}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star} \quad \text{K\_UNIT}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star} \quad \text{K\_NAT}$$

$$\frac{}{\Gamma \vdash ? : \star} \quad \text{K\_UNITYPE}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\, A : \star} \quad \text{K\_LIST}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \quad \text{K\_ARROW}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \text{K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A). B : \star} \quad \text{K\_FORALL}$$

$\boxed{\Gamma\, \mathrm{Ok}}$

$$\frac{}{\cdot\, \mathrm{Ok}} \quad \text{OK\_EMPTY}$$

$$\frac{\Gamma\, \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A)\, \mathrm{Ok}} \quad \text{OK\_TYPEVAR}$$

$$\frac{\Gamma\, \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\, \mathrm{Ok}} \quad \text{OK\_VAR}$$

$\boxed{\Gamma \vdash A \sim B}$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \sim A} \quad \text{C\_REFL}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \sim ?} \quad \text{C\_BOX}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash ? \sim A} \quad \text{C\_UNBOX}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\mathsf{List}\, A) \sim (\mathsf{List}\, B)} \quad \text{C\_LIST}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \to B_1) \sim (A_2 \to B_2)} \quad \text{C\_ARROW}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A). B_1) \sim (\forall (X <: A). B_2)} \quad \text{C\_FORALL}$$

$\boxed{G \vdash A \sqsubseteq B}$

$$\frac{\Gamma \vdash A : \star}{G \vdash A \sqsubseteq ?} \quad \text{P\_U}$$

$$\frac{\Gamma \vdash A : \star}{G \vdash A \sqsubseteq A} \quad \text{P\_REFL}$$

$$\frac{G \vdash A \sqsubseteq C \quad G \vdash B \sqsubseteq D}{G \vdash (A \to B) \sqsubseteq (C \to D)} \quad \textsc{P\_arrow}$$

$$\frac{G \vdash A \sqsubseteq C \quad G \vdash B \sqsubseteq D}{G \vdash (A \times B) \sqsubseteq (C \times D)} \quad \textsc{P\_prod}$$

$$\frac{G \vdash A \sqsubseteq B}{G \vdash (\mathsf{List}\, A) \sqsubseteq (\mathsf{List}\, B)} \quad \textsc{P\_list}$$

$$\frac{G \vdash B \sqsubseteq D}{G \vdash (\forall (X <: A).B) \sqsubseteq (\forall (X <: A).D)} \quad \textsc{P\_forall}$$

$\boxed{\Gamma \vdash A \lesssim B}$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim A} \quad \textsc{S\_Refl}$$

$$\frac{X <: A' \in \Gamma \quad \Gamma \vdash A' \sim A}{\Gamma \vdash X \lesssim A} \quad \textsc{S\_Var}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim \top} \quad \textsc{S\_Top}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim ?} \quad \textsc{S\_Box}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash ? \lesssim A} \quad \textsc{S\_Unbox}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} \lesssim \mathbb{S}} \quad \textsc{S\_NatSL}$$

$$\frac{\Gamma\, \mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} \lesssim \mathbb{S}} \quad \textsc{S\_UnitSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash \mathsf{List}\, A \lesssim \mathbb{S}} \quad \textsc{S\_ListSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \to B \lesssim \mathbb{S}} \quad \textsc{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \times B \lesssim \mathbb{S}} \quad \textsc{S\_ProdSL}$$

$$\frac{\Gamma \vdash A \lesssim B}{\Gamma \vdash (\mathsf{List}\, A) \lesssim (\mathsf{List}\, B)} \quad \textsc{S\_List}$$

$$\frac{\Gamma \vdash A_1 \lesssim A_2 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \times B_1) \lesssim (A_2 \times B_2)} \quad \textsc{S\_Prod}$$

$$\frac{\Gamma \vdash A_2 \lesssim A_1 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \to B_1) \lesssim (A_2 \to B_2)} \quad \textsc{S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 \lesssim B_2}{\Gamma \vdash (\forall (X <: A).B_1) \lesssim (\forall (X <: A).B_2)} \quad \textsc{S\_Forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma\, \mathrm{Ok}}{\Gamma \vdash x : A} \quad \textsc{T\_varP}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{box} : \forall(X <: \mathbb{S}).(X \to ?)}\ \textsc{T\_Box}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{unbox} : \forall(X <: \mathbb{S}).(? \to X)}\ \textsc{T\_Unbox}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}}\ \textsc{T\_unitP}$$

$$\frac{\Gamma\ \mathsf{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}}\ \textsc{T\_zeroP}$$

$$\frac{\Gamma \vdash t : A \quad \mathsf{nat}(A) = \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\ t : \mathsf{Nat}}\ \textsc{T\_succ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : C \quad \mathsf{nat}(C) = \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 : A}\ \textsc{T\_ncase}$$

$$\frac{\Gamma\ \mathsf{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: ?).\mathsf{List}\ X}\ \textsc{T\_empty}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : \mathsf{List}\ A_2 \quad \Gamma \vdash A_1 \lesssim A_2}{\Gamma \vdash t_1 :: t_2 : \mathsf{List}\ A_2}\ \textsc{T\_cons}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2}\ \textsc{T\_pair}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \to B}\ \textsc{T\_lam}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B}\ \textsc{T\_Lam}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash [A]t : [A/X]C}\ \textsc{T\_typeApp}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash t : B}\ \textsc{T\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : C \quad \mathsf{list}(C) = \mathsf{List}\ A \\ \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\ A \vdash t_2 : B\end{array}}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 : B}\ \textsc{T\_lcase}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 : C \quad \mathsf{fun}(C) = A_1 \to B_1 \\ \Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_2 \lesssim A_1\end{array}}{\Gamma \vdash t_1\ t_2 : B_2}\ \textsc{T\_app}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\ t : A_1}\ \textsc{T\_fst}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\ t : A_2}\ \textsc{T\_snd}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ Cast insertion

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A}\ \textsc{ci\_var}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \Rightarrow \mathsf{box} : S \to ?}\ \textsc{ci\_Box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \Rightarrow \mathsf{unbox} : ? \to S}\ \text{CI\_UNBOX}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \mathsf{Nat}}\ \text{CI\_ZERO}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \Rightarrow \mathsf{triv} : \mathsf{Unit}}\ \text{CI\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{succ}\, t_1 \Rightarrow \mathsf{succ}\,(\mathsf{unbox}_{\mathsf{Nat}}\, t_2) : \mathsf{Nat}}\ \text{CI\_SUCCU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t_1 \Rightarrow \mathsf{succ}\, t_2 : \mathsf{Nat}}\ \text{CI\_SUCC}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \Rightarrow t' : ?\\ \Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A\end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2) \Rightarrow (\mathsf{case}\,(\mathsf{unbox}_{\mathsf{Nat}}\, t')\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2') : A}\ \text{CI\_NCASEU}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \Rightarrow t' : \mathsf{Nat}\\ \Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A\end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2') : A}\ \text{CI\_NCASE}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2}\ \text{CI\_PAIR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\,(\mathsf{split}_{(? \times ?)}\, t_2) : ?}\ \text{CI\_FSTU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, t_2 : A_1}\ \text{CI\_FST}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\,(\mathsf{split}_{(? \times ?)}\, t_2) : ?}\ \text{CI\_SNDU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, t_2 : B}\ \text{CI\_SND}$$

$$\frac{}{\Gamma \vdash [] \Rightarrow [] : \forall (X <: ?).\mathsf{List}\, X}\ \text{CI\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_2' : \mathsf{List}\, A_2 \quad \Gamma \vdash A_1 \lesssim A_2 \quad \mathsf{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c\, t_1') :: t_2') : \mathsf{List}\, A}\ \text{CI\_CONS}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \Rightarrow t' : ?\\ \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : ?, y : \mathsf{List}\, ? \vdash t_2 \Rightarrow t_2' : B\end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\,(\mathsf{split}_{(\mathsf{List}\, ?)}\, t')\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B}\ \text{CI\_LCASEU}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \Rightarrow t : \mathsf{List}\, A\\ \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 \Rightarrow t_2' : B\end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B}\ \text{CI\_LCASE}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \to A_2}\ \text{CI\_LAM}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow t_1' : ?\\ \Gamma \vdash t_2 \Rightarrow t_2' : A_2 \quad \mathsf{caster}(A_2, ?) = c\end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow (\mathsf{split}_{(? \to ?)}\, t_1')\,(c\, t_2') : ?}\ \text{CI\_APPU}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_2 \Rightarrow t_2' : A_2\\ \Gamma \vdash t_1 \Rightarrow t_1' : A_1 \to B \quad \Gamma \vdash A_2 \lesssim A_1 \quad \mathsf{caster}(A_2, A_1) = c\end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow t_1'\,(c\, t_2') : B}\ \text{CI\_APP}$$

$$\frac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall(X <: A).B} \quad \textsc{ci\_Lam}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall(X <: B).C \quad \Gamma \vdash A \lesssim B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A]t_2) : [A/X]C} \quad \textsc{ci\_typeApp}$$

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Definition rules:        81 good    0 bad
Definition rule clauses: 160 good    0 bad
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