$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ s \quad ::=$

| x
| triv
| squash_U
| split_U
| Squash_S
| Split_S
| box_C
| unbox_C
| $\lambda x : A.t$
| $t_1\ t_2$
| (t_1, t_2)
| $\mathsf{fst}\ t$
| $\mathsf{snd}\ t$
| $\mathsf{succ}\ t$
| 0
| $\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2$
| $\mathsf{error}\ A\ B$
| (t) S

$n \quad ::=$

| 0
| $\mathsf{succ}\ n$

$v \quad ::=$

| triv
| $\lambda x : A.t$
| n
| split_U
| squash_U
| box_C
| unbox_C

$\mathcal{C} \quad ::=$

| $\square$
| $\lambda x : A.\mathcal{C}$
| $\mathcal{C}\ t_2$
| $t_1\ \mathcal{C}$
| $(\mathcal{C}, t_2)$
| $(t_1, \mathcal{C})$
| $\mathsf{fst}\ \mathcal{C}$
| $\mathsf{snd}\ \mathcal{C}$
| $\mathsf{succ}\ \mathcal{C}$
| $\mathsf{case}\ \mathcal{C}\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2$
| $\mathsf{case}\ t\ \mathsf{of}\ 0 \to \mathcal{C}, (\mathsf{succ}\ x) \to t_2$
| $\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to \mathcal{C}$

$$T \quad ::= $$
$$\quad | \quad \textsf{Unit}$$
$$\quad | \quad \textsf{Nat}$$

$$A,\ B,\ C,\ D,\ E,\ R,\ X,\ Y,\ U,\ S \quad ::=$$
$$\quad | \quad \textsf{Unit}$$
$$\quad | \quad \textsf{Nat}$$
$$\quad | \quad ?$$
$$\quad | \quad A_1 \to A_2$$
$$\quad | \quad A_1 \times A_2$$
$$\quad | \quad (A) \qquad \textsf{S}$$

$$\Gamma \quad ::=$$
$$\quad | \quad \cdot$$
$$\quad | \quad \Gamma, x : A$$

$$\boxed{\Gamma \vdash t : A}$$

$$\frac{x : A \,\in\, \Gamma}{\Gamma \vdash x : A} \ \ \text{VAR}$$

$$\frac{}{\Gamma \vdash \textsf{box}_T : T \to ?} \ \ \text{BOX}$$

$$\frac{}{\Gamma \vdash \textsf{unbox}_T : ? \to T} \ \ \text{UNBOX}$$

$$\frac{}{\Gamma \vdash \textsf{Box}_A : A \to ?} \ \ \text{BOXG}$$

$$\frac{}{\Gamma \vdash \textsf{Unbox}_A : ? \to A} \ \ \text{UNBOXG}$$

$$\frac{}{\Gamma \vdash \textsf{squash}_U : U \to ?} \ \ \text{SQUASH}$$

$$\frac{}{\Gamma \vdash \textsf{split}_U : ? \to U} \ \ \text{SPLIT}$$

$$\frac{}{\Gamma \vdash \textsf{Split}_S : S \to ?} \ \ \text{SPLITG}$$

$$\frac{}{\Gamma \vdash \textsf{Squash}_S : ? \to S} \ \ \text{SQUASHG}$$

$$\frac{}{\Gamma \vdash \textsf{triv} : \textsf{Unit}} \ \ \text{UNIT}$$

$$\frac{}{\Gamma \vdash 0 : \textsf{Nat}} \ \ \text{ZERO}$$

$$\frac{\Gamma \vdash t : \textsf{Nat}}{\Gamma \vdash \textsf{succ}\ t : \textsf{Nat}} \ \ \text{SUCC}$$

$$\frac{\Gamma \vdash t : \textsf{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \textsf{Nat} \vdash t_2 : A}{\Gamma \vdash \textsf{case}\ t\ \textsf{of}\ 0 \to t_1, (\textsf{succ}\ x) \to t_2 : A} \ \ \text{CASE}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \ \ \text{PAIR}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \textsf{fst}\ t : A_1} \ \ \text{FST}$$

$$\frac{\Gamma \vdash t : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2} \quad \text{SND}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A_1.t : A \to B} \quad \text{LAM}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\, t_2 : B} \quad \text{APP}$$

$$\frac{}{\Gamma \vdash \mathsf{error}\, A\, B : B} \quad \text{ERROR}$$

$$\boxed{\Gamma \vdash t_1 \rightsquigarrow t_2 : A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \rightsquigarrow x : A} \quad \text{RD\_VAR}$$

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash \mathsf{unbox}_T\,(\mathsf{box}_T\, t) \rightsquigarrow t : T} \quad \text{RD\_RETRACT}T$$

$$\frac{\Gamma \vdash t : T_1 \quad T_1 \neq T_2}{\Gamma \vdash \mathsf{unbox}_{T_2}\,(\mathsf{box}_{T_1}\, t) \rightsquigarrow \mathsf{error}\, T_1\, T_2 : T_2} \quad \text{RD\_RETRACT}E$$

$$\frac{}{\Gamma \vdash \mathcal{C}[\mathsf{error}\, T_1\, T_2] \rightsquigarrow \mathsf{error}\, T_1\, T_2 : T_2} \quad \text{RD\_ERROR}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : T}{\Gamma \vdash \mathsf{unbox}_T\, t_1 \rightsquigarrow \mathsf{unbox}_T\, t_2 : T} \quad \text{RD\_UNBOX}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathsf{Unbox}_A\,(\mathsf{Box}_A\, t) \rightsquigarrow t : A} \quad \text{RD\_RETRACT}TG$$

$$\frac{\Gamma \vdash t : S}{\Gamma \vdash \mathsf{Split}_S\,(\mathsf{Squash}_S\, t) \rightsquigarrow t : S} \quad \text{RD\_RETRACT}TSG$$

$$\frac{\Gamma \vdash t : U}{\Gamma \vdash \mathsf{split}_U\,(\mathsf{squash}_U\, t) \rightsquigarrow t : U} \quad \text{RD\_RETRACT}U$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_2 : U}{\Gamma \vdash \mathsf{split}_U\, t_1 \rightsquigarrow \mathsf{split}_U\, t_2 : U} \quad \text{RD\_SPLIT}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t' : \mathsf{Nat}} \quad \text{RD\_SUCC}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A}{\Gamma \vdash \mathsf{case}\, 0 \,\mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow t_1 : A} \quad \text{RD\_CASE}0$$

$$\frac{\Gamma \vdash t : \mathsf{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A}{\Gamma \vdash \mathsf{case}\,(\mathsf{succ}\, t)\,\mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow [t/x]t_2 : A} \quad \text{RD\_CASE}S\text{UCC}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : \mathsf{Nat} \quad \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A}{\Gamma \vdash \mathsf{case}\, t \,\mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow \mathsf{case}\, t' \,\mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A} \quad \text{RD\_CASE}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 : A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash (\lambda x : A_1.t_1)\, t_2 \rightsquigarrow [t_2/x]t_2 : A_2} \quad \text{RD\_BETA}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{fst}\,(t_1, t_2) \rightsquigarrow t_1 : A_1} \quad \text{RD\_PROJ}1$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash \mathsf{snd}\,(t_1, t_2) \rightsquigarrow t_2 : A_2} \quad \text{RD\_PROJ}2$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \rightarrow A_2 \quad \Gamma \vdash t_2 : A_1}{\Gamma \vdash t_1\, t_2 \rightsquigarrow t_1'\, t_2 : A_2} \quad \text{RD\_APP}1$$

$$\frac{\Gamma \vdash v : A_1 \rightarrow A_2 \quad \Gamma \vdash t \rightsquigarrow t' : A_1}{\Gamma \vdash v\, t \rightsquigarrow v\, t' : A_2} \quad \text{RD\_APP}2$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t \rightsquigarrow \mathsf{fst}\, t' : A_1} \quad \text{RD\_FST}$$

$$\frac{\Gamma \vdash t \rightsquigarrow t' : A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t \rightsquigarrow \mathsf{snd}\, t' : A_2} \quad \text{RD\_SND}$$

$$\frac{\Gamma \vdash t_1 \rightsquigarrow t_1' : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1', t_2) : A_1 \times A_2} \quad \text{RD\_PAIR}1$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 \rightsquigarrow t_2' : A_2}{\Gamma \vdash (t_1, t_2) \rightsquigarrow (t_1, t_2') : A_1 \times A_2} \quad \text{RD\_PAIR}2$$

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Definition rules:        41 good     0 bad
Definition rule clauses: 73 good     0 bad
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