```
termvar, \, x, \, y, \, z, \, f
  typevar, X, Y, Z
  index,\;i,\;j,\;k
  t, c, v, s, n
                                                         ::=
                                                                     \boldsymbol{x}
                                                                     triv
                                                                     box
                                                                     unbox
                                                                     \Lambda(X <: A).t
                                                                     [A]t
                                                                     \lambda(x:A).t
                                                                     t_1 t_2
                                                                    (t_1, t_2)
                                                                     \mathsf{fst}\;t
                                                                     \mathsf{snd}\; t
                                                                     \mathsf{succ}\ t
                                                                    case t of t_3 \rightarrow t_1, t_4 \rightarrow t_2
                                                                     t :: t'
                                                                                                                       S
                                                                     (t)
  K
                                                          ::=
  A, B, C, D, E, S, U
                                                                    \top
                                                                    \mathbb{S}
                                                                    X
                                                                    \mathsf{List}\,A
                                                                    \forall (X <: A).B
                                                                    Unit
                                                                     Nat
                                                                    A_1 \to A_2 \\ A_1 \times A_2
                                                                                                                       S
 Γ
                                                                    \begin{array}{l} \Gamma, X <: A \\ \Gamma, x : A \end{array}
\Gamma \vdash A : \star
                                                                \frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star}
                                                                                                   K_{\text{-}TOP}
                                                                               \overline{\Gamma \vdash \mathbb{S} : \star}
                                                                                                   K\_{\rm SL}
```

 $\Gamma \operatorname{Ok}$

$$\frac{\Gamma \text{ Ok} \quad \text{OK_EMPTY}}{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star} \quad \text{OK_TYPEVAR}$$

$$\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A) \text{ Ok}} \quad \text{OK_VAR}$$

 $\Gamma \vdash A \sim B$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \sim A} \quad \text{C_Refl}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash A \sim ?} \quad \text{C_Box}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash ? \sim A} \quad \text{C_Unbox}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\text{List } A) \sim (\text{List } B)} \quad \text{C_List}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \rightarrow B_1) \sim (A_2 \rightarrow B_2)} \quad \text{C_Arrow}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C_Prod}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C_Prod}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A).B_1) \sim (\forall (X <: A).B_2)} \quad \text{C_Forall}$$

 $A \sqsubseteq B$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{A \sqsubseteq ?} \quad P_{-}U$$

$$\frac{A \sqsubseteq A}{A \sqsubseteq A} \quad P_{-}REFL$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad P_{-}ARROW$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P_PROD}$$

$$\frac{A \sqsubseteq B}{(\text{List } A) \sqsubseteq (\text{List } B)} \quad \text{P_LIST}$$

$$\frac{B_1 \sqsubseteq B_2}{(\forall (X <: A).B_1) \sqsubseteq (\forall (X <: A).B_2)} \quad \text{P_FORALL}$$

 $t \sqsubseteq t'$

$$\begin{array}{c|c} \hline t \sqsubseteq t & \mathrm{TP_REFL} \\ \hline t_1 \sqsubseteq t_2 \\ \hline (\operatorname{succ} t_1) \sqsubseteq (\operatorname{succ} t_2) & \mathrm{TP_SUCC} \\ \hline \\ t_1 \sqsubseteq t_4 & t_2 \sqsubseteq t_5 & t_3 \sqsubseteq t_6 \\ \hline (\operatorname{case} t_1 \operatorname{of} 0 \to t_2, (\operatorname{succ} x) \to t_3) \sqsubseteq (\operatorname{case} t_4 \operatorname{of} 0 \to t_5, (\operatorname{succ} x) \to t_6) \\ \hline \\ \frac{t_1 \sqsubseteq t_3}{(t_1, t_2) \sqsubseteq (t_3, t_4)} & \mathrm{TP_PAIR} \\ \hline \\ \frac{t_1 \sqsubseteq t_2}{(\operatorname{fst} t_1) \sqsubseteq (\operatorname{fst} t_2)} & \mathrm{TP_FST} \\ \hline \\ \frac{t_1 \sqsubseteq t_2}{(\operatorname{snd} t_1) \sqsubseteq (\operatorname{snd} t_2)} & \mathrm{TP_SND} \\ \hline \\ \frac{t_1 \sqsubseteq t_3}{(\operatorname{tot} t_1) \sqsubseteq (\operatorname{snd} t_2)} & \mathrm{TP_CONS} \\ \hline \\ \frac{t_1 \sqsubseteq t_3}{(t_1 : : t_2) \sqsubseteq (t_3 : : t_4)} & \mathrm{TP_CONS} \\ \hline \\ \frac{t_1 \sqsubseteq t_4}{(t_1 : : t_2) \sqsubseteq (t_3 : : t_4)} & \mathrm{TP_CONS} \\ \hline \\ \overline{(\operatorname{case} t_1 \operatorname{of} [] \to t_2, (x : : y) \to t_3)} \sqsubseteq (\operatorname{case} t_4 \operatorname{of} 0 \to t_5, (x : : y) \to t_6)} \\ \hline \\ \frac{t_1 \sqsubseteq t_2}{(\operatorname{A} 1 \sqsubseteq t_2)} & \mathrm{TP_FUN} \\ \hline \\ \frac{t_1 \sqsubseteq t_3}{(\lambda(x : A_1).t)} \sqsubseteq (\lambda(x : A_2).t_2)} & \mathrm{TP_FUN} \\ \hline \\ \frac{t_1 \sqsubseteq t_3}{(\lambda(x : A).t_1)} \sqsubseteq (t_3 t_4)} & \mathrm{TP_APP} \\ \hline \\ \frac{t_1 \sqsubseteq t_2}{(\Lambda(X < : A).t_1)} \sqsubseteq (\Lambda(X < : A).t_2)} & \mathrm{TP_TFUN} \\ \hline \\ \frac{t_1 \sqsubseteq t_2}{[A]t_1 \sqsubseteq [B]t_2} & \mathrm{TP_TAPP} \\ \hline \\ \hline \\ \overline{[A]t_1 \sqsubseteq [B]t_2}} & \mathrm{TP_TAPP} \\ \hline \end{array}$$

 $\Gamma \vdash A \lesssim B$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim A} \quad \text{S_REFL}$$

$$\frac{X <: A' \in \Gamma \quad \Gamma \vdash A' \sim A}{\Gamma \vdash X \lesssim A} \quad \text{S_VAR}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim \top} \quad \text{S_TOP}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash A \lesssim ?} \quad \text{S_Box}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash ? \lesssim A} \quad \text{S_UNBOX}$$

$$\frac{\Gamma \text{ Ok}}{\Gamma \vdash ? \lesssim \mathbb{S}} \quad \text{S_USL}$$

$$\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Nat} \lesssim \mathbb{S}} \quad \text{S_NATSL}$$

$$\frac{\Gamma \text{ Ok}}{\Gamma \vdash \text{Unit} \lesssim \mathbb{S}} \quad \text{S_UNITSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash \text{List } A \lesssim \mathbb{S}} \quad \text{S_LISTSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \to B \lesssim \mathbb{S}} \quad \text{S_ARROWSL}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S} \quad \Gamma \vdash B \lesssim \mathbb{S}}{\Gamma \vdash A \times B \lesssim \mathbb{S}} \quad \text{S_PRODSL}$$

$$\frac{\Gamma \vdash A \lesssim B}{\Gamma \vdash (\text{List } A) \lesssim (\text{List } B)} \quad \text{S_LIST}$$

$$\frac{\Gamma \vdash A_1 \lesssim A_2 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \times B_1) \lesssim (A_2 \times B_2)} \quad \text{S_PROD}$$

$$\frac{\Gamma \vdash A_2 \lesssim A_1 \quad \Gamma \vdash B_1 \lesssim B_2}{\Gamma \vdash (A_1 \to B_1) \lesssim (A_2 \to B_2)} \quad \text{S_ARROW}$$

$$\frac{\Gamma, X \leqslant A \vdash B_1 \lesssim B_2}{\Gamma \vdash (\forall (X \leqslant : A).B_1) \lesssim (\forall (X \leqslant : A).B_2)} \quad \text{S_FORALL}$$

 $\Gamma \vdash_{\mathsf{SG}} t : A$

$$\frac{x:A\in\Gamma\ \Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}} x:A} \quad \text{T_-VARP}$$

$$\frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}} \mathsf{box}:\forall(X<:\mathbb{S}).(X\to?)} \quad \text{T_-Box}$$

$$\frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}} \mathsf{unbox}:\forall(X<:\mathbb{S}).(?\to X)} \quad \text{T_-UNBOX}$$

$$\frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}} \mathsf{unbox}:\forall(X<:\mathbb{S}).(?\to X)} \quad \text{T_-UNITP}$$

$$\frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}} \mathsf{triv}:\mathsf{Unit}} \quad \text{T_-ZEROP}$$

$$\frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}} 0:\mathsf{Nat}} \quad \text{T_-ZEROP}$$

$$\frac{\Gamma\vdash_{\mathsf{SG}} t:A \quad \mathsf{nat}(A)=\mathsf{Nat}}{\Gamma\vdash_{\mathsf{SG}} \mathsf{succ}\ t:\mathsf{Nat}} \quad \text{T_-SUCC}$$

$$\frac{\Gamma\vdash_{\mathsf{SG}} t:C \quad \mathsf{nat}(C)=\mathsf{Nat} \quad \Gamma\vdash_{\mathsf{A}1} \sim A}{\Gamma\vdash_{\mathsf{SG}} t_1:A_1 \quad \Gamma,x:\mathsf{Nat}\vdash_{\mathsf{SG}} t_2:A_2 \quad \Gamma\vdash_{\mathsf{A}2} \sim A} \quad \text{T_-NCASE}$$

$$\frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}}} \ \frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}}} \ \frac{\Gamma\ \text{Ok}}{\Gamma\vdash_{\mathsf{SG}}} \ \frac{\Gamma\ \text{CMPTY}}{\Gamma}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{SG}} t_2 : A_2 \quad \operatorname{inst}(A_2) - \operatorname{List} A_3 \quad \Gamma \vdash_{\mathsf{A}_1} \cdot A_3}{\Gamma \vdash_{\mathsf{SG}} t_1 : t_1 \quad \Gamma \vdash_{\mathsf{SG}} t_2 : A_2} \quad \text{T PAIR}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{SG}} t_2 : A_2}{\Gamma \vdash_{\mathsf{SG}} \lambda(x : t_1) : A_1 \cdot A_2} \quad \text{T PAIR}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} \lambda(x : A_1) : A_1 \cdot A_2}{\Gamma \vdash_{\mathsf{SG}} \lambda(x : A_1) : \forall (X < A_1) \cdot B} \quad \text{T.LAM}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} \lambda(x : A_1) : \forall (X < A_1) \cdot B}{\Gamma \vdash_{\mathsf{SG}} \lambda(x : A_1) : \forall (X < A_1) \cdot B} \quad \text{T.LAM}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_1) : \forall (X < A_1) \cdot B}{\Gamma \vdash_{\mathsf{SG}} \lambda(x : A_1) : \forall (X < A_1) \cdot B} \quad \text{T.LAM}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_1) : \forall (X < A_1) \cdot B}{\Gamma \vdash_{\mathsf{SG}} \lambda(x : A_1) : \forall (X < A_1) \cdot B} \quad \text{T.LAM}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_1) : (X \lor_{\mathsf{SG}} A_1) : (X \lor_{\mathsf{SG}} A_1)}{\Gamma \vdash_{\mathsf{SG}} t : B} \quad \text{T.Sub}} \quad \text{T.LCASE}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_1) : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_1) : (X \lor_{\mathsf{SG}} A_2)}{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2)} \quad \text{T.LCASE}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2)}{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2)} \quad \text{T.LCASE}} \quad \text{T.LCASE}}$$

$$\frac{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2)}{\Gamma \vdash_{\mathsf{SG}} t : (X \lor_{\mathsf{SG}} A_2) : (X \lor_{\mathsf{SG}} A_2)} \quad \text{T.LCASE}} \quad \text{T.LCASE}$$

6

0 bad

Definition rule clauses: 191 good