$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ s$	$::=$		
	$\mid$	x	
	$\mid$	triv	
	$\mid$	box	
	$\mid$	unbox	
	$\mid$	error_A	
	$\mid$	$\Lambda(X <: A).t$	
	$\mid$	$[A]t$	
	$\mid$	$\lambda(x : A).t$	
	$\mid$	$t_1\ t_2$	
	$\mid$	(t_1, t_2)	
	$\mid$	$\mathsf{fst}\ t$	
	$\mid$	$\mathsf{snd}\ t$	
	$\mid$	$\mathsf{succ}\ t$	
	$\mid$	0	
	$\mid$	$\mathsf{case}\ t :\ A\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
	$\mid$	$[]$	
	$\mid$	$t :: t'$	
	$\mid$	(t)	S
	$\mid$	squash	
	$\mid$	split	
$n,\ m$	$::=$		
	$\mid$	0	
	$\mid$	$\mathsf{succ}\ n$	
v	$::=$		
	$\mid$	triv	
	$\mid$	$[]$	
	$\mid$	squash_S	
	$\mid$	split_S	
	$\mid$	box_A	
	$\mid$	unbox_A	
	$\mid$	$\Lambda(X <: A).t$	
	$\mid$	$\lambda(x : A).t$	
	$\mid$	n	
Kd	$::=$		
	$\mid$	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U,\ K$	$::=$		
	$\mid$	X	
	$\mid$	$\top$	
	$\mid$	$\mathsf{List}\ A$	
	$\mid$	$\forall(X <: A).B$	
	$\mid$	$\mathbb{S}$	
	$\mid$	Unit	

$$
\begin{array}{lll}
 & | & \mathsf{Nat} \\
 & | & ? \\
 & | & A_1 \to A_2 \\
 & | & A_1 \times A_2 \\
 & | & (A) \qquad\qquad \mathsf{S}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma & ::= & \\
 & | & \cdot \\
 & | & \Gamma, X <: A \\
 & | & \Gamma, x : A
\end{array}
$$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star}\ \textsc{K\_var}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star}\ \textsc{K\_unit}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star}\ \textsc{K\_nat}$$

$$\frac{}{\Gamma \vdash ? : \star}\ \textsc{K\_unitype}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\ A : \star}\ \textsc{K\_list}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star}\ \textsc{K\_arrow}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star}\ \textsc{K\_prod}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A).B : \star}\ \textsc{K\_forall}$$

$\boxed{\Gamma\ \mathrm{Ok}}$

$$\frac{}{\cdot\ \mathrm{Ok}}\ \textsc{Ok\_empty}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A)\ \mathrm{Ok}}\ \textsc{Ok\_typeVar}$$

$$\frac{\Gamma\ \mathrm{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\ \mathrm{Ok}}\ \textsc{Ok\_var}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A <: A}\ \textsc{S\_Refl}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A <: \top}\ \textsc{S\_Top}$$

$$\frac{X <: A \in \Gamma \quad \Gamma\ \mathrm{Ok}}{\Gamma \vdash X <: A}\ \textsc{S\_Var}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash U <: \mathbb{S}}\ \textsc{S\_USL}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}}\ \text{S\_NatSL}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}}\ \text{S\_UnitSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{List}\ A <: \mathbb{S}}\ \text{S\_ListSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}}\ \text{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}}\ \text{S\_ProdSL}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash \mathsf{List}\ A <: \mathsf{List}\ B}\ \text{S\_List}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2}\ \text{S\_Prod}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2}\ \text{S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall (X <: A).B_1 <: \forall (X <: A).B_2}\ \text{S\_Forall}$$

$\boxed{A \sqsubseteq B}$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{A \sqsubseteq ?}\ \text{P\_U}$$

$$\frac{}{A \sqsubseteq A}\ \text{P\_refl}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)}\ \text{P\_arrow}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)}\ \text{P\_prod}$$

$$\frac{A \sqsubseteq B}{(\mathsf{List}\ A) \sqsubseteq (\mathsf{List}\ B)}\ \text{P\_list}$$

$$\frac{B_1 \sqsubseteq B_2}{(\forall (X <: A).B_1) \sqsubseteq (\forall (X <: A).B_2)}\ \text{P\_forall}$$

$\boxed{\Gamma \vdash t \sqsubseteq t'}$

$$\frac{x : A\ \in \Gamma \quad \Gamma\ \mathrm{Ok}}{\Gamma \vdash x \sqsubseteq x}\ \text{TP\_var}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash 0 \sqsubseteq 0}\ \text{TP\_nat}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash \mathsf{triv} \sqsubseteq \mathsf{triv}}\ \text{TP\_triv}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash [] \sqsubseteq []}\ \text{TP\_empty}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{succ}\ t_1) \sqsubseteq (\mathsf{succ}\ t_2)}\ \textsc{TP\_succ}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : \mathsf{Nat} \vdash t_3 \sqsubseteq t_6}{\Gamma \vdash (\mathsf{case}\ t_1 : \mathsf{Nat}\ \mathsf{of}\ 0 \to t_2, (\mathsf{succ}\ x) \to t_3) \sqsubseteq (\mathsf{case}\ t_4 : \mathsf{Nat}\ \mathsf{of}\ 0 \to t_5, (\mathsf{succ}\ x) \to t_6)}\ \textsc{TP\_Nate}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1, t_2) \sqsubseteq (t_3, t_4)}\ \textsc{TP\_pair}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{fst}\ t_1) \sqsubseteq (\mathsf{fst}\ t_2)}\ \textsc{TP\_fst}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{snd}\ t_1) \sqsubseteq (\mathsf{snd}\ t_2)}\ \textsc{TP\_snd}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 :: t_2) \sqsubseteq (t_3 :: t_4)}\ \textsc{TP\_cons}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : A_2, y : \mathsf{List}\ A_2 \vdash t_3 \sqsubseteq t_6 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\mathsf{case}\ t_1 : \mathsf{List}\ A_1\ \mathsf{of}\ [] \to t_2, (x :: y) \to t_3) \sqsubseteq (\mathsf{case}\ t_4 : \mathsf{List}\ A_2\ \mathsf{of}\ 0 \to t_5, (x :: y) \to t_6)}\ \textsc{TP\_Liste}$$

$$\frac{\Gamma, x : A_2 \vdash t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)}\ \textsc{TP\_Fun}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1\ t_2) \sqsubseteq (t_3\ t_4)}\ \textsc{TP\_app}$$

$$\frac{\Gamma, X <: A \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)}\ \textsc{TP\_tfun}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{\Gamma \vdash [A]t_1 \sqsubseteq [B]t_2}\ \textsc{TP\_tapp}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : B \quad A \sqsubseteq B}{\Gamma \vdash \mathsf{error}_A \sqsubseteq t}\ \textsc{TP\_error}$$

$\boxed{\Gamma \vdash_{\mathsf{CG}} t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} x : A}\ \textsc{T\_varP}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box} : \forall(X <: \mathbb{S}).(X \to ?)}\ \textsc{T\_Box}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox} : \forall(X <: \mathbb{S}).(? \to X)}\ \textsc{T\_Unbox}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{squash}_K : K \to ?}\ \textsc{T\_squash}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{split}_K : ? \to K}\ \textsc{T\_split}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{triv} : \mathsf{Unit}}\ \textsc{T\_unitP}$$

$$\frac{\Gamma\ \mathrm{Ok}}{\Gamma \vdash_{\mathsf{CG}} 0 : \mathsf{Nat}}\ \textsc{T\_zeroP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{succ}\, t : \mathsf{Nat}} \quad \text{T\_SUCC}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat} \\ \Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash_{\mathsf{CG}} t_2 : A\end{array}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t\, \mathsf{:\, Nat\, of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A} \quad \text{T\_NCASE}$$

$$\frac{\Gamma\, \mathrm{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash_{\mathsf{CG}} [] : \forall (X <: ?).\mathsf{List}\, X} \quad \text{T\_EMPTY}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma \vdash_{\mathsf{CG}} t_2 : \mathsf{List}\, A}{\Gamma \vdash_{\mathsf{CG}} t_1 :: t_2 : \mathsf{List}\, A} \quad \text{T\_CONS}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\mathsf{CG}} t : \mathsf{List}\, A \\ \Gamma \vdash_{\mathsf{CG}} t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash_{\mathsf{CG}} t_2 : B\end{array}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t\, \mathsf{:\, List}\, A\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 : B} \quad \text{T\_LCASE}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A_2}{\Gamma \vdash_{\mathsf{CG}} (t_1, t_2) : A_1 \times A_2} \quad \text{T\_PAIR}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{fst}\, t : A_1} \quad \text{T\_FST}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{snd}\, t : A_2} \quad \text{T\_SND}$$

$$\frac{\Gamma, x : A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \lambda (x : A).t : A \to B} \quad \text{T\_LAM}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \to B \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A}{\Gamma \vdash_{\mathsf{CG}} t_1\, t_2 : B} \quad \text{T\_APP}$$

$$\frac{\Gamma, X <: A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \Lambda (X <: A).t : \forall (X <: A).B} \quad \text{T\_LAM}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \forall (X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} [A]t : [A/X]C} \quad \text{T\_TYPEAPP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} t : B} \quad \text{T\_SUB}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{error}_A : A} \quad \text{T\_ERROR}$$

$\boxed{t_1 \rightsquigarrow t_2}$ call by name

$$\frac{}{\mathsf{unbox}_A\, (\mathsf{box}_A\, t) \rightsquigarrow t} \quad \text{RD\_RETRACT}$$

$$\frac{A \neq B}{\mathsf{unbox}_A\, (\mathsf{box}_B\, t) \rightsquigarrow \mathsf{error}_A} \quad \text{RD\_RETRACTE}$$

$$\frac{}{\mathsf{split}_U\, (\mathsf{squash}_U\, t) \rightsquigarrow t} \quad \text{RD\_RETRACTU}$$

$$\frac{U_1 \neq U_2}{\mathsf{split}_{U_1}\, (\mathsf{squash}_{U_2}\, t) \rightsquigarrow \mathsf{error}_{U_1}} \quad \text{RD\_RETRACTUE}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t'} \quad \text{RD\_SUCC}$$

$$\frac{}{\mathsf{case}\,0\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2 \rightsquigarrow t_1}\ \textsc{rd\_ncase0}$$

$$\frac{}{\mathsf{case}\,(\mathsf{succ}\,t)\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2 \rightsquigarrow [t/x]t_2}\ \textsc{rd\_ncaseSucc}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\,t\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2 \rightsquigarrow \mathsf{case}\,t'\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2}\ \textsc{rd\_ncase1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\,t\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2 \rightsquigarrow \mathsf{case}\,t\colon \mathsf{Nat\ of}\ 0 \to t_1', (\mathsf{succ}\,x) \to t_2}\ \textsc{rd\_ncase2}$$

$$\frac{t_2 \rightsquigarrow t_2'}{\mathsf{case}\,t\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2 \rightsquigarrow \mathsf{case}\,t\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\,x) \to t_2'}\ \textsc{rd\_ncase3}$$

$$\frac{}{\mathsf{case}\,[]\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow t_1}\ \textsc{rd\_lcaseEmpty}$$

$$\frac{}{\mathsf{case}\,(t_1 :: t_2)\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_3, (x :: y) \to t_4 \rightsquigarrow [t_1/x][t_2/y]t_4}\ \textsc{rd\_lcaseCons}$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1 :: t_2 \rightsquigarrow t_1' :: t_2}\ \textsc{rd\_head}$$

$$\frac{t_2 \rightsquigarrow t_2'}{t_1 :: t_2 \rightsquigarrow t_1 :: t_2'}\ \textsc{rd\_tail}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\,t\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow \mathsf{case}\,t'\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2}\ \textsc{rd\_lcase1}$$

$$\frac{t_1 \rightsquigarrow t_1'}{\mathsf{case}\,t\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow \mathsf{case}\,t\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1', (x :: y) \to t_2}\ \textsc{rd\_lcase2}$$

$$\frac{t_2 \rightsquigarrow t_2'}{\mathsf{case}\,t\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow \mathsf{case}\,t\colon \mathsf{List}\,A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2'}\ \textsc{rd\_lcase3}$$

$$\frac{}{(\lambda(x : A_1).t_2)\,t_1 \rightsquigarrow [t_1/x]t_2}\ \textsc{rd\_beta}$$

$$\frac{x \notin \mathsf{FV}(t)}{\lambda(x : A_1).t\,x \rightsquigarrow t}\ \textsc{rd\_eta}$$

$$\frac{}{\mathsf{fst}\,(t_1, t_2) \rightsquigarrow t_1}\ \textsc{rd\_proj1}$$

$$\frac{}{\mathsf{snd}\,(t_1, t_2) \rightsquigarrow t_2}\ \textsc{rd\_proj2}$$

$$\frac{}{(\mathsf{fst}\,t, \mathsf{snd}\,t) \rightsquigarrow t}\ \textsc{rd\_etaP}$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1\,t_2 \rightsquigarrow t_1'\,t_2}\ \textsc{rd\_app1}$$

$$\frac{t_2 \rightsquigarrow t_2'}{t_1\,t_2 \rightsquigarrow t_1\,t_2'}\ \textsc{rd\_app2}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{fst}\,t \rightsquigarrow \mathsf{fst}\,t'}\ \textsc{rd\_fst}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{snd}\,t \rightsquigarrow \mathsf{snd}\,t'}\ \textsc{rd\_snd}$$

$$\frac{t_1 \rightsquigarrow t_1'}{(t_1, t_2) \rightsquigarrow (t_1', t_2)}\ \textsc{rd\_pair1}$$

$$\frac{t_2 \rightsquigarrow t_2'}{(t_1, t_2) \rightsquigarrow (t_1, t_2')} \quad \text{RD\_PAIR2}$$

$$\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t} \quad \text{RD\_TYPEBETA}$$

$$\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2} \quad \text{RD\_TYPEAPP}$$

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Definition rules:         97 good    0 bad
Definition rule clauses: 179 good    0 bad
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