```
termvar, \, x, \, y, \, z, \, f
  typevar, X, Y, Z
  index,\;i,\;j,\;k
  t, c, v, s, n
                                                         ::=
                                                                     \boldsymbol{x}
                                                                     triv
                                                                     box
                                                                     unbox
                                                                     \Lambda(X <: A).t
                                                                     [A]t
                                                                     \lambda(x:A).t
                                                                     t_1 t_2
                                                                    (t_1, t_2)
                                                                     \mathsf{fst}\;t
                                                                     \mathsf{snd}\; t
                                                                     \mathsf{succ}\ t
                                                                    case t of t_3 \rightarrow t_1, t_4 \rightarrow t_2
                                                                     t :: t'
                                                                                                                       S
                                                                     (t)
  K
                                                          ::=
  A, B, C, D, E, S, U
                                                                    \top
                                                                    \mathbb{S}
                                                                    X
                                                                    \mathsf{List}\,A
                                                                    \forall (X <: A).B
                                                                    Unit
                                                                     Nat
                                                                    A_1 \to A_2 \\ A_1 \times A_2
                                                                                                                       S
 Γ
                                                                    \begin{array}{l} \Gamma, X <: A \\ \Gamma, x : A \end{array}
\Gamma \vdash A : \star
                                                                \frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star}
                                                                                                   K_{\text{-}TOP}
                                                                               \overline{\Gamma \vdash \mathbb{S} : \star}
                                                                                                   K\_{\rm SL}
```

 $\Gamma \, \mathrm{Ok}$ 

$$\frac{\Gamma \text{ Ok} \quad \text{OK\_EMPTY}}{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star} \quad \text{OK\_TYPEVAR}$$

$$\frac{\Gamma \text{ Ok} \quad \Gamma \vdash A : \star}{\Gamma \text{ ($\Gamma$, $X$ : $A$) Ok}} \quad \text{OK\_VAR}$$

 $\Gamma \vdash A \sim B$ 

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \sim A} \quad \text{C_Refl}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash A \sim ?} \quad \text{C_Box}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash ? \sim A} \quad \text{C_Unbox}$$

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{\Gamma \vdash ? \sim A} \quad \text{C_Split}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash \text{(List } A) \sim \text{(List } B)} \quad \text{C_List}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (A_1 \rightarrow B_1) \sim (A_2 \rightarrow B_2)} \quad \text{C_Arrow}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C_Prod}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C_Prod}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A).B_1) \sim (\forall (X <: A).B_2)} \quad \text{C_Forall}$$

 $A \sqsubseteq B$ 

$$\frac{\Gamma \vdash A \lesssim \mathbb{S}}{A \vdash ?} \quad P_{-}U$$

$$\overline{S \sqsubseteq ?} \quad P_{\_SKELETON}$$

$$\overline{A \sqsubseteq A} \quad P_{\_REFL}$$

$$\overline{A \sqsubseteq C} \quad B \sqsubseteq D$$

$$\overline{(A \to B) \sqsubseteq (C \to D)} \quad P_{\_ARROW}$$

$$\overline{A \sqsubseteq C} \quad B \sqsubseteq D$$

$$\overline{(A \times B) \sqsubseteq (C \times D)} \quad P_{\_PROD}$$

$$\overline{(A \times B) \sqsubseteq (List B)} \quad P_{\_LIST}$$

$$\overline{A \sqsubseteq B \atop (List A) \sqsubseteq (List B)} \quad P_{\_LIST}$$

$$\overline{B_1 \sqsubseteq B_2}$$

$$\overline{(\forall (X <: A).B_1) \sqsubseteq (\forall (X <: A).B_2)} \quad P_{\_FORALL}$$

 $t \sqsubseteq t'$ 

 $\Gamma \vdash A \lesssim B$ 

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \leq A} \quad \text{S\_Refl}$$

$$\frac{X <: A' \in \Gamma \quad \Gamma \vdash A' \sim A}{\Gamma \vdash X \lesssim A} \quad \text{S-VAR}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \lesssim \top} \quad \text{S-TOP}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash A \lesssim ?} \quad \text{S-Box}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash A \lesssim ?} \quad \text{S-DIT}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash R \lesssim ?} \quad \text{S-SPLIT}$$

$$\frac{\Gamma \lor A \lesssim \$}{\Gamma \vdash A \lesssim \$} \quad \text{S-NATSL}$$

$$\frac{\Gamma \lor A \leqslant \$}{\Gamma \vdash A \Leftrightarrow \$} \quad \text{S-NATSL}$$

$$\frac{\Gamma \lor A \leqslant \$}{\Gamma \vdash A \Leftrightarrow \$} \quad \text{S-LISTSL}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash A \Rightarrow B \lesssim \$} \quad \text{S-LISTSL}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash A \Rightarrow B \lesssim \$} \quad \text{S-ARROWSL}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash A \times B \lesssim \$} \quad \text{S-PRODSL}$$

$$\frac{\Gamma \vdash A \lesssim \$}{\Gamma \vdash (\text{List } A) \lesssim (\text{List } B)} \quad \text{S-LIST}$$

$$\frac{\Gamma \vdash A \lesssim A_2}{\Gamma \vdash (A_1 \times B_1) \lesssim (A_2 \times B_2)} \quad \text{S-PROD}$$

$$\frac{\Gamma \vdash A_2 \lesssim A_1}{\Gamma \vdash (A_1 \to B_1) \lesssim (A_2 \to B_2)} \quad \text{S-PROD}$$

$$\frac{\Gamma \vdash A_2 \lesssim A_1}{\Gamma \vdash (A_1 \to B_1) \lesssim (A_2 \to B_2)} \quad \text{S-ARROW}$$

$$\frac{\Gamma, X \leqslant A \vdash B_1 \lesssim B_2}{\Gamma \vdash (\forall (X \leqslant A) \land B_1) \lesssim (\forall (X \leqslant A) \land B_2)} \quad \text{S-FORALL}$$

 $\Gamma \vdash_{\mathsf{SG}} t : A$ 

$$\begin{array}{c} x:A\in\Gamma\quad\Gamma\,\mathrm{Ok}\\ \hline \Gamma\vdash_{\mathsf{SG}}x:A & & & \\ \hline \Gamma\vdash_{\mathsf{SG}}t:A & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{box}_{A}t:? & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{box}_{A}t:? & & \\ \hline \Gamma\vdash_{\mathsf{SG}}t:A & & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{unbox}_{A}t:A & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{t}:S & & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{squash}_{S}t:? & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{squash}_{S}t:? & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{split}_{S}t:S & & \\ \hline \Gamma\vdash_{\mathsf{SG}}\mathrm{split}_{S}t:S & & \\ \hline \end{array}$$

$$\frac{\Gamma \text{ Ok}}{\Gamma \vdash_{\text{SG}} \text{ triv} : \text{Unit}} \quad \text{T_UNITP}$$

$$\frac{\Gamma \text{ Ok}}{\Gamma \vdash_{\text{SG}} t : A \quad \text{nat}(A) = \text{Nat}} \quad \text{T_SCOP}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : A \quad \text{nat}(A) = \text{Nat}}{\Gamma \vdash_{\text{SG}} \text{ succ} : \text{Nat}} \quad \text{T_SUCC}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C \quad \text{nat}(C) = \text{Nat} \quad \Gamma \vdash_{\text{A1}} \sim A}{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma_{\text{A1}} = A_1 \quad \Gamma_{\text{A2}} \sim A} \quad \text{T_NCASE}$$

$$\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{list}(A_2) = \text{List}(A_3)}{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{list}(A_2) = \text{List}(A_3)} \quad \Gamma \vdash_{\text{A1}} \sim A_3} \quad \text{T_-CONS}$$

$$\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{list}(A_2) = \text{List}(A_3)}{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{list}(A_2) = \text{List}(A_3)} \quad \text{T_-PAIR}$$

$$\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t_2 : A_2 \quad \text{T_-PAIR}}{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t : B} \quad \text{T_-LAM}$$

$$\frac{\Gamma \vdash_{\text{SG}} t_1 : A_1 \quad \Gamma \vdash_{\text{SG}} t : B}{\Gamma \vdash_{\text{SG}} A(X < A) . t : \forall (X < A) . B} \quad \text{T_-LAM}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : \nabla (X < B) . C \quad \Gamma \vdash_{\text{A}} \lesssim B}{\Gamma \vdash_{\text{SG}} t_1 : A \quad \Gamma \vdash_{\text{A}} \lesssim B} \quad \text{T_-TYPEAPP}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t_1 : A \quad \Gamma \vdash_{\text{A}} \lesssim B} \quad \text{T_-SUB}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t_1 : A \quad \Gamma \vdash_{\text{A}} \lesssim B} \quad \text{T_-SUB}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t_1 : C} \quad \text{T_-LA} = B \quad \Gamma \vdash_{\text{B_2}} \sim B \quad \text{T_-LAPP}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t_1 : C} \quad \text{T_-LAPP} \quad \text{T_-ESG} t_1 : C}{\Gamma \vdash_{\text{SG}} t_1 : C} \quad \text{T_-LAPP} \quad \text{T_-ESG} t_1 : C}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t_1 : A} \quad \text{T_-FST}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t_1 : A} \quad \text{T_-FST}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-DND}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-DND}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\Gamma \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text{T_-NDD}$$

$$\frac{\tau \vdash_{\text{SG}} t : C}{\Gamma \vdash_{\text{SG}} t : A} \quad \text$$

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\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\ t_2 : \mathsf{Nat}} \quad \text{CL_SUCC}
                                          \Gamma \vdash t \Rightarrow t' : ? \quad \Gamma \vdash A_1 \sim A \quad \mathsf{caster}(A_1, A) = c_1
\frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A_2 \quad \Gamma \vdash A_2 \sim A \quad \mathsf{caster}(A_2, A) = c_2}{\Gamma \vdash (\mathsf{case} \ t \ \mathsf{of} \ 0 \rightarrow t_1, (\mathsf{succ} \ x) \rightarrow t_2) \Rightarrow (\mathsf{case} \ (\mathsf{unbox}_{\mathsf{Nat}} \ t') \ \mathsf{of} \ 0 \rightarrow (c_1 \ t_1'), (\mathsf{succ} \ x) \rightarrow (c_2 \ t_2')) : A_1 \rightarrow (c_2 \ t_2') \rightarrow
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     CI_NCASEU
                            \Gamma \vdash t \Rightarrow t' : \mathsf{Nat} \quad \Gamma \vdash A_1 \sim A \quad \mathsf{caster}(A_1, A) = c_1
                          \frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A_2 \quad \Gamma \vdash A_2 \sim A \quad \mathsf{caster}(A_2, A) = c_2}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2) \Rightarrow (\mathsf{case}\ t'\ \mathsf{of}\ 0 \to t_1', (\mathsf{succ}\ x) \to t_2') : A}
                                                                                                                                          \frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \quad \text{CI\_PAIR}
                                                                                                                                                    \frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{fst}\ t_1 \Rightarrow \mathsf{fst}\ (\mathsf{split}_{(? \times ?)}\ t_2) : ?} \quad \text{CI\_FSTU}
                                                                                                                                                                               \frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \mathsf{fst} \ t_1 \Rightarrow \mathsf{fst} \ t_2 : A_1} \quad \text{CL_FST}
                                                                                                                                               \frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{snd}\ t_1 \Rightarrow \mathsf{snd}\ (\mathsf{split}_{(? \times ?)}\ t_2) : ?} \quad \text{CI\_SNDU}
                                                                                                                                                                                \frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\ t_1 \Rightarrow \mathsf{snd}\ t_2 : B} \quad \text{CI\_SND}
                                                                                                                                                    \frac{1 - 1}{\Gamma \vdash [] \Rightarrow [] : \forall (X <: \top).\mathsf{List}\,X} \quad \text{CI\_EMPTY}
                                                                                             \frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_2' : ?}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((\mathsf{box}_{A_1} t_1') :: (\mathsf{split}_{(\mathsf{List}\,?)} t_2')) : \mathsf{List}\,?} \quad \text{CI\_CONSU}
                                \frac{\Gamma \vdash t_1 \Rightarrow t_1' : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_2' : \mathsf{List}\, A_2 \quad \Gamma \vdash A_1 \lesssim A_2 \quad \mathsf{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c \ t_1') :: t_2') : \mathsf{List}\, A_2}
                                          \Gamma \vdash t \Rightarrow t' : ? caster(B_1, B) = c_1 caster(B_2, B) = c_2
                                          \Gamma \vdash t_1 \Rightarrow t_1': B_1 \quad \Gamma, x: ?, y: \mathsf{List}\,? \vdash t_2 \Rightarrow t_2': B_2 \quad \Gamma \vdash B_1 \sim B \quad \Gamma \vdash B_2 \sim B
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       CI_LCASEU
\overline{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2)} \Rightarrow (\mathsf{case}\ (\mathsf{split}_{(\mathsf{List}\ ?)}\ t')\ \mathsf{of}\ [] \to (c_1\ t_1'), (x :: y) \to (c_2\ t_2')) : B
                         \Gamma \vdash t \Rightarrow t : \mathsf{List}\,A \quad \mathsf{caster}(B_1,B) = c_1 \quad \mathsf{caster}(B_2,B) = c_2
                \frac{\Gamma \vdash t_1 \Rightarrow t_1' : B_1 \quad \Gamma, x : A, y : \mathsf{List} \ A \vdash t_2 \Rightarrow t_2' : B_2 \quad \Gamma \vdash B_1 \sim B \quad \Gamma \vdash B_2 \sim B}{\Gamma \vdash (\mathsf{case} \ t \ \mathsf{of} \ [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case} \ t' \ \mathsf{of} \ [] \to (c_1 \ t_1'), (x :: y) \to (c_2 \ t_2')) : B}
                                                                                                                            \frac{\Gamma, x: A_1 \vdash t_1 \Rightarrow t_2: A_2}{\Gamma \vdash \lambda(x:A_1).t_1 \Rightarrow \lambda(x:A_1).t_2: A_1 \rightarrow A_2} \quad \text{CI\_LAM}
                                                                                                                                           \Gamma \vdash t_1 \Rightarrow t_1' : ?
                                                                                                                                     \frac{\Gamma \vdash t_2 \Rightarrow t_2' : A_2 \quad \mathsf{caster}(A_2, ?) = c}{\Gamma \vdash t_1 \ t_2 \Rightarrow (\mathsf{split}_{(? \to ?)} \ t_1') \ (c \ t_2') : ?} \quad \mathsf{CI\_APPU}
                                                                                       \Gamma \vdash t_2 \Rightarrow t_2' : A_2
                                                                                  \frac{\Gamma \vdash t_1 \Rightarrow t_1^{\overline{t}} : A_1 \to B \quad \Gamma \vdash A_2 \sim A_1 \quad \mathsf{caster}(A_2, A_1) = c}{\Gamma \vdash t_1 \ t_2 \Rightarrow t_1' \ (c \ t_2') : B} \quad \text{CI\_APP}
                                                                                                                                                                          \Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B
                                                                                                                                                                                                                                                                                                                                                                                                  CI_LAM
                                                                                              \overline{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall (X <: A).B}
                                                                            \frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall (X <: B).C \quad \Gamma \vdash A \sim A' \quad \Gamma \vdash A' <: B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A']t_2) : [A'/X]C} \quad \text{CI_TYPEAPP}
```

Definition rules: 99 good 0 bad Definition rule clauses: 194 good 0 bad