

*termvar*,  $x$

*index*,  $k$

$t$	::=	term
	$x$	variable
	<b>triv</b>	unit
	$\lambda x : T. t$	$\lambda$ -abstraction
	$t_1 t_2$	function application
	$(t_1, t_2)$	pair constructor
	<b>fst</b> $t$	first projection
	<b>snd</b> $t$	second projection
	<b>succ</b> $t$	successor function
	0	zero
	$(t)$	S

$T$	::=	type
	1	unit type
	$\mathbb{N}$	natural number type
	?	untyped universe
	$T_1 \rightarrow T_2$	function type
	$T_1 \times T_2$	cartesian product type
	$(T)$	S

$\Gamma$	::=	typing context
	.	empty context
	$\Gamma, x : T$	cons

$\boxed{T_1 \sim_U T_2}$   $T_1$  can can be converted into  $T_2$

$\overline{T \sim_U T}$	CV_REFL
$\frac{T_1 \sim_U T_2 \quad T_2 \sim_U T_3}{T_1 \sim_U T_3}$	CV_TRANS
$\overline{(? \rightarrow ?) \sim_U ?}$	CV_INJ
$\overline{? \sim_U (? \rightarrow ?)}$	CV_SURJ
$\frac{T_1 \sim_U T'_1}{(T_1 \times T_2) \sim_U (T'_1 \times T_2)}$	CV_PAIR1
$\frac{T_2 \sim_U T'_2}{(T_1 \times T_2) \sim_U (T_1 \times T'_2)}$	CV_PAIR2
$\frac{T_1 \sim_U T'_1}{(T_1 \rightarrow T_2) \sim_U (T'_1 \rightarrow T_2)}$	CV_FUN1
$\frac{T_2 \sim_U T'_2}{(T_1 \rightarrow T_2) \sim_U (T_1 \rightarrow T'_2)}$	CV_FUN2

$\boxed{T_1 \sim T_2}$   $T_1$  is consistent with  $T_2$

$\overline{T \sim T}$	CS_REFL
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$$\begin{array}{c}
\frac{}{? \sim T} \quad \text{CS\_UL} \\
\frac{}{T \sim ?} \quad \text{CS\_UR} \\
\frac{T_1 \sim_U T_2}{T_1 \sim T_2} \quad \text{CS\_CONV} \\
\frac{T_1 \sim T'_1 \quad T_2 \sim T'_2}{(T_1 \times T_2) \sim (T'_1 \times T'_2)} \quad \text{CS\_PAIR} \\
\frac{T_1 \sim T'_1 \quad T_2 \sim T'_2}{(T_1 \rightarrow T_2) \sim (T'_1 \rightarrow T'_2)} \quad \text{CS\_ARROW}
\end{array}$$

$\boxed{\Gamma \vdash t : T}$   $t$  has type  $T$  in context  $\Gamma$

$$\begin{array}{c}
\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{VAR} \\
\frac{}{\Gamma \vdash \text{triv} : 1} \quad \text{UNIT} \\
\frac{}{\Gamma \vdash 0 : \mathbb{N}} \quad \text{ZERO} \\
\frac{\Gamma \vdash t : \mathbb{N}}{\Gamma \vdash \text{succ } t : \mathbb{N}} \quad \text{SUCC} \\
\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2} \quad \text{PAIR} \\
\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \text{fst } t : T_1} \quad \text{FST} \\
\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \text{snd } t : T_2} \quad \text{SND} \\
\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x : T_1. t : T_1 \rightarrow T_2} \quad \text{ABS} \\
\frac{\Gamma \vdash t : T_1 \quad T_1 \sim_U T_2}{\Gamma \vdash t : T_2} \quad \text{U} \\
\frac{\Gamma \vdash t_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash t_2 : T_3 \quad T_3 \sim T_1}{\Gamma \vdash t_1 t_2 : T_2} \quad \text{APP}
\end{array}$$

$\boxed{t_1 \rightsquigarrow t_2}$   $t_1$  reduces to  $t_2$

$$\begin{array}{c}
\frac{}{(\lambda x : T. t_2) t_1 \rightsquigarrow [t_1/x] t_2} \quad \text{RD\_BETA} \\
\frac{}{(\lambda x : T. t x) \rightsquigarrow t} \quad \text{RD\_ETA} \\
\frac{}{\text{fst } (t_1, t_2) \rightsquigarrow t_1} \quad \text{RD\_PROJ1} \\
\frac{}{\text{snd } (t_1, t_2) \rightsquigarrow t_2} \quad \text{RD\_PROJ2} \\
\frac{}{(\text{fst } t, \text{snd } t) \rightsquigarrow t} \quad \text{RD\_ETAP}
\end{array}$$

Definition rules: 29 good 0 bad  
Definition rule clauses: 45 good 0 bad