$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ v,\ s,\ n$ ::=

	x
	triv
	box
	unbox
	$\Lambda(X <: A).t$
	$[A]t$
	$\lambda(x : A).t$
	$t_1\ t_2$
	(t_1, t_2)
	fst t
	snd t
	succ t
	0
	case t of $t_3 \rightarrow t_1, t_4 \rightarrow t_2$
	[]
	$t :: t'$
	(t) S

K ::=

| | $\star$ |

$A,\ B,\ C,\ D,\ E,\ S,\ U$ ::=

	X
	List A
	$\forall(X <: A).B$
	SL
	Unit
	Nat
	?
	$A_1 \rightarrow A_2$
	$A_1 \times A_2$
	(A) S

Γ ::=

	.
	$\Gamma, X <: A$
	$\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} : \star} \quad \text{K\_UNIT}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} : \star} \quad \text{K\_NAT}$$

$$\frac{}{\Gamma \vdash\ ? : \star} \quad \text{K\_UNITYPE}$$

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash \mathsf{List}\, A : \star} \quad \text{K\_LIST}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \to B : \star} \quad \text{K\_ARROW}$$

$$\frac{\Gamma \vdash A : \star \quad \Gamma \vdash B : \star}{\Gamma \vdash A \times B : \star} \quad \text{K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B : \star}{\Gamma \vdash \forall (X <: A).B : \star} \quad \text{K\_FORALL}$$

$\boxed{\Gamma\, \mathsf{Ok}}$

$$\frac{}{\cdot\, \mathsf{Ok}} \quad \text{OK\_EMPTY}$$

$$\frac{\Gamma\, \mathsf{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, X <: A)\, \mathsf{Ok}} \quad \text{OK\_TYPEVAR}$$

$$\frac{\Gamma\, \mathsf{Ok} \quad \Gamma \vdash A : \star}{(\Gamma, x : A)\, \mathsf{Ok}} \quad \text{OK\_VAR}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma\, \mathsf{Ok}}{\Gamma \vdash A <: A} \quad \text{S\_REFL}$$

$$\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C} \quad \text{S\_TRANS}$$

$$\frac{X <: A \in \Gamma \quad \Gamma\, \mathsf{Ok}}{\Gamma \vdash X <: A} \quad \text{S\_VAR}$$

$$\frac{\Gamma\, \mathsf{Ok}}{\Gamma \vdash A <: ?} \quad \text{S\_U}$$

$$\frac{\Gamma\, \mathsf{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathsf{SL}} \quad \text{S\_NATS}$$

$$\frac{\Gamma\, \mathsf{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathsf{SL}} \quad \text{S\_UNITS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL}}{\Gamma \vdash \mathsf{List}\, A <: \mathsf{SL}} \quad \text{S\_LISTS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL} \quad \Gamma \vdash B <: \mathsf{SL}}{\Gamma \vdash (A \to B) <: \mathsf{SL}} \quad \text{S\_ARROWS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL} \quad \Gamma \vdash B <: \mathsf{SL}}{\Gamma \vdash (A \times B) <: \mathsf{SL}} \quad \text{S\_PRODS}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash (\mathsf{List}\, A) <: (\mathsf{List}\, B)} \quad \text{S\_LIST}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \times B_1) <: (A_2 \times B_2)} \quad \text{S\_PROD}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \to B_1) <: (A_2 \to B_2)} \quad \text{S\_ARROW}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash (\forall(X <: A).B_1) <: (\forall(X <: A).B_2)} \quad \text{S\_Forall}$$

$\boxed{\Gamma \vdash A \sim B}$

$$\frac{}{\Gamma \vdash A \sim A} \quad \text{C\_Refl}$$

$$\frac{}{\Gamma \vdash A \sim ?} \quad \text{C\_Box}$$

$$\frac{}{\Gamma \vdash ? \sim A} \quad \text{C\_Unbox}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\mathsf{List}\,A) \sim (\mathsf{List}\,B)} \quad \text{C\_List}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \to B_1) \sim (A_2 \to B_2)} \quad \text{C\_Arrow}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C\_Prod}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall(X <: A).B_1) \sim (\forall(X <: A).B_2)} \quad \text{C\_Forall}$$

$\boxed{A \sqsubseteq B}$

$$\frac{}{A \sqsubseteq ?} \quad \text{P\_U}$$

$$\frac{}{A \sqsubseteq A} \quad \text{P\_refl}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad \text{P\_arrow}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P\_prod}$$

$$\frac{A \sqsubseteq B}{(\mathsf{List}\,A) \sqsubseteq (\mathsf{List}\,B)} \quad \text{P\_list}$$

$$\frac{B \sqsubseteq D}{(\forall(X <: A).B) \sqsubseteq (\forall(X <: A).D)} \quad \text{P\_forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma\,\mathsf{Ok}}{\Gamma \vdash x : A} \quad \text{T\_varP}$$

$$\frac{}{\Gamma \vdash \mathsf{box} : \forall(X <: \mathsf{SL}).(X \to ?)} \quad \text{T\_Box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} : \forall(X <: \mathsf{SL}).(? \to X)} \quad \text{T\_Unbox}$$

$$\frac{\Gamma\,\mathsf{Ok}}{\Gamma \vdash \mathsf{triv} : \mathsf{Unit}} \quad \text{T\_unitP}$$

$$\frac{\Gamma\,\mathsf{Ok}}{\Gamma \vdash 0 : \mathsf{Nat}} \quad \text{T\_zeroP}$$

$$\frac{\Gamma \vdash t : A \quad \mathsf{nat}(A) = \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\,t : \mathsf{Nat}} \quad \text{T\_succ}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : C \quad \mathsf{nat}(C) = \mathsf{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 : A\end{array}}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ 0 \to t_1, (\mathsf{succ}\ x) \to t_2 : A} \quad \text{T\_NCASE}$$

$$\frac{\Gamma\ \mathsf{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall(X <: ?).\mathsf{List}\ X} \quad \text{T\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : \mathsf{List}\ A_2 \quad \Gamma \vdash A_1 \sim A_2}{\Gamma \vdash t_1 :: t_2 : \mathsf{List}\ A_2} \quad \text{T\_CONS}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \quad \text{T\_PAIR}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda(x : A).t : A \to B} \quad \text{T\_LAM}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda(X <: A).t : \forall(X <: A).B} \quad \text{T\_LAM}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C} \quad \text{T\_TYPEAPP}$$

$$\frac{\Gamma \vdash t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash t : B} \quad \text{T\_SUB}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : C \quad \mathsf{list}(C) = \mathsf{List}\ A \\ \Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\ A \vdash t_2 : B\end{array}}{\Gamma \vdash \mathsf{case}\ t\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 : B} \quad \text{T\_LCASE}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 : C \quad \mathsf{fun}(C) = A_1 \to B_1 \\ \Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_2 \sim A_1\end{array}}{\Gamma \vdash t_1\ t_2 : B_2} \quad \text{T\_APP}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\ t : A_1} \quad \text{T\_FST}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\ t : A_2} \quad \text{T\_SND}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ \quad Cast insertion

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CI\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \Rightarrow \mathsf{box} : \forall(X <: \mathsf{SL}).(X \to ?)} \quad \text{CI\_BOX}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \Rightarrow \mathsf{unbox} : \forall(X <: \mathsf{SL}).(? \to X)} \quad \text{CI\_UNBOX}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \mathsf{Nat}} \quad \text{CI\_ZERO}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \Rightarrow \mathsf{triv} : \mathsf{Unit}} \quad \text{CI\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\ (\mathsf{unbox}_{\mathsf{Nat}}\ t_2) : \mathsf{Nat}} \quad \text{CI\_SUCCU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\ t_1 \Rightarrow \mathsf{succ}\ t_2 : \mathsf{Nat}} \quad \text{CI\_SUCC}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash t \Rightarrow t' : \,? \\ \Gamma \vdash t_1 \Rightarrow t'_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t'_2 : A\end{array}}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ 0 \rightarrow t_1, (\mathsf{succ}\ x) \rightarrow t_2) \Rightarrow (\mathsf{case}\ (\mathsf{unbox_{Nat}}\ t')\ \mathsf{of}\ 0 \rightarrow t'_1, (\mathsf{succ}\ x) \rightarrow t'_2) : A}\ \text{CI\_NCASEU}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash t \Rightarrow t' : \mathsf{Nat} \\ \Gamma \vdash t_1 \Rightarrow t'_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t'_2 : A\end{array}}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ 0 \rightarrow t_1, (\mathsf{succ}\ x) \rightarrow t_2) \Rightarrow (\mathsf{case}\ t'\ \mathsf{of}\ 0 \rightarrow t'_1, (\mathsf{succ}\ x) \rightarrow t'_2) : A}\ \text{CI\_NCASE}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2}\ \text{CI\_PAIR}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t_2 : \,?}{\Gamma \vdash \mathsf{fst}\ t_1 \Rightarrow \mathsf{fst}\ (\mathsf{split}_{(? \times ?)}\ t_2) : \,?}\ \text{CI\_FSTU}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\ t_1 \Rightarrow \mathsf{fst}\ t_2 : A_1}\ \text{CI\_FST}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t_2 : \,?}{\Gamma \vdash \mathsf{snd}\ t_1 \Rightarrow \mathsf{snd}\ (\mathsf{split}_{(? \times ?)}\ t_2) : \,?}\ \text{CI\_SNDU}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\ t_1 \Rightarrow \mathsf{snd}\ t_2 : B}\ \text{CI\_SND}$$

$$\dfrac{}{\Gamma \vdash [] \Rightarrow [] : \forall(X <:\,?).\mathsf{List}\ X}\ \text{CI\_EMPTY}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t'_2 : \mathsf{List}\ A_2 \quad \Gamma \vdash A_1 \sim A_2 \quad \mathsf{caster}(A_1, A_2) = c}{\Gamma \vdash (t_1 :: t_2) \Rightarrow ((c\ t'_1) :: t'_2) : \mathsf{List}\ A}\ \text{CI\_CONS}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash t \Rightarrow t' : \,? \\ \Gamma \vdash t_1 \Rightarrow t'_1 : B \quad \Gamma, x : \,?, y : \mathsf{List}\ ? \vdash t_2 \Rightarrow t'_2 : B\end{array}}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ [] \rightarrow t_1, (x :: y) \rightarrow t_2) \Rightarrow (\mathsf{case}\ (\mathsf{split}_{(\mathsf{List}\ ?)}\ t')\ \mathsf{of}\ [] \rightarrow t'_1, (x :: y) \rightarrow t'_2) : B}\ \text{CI\_LCASEU}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash t \Rightarrow t : \mathsf{List}\ A \\ \Gamma \vdash t_1 \Rightarrow t'_1 : B \quad \Gamma, x : A, y : \mathsf{List}\ A \vdash t_2 \Rightarrow t'_2 : B\end{array}}{\Gamma \vdash (\mathsf{case}\ t\ \mathsf{of}\ [] \rightarrow t_1, (x :: y) \rightarrow t_2) \Rightarrow (\mathsf{case}\ t'\ \mathsf{of}\ [] \rightarrow t'_1, (x :: y) \rightarrow t'_2) : B}\ \text{CI\_LCASE}$$

$$\dfrac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \rightarrow A_2}\ \text{CI\_LAM}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash t_1 \Rightarrow t'_1 : \,? \\ \Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \quad \mathsf{caster}(A_2, ?) = c\end{array}}{\Gamma \vdash t_1\ t_2 \Rightarrow (\mathsf{split}_{(? \rightarrow ?)}\ t'_1)\ (c\ t'_2) : \,?}\ \text{CI\_APPU}$$

$$\dfrac{\begin{array}{c}\Gamma \vdash t_2 \Rightarrow t'_2 : A_2 \\ \Gamma \vdash t_1 \Rightarrow t'_1 : A_1 \rightarrow B \quad \Gamma \vdash A_2 \sim A_1 \quad \mathsf{caster}(A_2, A_1) = c\end{array}}{\Gamma \vdash t_1\ t_2 \Rightarrow t'_1\ (c\ t'_2) : B}\ \text{CI\_APP}$$

$$\dfrac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall(X <: A).B}\ \text{CI\_LAM}$$

$$\dfrac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A]t_2) : [A/X]C}\ \text{CI\_TYPEAPP}$$

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Definition rules:          78 good      0 bad
Definition rule clauses: 149 good       0 bad
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