$termvar$, x, y, z, f, r, ys
$typevar$, X, Y, Z
$index$, i, j, k

$t,\ c,\ s$	$::=$		
	$\vert$	x	
	$\vert$	triv	
	$\vert$	box	
	$\vert$	unbox	
	$\vert$	error_A	
	$\vert$	error	
	$\vert$	$\Lambda(X <: A).t$	
	$\vert$	$[A]t$	
	$\vert$	$\lambda(x : A).t$	
	$\vert$	$t_1\ t_2$	
	$\vert$	(t_1, t_2)	
	$\vert$	$\mathsf{fst}\ t$	
	$\vert$	$\mathsf{snd}\ t$	
	$\vert$	$\mathsf{succ}\ t$	
	$\vert$	0	
	$\vert$	$\mathsf{case}\ t\colon A\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
	$\vert$	$[]$	
	$\vert$	$t :: t'$	
	$\vert$	(t)	S
	$\vert$	squash	
	$\vert$	split	
$n,\ m$	$::=$		
	$\vert$	0	
	$\vert$	$\mathsf{succ}\ n$	
v	$::=$		
	$\vert$	triv	
	$\vert$	$[]$	
	$\vert$	squash_S	
	$\vert$	split_S	
	$\vert$	box_A	
	$\vert$	unbox_A	
	$\vert$	$\Lambda(X <: A).t$	
	$\vert$	$\lambda(x : A).t$	
	$\vert$	n	
	$\vert$	$\mathsf{case}\ t\colon A\ \mathsf{of}\ t_3 \to t_1, t_4 \to t_2$	
Kd	$::=$		
	$\vert$	$\star$	
$A,\ B,\ C,\ D,\ E,\ S,\ U,\ K,\ T$	$::=$		
	$\vert$	X	
	$\vert$	$\top$	
	$\vert$	$\mathsf{List}\ A$	
	$\vert$	$\forall(X <: A).B$	

$$
\begin{array}{lll}
 & | & \mathbb{S} \\
 & | & \mathsf{Unit} \\
 & | & \mathsf{Nat} \\
 & | & ? \\
 & | & A_1 \to A_2 \\
 & | & A_1 \times A_2 \\
 & | & (A) \qquad\qquad \mathsf{S}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma & ::= & \\
 & | & . \\
 & | & \Gamma, X <: A \\
 & | & \Gamma, x : A
\end{array}
$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{}{\Gamma \vdash A <: A}\ \textsc{S\_Refl}$$

$$\frac{}{\Gamma \vdash A <: \top}\ \textsc{S\_Top}$$

$$\frac{X <: A \in \Gamma}{\Gamma \vdash X <: A}\ \textsc{S\_Var}$$

$$\frac{}{\Gamma \vdash \top <: \mathbb{S}}\ \textsc{S\_TopSL}$$

$$\frac{}{\Gamma \vdash \mathsf{Nat} <: \mathbb{S}}\ \textsc{S\_NatSL}$$

$$\frac{}{\Gamma \vdash \mathsf{Unit} <: \mathbb{S}}\ \textsc{S\_UnitSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \mathsf{List}\, A <: \mathbb{S}}\ \textsc{S\_ListSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}}\ \textsc{S\_ArrowSL}$$

$$\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}}\ \textsc{S\_ProdSL}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash \mathsf{List}\, A <: \mathsf{List}\, B}\ \textsc{S\_List}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2}\ \textsc{S\_Prod}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2}\ \textsc{S\_Arrow}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2}\ \textsc{S\_Forall}$$

$\boxed{\Gamma_1 \sqsubseteq \Gamma_2}$

$$\frac{}{\Gamma \sqsubseteq \Gamma}\ \textsc{CtxP\_refl}$$

$$\frac{\Gamma_1 \sqsubseteq \Gamma_2 \quad A \sqsubseteq A' \quad \Gamma_3 \sqsubseteq \Gamma_4}{\Gamma_1, x : A, \Gamma_3 \sqsubseteq \Gamma_2, x : A', \Gamma_4}\ \textsc{CtxP\_ext}$$

$\boxed{A \sqsubseteq B}$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{A \sqsubseteq ?} \quad \text{P\_U}$$

$$\frac{}{A \sqsubseteq A} \quad \text{P\_REFL}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad \text{P\_ARROW}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P\_PROD}$$

$$\frac{A \sqsubseteq B}{(\mathsf{List}\, A) \sqsubseteq (\mathsf{List}\, B)} \quad \text{P\_LIST}$$

$$\frac{B_1 \sqsubseteq B_2}{(\forall(X <: A).B_1) \sqsubseteq (\forall(X <: A).B_2)} \quad \text{P\_FORALL}$$

$\boxed{\Gamma \vdash t \sqsubseteq t'}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \sqsubseteq x} \quad \text{TP\_VAR}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \mathsf{split}_{S_1} \sqsubseteq \mathsf{split}_{S_2}} \quad \text{TP\_SPLIT}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \mathsf{squash}_{S_1} \sqsubseteq \mathsf{squash}_{S_2}} \quad \text{TP\_SQUASH}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \sqsubseteq \mathsf{box}} \quad \text{TP\_BOX}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \sqsubseteq \mathsf{unbox}} \quad \text{TP\_UNBOX}$$

$$\frac{}{\Gamma \vdash 0 \sqsubseteq 0} \quad \text{TP\_NAT}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \sqsubseteq \mathsf{triv}} \quad \text{TP\_TRIV}$$

$$\frac{}{\Gamma \vdash [] \sqsubseteq []} \quad \text{TP\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{succ}\, t_1) \sqsubseteq (\mathsf{succ}\, t_2)} \quad \text{TP\_SUCC}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : \mathsf{Nat} \vdash t_3 \sqsubseteq t_6}{\Gamma \vdash (\mathsf{case}\, t_1 : \mathsf{Nat}\,\mathsf{of}\, 0 \to t_2, (\mathsf{succ}\, x) \to t_3) \sqsubseteq (\mathsf{case}\, t_4 : \mathsf{Nat}\,\mathsf{of}\, 0 \to t_5, (\mathsf{succ}\, x) \to t_6)} \quad \text{TP\_NATE}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1, t_2) \sqsubseteq (t_3, t_4)} \quad \text{TP\_PAIR}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{fst}\, t_1) \sqsubseteq (\mathsf{fst}\, t_2)} \quad \text{TP\_FST}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{snd}\, t_1) \sqsubseteq (\mathsf{snd}\, t_2)} \quad \text{TP\_SND}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP\_CONS}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : A_2, y : \mathsf{List}\, A_2 \vdash t_3 \sqsubseteq t_6 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\mathsf{case}\, t_1 : \mathsf{List}\, A_1\, \mathsf{of}\, [] \to t_2, (x :: y) \to t_3) \sqsubseteq (\mathsf{case}\, t_4 : \mathsf{List}\, A_2\, \mathsf{of}\, 0 \to t_5, (x :: y) \to t_6)} \text{ TP\_LISTE}$$

$$\frac{\Gamma, x : A_2 \vdash t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \text{ TP\_FUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1\, t_2) \sqsubseteq (t_3\, t_4)} \text{ TP\_APP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : ?}{\Gamma \vdash (\mathsf{unbox}_A\, t) \sqsubseteq t} \text{ TP\_UNBOXING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A}{\Gamma \vdash t \sqsubseteq (\mathsf{box}_A\, t)} \text{ TP\_BOXING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : ?}{\Gamma \vdash (\mathsf{split}_S\, t) \sqsubseteq t} \text{ TP\_SPLITING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : S}{\Gamma \vdash t \sqsubseteq (\mathsf{squash}_S\, t)} \text{ TP\_SQUASHING}$$

$$\frac{\Gamma, X <: A \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \text{ TP\_TFUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{\Gamma \vdash [A]t_1 \sqsubseteq [B]t_2} \text{ TP\_TAPP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : B \quad A \sqsubseteq B}{\Gamma \vdash \mathsf{error}_A \sqsubseteq t} \text{ TP\_ERROR}$$

$\boxed{\Gamma \vdash_{\mathsf{CG}} t : A}$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{CG}} x : A} \text{ T\_VARP}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{CG}} x : A} \text{ T\_VAR}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box}_A : A \to ?} \text{ T\_BOX}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox}_A : ? \to A} \text{ T\_UNBOX}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box} : \forall(X <: \mathbb{S}).(X \to ?)} \text{ T\_BOXP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox} : \forall(X <: \mathbb{S}).(? \to X)} \text{ T\_UNBOXP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{squash}_S : S \to ?} \text{ T\_SQUASH}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{split}_S : ? \to S} \text{ T\_SPLIT}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{triv} : \mathsf{Unit}} \text{ T\_UNITP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} 0 : \mathsf{Nat}} \text{ T\_ZEROP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{succ}\, t : \mathsf{Nat}} \text{ T\_SUCC}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat} \quad \Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash_{\mathsf{CG}} t_2 : A}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t \colon \mathsf{Nat}\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 : A} \ \textsc{T\_ncase}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} [] : \forall(X <: ?).\mathsf{List}\, X} \ \textsc{T\_empty}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma \vdash_{\mathsf{CG}} t_2 : \mathsf{List}\, A}{\Gamma \vdash_{\mathsf{CG}} t_1 :: t_2 : \mathsf{List}\, A} \ \textsc{T\_cons}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{List}\, A \quad \Gamma \vdash_{\mathsf{CG}} t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash_{\mathsf{CG}} t_2 : B}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t \colon \mathsf{List}\, A\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 : B} \ \textsc{T\_lcase}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A_2}{\Gamma \vdash_{\mathsf{CG}} (t_1, t_2) : A_1 \times A_2} \ \textsc{T\_pair}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{fst}\, t : A_1} \ \textsc{T\_fst}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{snd}\, t : A_2} \ \textsc{T\_snd}$$

$$\frac{\Gamma, x : A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \lambda(x : A).t : A \to B} \ \textsc{T\_lam}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \to B \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A}{\Gamma \vdash_{\mathsf{CG}} t_1\, t_2 : B} \ \textsc{T\_app}$$

$$\frac{\Gamma, X <: A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \Lambda(X <: A).t : \forall(X <: A).B} \ \textsc{T\_Lam}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} [A]t : [A/X]C} \ \textsc{T\_typeApp}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} t : B} \ \textsc{T\_Sub}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{error}_A : A} \ \textsc{T\_error}$$

$\boxed{t_1 \rightsquigarrow t_2}$ call by name

$$\frac{}{\mathsf{unbox}_A\, (\mathsf{box}_A\, t) \rightsquigarrow t} \ \textsc{rd\_retrac}\mathrm{T}$$

$$\frac{A \neq B}{\mathsf{unbox}_A\, (\mathsf{box}_B\, t) \rightsquigarrow \mathsf{error}_A} \ \textsc{rd\_retrac}\mathrm{TE}$$

$$\frac{}{\mathsf{split}_S\, (\mathsf{squash}_S\, t) \rightsquigarrow t} \ \textsc{rd\_retract}\mathrm{U}$$

$$\frac{S_1 \neq S_2}{\mathsf{split}_{S_1}\, (\mathsf{squash}_{S_2}\, t) \rightsquigarrow \mathsf{error}_{S_1}} \ \textsc{rd\_retract}\mathrm{UE}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{succ}\, t \rightsquigarrow \mathsf{succ}\, t'} \ \textsc{rd\_succ}$$

$$\frac{}{\mathsf{case}\, 0 \colon \mathsf{Nat}\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow t_1} \ \textsc{rd\_ncase}0$$

$$\frac{}{\mathsf{case}\, (\mathsf{succ}\, t) \colon \mathsf{Nat}\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow [t/x]t_2} \ \textsc{rd\_ncase}\mathrm{Succ}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\ t \colon \mathsf{Nat\ of}\ 0 \rightarrow t_1, (\mathsf{succ}\ x) \rightarrow t_2 \rightsquigarrow \mathsf{case}\ t' \colon \mathsf{Nat\ of}\ 0 \rightarrow t_1, (\mathsf{succ}\ x) \rightarrow t_2} \quad \textsc{rd\_ncase}$$

$$\frac{}{\mathsf{case}\ [] \colon \mathsf{List}\ A\ \mathsf{of}\ [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow t_1} \quad \textsc{rd\_lcaseEmpty}$$

$$\frac{}{\mathsf{case}\ (t_1 :: t_2) \colon \mathsf{List}\ A\ \mathsf{of}\ [] \rightarrow t_3, (x :: y) \rightarrow t_4 \rightsquigarrow [t_1/x][t_2/y]t_4} \quad \textsc{rd\_lcaseCons}$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1 :: t_2 \rightsquigarrow t_1' :: t_2} \quad \textsc{rd\_head}$$

$$\frac{t_2 \rightsquigarrow t_2'}{t_1 :: t_2 \rightsquigarrow t_1 :: t_2'} \quad \textsc{rd\_tail}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{case}\ t \colon \mathsf{List}\ A\ \mathsf{of}\ [] \rightarrow t_1, (x :: y) \rightarrow t_2 \rightsquigarrow \mathsf{case}\ t' \colon \mathsf{List}\ A\ \mathsf{of}\ [] \rightarrow t_1, (x :: y) \rightarrow t_2} \quad \textsc{rd\_lcase}$$

$$\frac{}{(\lambda(x : A_1).t_2)\ t_1 \rightsquigarrow [t_1/x]t_2} \quad \textsc{rd\_beta}$$

$$\frac{}{\mathsf{fst}\ (t_1, t_2) \rightsquigarrow t_1} \quad \textsc{rd\_proj}1$$

$$\frac{}{\mathsf{snd}\ (t_1, t_2) \rightsquigarrow t_2} \quad \textsc{rd\_proj}2$$

$$\frac{t_1 \rightsquigarrow t_1'}{t_1\ t_2 \rightsquigarrow t_1'\ t_2} \quad \textsc{rd\_app}$$

$$\frac{t_2 \rightsquigarrow t_2'}{v\ t_2 \rightsquigarrow v\ t_2'} \quad \textsc{rd\_app}2$$

$$\frac{t \rightsquigarrow t'}{\mathsf{split}_S\ t \rightsquigarrow \mathsf{split}_S\ t'} \quad \textsc{rd\_split}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{unbox}_A\ t \rightsquigarrow \mathsf{unbox}_A\ t'} \quad \textsc{rd\_unbox}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{fst}\ t \rightsquigarrow \mathsf{fst}\ t'} \quad \textsc{rd\_fst}$$

$$\frac{t \rightsquigarrow t'}{\mathsf{snd}\ t \rightsquigarrow \mathsf{snd}\ t'} \quad \textsc{rd\_snd}$$

$$\frac{t_1 \rightsquigarrow t_1'}{(t_1, t_2) \rightsquigarrow (t_1', t_2)} \quad \textsc{rd\_pair}1$$

$$\frac{t_2 \rightsquigarrow t_2'}{(t_1, t_2) \rightsquigarrow (t_1, t_2')} \quad \textsc{rd\_pair}2$$

$$\frac{}{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t} \quad \textsc{rd\_typeBeta}$$

$$\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2} \quad \textsc{rd\_typeApp}$$

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Definition rules:        95 good    0 bad
Definition rule clauses: 160 good    0 bad
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