$termvar,\ x,\ y,\ z,\ f$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ v,\ s,\ n$ ::=
| x
| triv
| box
| unbox
| $\Lambda(X <: A).t$
| $[A]t$
| $\lambda(x : A).t$
| $t_1\ t_2$
| (t_1, t_2)
| fst t
| snd t
| succ t
| 0
| case t of $t_3 \rightarrow t_1, t_4 \rightarrow t_2$
| $[]$
| $t :: t'$
| (t) S

K ::=
| $\star$

$A,\ B,\ C,\ D,\ E,\ S,\ U$ ::=
| X
| List A
| $\forall(X <: A).B$
| $\top$
| SL
| Unit
| Nat
| ?
| $A_1 \rightarrow A_2$
| $A_1 \times A_2$
| (A) S

Γ ::=
| $\cdot$
| $\Gamma, X <: A$
| $\Gamma, x : A$

$\boxed{\Gamma \vdash A : \star}$

$$\frac{\Gamma_1 \vdash A : \star}{\Gamma_1, X <: A, \Gamma_2 \vdash X : \star} \quad \text{K\_VAR}$$

$$\frac{}{\Gamma \vdash \text{Unit} : \star} \quad \text{K\_UNIT}$$

$$\frac{}{\Gamma \vdash \text{Nat} : \star} \quad \text{K\_NAT}$$

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$$\frac{}{\Gamma \vdash\ ?:\star}\quad \text{K\_UNITYPE}$$

$$\frac{\Gamma \vdash A:\star}{\Gamma \vdash \mathsf{List}\ A:\star}\quad \text{K\_LIST}$$

$$\frac{\Gamma \vdash A:\star \quad \Gamma \vdash B:\star}{\Gamma \vdash A \rightarrow B:\star}\quad \text{K\_ARROW}$$

$$\frac{\Gamma \vdash A:\star \quad \Gamma \vdash B:\star}{\Gamma \vdash A \times B:\star}\quad \text{K\_PROD}$$

$$\frac{\Gamma, X <: A \vdash B:\star}{\Gamma \vdash \forall(X <: A).B:\star}\quad \text{K\_FORALL}$$

$\boxed{\Gamma\,\mathsf{Ok}}$

$$\frac{}{\cdot\,\mathsf{Ok}}\quad \text{OK\_EMPTY}$$

$$\frac{\Gamma\,\mathsf{Ok} \quad \Gamma \vdash A:\star}{(\Gamma, X <: A)\,\mathsf{Ok}}\quad \text{OK\_TYPEVAR}$$

$$\frac{\Gamma\,\mathsf{Ok} \quad \Gamma \vdash A:\star}{(\Gamma, x : A)\,\mathsf{Ok}}\quad \text{OK\_VAR}$$

$\boxed{\Gamma \vdash A <: B}$

$$\frac{\Gamma\,\mathsf{Ok}}{\Gamma \vdash A <: A}\quad \text{S\_REFL}$$

$$\frac{\Gamma \vdash A <: B \quad \Gamma \vdash B <: C}{\Gamma \vdash A <: C}\quad \text{S\_TRANS}$$

$$\frac{X <: A \in \Gamma \quad \Gamma\,\mathsf{Ok}}{\Gamma \vdash X <: A}\quad \text{S\_VAR}$$

$$\frac{\Gamma\,\mathsf{Ok}}{\Gamma \vdash A <: \top}\quad \text{S\_TOP}$$

$$\frac{\Gamma\,\mathsf{Ok}}{\Gamma \vdash \mathsf{Nat} <: \mathsf{SL}}\quad \text{S\_NATS}$$

$$\frac{\Gamma\,\mathsf{Ok}}{\Gamma \vdash \mathsf{Unit} <: \mathsf{SL}}\quad \text{S\_UNITS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL}}{\Gamma \vdash \mathsf{List}\ A <: \mathsf{SL}}\quad \text{S\_LISTS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL} \quad \Gamma \vdash B <: \mathsf{SL}}{\Gamma \vdash (A \rightarrow B) <: \mathsf{SL}}\quad \text{S\_ARROWS}$$

$$\frac{\Gamma \vdash A <: \mathsf{SL} \quad \Gamma \vdash B <: \mathsf{SL}}{\Gamma \vdash (A \times B) <: \mathsf{SL}}\quad \text{S\_PRODS}$$

$$\frac{\Gamma \vdash A <: B}{\Gamma \vdash (\mathsf{List}\ A) <: (\mathsf{List}\ B)}\quad \text{S\_LIST}$$

$$\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \times B_1) <: (A_2 \times B_2)}\quad \text{S\_PROD}$$

$$\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash (A_1 \rightarrow B_1) <: (A_2 \rightarrow B_2)}\quad \text{S\_ARROW}$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash (\forall (X <: A).B_1) <: (\forall (X <: A).B_2)} \quad \text{S\_Forall}$$

$\boxed{\Gamma \vdash A \sim B}$

$$\frac{}{\Gamma \vdash A \sim A} \quad \text{C\_Refl}$$

$$\frac{}{\Gamma \vdash A \sim ?} \quad \text{C\_Box}$$

$$\frac{}{\Gamma \vdash ? \sim A} \quad \text{C\_Unbox}$$

$$\frac{\Gamma \vdash A \sim B}{\Gamma \vdash (\text{List } A) \sim (\text{List } B)} \quad \text{C\_List}$$

$$\frac{\Gamma \vdash A_2 \sim A_1 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \to B_1) \sim (A_2 \to B_2)} \quad \text{C\_Arrow}$$

$$\frac{\Gamma \vdash A_1 \sim A_2 \quad \Gamma \vdash B_1 \sim B_2}{\Gamma \vdash (A_1 \times B_1) \sim (A_2 \times B_2)} \quad \text{C\_Prod}$$

$$\frac{\Gamma, X <: A \vdash B_1 \sim B_2}{\Gamma \vdash (\forall (X <: A).B_1) \sim (\forall (X <: A).B_2)} \quad \text{C\_Forall}$$

$\boxed{\Gamma \vdash t : A}$

$$\frac{x : A \in \Gamma \quad \Gamma \, \text{Ok}}{\Gamma \vdash x : A} \quad \text{varP}$$

$$\frac{}{\Gamma \vdash \text{box} : \forall (X <: \text{SL}).(X \to ?)} \quad \text{Box}$$

$$\frac{}{\Gamma \vdash \text{unbox} : \forall (X <: \text{SL}).(? \to X)} \quad \text{Unbox}$$

$$\frac{\Gamma \, \text{Ok}}{\Gamma \vdash \text{triv} : \text{Unit}} \quad \text{unitP}$$

$$\frac{\Gamma \, \text{Ok}}{\Gamma \vdash 0 : \text{Nat}} \quad \text{zeroP}$$

$$\frac{\Gamma \vdash t : \text{Nat}}{\Gamma \vdash \text{succ } t : \text{Nat}} \quad \text{succ}$$

$$\frac{\begin{array}{c} \Gamma \vdash t : C \quad \text{nat}(C) = \text{Nat} \\ \Gamma \vdash t_1 : A \quad \Gamma, x : \text{Nat} \vdash t_2 : A \end{array}}{\Gamma \vdash \text{case } t \text{ of } 0 \to t_1, (\text{succ } x) \to t_2 : A} \quad \text{ncase}$$

$$\frac{\Gamma \, \text{Ok} \quad \Gamma \vdash A : \star}{\Gamma \vdash [] : \forall (X <: \top).\text{List } X} \quad \text{empty}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Gamma \vdash t_2 : \text{List } A}{\Gamma \vdash t_1 :: t_2 : \text{List } A} \quad \text{cons}$$

$$\frac{\Gamma \vdash t_1 : A_1 \quad \Gamma \vdash t_2 : A_2}{\Gamma \vdash (t_1, t_2) : A_1 \times A_2} \quad \text{pair}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda (x : A).t : A \to B} \quad \text{lam}$$

$$\frac{\Gamma, X <: A \vdash t : B}{\Gamma \vdash \Lambda (X <: A).t : \forall (X <: A).B} \quad \text{Lam}$$

$$\frac{\Gamma \vdash t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash [A]t : [A/X]C} \quad \text{TYPEAPP}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash t : B} \quad \text{SUB}$$

$$\frac{\Gamma \vdash t : C \quad \mathsf{list}(C) = \mathsf{List}\, A}{\Gamma \vdash t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 : B}{\Gamma \vdash \mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2 : B} \quad \text{LCASE}$$

$$\frac{\Gamma \vdash t_1 : C \quad \mathsf{fun}(C) = A_1 \to B}{\Gamma \vdash t_2 : A_2 \quad \Gamma \vdash A_1 \sim A_2}{\Gamma \vdash t_1\, t_2 : B} \quad \text{APP}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t : A_1} \quad \text{FST}$$

$$\frac{\Gamma \vdash t : B \quad \mathsf{prod}(B) = A_1 \times A_2}{\Gamma \vdash \mathsf{snd}\, t : A_2} \quad \text{SND}$$

$\boxed{\Gamma \vdash t_1 \Rightarrow t_2 : A}$ Cast insertion

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow x : A} \quad \text{CI\_VAR}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \Rightarrow \mathsf{box} : \forall(X <: \mathsf{SL}).(X \to ?)} \quad \text{CI\_BOX}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \Rightarrow \mathsf{unbox} : \forall(X <: \mathsf{SL}).(? \to X)} \quad \text{CI\_UNBOX}$$

$$\frac{}{\Gamma \vdash 0 \Rightarrow 0 : \mathsf{Nat}} \quad \text{CI\_ZERO}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \Rightarrow \mathsf{triv} : \mathsf{Unit}} \quad \text{CI\_TRIV}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}\, t_1 \Rightarrow \mathsf{succ}\, t_2 : \mathsf{Nat}} \quad \text{CI\_SUCC}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : ?}{\Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2) \Rightarrow (\mathsf{case}\, (\mathsf{unbox}_{\mathsf{Nat}}\, t')\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2') : A} \quad \text{CI\_NCASEU}$$

$$\frac{\Gamma \vdash t \Rightarrow t' : \mathsf{Nat}}{\Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma, x : \mathsf{Nat} \vdash t_2 \Rightarrow t_2' : A}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, 0 \to t_1, (\mathsf{succ}\, x) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, 0 \to t_1', (\mathsf{succ}\, x) \to t_2') : A} \quad \text{CI\_NCASE}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_3 : A_1 \quad \Gamma \vdash t_2 \Rightarrow t_4 : A_2}{\Gamma \vdash (t_1, t_2) \Rightarrow (t_3, t_4) : A_1 \times A_2} \quad \text{CI\_PAIR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, (\mathsf{split}_{(? \times ?)}\, t_2) : ?} \quad \text{CI\_FSTU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A_1 \times A_2}{\Gamma \vdash \mathsf{fst}\, t_1 \Rightarrow \mathsf{fst}\, t_2 : A_1} \quad \text{CI\_FST}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : ?}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, (\mathsf{split}_{(? \times ?)}\, t_2) : ?} \quad \text{CI\_SNDU}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : A \times B}{\Gamma \vdash \mathsf{snd}\, t_1 \Rightarrow \mathsf{snd}\, t_2 : B} \quad \text{CI\_SND}$$

$$\frac{}{\Gamma \vdash [] \Rightarrow [] : \forall(X <: \top).\mathsf{List}\, X} \quad \text{CI\_EMPTY}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_1' : A \quad \Gamma \vdash t_2 \Rightarrow t_2' : \mathsf{List}\, A}{\Gamma \vdash (t_1 :: t_2) \Rightarrow (t_1' :: t_2') : \mathsf{List}\, A} \quad \text{CI\_CONS}$$

$$\frac{\begin{array}{c} \Gamma \vdash t \Rightarrow t' : ? \\ \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : ?, y : \mathsf{List}\, ? \vdash t_2 \Rightarrow t_2' : B \end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, (\mathsf{split}_{(\mathsf{List}\, ?)}\, t')\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B} \quad \text{CI\_LCASEU}$$

$$\frac{\begin{array}{c} \Gamma \vdash t \Rightarrow t : \mathsf{List}\, A \\ \Gamma \vdash t_1 \Rightarrow t_1' : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash t_2 \Rightarrow t_2' : B \end{array}}{\Gamma \vdash (\mathsf{case}\, t\, \mathsf{of}\, [] \to t_1, (x :: y) \to t_2) \Rightarrow (\mathsf{case}\, t'\, \mathsf{of}\, [] \to t_1', (x :: y) \to t_2') : B} \quad \text{CI\_LCASE}$$

$$\frac{\Gamma, x : A_1 \vdash t_1 \Rightarrow t_2 : A_2}{\Gamma \vdash \lambda(x : A_1).t_1 \Rightarrow \lambda(x : A_1).t_2 : A_1 \to A_2} \quad \text{CI\_LAM}$$

$$\frac{\begin{array}{c} \Gamma \vdash t_1 \Rightarrow t_1' : ? \\ \Gamma \vdash t_2 \Rightarrow t_2' : A_2 \quad \mathsf{caster}(A_2, ?) = c \end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow (\mathsf{split}_{(? \to ?)}\, t_1')\, (c\, t_2') : ?} \quad \text{CI\_APPU}$$

$$\frac{\begin{array}{c} \Gamma \vdash t_2 \Rightarrow t_2' : A_2 \\ \Gamma \vdash t_1 \Rightarrow t_1' : A_1 \to B \quad \Gamma \vdash A_2 \sim A_1 \quad \mathsf{caster}(A_2, A_1) = c \end{array}}{\Gamma \vdash t_1\, t_2 \Rightarrow t_1'\, (c\, t_2') : B} \quad \text{CI\_APP}$$

$$\frac{\Gamma, X <: A \vdash t_1 \Rightarrow t_2 : B}{\Gamma \vdash (\Lambda(X <: A).t_1) \Rightarrow (\Lambda(X <: A).t_2) : \forall(X <: A).B} \quad \text{CI\_LAM}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow t_2 : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash ([A]t_1) \Rightarrow ([A]t_2) : [A/X]C} \quad \text{CI\_TYPEAPP}$$

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Definition rules:          71 good     0 bad
Definition rule clauses:  137 good     0 bad
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