On gradual LNL-models

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1 Gradual LNL model

Definition 1. *A **gradual LNL-model** consists of*

1. *a gradual λ-model $G\lambda^* = (\mathcal{T}, C, *_C, \mathsf{T}, \mathsf{split}, \mathsf{squash}, \mathsf{box}, \mathsf{unbox}, \mathsf{error})$*
2. *a symmetric monoidal closed category $(\mathcal{L}, I, \otimes, \multimap)$ with distinguished object $*_{\mathcal{L}}$*
3. *a pair of strong closed monoidal functors $(G, n) : \mathcal{L} \longrightarrow C$ and $(F, \hat{F}) : C \longrightarrow \mathcal{L}$ forming a monoidal adjunction with $F \dashv G$.*

Theorem 2. *A gradual LNL-model is an LNL model.*

The subscript on $*$ shall be omitted below where the meaning is clear from the context.

We now prepare to show that FG preserves the retract properties of squash_S and split_S in C for $* \Rightarrow *$ and $* \times *$. That is, we shall show that $FG(*) \multimap FG(*)$ is a retract of $FG*$, as is $FG* \otimes FG*$.

Lemma 3. *$FG(*) \multimap FG(*)$ is a retract of $FG*$ with morphisms*

$$\mathcal{L}\mathsf{squash}_{FG* \multimap FG*} \coloneqq \hat{F}_{*,*}; F(\mathsf{Unbox}_{G*} \Rightarrow \Box G*); F\mathsf{squash}_{* \Rightarrow *}; F(\mathsf{Unbox}_{G*})$$

and

$$\mathcal{L}\mathsf{split}_{FG* \multimap FG*} \coloneqq F(\mathsf{Box}_{G*}); F\mathsf{split}_{* \Rightarrow *}; F(\mathsf{Box}_{G*} \Rightarrow \mathsf{Unbox}_{G*}); \hat{F}^{-1}_{*,*}$$

Proof. Observe that:

$$\mathcal{L}\mathsf{squash}; \mathcal{L}\mathsf{split} = m_{*,*}; F\mathsf{squash}_{* \Rightarrow *}; F\mathsf{split}_{* \Rightarrow *}; p_{*,*}$$

□

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