On gradual LNL-models

Harley Eades III and Richard Blair

Computer Science, Augusta University
Computer Science, University of Iowa

1 Gradual LNL model

Definition 1. *A **gradual LNL-model** consists of*

1. *a gradual λ-model $G\lambda^* = (\mathcal{T}, C, *_C, \mathsf{T}, \mathsf{split}, \mathsf{squash}, \mathsf{box}, \mathsf{unbox}, \mathsf{error})$*
2. *a symmetric monoidal closed category $(\mathcal{L}, I, \otimes, \multimap)$ with distinguished object $*_{\mathcal{L}}$*
3. *a pair of symmetric monoidal functors $(G, n) : \mathcal{L} \longrightarrow C$ and $(F, m) : C \longrightarrow \mathcal{L}$ forming a symmetric monoidal adjunction with $F \dashv G$.*

that satisfy the following additional conditions:

1. *$F* \multimap F*$ is a retract of $F(* \Rightarrow *)$. That is, there exist morphisms $m_r : F(*) \multimap F(*) \longrightarrow F(* \Rightarrow *)$ and $m_l : F(* \Rightarrow *) \longrightarrow F(*) \multimap F(*)$ with $m_r; m_l = \mathsf{id}_{F(*) \multimap F(*)}$.*
2. *$G(*) \cong *$*

The subscript on $*$ shall be omitted below where the meaning is clear from the context.

We now show that F preserves the retract properties of squash_S and split_S in C for $* \Rightarrow *$ and $* \times *$. That is, we shall show that $F(*) \multimap F(*)$ is a retract of $F*$, as is $F * \otimes F*$.

Lemma 2. *$F(*) \multimap F(*)$ is a retract of $F*$ with morphisms $\mathcal{L}\mathsf{squash}_{F* \multimap F*} := m_{*,*}; F\mathsf{squash}_{* \Rightarrow *}$ and $\mathcal{L}\mathsf{split} := F\mathsf{split}_{* \Rightarrow *}; p_{*,*}$.*

Proof. Observe that:

$$\begin{aligned}
\mathcal{L}\mathsf{squash}; \mathcal{L}\mathsf{split} &= m_{*,*}; F\mathsf{squash}_{* \Rightarrow *}; F\mathsf{split}_{* \Rightarrow *}; p_{*,*} \\
&= m_{*,*}; F(\mathsf{squash}_{* \Rightarrow *}; \mathsf{split}_{* \Rightarrow *}); p_{*,*} \\
&= m_{*,*}; F(id_{* \Rightarrow *}); p_{*,*} \\
&= m_{*,*}; id_{F(* \Rightarrow *)}; p_{*,*} \\
&= m_{*,*}; p_{*,*} \\
&= \mathsf{id}_{F(*) \otimes F(*)}
\end{aligned}$$

□

References

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