I. P. Castro & C. Vanderwel, *Turbulent Flows: An Introduction*, IOP, 2021.

Chapter 6 Sample Exercises

ERRATA: Please note there is a typo in the eBook in Question 6.3(d) in the first mathematical which has been corrected below.

6.1 Deduce the momentum integral equation (6.15) from the mean streamwise momentum equation (6.10) by taking its integral with respect to y.

6.2 The file "MixingLayerData.txt" contains properties of a planar mixing layer, interpolated from the hot-wire measurements of Delville & Bonnet (1997). This dataset focusses on the results taken a distance of $x = 800$ mm downstream of the splitting plate, which separates two flows with freestream velocities of $U_B = 41.54$ m s^{-1} (upper) and $U_A = 22.40$ m s^{-1} (lower) in a wind tunnel. The data include y (mm), U (m/s), $\overline{u'v'}$ (m^2 s^{-2}), k (m^2 s^{-2}), and ϵ (m^2 s^{-3}).

a. Plot the velocity profile along with the approximate self-similar solution given by equation (6.50) on the same axes. Note that the centreline drifts away from $y = 0$ as a mixing layer develops.

b. One way of defining the width of the mixing layer is the distance between the points where the velocity equals $U_A + 0.9(U_B - U_A)$ and the point where it equals $U_A + 0.1(U_B - U_A)$. Determine the width of this mixing layer. Assuming self-similarity, how far downstream would you expect the width of the mixing layer to double?

c. Plot the Reynolds shear stress and turbulent kinetic energy profiles and explain why they are shaped as they are.

d. Compute and plot the production and dissipation profiles and explain why they are shaped as they are.

6.3 The file "WakeData.txt" contains measurements in the planar wake behind a 2D aerofoil from the experiment of Nakayama (1985). The file contains hot-wire measurements of the mean velocity, U, and the Reynolds stress, $\overline{u'v'}$, among other data, obtained as vertical profiles behind the aerofoil at zero degrees angle of attack. The data only cover the near-wake behind the airfoil (ie. distances ranging from 0.01-2.00 chord lengths behind the trailing edge of the aerofoil), so in this exercise one can evaluate whether this flow can be described as self-similar.

a. Plot the mean streamwise velocity profile U/U_∞ versus the vertical position y/c. Notice how the wake spreads and the velocity deficit decreases with downstream distance.

b. Now, define a new variable equal to the velocity deficit function (i.e. $(1 - U/U_\infty)$) and plot that as a function y/c.

c. The self-similar profile for a planar wake is given by $f = e^{-a\eta^2}$ (equation (6.46)). Fit a function of this form to the velocity deficit data (i.e. $1 - U/U_\infty = c_1 e^{c_2(x-c_3)^2}$, where c_1, c_2, and c_3 are tuneable coefficients). Hint: either use a curve fitting tool to evaluate the coefficients or guess them by trial and error. Note that the best fit of c_1 represents the velocity scale U_S, the best fit of c_2 is related to the halfwidth (see (d)) and the best fit of c_3 accounts for any shift in the centreline of the wake.

d. What is the halfwidth of this wake? One way to determine the halfwidth is from the second-central moment of the profile. This should be roughly equal to $\sqrt{-1/(2c_2)}$, where c_2 is the coefficient from curve fit in (c). From these estimates, determine the halfwidth at half-maximum velocity (HWHM), which is the definition of δ that we use in this chapter, by multiplying by a factor of $\sqrt{2\ln(2)}$.

e. Replot the velocity data using the self-similar variables determined above (i.e. $f = (1 - U/U_\infty)/(U_S/U_\infty)$ and $\eta = (y - c_3)/\delta$). Do the data collapse?

f. Check whether $U_S\delta$ and $\beta = U_\infty/U_S \; \mathrm{d}\delta/\mathrm{d}x$ are constant, whether the halfwidth grows as $\delta \sim x^{1/2}$ and whether the maximum velocity deficit decays as $U_S \sim x^{-1/2}$, as predicted by the similarity solutions. If these relations are violated, it indicates that the assumptions for self-similarity are perhaps not applicable in this near-wake. Why might this be the case?

g. Plot the Reynolds shear stress profiles, also using self-similar variables (i.e. $g = -(\overline{u'v'}/U_\infty^2)/(U_S/U_\infty)^2$). Do the data collapse? Often while the mean properties look self-similar, it takes longer for the turbulence properties to truly collapse.