

## Chapter 6 Sample Exercises

- 6.1. The file “MixingLayerData.txt” contains properties of a planar mixing layer, interpolated from the hot-wire measurements of Delville & Bonnet (1995). This dataset focusses on the results taken a distance of  $x = 800$  mm downstream of the splitting plate, which separates two flows with freestream velocities of  $U_B = 41.54 \text{ m s}^{-1}$  (upper) and  $U_A = 22.40 \text{ m s}^{-1}$  (lower) in a wind tunnel. The data includes  $y$  (mm),  $U$  (m/s),  $\overline{u'v'}$  ( $\text{m}^2 \text{ s}^{-2}$ ),  $k$  ( $\text{m}^2 \text{ s}^{-2}$ ), and  $\epsilon$  ( $\text{m}^2 \text{ s}^{-3}$ ).
- Plot the velocity profile along with the approximate self-similar solution given by Equation (7.50) on the same axes. Note that the centreline drifts away from  $y = 0$  as a mixing layer develops.
  - One way of defining the width of the mixing layer is the distance between the points where the velocity equals  $U_A + 0.9(U_B - U_A)$  and the point where it equals  $U_A + 0.1(U_B - U_A)$ . Determine the width of this mixing layer. Assuming self-similarity, how far downstream would you expect the width of the mixing layer to double?
  - Plot the Reynolds shear stress and turbulent kinetic energy profiles and explain why they are shaped as they are.
  - Compute and plot the production and dissipation profiles and explain why they are shaped as they are.
- 6.2. The file “WakeData.txt” contains measurements of the planar wake behind a 2D aerofoil from the experiment of Nakamura (1985). The file contains hot-wire measurements of the mean velocity,  $U$ , and the Reynolds stress,  $u'v'$ , among other data obtained in vertical profiles behind the aerofoil at zero degrees angle of attack. The data only covers the near-wake behind the airfoil (ie. distances ranging from 0.01-2.00 chord lengths behind the trailing edge of the aerofoil), so in this exercise we will evaluate whether this flow can be described as self-similar.
- Plot the mean streamwise velocity profile  $U/U_\infty$  versus the vertical position  $y/c$ . Notice how the wake spreads and the velocity deficit decreases with downstream distance.
  - Now, define a new variable equal to the velocity deficit function (i.e.  $(1 - U/U_\infty)$ ) and plot that as a function  $y/c$ .
  - The self-similar profile for a planar wake is given by  $f = e^{a\eta^2}$  (Eq. 7.46). See if you can fit a function of this form to the velocity deficit data (ie.  $(1 - U/U_\infty) = c_1 \exp(c_2(x - c_3)^2)$ , where  $c_1$ ,  $c_2$ , and  $c_3$  are tuneable coefficients). Hint: you can either use a curve fitting tool to evaluate the coefficients or you can guess them by trial and error. Note, the best fit of  $c_1$  represents the velocity scale  $U_S$ , the best fit of  $c_2$  is related to the halfwidth (see (d)), and the best fit of  $c_3$  accounts for any shift in the centreline of the wake.

- d. What is the halfwidth of this wake? One way to determine the halfwidth is from the second-central moment of the profile. This should be roughly equal to  $\sqrt{-1/(2c_2)}$ , where  $c_2$  is the coefficient from curve fit in (c). From these estimates, determine the halfwidth at half-max (HWHM), which is the definition of  $\delta$  that we use in this chapter, by multiplying by a factor of  $\sqrt{2 \ln(2)}$ .
- e. Replot the velocity data using the self-similar variables determined above (ie.  $f = (1 - U/U_\infty)/(U_S/U_\infty)$  and  $\eta = (y - c_3)/\delta$ ). Does the data collapse?
- f. Check whether  $U_S \delta$  and  $\beta = U_\infty/U_S \, d\delta/dx$  are constant, whether the halfwidth grows as  $\delta \propto x^{1/2}$ , and whether the maximum velocity deficit decays as  $U_S \propto x^{-1/2}$  as predicted by the similarity solutions. If these relations are violated, it indicates that our assumptions for self-similarity are perhaps not yet applicable in this near-wake.
- g. Plot the shear stress profiles using self-similar variables as well (ie.  $g = (-uv/U_\infty^2)/(U_S/U_\infty)^2$ ). Does the data collapse? Often while the mean properties look self-similar, it takes longer for the turbulence properties to truly collapse.