



Chapter 6 Sample Exercises

6.1 Deduce the momentum integral equation (6.15) from the mean streamwise momentum equation (6.10) by taking its integral with respect to y .

 6.2 The file “[MixingLayerData.txt](#)” contains properties of a planar mixing layer, interpolated from the hot-wire measurements of Delville & Bonnet (1997). This dataset focusses on the results taken a distance of $x = 800$ mm downstream of the splitting plate, which separates two flows with freestream velocities of $U_B = 41.54$ m s⁻¹ (upper) and $U_A = 22.40$ m s⁻¹ (lower) in a wind tunnel. The data include y (mm), U (m/s), $\overline{u'v'}$ (m² s⁻²), k (m² s⁻²), and ϵ (m² s⁻³).

- Plot the velocity profile along with the approximate self-similar solution given by equation (6.50) on the same axes. Note that the centreline drifts away from $y = 0$ as a mixing layer develops.
- One way of defining the width of the mixing layer is the distance between the points where the velocity equals $U_A + 0.9(U_B - U_A)$ and the point where it equals $U_A + 0.1(U_B - U_A)$. Determine the width of this mixing layer. Assuming self-similarity, how far downstream would you expect the width of the mixing layer to double?
- Plot the Reynolds shear stress and turbulent kinetic energy profiles and explain why they are shaped as they are.
- Compute and plot the production and dissipation profiles and explain why they are shaped as they are.

 6.3 The file “[WakeData.txt](#)” contains measurements in the planar wake behind a 2D aerofoil from the experiment of Nakayama (1985). The file contains hot-wire measurements of the mean velocity, U , and the Reynolds stress, $\overline{u'v'}$, among other data, obtained as vertical profiles behind the aerofoil at zero degrees angle of attack. The data only cover the near-wake behind the airfoil (ie. distances ranging from 0.01-2.00 chord lengths behind the trailing edge of the aerofoil), so in this exercise one can evaluate whether this flow can be described as self-similar.

- Plot the mean streamwise velocity profile U/U_∞ versus the vertical position y/c . Notice how the wake spreads and the velocity deficit decreases with downstream distance.
- Now, define a new variable equal to the velocity deficit function (i.e. $(1 - U/U_\infty)$) and plot that as a function y/c .
- The self-similar profile for a planar wake is given by $f = e^{-a\eta^2}$ (equation (6.46)). Fit a function of this form to the velocity deficit data (i.e. $1 - U/U_\infty = c_1 e^{c_2(x-c_3)^2}$, where c_1 , c_2 , and c_3 are tuneable coefficients). Hint: either use a curve fitting tool to evaluate the coefficients or guess them by

trial and error. Note that the best fit of c_1 represents the velocity scale U_S , the best fit of c_2 is related to the halfwidth (see (d)) and the best fit of c_3 accounts for any shift in the centreline of the wake.

- d. What is the halfwidth of this wake? One way to determine the halfwidth is from the second-central moment of the profile. This should be roughly equal to $\sqrt{1 - 1/(2c_2)}$, where c_2 is the coefficient from curve fit in (c). From these estimates, determine the halfwidth at half-maximum velocity (HWHM), which is the definition of δ that we use in this chapter, by multiplying by a factor of $\sqrt{2 \ln(2)}$.
- e. Replot the velocity data using the self-similar variables determined above (i.e. $f = (1 - U/U_\infty)/(U_S/U_\infty)$ and $\eta = (y - c_3)/\delta$). Do the data collapse?
- f. Check whether $U_S \delta$ and $\beta = U_\infty/U_S \, d\delta/dx$ are constant, whether the halfwidth grows as $\delta \sim x^{1/2}$ and whether the maximum velocity deficit decays as $U_S \sim x^{-1/2}$, as predicted by the similarity solutions. If these relations are violated, it indicates that the assumptions for self-similarity are perhaps not applicable in this near-wake. Why might this be the case?
- g. Plot the Reynolds shear stress profiles, also using self-similar variables (i.e. $g = -(\overline{u'v'}/U_\infty^2)/(U_S/U_\infty)^2$). Do the data collapse? Often while the mean properties look self-similar, it takes longer for the turbulence properties to truly collapse.