

Boolean chains *

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Abstract

A novel algorithm for computation of Boolean operations between cellular complexes is given in this module. It is based on bucketing of possibly interacting geometry using a box-extension of kd-trees, normally used for point proximity queries. Such kd-tree representation of containment boxes of cells, allow us to compute a number of independent buckets of data to be used for local intersection, followed by elimination of duplicated data. Actually we reduce the intersection of boundaries in 3D to the independent intersections of the buckets of (transformed) faces with the 2D subspace $z = 0$, in order to reconstruct each splitted facet of boolean arguments, suitably transformed ther together with the bucket of indent facets. A final tagging of cells as either belonging or not to each operand follows, allowing for fast extraction of Boolean results between any pair of chains (subsets of cells). This Boolean algorithm can be considered of a *Map-Reduce* kind, and hence suitable of a distributed implementation over big datasets. The actual engineered implementation will follow the present prototype, using some distributed NoSQL database, like MongoDB or Riak.

Contents

1 Introduction

2 Preview of the algorithm

The whole Boolean algorithm is composed by four stages in sequence, denoted in the following as *Unification*, *Bucketing*, *Intersection*, and *Reconstruction*. The algorithm described here is both multidimensional and variadic. Multidimensional means that the arguments are solid in Euclidean space of dimension d , with d small integer. The *arity* of a function or operation is the number of arguments or operands the function or operation accepts. In computer science, a function accepting a variable number of arguments is called *variadic*.

*This document is part of the *Linear Algebraic Representation with CoChains* (LAR-CC) framework [?].
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2.1 Unification

In this first step the boundaries of the n Boolean arguments are computed and merged together as a set of chains defined in the discrete set V made by the union of their vertices, and possibly by a discrete set of points generated by intersection of cells of complementary dimension, i.e. whose dimensions add up to the dimension of the ambient space. Actually, only the (*oriented*) boundaries V, FV_i ($1 \leq i \leq n$) of the varius arguments are retained here, and used by the following steps of the algorithm.

2.2 Bucketing

The bounding boxes of facets FV_i are computed, and their *box-kd-tree* is worked-out, so providing a group of buckets of close cells, that can be elaborated independently, and possibly in parallel, to compute the intersections of the boundary cells.

2.3 Intersection

For each facet f of one of Boolean arguments, the subset $F(f)$ of incident or intersecting facets of boundaries of the other arguments were computed in the previous *bucketing* step. So, each F is transformed by the affine map that sends f into the $z = 0$ subspace, and there is intersected with this subspace, generating a subset $E(f)$ of coplanar edges. This one is projected in 2D, and the *regularized* cellular 2-complex $G(f)$ induced by it is computed, and mapped back to the original space position and orientation of f (providing a partition of it induced by the other boundaries).

2.4 Reconstruction

Like for in the reconstruction of 2D solid cells using the angular ordering of edges around the vertices, the coincident edges are identified in 3D, and used to sort the incident faces sing vhe falues of solid angles given with one reference face. The 3D space partition induced by $\cup_f G(f)$ is finally reconstructed, possibly in parallel, by traversing the adjacent sets of facets on the boundary of each solid cell.

3 Implementation

3.1 Box-kd-tree

```
Remove subsets from bucket list @D Remove subsets from bucket list @ """
Remove subsets from bucket list """
def removeSubsets(buckets):
    n = len(buckets)
    A = zeros((n,n))
    for i,bucket in enumerate(buckets):
        for j,bucket1 in enumerate(buckets):
            if set(bucket).issubset(set(bucket1)):
                A[i,j] = 1
    B = AA(sum)(A.tolist())
    out = [bucket for i,bucket in enumerate(buckets) if B[i]==1]
    return out @
```

```

Iterate the splitting until splittingStack is empty @D Iterate the splitting until
splittingStack is empty @"""
Iterate the splitting until splittingStack is empty """
def
boxTest(boxes,h,k): if len(boxes[0])==4: B1,B2,B3,B4 = boxes[k] b1,b2,b3,b4 = boxes[h]
return not (b3|B1 or B3|b1 or b4|B2 or B4|b2) else: B1,B2,B3,B4,B5,B6,=boxes[k]b1,b2,b3,b4,b5,b6,=boxes[h]
B1orB4 < b1orb5 < B2orB5 < b2orb6 < B3orB6 < b3)

def boxBuckets3d(boxes): bucket = range(len(boxes)) splittingStack = [bucket] final-
Buckets = [] while splittingStack != []: bucket = splittingStack.pop() below,above =
splitOnThreshold(boxes,bucket,1) below1,above1 = splitOnThreshold(boxes,above,2) be-
low2,above2 = splitOnThreshold(boxes,below,2)
below11,above11 = splitOnThreshold(boxes,above1,3) below21,above21 = splitOnThresh-
old(boxes,below1,3) below12,above12 = splitOnThreshold(boxes,above2,3) below22,above22
= splitOnThreshold(boxes,below2,3)
splitting(above1,below11,above11, finalBuckets,splittingStack) splitting(below1,below21,above21,
finalBuckets,splittingStack) splitting(above2,below12,above12, finalBuckets,splittingStack)
splitting(below2,below22,above22, finalBuckets,splittingStack)

finalBuckets = list(set(AA(tuple)(finalBuckets))) parts = geomPartitionate(boxes,finalBuckets)
parts = [[h for h in part if boxTest(boxes,h,k)] for k,part in enumerate(parts)] return
AA(sorted)(parts) @

aaaaaaa @D aaaaaaa @""" aaaaaa """
@
```

3.2 Merging the boundaries

3.3 Elementary splitting

In this section we implement the splitting of $(d - 1)$ -faces, stored in `FV`, induced by the buckets of $(d - 1)$ -faces, stored in `parts`, and one-to-one associated to them. Of course, (a) both such arrays have the same number of elements, and (b) whereas `FV` contains the indices of incident vertices for each face, `parts` contains the indices of adjacent faces for each face, with the further constraint that $i \notin \text{parts}(i)$.

Submanifold mapping computation The 4×4 (affine) `scipy` matrix `transform` of type `mat` is computed by the function `submanifoldMapping`, using as input the array `pivotFace` that contains the vertices of the so-called *pivot* face, i.e. of the face to be mapped to the coordinate subspace $z = 0$ (in 3D).

```

@D Submanifold mapping computation @""" Submanifold mapping computation """
def submanifoldMapping(pivotFace): tx,ty,tz = pivotFace[0] transl = mat([[1,0,0,-tx],[0,1,0,-
ty],[0,0,1,-tz],[0,0,0,1]]) facet = [ VECTDIFF([v,pivotFace[0]]) for v in pivotFace ] m = face-
Transformations(facet) mapping = mat([[m[0,0],m[0,1],m[0,2],0],[m[1,0],m[1,1],m[1,2],0],[m[2,0],
m[2,1],m[2,2],0],[0,0,0,1]]) transform = mapping * transl return transform @
```

```

Helper functions for spacePartition @D Helper functions for spacePartition @"""
Helper functions for spacePartition """
def submodel(V,FV,EV): FE = crossRelation(V,FV,EV)
def submodel0(f,F): fE = list(set(FE[f] + CAT([FE[g] for g in F]))) fF = [f]+F return fF,fE
return submodel0
def meetZero( sW, (w1,w2) ): testValue = sW[w1][2] * sW[w2][2] if testValue < 10**-4:
return False else: return True
def segmentIntersection(p1,p2): (x1,y1,z1),(x2,y2,z2) = p1,p2 if abs(z1-z2) != 0.0: alpha = z1/(z1-z2) x = x1+alpha*(x2-x1) y = y1+alpha*(y2-y1) return x,y,0.0 else: return None @

```

Computation of face transformations The faces in every $\text{parts}(i)$ must be affinely transformed into the subspace $x_d = 0$, in order to compute the intersection of its elements with this subspace, that are submanifolds of dimension $d - 2$.

A much better solution would be to compute a LU factorization of the the first columns of the U upper triangular or trapezoidal matrix ... see [gauss-elimination-in-python](#)

```

@D Computation of face transformations @""" Computation of affine face transformations """
from numpy import array from scipy.linalg import lu from scipy.linalg.basic import det

```

```

def COVECTOR(points): pointdim = len(points[0]) plane = Planef.bestFittingPlane(pointdim,
[item for sublist in points for item in sublist]) return [plane.get(I) for I in range(0,pointdim+1)]
def faceTransformations(facet): covector = COVECTOR(facet) translVector = facet[0]
translation newFacet = [ VECTDIFF([v,translVector]) for v in facet ] linear transformation:
boundaryFacet -> standard (d-1)-simplex d = len(facet[0]) m = mat( newFacet[1:d]
+ [covector[1:]]) if abs(det(m))<0.0001: for k in range(len(facet)-2): m = mat( new-
Facet[1+k+1:d+k+1] + [covector[1:]]) if abs(det(m))<0.0001: break transformMat =
m.T.I transformation in the subspace  $x_d = 0$  out = (transformMat*(mat(newFacet).T)).T.tolist() return transformMat

```

Sorting of points along a line Parametric sorting of aligned points along the array line.

```

@D Sorting of points along a line @""" Sorting of points along a line """
def computeSegments(line): p0 = line[0] p1 = line[1] p1p0 = VECTDIFF([p1,p0]) h = sorted([(Xk,k) for k, Xk in enumerate(line)], reverse = True)[0][1] params = [(p[h] - p0[h])/p1p0[h] for p in line] sortedPoints = TRANS(sorted(zip(params, line)))[1] segments = TRANS([sortedPoints[:-1], sortedPoints[1 :]]) return segments@

```

Remove intersection points external to a submanifold face When computing the LAR of each submanifold face embedded in $z = 0$ hyperplane, it is necessary to remove every vertex that was generated, outside the considered input face, by intersection of (incident) edges. This would produce spurious topology, i.e. cells that do not find adjacents cells incident to their boundaries.

```

@D Remove intersection points external to a submanifold face @"""
Remove intersection points external to a submanifold face """
def removeExternals(M,V,EV,fe, z,fz,ez):
    inputFace = Struct([(V,[EV[e] for e in fe])])
    v,ev = larApply(M)(struct2lar(inputFace))
    pol = [eval(vcode(w[:-1])) for w in v],ev
    out = []
    classify = pointInPolygonClassification(pol)
    for k,point in enumerate(z):
        if classify(point) == "p_out": out += [k]
    fz = [f for f in fz if not any([v in out for v in f])][0]
    if [e for e in ez if not any([v in out for v in e])]: return z, fz, ez

```

Space partitioning via submanifold mapping the function `spacePartition`, given in the below script, takes as input a *non-valid* (with the meaning used in solid modeling field — see [?]) LAR model of dimension $d - 1$, i.e. a triple (V, FV, EV) , and an array `parts` indexed on faces, and containing the subset of faces with greatest probability of intersecting each indexing face, respectively. The `spacePartition` function returns the *valid* LAR boundary model (W, FW, EW) of the space partition induced by FV .

```

@D Space partitioning via submanifold mapping @"""
Space partitioning via submanifold mapping """
def spacePartition(V,FV,EV, parts):
    FE = crossRelation(V,FV,EV)
    submodel0 = submodel(V,FV,EV)
    out = []
    input: face index f; candidate incident faces F[f];
    for f,F in enumerate(parts):
        Selection of the LAR submodel S(f) := (V,FV,EV)(f) restricted to [f]+F[f]
        fF,FE = submodel0(f,F)
        subModel = Struct([(V,[FV[g] for g in fF],[EV[h] for h in fE])])
        sV,sFV,sEV = struct2lar(subModel)
        Computation of submanifold map M moving f to z=0
        pivotFace = [V[v] for v in FV[f]]
        M = submanifoldMapping(pivotFace)
        submanifold transformation
        Transformation of S(f) by M, giving S = (sW,sEW) := M(S(f))
        sW,sFW,sEW = larApply(M)((sV,sFV,sEV))
        filtering of EW edges traversing z=0, giving EZ edges and incident faces FZEZ
        sFE = crossRelation(V,sFW,sEW)
        edges = list(set([e for k,face in enumerate(sFW) for e in sFE[k] if meetZero(sW, sEW[e])]))
        edgesPerFace = [e for e in sFE[k] if meetZero(sW, sEW[e]) for k,face in enumerate(sFW)]
        edges = list(set(CAT(edgesPerFace)))
        WW = AA(LIST)(range(len(sW)))
        wires = [sEW[e] for e in edges]
        for each face in FZEZ, computation of the aligned set of points p(z=0)
        points = collections.OrderedDict()
        lines = [[sW[w1],sW[w2]] for w1,w2 in wires]
        for k,(p,q) in enumerate(lines):
            point = segmentIntersection(p,q)
            if point != None:
                points[edges[k]] = point
                pointsPerFace = [set(face).intersection(points.keys()) for face in edgesPerFace]
                lines = [[points[e][:2] for e in face] for face in pointsPerFace]
                lines = [line for line in lines if line != []]
                vpoints = [(vcode(point),k) for k,point in enumerate(line)]
                for line in lines]:
                lines = [AA(eval)(dict(line).keys()) for line in vpoints]
        sorting of every aligned set FX, where X is the parameter along the intersection line
        theLines = []
        for line in lines:
            if len(line) > 2:
                segments = computeSegments(line)
                theLines += segments
            else:
                theLines += [line]
        Check that every set FX has even cardinality
        Construction of the planar set FX,EX of faces and lines
        lar = larFromLines(theLines)
        if lar != None:
            z,fz,ez,polygons = larFromLines(theLines)
            Remove external vertices
            z,fz,ez = removeExternals(M,V,EV,FE[f], z,fz,ez)
            w,fw,ew = larApply(M.I)(([v+[0.0] for v in z],fz,ez))
            out += [Struct([(w, fw, ew)])]
    return struct2lar(Struct(out))

```

3.4 Circular ordering of faces around edges

3D boundary triangulation of the space partition The function `boundaryTriangulation` given below is used to guarantee that there is a unique (simple) facet incident to an edge and contained in one LAR facet. More clearly, the Boolean decompositions generated by LAR allow for non convex cells, and in particular for nonconvex boundary facets of d -cells. This fact may induce errors in the computation of circularly sorted faces around edges. Conversely, by decomposing the faces into triangles, such ordering problems cannot appear. We also note that whereas every $(d - 1)$ -facet is made by coherently oriented triangles, it is not possible to give—a priori—a coherently orientation to all the facets, since the object interior and exterior are not defined (for now).

@D 3D boundary triangulation of the space partition @""” 3D boundary triangulation of the space partition ”””

```

def orientTriangle(pointTriple): v1 = array(pointTriple[1])-pointTriple[0] v2 = array(pointTriple[2])-pointTriple[0] if cross(v1,v2)[2] > 0: return REVERSE(pointTriple) else: return pointTriple
from copy import copy
def boundaryTriangulation(V,FV,EV,FE): triangleSet = [] print "=",V print "FV =",FV
print "EV =",EV print "FE =",FE
def mapVerts(inverseMap): def mapVerts0(mappedVerts): return (inverseMap * (mat(mappedVerts).T)).T
return mapVerts0
for f,face in enumerate(FV): print "f,face =",f,face triangledFace = [] EW = [EV[e]
for e in FE[f]] pivotFace = [V[v] for v in face] vertdict = dict([(w,v) for v,w in enumerate(face)]) EW = [[vertdict[w] for w in edge] for edge in EW] transform = submanifoldMapping(pivotFace) mappedVerts = (transform * (mat([p+[1.0] for p in pivotFace]).T)).T.tolist() verts2D = [point[:-2] for point in mappedVerts]

Construction of CDT (Constrained Delaunay Triangulation) for LAR face model =
(verts2D,[range(len(verts2D))],EW) struct = Struct([model]) U,EU = structBoundaryModel(struct) W,FW,EW,polygons = larFromLines([[U[u],U[v]] for u,v in EU]) triangles =
polygons2TriangleSet(W,polygons) trias = [[p+[1],q+[1],r+[1]] for p,q,r in CAT(triangles)]
inverseMap = transform.I trias = AA(mapVerts(inverseMap))(trias) triangledFace += [[v[:-1] for v in triangle] for triangle in trias] triangleSet += [triangledFace] return triangleSet

def triangleIndices(triangleSet,W): tree = spatial.cKDTree(W) TV,FT,t = [],[],-1
for face in triangleSet: ft = [] for triangle in face: print "triangle =",triangle points =
tree.query(triangle,1)[1].tolist() print "points =",points t += 1 TV += [points] ft += [t]
FT += [ft] print "TV =",TV VIEW(EXplode(1.2,1.2,1.2)(MKPOLS((W,TV)))) return TV,FT @

```

Computation of incidence between edges and 3D triangles @D Computation of incidence between edges and 3D triangles @ def edgesTriangles(EF, FW, TW, EW): ET = [None for k in range(len(EF))] for e,edgeFaces in enumerate(EF): ET[e] = [] for f in

```

edgeFaces: for t in TW[f]: if set(EW[e]).intersection(t)==set(EW[e]): ET[e] += [t] return
ET @

```

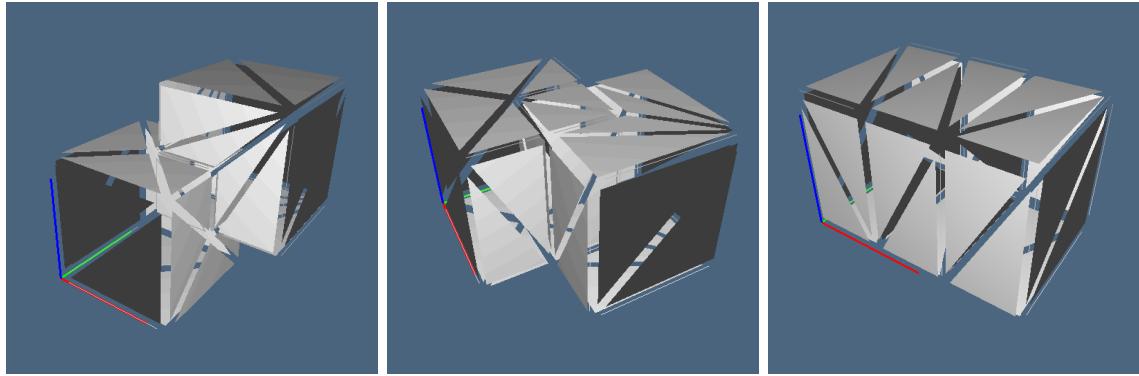


Figure 1: The triangulated boundaries of the space partition induced by two cubes (one is variously translated).

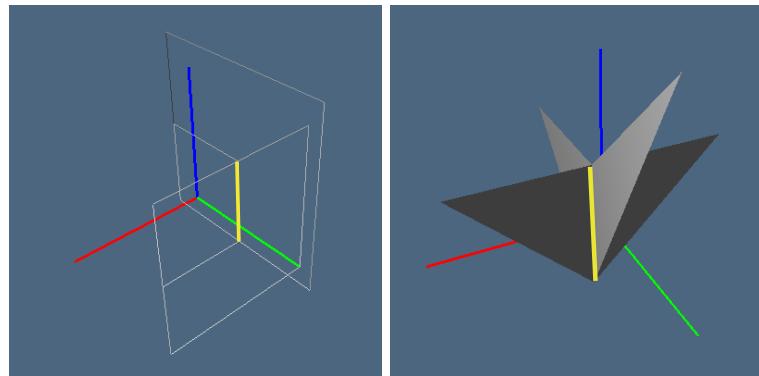


Figure 2: The triangles around an edge: `VIEW(STRUCT(MKPOLS((W,ET[35]))))`.

Example

```

In [2]: ET[35]
Out[2]: [[19, 7, 8], [6, 8, 7], [8, 7, 16], [4, 7, 8]]

In [3]: EF[35]
Out[3]: [4, 10, 11, 14]

In [4]: [FW[f] for f in EF[35]]
Out[4]: [(19, 7, 8, 12), (6, 10, 8, 7), (12, 8, 7, 16, 1, 2), (4, 5, 6, 7, 8, 9)]

```

```
In [5]: EW[35]
Out[5]: (7, 8)
```

Slope of edges The `faceSlopeOrdering` function, given in the script below, return the list `EF_angle` of lists of faces incident to the model edges, counterclockwise ordered with respect to the orientation of the edge. Let us remember that the edges are naturally oriented from the vertex of lesser index to that of greater index.

```
@D Slope of edges """
def planeProjection(normals): V = mat(normals) if all(V[:,0]==0): V = np.delete(V, 0, 1) elif all(V[:,1]==0): V = np.delete(V, 1, 1) elif all(V[:,2]==0): V = np.delete(V, 2, 1) return V

def faceSlopeOrdering(model,FE): V,FV,EV = model triangleSet = boundaryTriangulation(V,FV,EV,FE) print "=" ,triangleSet VIEW(EXPLODE(1.2,1.2,1.2)(AA(JOIN)(AA(POLYLINE)(CAT(triangleSet))))) TV,FT = triangleIndices(triangleSet,V) TE = crossRelation(V,TV,EV) ET,ET_angle = invertRelation(TE),[] import pdb; pdb.settrace() fore, etin enumerate(ET) v1, v2 = EV[e] v1v2 = set([v1, v2]) et_angle = [] t0 = et[0] tverts = [v1, v2]+list(set(TV[t0]).difference(v1v2)) e1 = UNITVECT(VECTDIFF([V[tverts[1]], V[tverts[0]]])) e1 = UNITVECT(VECTDIFF([V[tverts[2]], V[tverts[1]]])) cross(array(e1), e3).tolist() basis = mat([e1, e2, e3]).T transform = basis.Inormals = [] Tvs = [] for triangle in ET: verts = TV[triangle].vertSet = set(verts).difference(v1v2) tvs = [v1, v2]+list(vertSet) Tvs+=[tvs] w1 = UNITVECT(VECTDIFF([V[tvs[2]], V[tvs[0]]])) w2 = (transform*mat([w1]).T).Tw3 = cross(array([0, 0, 1]), w2).tolist()[0] normals+=[w3] normals = mat(normals).fork, tin enumerate(et): angle = math.atan2(normals[k, 1], normals[k, 0]) et_angle+=[angle] pairs = sorted(zip(et_angle, et, Tvs)) sortedTrias = [pair[1] for pair in pairs] triasVerts = [pair[2] for pair in pairs] ET_angle+=[sortedTrias] EF_angle = ET_to_EF_incidence(TV, FV, ET_angle) return EF_angle
```

Edge-triangles to Edge-faces incidence In the function `ET_to_EF_incidence` below, we convert the Edge-triangles incidence table `ET_angle` to a Edge-faces incidence table `EF_angle`. The input data to the algorithm are the relations `TW`, `FW`, and, of course, the incidence `ET_angle`. It works by computing two translationa tables `tableFT` and `tableTF` from face indices to triangle indices and viceversa. Of course, `assert(len(EF_angle) == 2*len(FW))` must be `True`.

```
@D Edge-triangles to Edge-faces incidence """
def ET_to_EF_incidence(TW, FW, ET_angle): tableFT = [None for _ in range(len(FW))] t = 0 for f, trias in enumerate(TW): tableFT[f] = range(t, t + len(trias)) t += len(trias) tableTF = invertRelation(tableFT) EF_angle = [[tableTF[t][0] for t in trias] for trias in ET_angle] assert(len(EF_angle) == 2 * len(FW)) return EF_angle @
```

Cells from $(d-1)$ -dimensional LAR model Since faces in the space partition induced by overlaping 3-coverings are $(d-1)$ -cells, they are located on the boundary of *two* d -cells of the partition. Hence, the traversal algorithm of the data structure storing the relevant

information may be driven by signing the two cofaces of each face as being either already visited or not.

3.5 Progressive reconstruction of 3-cell boundaries

The input to this stage is a 2-complex embedded in 3D, with 2-cells non necessarily convex. The output is the 3-space partition defined by the cellular 3-complex, whose 2-skeleton is the inpiut complex. In other words, we mu reconstruct the 3-cells induced by the 2-cells of the input complex. This is done reconstructing the 3-cells stepwise. Each 3-cell reconstruction is done starting from one `face` two-dimensional previously taken into account no more than one single time, so that every 2-face is used at most exaclty twice. An example of use of the functions implemented in this section is given in example `test12.py`

Edge cycles associated to a closed chain of edges The problem here is to conserve in the new `cycles` the same orientation of the previous ones, passed through the `orientedEdges` variable. We can formalize the problem as follows. Let call `pcycles` (for “previous cycles”) and `fcycle` (for “face cycle”) the algorithm input. the output is the *coherently oriented* `outcycles`. First, an orientation is given to `fcycle`; then this one is compared with the `pcycles` orientation, and it is possibly reversed, in order to get them coherently oriented. Finally, the direct sum of `pcycles` and `fcycle` is executed, giving the `outcycles`.

```
@D Cycles orientation @"""
Cycles orientation """
def cyclesOrient(pcycles,fcycle,EV):
if set(AA(ABS)(pcycles)).difference(fcycle)==set(): return []
ofcycle=_boundaryCycles(fcycle,EV)orientedo
ofcycle[0]orientediftype(pcycles[0]) == list : opcycle = CAT(pcycles)else : opcycle =
pcyclesint = set(opcycle).intersection(ofcycle)if int!= set() : ofcycle = reverseOrientation(ofcycle)outCh
[eeforeinofcycleifnot(-einopcycle)]outChain+ = [eeforeinopcycleifnot(-einofcycle)]return outChain
if name=="main":pcycles=[[-19,13,22,23]]fcycle=[30,20,18,2,26,19]cyclesOrientation(pcycles,fcycle)@
```

The 3-cell traversal algorithm Initially, the list of counterclockwise ordered faces around the oriented edges are computed, and stored as indexed by edges in the `EF_angle` list of lists. This information is stored in the compressed sparse row matrix `csrEF`, whose element (e, f) provides the *next* face index incident on edge e , after f .

Also, a list of list of zeros is stored in the `visitedFE` variable, in order to memorize the visited pairs (f, e) by writing one in their corresponding positions. The `firstSearch` function will so retrieve the first non visited pair, in order to start the extraction of a new 3-cell. The `cv` variable accumulates the vertex indices of the current 3-cell. When the 3-cell is completely extracted (how-to test?), will be stored as a new row in the `CV` relation.

The test for completeness of the extraction is done by computing the current boundary of the cell as a set of edges of faces, by python `XOR` of the edges of every accumulated face-edge relation. When this set it becomes empty, the 3-cell extraction is completed.

@D Cells from $(d - 1)$ -dimensional LAR model @””” Cells from $(d - 1)$ -dimensional LAR model ”””

```
def facesFromComponents(model,FE,EFangle) : initialization
V,FV,EV = model
visitedCell = [[None, None] for i in range(len(FV))]
face = 0
boundaryLoop, boundaryCycles = FE[face], EV
boundaryLoop[0].firstEdge = boundaryLoop[0].importPDB; PDB.set_trace()
cf, coe = getSolidCell(FE, face, visitedCell)
if visitedCell[face][0] == None : visitedCell[face][0] = edge
else : visitedCell[face][1] = edge
cv, ce = set(), set()
cv = cv.union(CAT([FV[f].forfincf])) ce = ce.union(CAT([FE[f].forfincf]))
CF, CV = [cf], [list(cv)], [list(ce)], [coe]

main loop
import PDB; PDB.set_trace()
while True :
    face, edge = startCell(visitedCell, FE, EV)
    if face == -1 : break
    boundaryLoop, boundaryCycles = FE[face], EV
    boundaryLoop = boundaryLoop[0].ifedgenotinboundary
    boundaryLoop = reverseOrientation(boundaryLoop)
    cf, coe = getSolidCell(FE, face, visitedCell, boundaryLoop)
    COE += [coe]
    for face, edge in zip(cf, coe) :
        if visitedCell[face][0] == None : visitedCell[face][0] = edge
        else : visitedCell[face][1] = edge
    cv, ce = set(), set()
    cv = cv.union(CAT([FV[f] for f in cf])) ce = ce.union(CAT([FE[f] for f in cf]))
    CV += [list(cv)]
    CE += [list(ce)]
return V, CV, FV, EV, CF, CE, COE
```

Start a new 3-cell The function `startCell` below is used to begin the extraction of a new 3-cell (after the first one was already extracted). Therefore its aim is to choose as first face one already previously extracted, in order to begin the current boundary with one cycle coherently oriented. This will be implemented by looking for a “`face`” position stored in `visitedCell` with just one `None` value in its row.

@D Start a new 3-cell @””” Start a new 3-cell ””” def startCell(visitedCell,FE,EV): if len([term for cell in visitedCell for term in cell if term==None])==1: return -1,-1 for face in range(len(visitedCell)): if len([term for term in visitedCell[face] if term==None])==1: edge = visitedCell[face][0] break else: pass TODO: implement search for isolated shells face,edge = -1,-1 return face,edge @

Face orientations storage In order to correctly accomplish the extraction of 3-cells from the 2-complex partition of the arguments’ space, it is necessary to use twice every 2-face, belonging with opposite orientations to the boundaries of two adjacent 3-cells. The array `faceOrientations`, initialized to $n \times 2$ zeros, with n equal to the number of 2-cells, is so used to store the orientations of faces considered as 2-cycles of edges.

In particular, the orientation of the 2-face is equivalent to the embedded orientation of one of its edges, corresponding either to the intrinsic orientation of this one, or to its opposite orientation. Hence, every time a face is used during the extraction of a 3-cell, (the elementary 1-chain of) one of its oriented edges is stored in `faceOrientations`, to remember its orientation, and eventually reverse the orientation of the face the next time it is used again. At the very end of the extraction algorithm, all the faces must be used twice, with opposite orientations.

```

@D Face orientations storage @"""
Face orientations storage """
def reverseOrientation(chain):
    return REVERSE([-cell for cell in chain])

def faceOrientation(boundaryLoop,face,FE,EV,cf):
    theBoundary = set(AA(ABS)(boundaryLoop))
    if theBoundary.intersection(FE[face]) == set() and theBoundary.difference(FE[face]) != set():
        BOH!!
        coboundaryFaces = [f for f in cf if set(FE[f]).intersection(theBoundary) != set()]
        face = coboundaryFaces[0]
        faceLoop,_boundaryCycles(FE[face],EV)
        faceLoop = faceLoop[0]
        commonEdges = set(faceLoop).intersection(boundaryLoop)
        if commonEdges == set() or commonEdges == 0:
            faceLoop = reverseOrientation(faceLoop)
            commonEdges = set(faceLoop).intersection(boundaryLoop)
            list(commonEdges)[0]
        if theEdge == 0:
            theEdge = list(commonEdges)[1]
        return -theEdge, face

```

Get single solid cell @D Get single solid cell @"""
Get single solid cell """
def getSolidCell(FE,face,visitedCells):
 def orientFace(face,boundaryLoop):
 for e in boundaryLoop:
 if ABS(e) in FE[face]:
 return -e
 coe = [orientFace(face,boundaryLoop)]
 cf = [face]
 import pdb; pdb.set_trace()
 while boundaryLoop != []:
 edge, face = faceOrientation(boundaryLoop, face, FE, EV, cf)
 if edge > 0:
 edgeFaces = FE_angle[edge]
 if edge < 0:
 edgeFaces = REVERSE(FE_angle[-edge])
 e = ABS(edge)
 n = len(edgeFaces)
 ind = (edgeFaces.index(face) + 1)
 nextFace = edgeFaces[ind]
 coe += [-orientFace(nextFace, boundaryLoop)]
 boundaryLoop = cyclesOrient(boundaryLoop, FE[nextFace], EV)
 face = nextFace
 if DEBUG:
 passVIEW(EXPLODE(1.2, 1.2, 1.2)(MKTRIANGLES((V,[FV]))))

Double check the faces boundaries made of edges Let us notice that a systematic use of the FE relation to compute the edges on the boundary of a face is *not* reliable when the faces are non-convex (WHY ??). A better solution is to double-check the result `FE[f]` when `len(FE[f]) > len(FV[f])`, in order to filter out the spurious edges ... The function below works with the precondition that vertices in `FV[f]` are spatially ordered along the face boundary.

```

@D Double check the faces boundaries made of edges @"""
Double check the faces boundaries made of edges """
def doubleCheckFaceBoundaries(FE,V,FV,EV):
    FEout = []
    for f,face in enumerate(FE):
        n = len(FV[f])
        if len(FE[f]) > n:
            contains both edges coded 0 and 1 ...
            (how to solve?) FEout += [list(set(face).difference([0]))]
        else:
            FEout += [face]
    return FEout

```

3.5.1 Main procedure of arrangement partitioning

@D Main procedure of arrangement partitioning @"""
Main procedure of arrangement partitioning """

```

@i Double check the faces boundaries made of edges @i
def thePartition(W,FW,EW):
    quadArray = [[W[v] for v in face] for face in FW]
    parts = boxBuckets3d(containmentBoxes(quadArray))
    Z,FZ,EZ = spacePartition(W,FW,EW,parts)
    Z,FZ,EZ = larSimplify((Z,FZ,EZ),radius=0.001)
    model = Z,FZ,EZ
    ZZ = AA(LIST)(range(len(Z)))
    submodel = STRUCT(MKPOLS((Z,EZ)))
    VIEW(larModelNumbering(1,1))

```

```

FE = crossRelation(Z,FZ,EZ) to be double checked !! EFangle = faceSlopeOrdering(model,FE)
V,CV,FV,EV,CF,CE,COE = facesFromComponents((Z,FZ,EZ),FE,EFangle)returnV,CV,FV,EV,CF,CE

```

3.6 Boolean chains

4 Exporting the Library

```

@O larlib/larlib/bool.py @"""
Module for Boolean computations between geometric objects """
from larlib import * from copy import copy DEBUG = False

```

```

@j Coding utilities @j Remove subsets from bucket list @j @j Iterate the splitting until splittingStack is empty @j @j Computation of face transformations @j @j Computation of affine face transformations @j @j Submanifold mapping computation @j @j Helper functions for spacePartition @j @j Sorting of points along a line @j @j Remove intersection points external to a submanifold face @j @j Space partitioning via submanifold mapping @j @j 3D boundary triangulation of the space partition @j @j Computation of incidence between edges and 3D triangles @j @j Directional and orthogonal projection operators @j @j Check edge-face ordering @j @j Slope of edges @j @j Oriented cycle of vertices from a 1-cycle of unoriented edges @j @j Edge-triangles to Edge-faces incidence @j @j Cells from ( $d - 1$ )-dimensional LAR model @j @j Permutation of edges defined by edge cycles @j @j Cycles orientation @j @j Start a new 3-cell @j @j Face orientations storage @j @j Check and store the orientation of faces @j @j Get single solid cell @j @j Main procedure of arrangement partitioning @j @

```

5 Test examples

5.1 Random triangles

Generation of random triangles and their boxes @O test/py/bool/test01.py @"""
Generation of random triangles and their boxes """ from larlib import * glass = MATERIAL([1,0,0,0.1, 0,1,0,0.1, 0,0,1,0.1, 0,0,0,0.1, 100])

```

randomTriaArray = randomTriangles(10,0.99) VIEW(STRUCT(AA(MKPOL)([[verts, [[1,2,3]], None] for verts in randomTriaArray])))

```

```

boxes = containmentBoxes(randomTriaArray) hexas = AA(box2exa)(boxes) cyan = COLOR(CYAN)(STRUCT(AA(MKPOL)([[verts, [[1,2,3]], None] for verts in randomTriaArray]))) yellow = STRUCT(AA(glass)(AA(MKPOL)([hex for hex,qualifier in hexas]))) VIEW(STRUCT([cyan,yellow])) @

```

Generation of random quadrilaterals and their boxes @O test/py/bool/test02.py @"""
Generation of random quadrilaterals and their boxes """ from larlib import * glass = MATERIAL([1,0,0,0.1, 0,1,0,0.1, 0,0,1,0.1, 0,0,0,0.1, 100])

```

randomQuadArray = randomQuads(10,1) VIEW(STRUCT(AA(MKPOL)([[verts, [[1,2,3,4]],
None] for verts in randomQuadArray])))

boxes = containmentBoxes(randomQuadArray) hexas = AA(box2exa)(boxes) cyan =
COLOR(CYAN)(STRUCT(AA(MKPOL)([[verts, [[1,2,3,4]], None] for verts in randomQuadAr-
ray]))) yellow = STRUCT(AA(glass)(AA(MKPOL)([hex for hex,qualifier in hexas]))) VIEW(STRUCT([cyan,y
@)

```

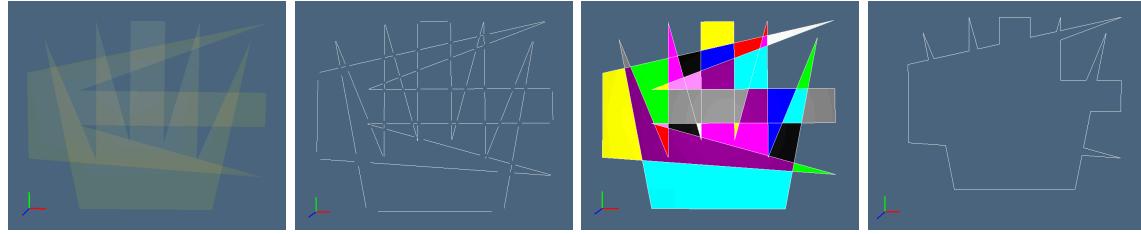


Figure 3: LAR complex from two polygons. (a) the input polygons; (b) the intersection of boundary lines; (c) the extracted *regularized* 2-complex; (d) the boundary LAR.

```

@O test/py/bool/test03.py @"""
Boolean complex generated by boundaries of two
complexes """
from larlib import *
glass = MATERIAL([1,0,0,0.1, 0,1,0,0.1, 0,0,1,0.1,
0,0,0,0.1, 100])

V1 = [[3,0],[11,0],[13,10],[10,11],[8,11],[6,11],[4,11],[1,10],[4,3],[6,4], [8,4],[10,3]] FV1 =
[[0,1,8,9,10,11],[1,2,11],[3,10,11],[4,5,9,10],[6,8,9],[0,7,8]] EV1=[[0,1],[0,7],[0,8],[1,2],[1,11],[2,11],[3,10],[3,11],[4,5
9],[6,8],[6,9],[7,8],[8,9],[9,10],[10,11]] BE1 = boundaryCells(FV1,EV1) lines1 = [[V1[v] for v
in EV1[edge]] for edge in BE1]

V2 = [[0,3],[14,2],[14,5],[14,7],[14,11],[0,8],[3,7],[3,5]] FV2 = [[0,5,6,7],[0,1,7],[4,5,6],[2,3,6,7]]
EV2 = [[0,1],[0,5],[0,7],[1,7],[2,3],[2,7],[3,6],[4,5],[4,6],[5,6],[6,7]] BE2 = boundaryCells(FV2,EV2)
lines2 = [[V2[v] for v in EV2[edge]] for edge in BE2]

VIEW(STRUCT([ glass(STRUCT(MKPOLS((V1,FV1))), glass(STRUCT(MKPOLS((V2,FV2)))) )
])) lines = lines1 + lines2 VIEW(STRUCT(AA(POLYLINE)(lines)))

global precision PRECISION -= 2 V,FV,EV,polygons = larFromLines(lines) VIEW(EXPLODE(1.2,1.2,1)(

VV = AA(LIST)(range(len(V))) submodel = STRUCT(MKPOLS((V,EV))) VIEW(larModelNumbering(1,
1]),submodel,1))

polylines = [[V[v] for v in face+[face[0]]] for face in FV[-1]] colors = [CYAN, MAGENTA, WHITE, RED, YELLOW, GREEN, GRAY, ORANGE, BLACK, BLUE, PURPLE, BROWN] sets = [COLOR(colors[k]VIEW(STRUCT([ T(3)(0.02)(STRUCT(AA(POLYLINE)(lines))), STRUCT(sets)])) )

VIEW(EXPLODE(1.2,1.2,1)((AA(POLYLINE)(polylines)))) polylines = [ [V[v] for v
in FV[-1]+[FV[-1][0]]] ] VIEW(EXPLODE(1.2,1.2,1)((AA(POLYLINE)(polylines)))) @

```

5.2 Testing the box-kd-trees

```
Visualizing with different colors the buckets of box-kd-tree @O test/py/bool/test04.py
@ """ Visualizing with different colors the buckets of box-kd-tree """
from larlib import *
randomQuadArray = randomQuads(30,0.8) VIEW(STRUCT(AA(MKPOL)([[verts, [[1,2,3,4]], None] for verts in randomQuadArray])))
V,[VV,EV,FV,CV] = larCuboids([2,2,1],True) cubeGrid = Struct([(V,FV,EV)],"cubeGrid")
cubeGrids = Struct(2*[cubeGrid,s(.5,.5,.5)]) V,FV,EV = struct2lar(cubeGrids) boxes =
containmentBoxes([[V[v] for v in f] for f in FV]) VV = AA(LIST)(range(len(V))) submodel =
STRUCT(MKPOLS((V,EV))) VIEW(larModelNumbering(1,1,1)(V,[VV,EV,FV],submodel,0.6))
parts = boxBuckets3d(boxes) V,FV,EV = spacePartition(V,FV,EV, parts) VV = AA(LIST)(range(len(V)))
submodel = STRUCT(MKPOLS((V,EV))) VIEW(larModelNumbering(1,1,1)(V,[VV,EV,FV],submodel,0.6))
boxes = containmentBoxes(randomQuadArray) hexas = AA(box2exa)(boxes) glass =
MATERIAL([1,0,0,0.1, 0,1,0,0.1, 0,0,1,0.1, 0,0,0,0.1, 100]) yellow = STRUCT(AA(glass)(AA(MKPOL)([hex
for hex,data in hexas]))) VIEW(STRUCT([cyan, yellow]))
parts = boxBuckets3d(boxes) for k,part in enumerate(parts): bunch = [glass(STRUCT(
[MKPOL(hexas[h][0]) for h in part]))] bunch += [COLOR(RED)(MKPOL(hexas[k][0]))] if
k==30: VIEW(STRUCT(bunch)) @
```

5.3 Intersection of geometry subsets

```
Two unit cubes @D Two unit cubes @""" Two unit cubes """
from larlib import *
V,[VV,EV,FV,CV] = larCuboids([2,2,2],True) cube1 = Struct([(V,FV,EV)],"cube1")
twoCubes = Struct([cube1,t(.5,.5,.5),cube1])
glass = MATERIAL([1,0,0,0.1, 0,1,0,0.1, 0,0,1,0.1, 0,0,0,0.1, 100])
twoCubes = Struct([cube1,t(-1,.5,1),cube1]) other test example twoCubes = Struct([cube1,t(.5,.5,0),cube1])
other test example twoCubes = Struct([cube1,t(.5,0,0),cube1]) other test example
V,FV,EV = struct2lar(twoCubes) VIEW(EXplode(1.2,1.2,1.2)(MKPOLS((V,FV))))
quadArray = [[V[v] for v in face] for face in FV] boxes = containmentBoxes(quadArray)
hexas = AA(box2exa)(boxes) parts = boxBuckets3d(boxes) @
def POLYGONS((V,FV)):
```

```
Face (and incident faces) transformation @O test/py/bool/test05.py @"""
non-
valid -i valid solid representation of a space partition """
from larlib import *
@i Two unit cubes @i
W,FW,EW = spacePartition(V,FV,EV, parts) polylines = lar2polylines((W,FW)) VIEW(EXplode(1.2,1,
WW = AA(LIST)(range(len(W))) submodel = STRUCT(MKPOLS((W,EW))) VIEW(larModelNumbering
@
```

```
3-cell reconstruction from LAR space partition @O test/py/bool/test06.py @"""
3-cell reconstruction from LAR space partition """
from larlib import * @i Two unit cubes
```

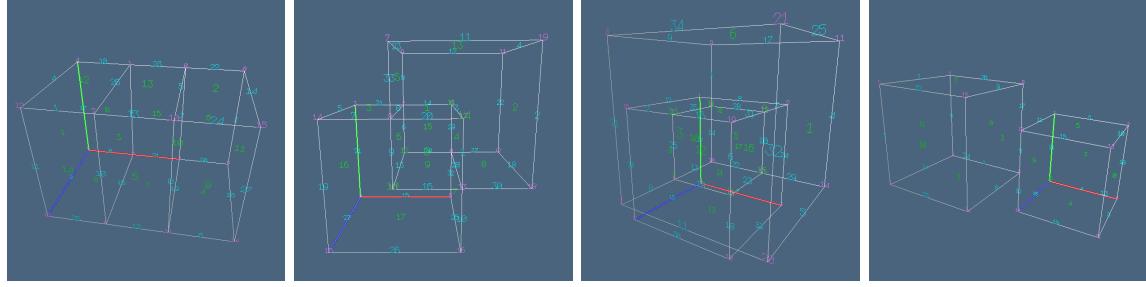


Figure 4: LAR complex of the space decomposition generated by two cubes in special positions. (a) translation on one coordinate; (b) translation on two coordinates; (c) translation on three coordinates; (d) non-manifold position along an edge.

```
@_ W,FW,EW = spacePartition(V,FV,EV, parts) WW = AA(LIST)(range(len(W))) submodel = STRUCT(MKPOLS((W,EW))) VIEW(larModelNumbering(1,1,1)(W,[WW,EW,FW],submodel,0.6)) @
```

2D polygon triangulation Here a 2D polygon is imported from an SVG file made of boundary lines, and the V,FV,EV LAR model is generated.

```
@O test/py/bool/test07.py """ 2D polygon triangulation """ from larlib import *  
from support import PolygonTessellator,vertex  
filename = "test/svg/bool/interior.svg" lines = svg2lines(filename) V,FV,EV,polygons  
= larFromLines(lines) VIEW(EXPLODE(1.2,1.2,1)(MKPOLIS((V,FV[:-1]+EV)) + AA(MK)(V)))  
VIEW(EXPLODE(1.2,1.2,1)(MKTRIANGLES((V,FV,EV)) )) @
```

From triples of points to LAR model of boundary triangulation @O test/py/bool/test08.py
@ import sys from larlib import *
sys.path.insert(0, 'test/py/bool/') from test06 import *
""" From triples of points to LAR model """ FE = crossRelation(W,FW,EW) triangleSet = boundaryTriangulation(W,FW,EW,FE) TW,FT = triangleIndices(triangleSet,W)
VIEW(EXPLODE(1.2,1.2,1.2)(MKPOLIS((W,CAT(TW))))) @

Visualization of of incidence between edges and 3D triangles @O test/py/bool/test09.py
@ """ Visualization of of incidence between edges and 3D triangles """ import sys from larlib import *
sys.path.insert(0, 'test/py/bool/') from test08 import *
model = W,FW,EW FE = crossRelation(W,FW,EW) EF = invertRelation(FE)
triangleSet = boundaryTriangulation(W,FW,EW,FE) TW, = triangleIndices(triangleSet,W)
VIEW(EXPLODE(1.2,1.2,1.2)(MKPOLIS((W,CAT(TW)))))

```

ET = edgesTriangles(EF,FW,TW,EW) VIEW(EXPLODE(1.2,1.2,1.2)(MKPOL((W,CAT(ET))))))  

VIEW(STRUCT(MKPOL((W,ET[35]))))

```

```

from larlib.iot3d import polyline2lar V,FV,EV = polyline2lar([[W[v] for v in FW[f]] for  

f in EF[35]]) ) VIEW(STRUCT(MKPOL((V,EV)))) @

```

Visualization of indices of the boundary triangulation @O test/py/bool/test10.py
@ """ Visualization of indices of the boundary triangulation """ from larlib import *
 sys.path.insert(0, 'test/py/bool/') from test09 import *
model = W,FW,EW FE = crossRelation(W,FW,EW) EF_angle = faceSlopeOrdering(model,FE)
WW = AA(LIST)(range(len(W))) submodel = SKEL1(STRUCT(MKPOL((W,CAT(TW))))))VIEW(l

Visualization after sorted edge-faces incidence computation @O test/py/bool/test11a.py
@ from larlib import * @; testing example @((t(.5,.5,.5),r(0,0,PI/6))@) @; @
 @O test/py/bool/test11b.py @ from larlib import * @; testing example @((t(.5,.5,0),r(0,0,PI/6))@)
@; @
 @O test/py/bool/test11c.py @ from larlib import * @; testing example @((t(.5,0,0),r(0,0,PI/6))@)
@; @
 @O test/py/bool/test11d.py @ from larlib import * @; testing example @((t(0,0,0),r(0,0,PI/6))@)
@; @
 @O test/py/bool/test11e.py @ from larlib import * @; testing example @((s(.5,.5,.5),r(0,0,PI/6))@)
@; @
 @O test/py/bool/test11f.py @ from larlib import * @; testing example @((t(.25,.25,.25),s(.5,.5,.5),r(0,0,PI/6))@)
@; @
 @O test/py/bool/test11g.py @ from larlib import * @; testing example @((t(.25,.25,.75),s(.5,.5,.5),r(0,0,PI/6))@)
@; @
 @O test/py/bool/test11h.py @ from larlib import * @; testing example @((t(1.5,1.5,0),r(0,0,PI/6))@)
@; @
 @D testing example @ """ Visualization of indices of the boundary triangulation """
 V,[VV,EV,FV,CV] = larCuboids([2,2,1],True) cubeGrid = Struct([(V,FV,EV)],"cubeGrid")
cubeGrids = Struct(3*[cubeGrid,@1])
 V,FV,EV = struct2lar(cubeGrids) VIEW(EXPLODE(1.2,1.2,1.2)(MKPOL((V,FV))))
V,CV,FV,EV,CF,CE,COE,FE = thePartition(V,FV,EV)
 cellLengths = AA(len)(CF) boundaryPosition = cellLengths.index(max(cellLengths))
BF = CF[boundaryPosition]; del CF[boundaryPosition]; del CE[boundaryPosition] BE =
list(e for f in BF for e in FE[f])
 Volume((V,[FV[f] for f in CF[0]]))
 VIEW(EXPLODE(1.2,1.2,1.2)(MKTRIANGLES((V,[FV[f] for f in BF],[EV[e] for e in
BE])))) VIEW(EXPLODE(1.2,1.2,1.2)([MKSOLID(V,[FV[f] for f in cell],[EV[e] for e in
set(CAT([FE[f] for f in cell]))]) for cell in CF])) VIEW(EXPLODE(1.2,1.2,1.2)([STRUCT(MKPOL((V,[EV[e]
for e in cell)))) for cell in CE])) @

```

@D testing the intersection of B-reps @"""
testing the intersection of B-reps """
BF = [FV[f] for f in boundaryCells(CV,FV)] VIEW(EXPLODE(1.2,1.2,1.2)(MKPOL((V,BF))))
,BE = larFacets((V,BF),dim = 2)cubeGrid = Struct([(V,BF,BE)],"cubeGrid")@
@O test/py/bool/test12.py @"""
Testing of point-in-polygon classification algorithm
"""
from larlib import *
filename = "test/svg/inters/tile.svg" lines = svg2lines(filename) V,EV = lines2lar(lines)
@i Point in polygon testing @i
result = [] classify = pointInPolygonClassification((V,EV)) for k in range(10000): query-
Point = [random.random(),random.random()] inOut = classify(queryPoint) print k,queryPoint,inOut
if inOut == "pin" : result += [MK(queryPoint)] elif inOut == "pout" : result += [
COLOR(RED)(MK(queryPoint))]
VIEW(STRUCT(MKPOL((V,EV))+result)) @

```

Testing of point-in-polygon classification on a random polygon

```

@O test/py/bool/test14.py
@"""
Testing of point-in-polygon classification on a random polygon """
from larlib import *
filename = "test/svg/inters/tile.svg" lines = svg2lines(filename) V,EV = lines2lar(lines)
@i Point in polygon testing @i
result = [] classify = pointInPolygonClassification((V,EV)) for k in range(10000): query-
Point = [random.random(),random.random()] inOut = classify(queryPoint) print k,queryPoint,inOut
if inOut == "pin" : result += [MK(queryPoint)] elif inOut == "pout" : result += [
COLOR(RED)(MK(queryPoint))]
VIEW(STRUCT(MKPOL((V,EV))+result)) @

```

5.4 Polygon triangulation

```

@O test/py/bool/test13.py @"""
Polygon triangulation and importing to LAR """
from larlib import * from p2t import *
@i load points @i @i input polyline visualization @i @i CDT triangulation with poly2tri
@i @i conversion of triangulation to LAR @i @i visualization of LAR model of triangulation
@i @i reconstruction of boundary polyline for LAR model @i @i visualization of LAR
generated boundary polyline @i @

```

load points @D load points @"""
load points """
def load_points(file_name) : infile =
open(file_name,"r") points = [] while infile : line = infile.readline() s = line.split() if len(s) ==
0 : breakpoints.append([float(s[0]), float(s[1])]) return points @

input polyline visualization @D input polyline visualization @"""
input polyline visualization """
points = load_points("test/data/nazca_mountainkey.dat") VIEW(POLYLINE(points)) @

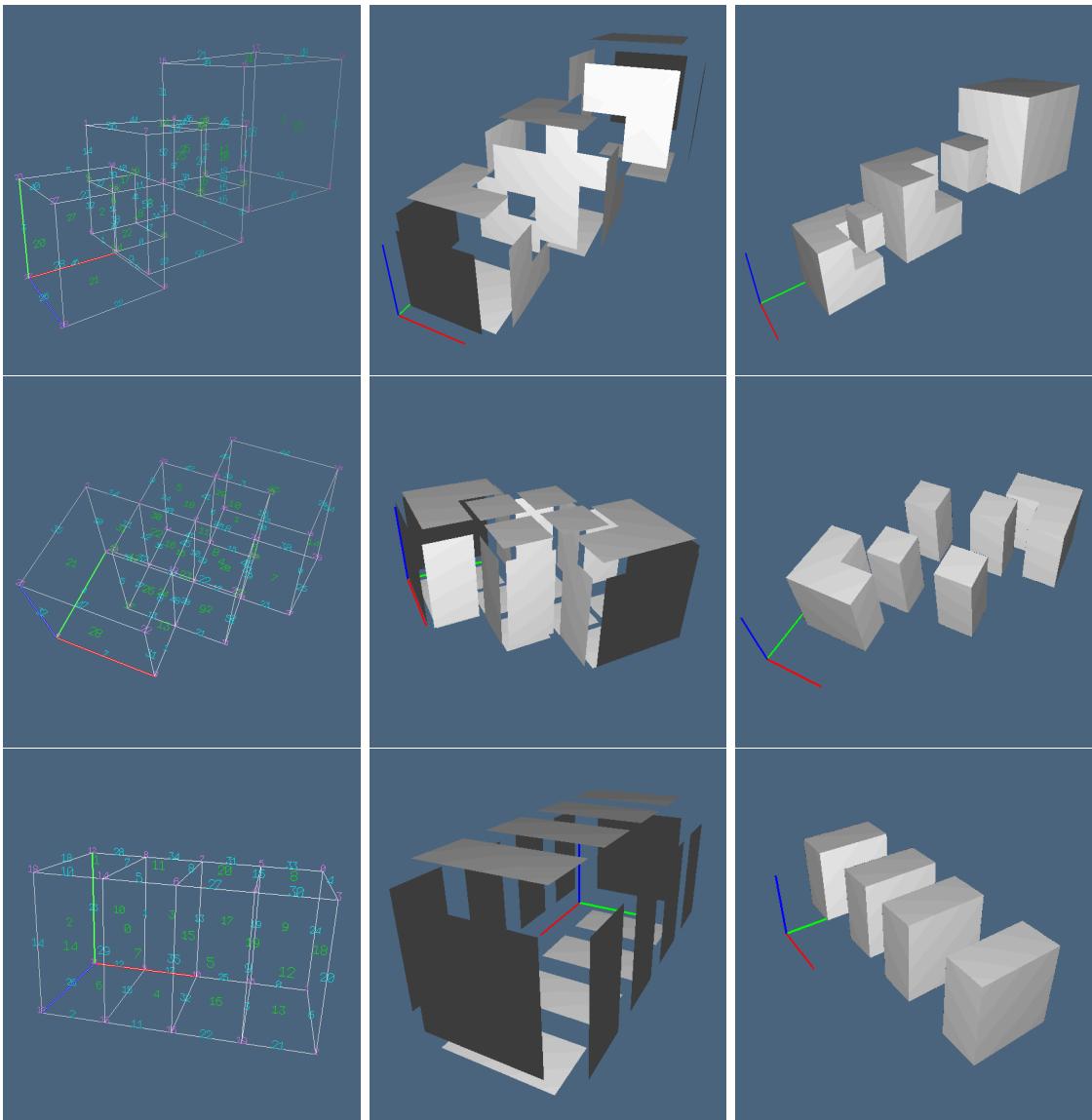


Figure 5: Examples of 3-cell extraction of two simple Boolean 2-complex, and their boundaries. Notice the numbers of (solid) 3-cells.

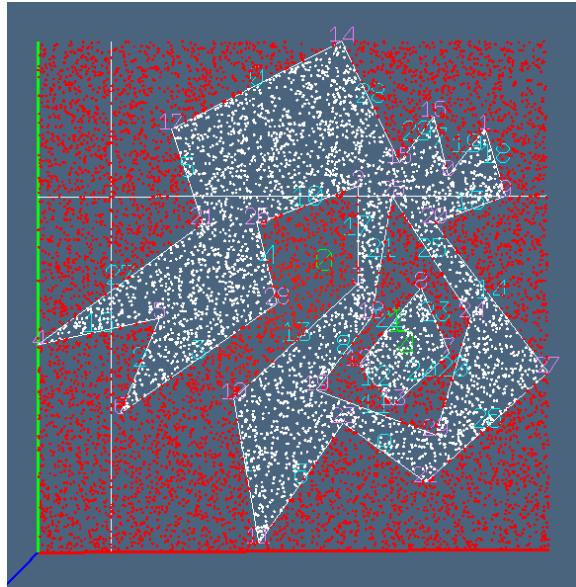


Figure 6: Testing the point-in-polygon algorithm.

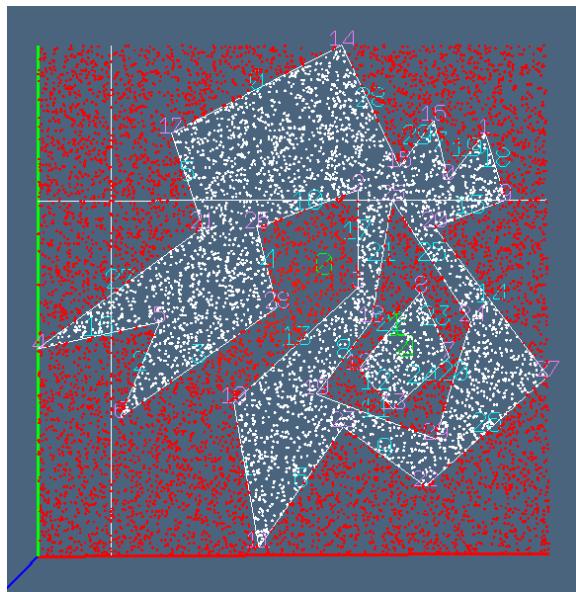


Figure 7: Testing the point-in-polygon algorithm.

CDT triangulation with poly2tri @D CDT triangulation with poly2tri @"" CDT triangulation with poly2tri """ polyline = [Point(p[0],p[1]) for p in points] cdt = CDT(polyline) triangles = cdt.triangulate() @

conversion of triangulation to LAR @D conversion of triangulation to LAR @"" conversion of triangulation to LAR """ trias = [[[t.a.x,t.a.y],[t.b.x,t.b.y],[t.c.x,t.c.y]] for t in triangles] vdict = defaultdict(list) for k,point in enumerate(CAT(trias)): vdict[vcode(point)] += [k] vdict = OrderedDict(zip(vdict.keys(),range(len(vdict.keys())))) FV = [(vdict[vcode(a)], vdict[vcode(b)], vdict[vcode(c)]) for [a,b,c] in trias] repeatedEdges = CAT([[v1,v2],[v2,v3],[v3,v1]] for [v1,v2,v3] in FV) EV = list(set(AA(tuple)(AA(sorted)(repeatedEdges)))) V = [eval(vect) for vect in vdict] @

visualization of LAR model of triangulation @D visualization of LAR model of triangulation @"" visualization of LAR model of triangulation """ VIEW(EXPLODE(1.2,1.2,1)(MKPOLS((V,F VIEW(EXPLODE(1.2,1.2,1)(MKPOLS((V,EV)))) @

reconstruction of boundary polyline for LAR model @D reconstruction of boundary polyline for LAR model @"" reconstruction of boundary polyline for LAR model """ EW = boundaryCells(FV,EV) VIEW(EXPLODE(1.2,1.2,1)(MKPOLS((V,[EV[e] for e in EW]))) model = (V,FV,[EV[e] for e in EW]) struct = Struct([model]) @

visualization of LAR generated boundary polyline @D visualization of LAR generated boundary polyline @"" visualization of LAR generated boundary polyline """ poly = boundaryModel2polylines(structBoundaryModel(struct)) VIEW(POLYLINE(poly[0][:-1])) @

A Code utilities

A.1 Generation of random data

Some utility fuctions used by the module are collected in this appendix. Their macro names can be seen in the below script.

@D Coding utilities @"" Coding utilities """ global count @j Generation of a random 3D point @j @j Generation of random 3D triangles @j @j Generation of random 3D quadrilaterals @j @j Generation of a single random triangle @j @j Containment boxes @j @j Transformation of a 3D box into an hexahedron @j @j Generation of a list of HPCs from a LAR model with non-convex faces @j @

Generation of a list of HPCs from a LAR model with non-convex faces @D Generation of a list of HPCs from a LAR model with non-convex faces @"" Generation of

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a list of HPCs from a LAR model with non-convex faces """
    def MKTRIANGLES(model):
        V,FV,EV = model
        FE = crossRelation(V,FV,EV)
        if len(V[0]) == 2:
            V=[v+[0] for v in V]
        triangleSets = boundaryTriangulation(V,FV,EV,FE)
        return [STRUCT([MKPOL([verts,[[3,2,1]],None])
                      for verts in triangleSets]) for triangledFace in triangleSets]

    def MKSOLID(*model):
        V,FV,EV = model
        FE = crossRelation(V,FV,EV)
        pivot = V[0]
        VF = invertRelation(FV)
        faces = [face for face in FV if face not in VF[0]]
        faces = [face for face in FV]
        triangleSets = boundaryTriangulation(V,faces,EV,FE)
        return XOR([
                    MKPOL([face+[pivot], [range(1,len(face)+2)],None])
                    for face in CAT(triangleSets)])

```

Generation of random triangles The function `randomTriangles` returns the array `randomTriaArray` with a given number of triangles generated within the unit 3D interval. The `scaling` parameter is used to scale every such triangle, generated by three random points, that could be possibly located to far from each other, even at the distance of the diagonal of the unit cube.

The arrays `xs`, `ys` and `zs`, that contain the x, y, z coordinates of triangle points, are used to compute the minimal translation `v` needed to transport the entire set of data within the positive octant of the 3D space.

```

@D Generation of random 3D triangles @"""
    def randomTriangles(numberOfTriangles=400,scaling=0.3):
        randomTriaArray = [rtriangle(scaling) for k in range(numberOfTriangles)]
        [xs,ys,zs] = TRANS(CAT(randomTriaArray))
        xmin, ymin, zmin = min(xs), min(ys), min(zs)
        v = array([-xmin,-ymin, -zmin])
        randomTriaArray = [[list(v1+v), list(v2+v), list(v3+v)] for v1,v2,v3 in randomTriaArray]
        return randomTriaArray

```

Generation of random 3D quadrilaterals @D Generation of random 3D quadrilaterals @"""
 Generation of random 3D quadrilaterals """
 def randomQuads(numberOfQuads=400,scaling=0.3):
 randomTriaArray = [rtriangle(scaling) for k in range(numberOfQuads)]
 [xs,ys,zs] = TRANS(CAT(randomTriaArray))
 xmin, ymin, zmin = min(xs), min(ys), min(zs)
 v = array([-xmin,-ymin, -zmin])
 randomQuadArray = [AA(list)([v1+v, v2+v, v3+v, v+v2-v1+v3]) for v1,v2,v3 in randomTriaArray]
 return randomQuadArray

Generation of a random 3D point A single random point, codified in floating point format, and with a fixed (quite small) number of digits, is returned by the `rpoint3d()` function, with no input parameters. @D Generation of a random 3D point @"""
 Generation of a random 3D point """
 def rpoint3d():
 return eval('vcode([' + random.random().__str__() + ','
 + random.random().__str__() + ','
 + random.random().__str__()])

Generation of a single random triangle A single random triangle, scaled about its centroid by the `scaling` parameter, is returned by the `rtriangle()` function, as a tuple of two random points in the unit square. @D Generation of a single random triangle

```

@""” Generation of a single random triangle ””” def rtriangle(scaling): v1,v2,v3 = array(rpoint3d()), array(rpoint3d()), array(rpoint3d()) c = (v1+v2+v3)/3 pos = rpoint3d() v1 = (v1-c)*scaling + pos v2 = (v2-c)*scaling + pos v3 = (v3-c)*scaling + pos return tuple(eval(vcode(v1))), tuple(eval(vcode(v2))), tuple(eval(vcode(v3))) @

```

Containment boxes Given as input a list `randomTriaArray` of pairs of 2D points, the function `containmentBoxes` returns, in the same order, the list of *containment boxes* of the input lines. A *containment box* of a geometric object of dimension d is defined as the minimal d -cuboid, equioriented with the reference frame, that contains the object. For a 2D line it is given by the tuple (x_1, y_1, x_2, y_2) , where (x_1, y_1) is the point of minimal coordinates, and (x_2, y_2) is the point of maximal coordinates.

```

@D Containment boxes @""” Containment boxes ””” def containmentBoxes(randomPointArray,qualifier=0)
if len(randomPointArray[0])==2: boxes = [eval(vcode([min(x1,x2), min(y1,y2), min(z1,z2),
max(x1,x2), max(y1,y2), max(z1,z2)]))]+[qualifier] for ((x1,y1,z1),(x2,y2,z2)) in random-
PointArray] elif len(randomPointArray[0])==3: boxes = [eval(vcode([min(x1,x2,x3), min(y1,y2,y3),
min(z1,z2,z3), max(x1,x2,x3), max(y1,y2,y3), max(z1,z2,z3)]))]+[qualifier] for ((x1,y1,z1),(x2,y2,z2),(x3,y3,z3)) in randomPointArray] elif len(randomPointArray[0])==4: boxes = [eval(vcode([min(x1,x2,x3,x4),
min(y1,y2,y3,y4), min(z1,z2,z3,z4), max(x1,x2,x3,x4), max(y1,y2,y3,y4), max(z1,z2,z3,z4)]))]+[qualifier]
for ((x1,y1,z1),(x2,y2,z2),(x3,y3,z3),(x4,y4,z4)) in randomPointArray] return boxes @

```

Transformation of a 3D box into an hexahedron The transformation of a 2D box into a closed rectangular polyline, given as an ordered sequwncw of 2D points, is produced by the function `box2exa` @D Transformation of a 3D box into an hexahedron @""” Transformation of a 3D box into an hexahedron ””” def box2exa(box): x1,y1,z1,x2,y2,z2,type = box verts = [[x1,y1,z1], [x1,y1,z2], [x1,y2,z1], [x1,y2,z2], [x2,y1,z1], [x2,y1,z2], [x2,y2,z1],
[x2,y2,z2]] cell = [range(1,len(verts)+1)] return [verts,cell,None],type

```

def lar2boxes(model,qualifier=0): V,CV = model boxes = [] for k,cell in enumerate(CV): verts = [V[v] for v in cell] x1,y1,z1 = [min(coord) for coord in TRANS(verts)] x2,y2,z2 = [max(coord) for coord in TRANS(verts)] boxes += [eval(vcode([min(x1,x2),min(y1,y2),min(z1,z2),n
return boxes @

```