

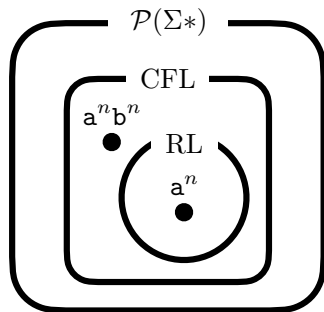
Lecture 15 – Examples of Pushdown Automata

COSE215: Theory of Computation

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Languages	Automata	Grammars
Context-Free Language (CFL)	Pushdown Automata (PDA)	Context-Free Grammar (CFG)
Regular Language (RL)	Finite Automata (FA)	Regular Expression (RE)

1. Examples of Pushdown Automata

Example 1: $a^n b^n$

Example 2: $a^n b^{2n}$

Example 3: ww^R

Example 4: Equal Number of a's and b's

Example 5: Unequal Number of a's and b's

Example 6: Not of the Form ww

Example 1: $a^n b^n$

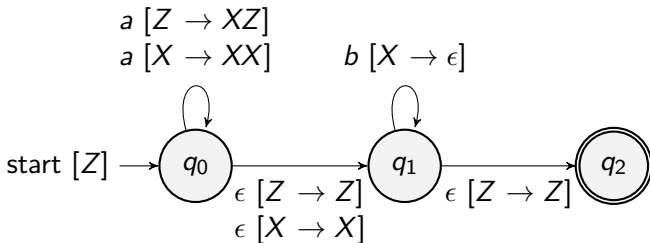
Construct a PDA that accepts the language by final states:

$$L = \{a^n b^n \mid n \geq 0\}$$

Example 1: $a^n b^n$

Construct a PDA that accepts the language by final states:

$$L = \{a^n b^n \mid n \geq 0\}$$



Example 2: $a^n b^{2n}$

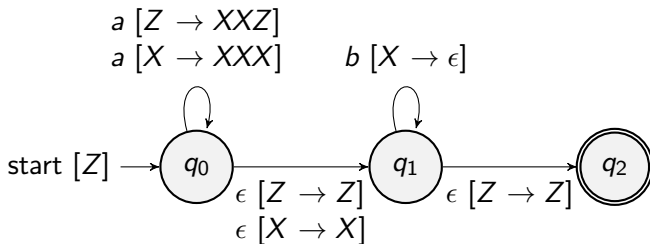
Construct a PDA that accepts the language by final states:

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Construct a PDA that accepts the language by final states:

$$L = \{a^n b^{2n} \mid n \geq 0\}$$



Example 3: ww^R

Construct a PDA that accepts the language by final states:

$$L = \{ww^R \mid w \in \{a,b\}^*\}$$

Example 3: ww^R

Construct a PDA that accepts the language by final states:

$$L = \{ww^R \mid w \in \{a,b\}^*\}$$

$a [Z \rightarrow XZ]$

$a [X \rightarrow XX]$

$a [Y \rightarrow XY]$

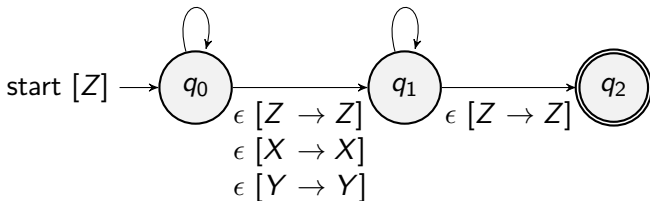
$b [Z \rightarrow YZ]$

$b [X \rightarrow YX]$

$b [Y \rightarrow YY]$

$a [X \rightarrow \epsilon]$

$b [Y \rightarrow \epsilon]$



Example 4: Equal Number of a's and b's

Construct a PDA that accepts the language by empty stacks:

$$L = \{w \in \{a, b\}^* \mid N_a(w) = N_b(w)\}$$

where $N_a(w)$ and $N_b(w)$ are the number of a's and b's in w , respectively.

Example 4: Equal Number of a's and b's

Construct a PDA that accepts the language by empty stacks:

$$L = \{w \in \{a, b\}^* \mid N_a(w) = N_b(w)\}$$

where $N_a(w)$ and $N_b(w)$ are the number of a's and b's in w , respectively.

$a [Z \rightarrow PZ]$

$a [P \rightarrow PP]$

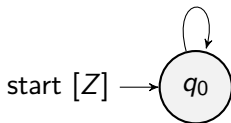
$a [N \rightarrow \epsilon]$

$b [Z \rightarrow NZ]$

$b [P \rightarrow \epsilon]$

$b [N \rightarrow NN]$

$\epsilon [Z \rightarrow \epsilon]$



Example 5: Unequal Number of a's and b's

Construct a PDA that accepts the language by empty stacks:

$$L = \{w \in \{a, b\}^* \mid N_a(w) \neq N_b(w)\}$$

where $N_a(w)$ and $N_b(w)$ are the number of a's and b's in w , respectively.

Example 5: Unequal Number of a's and b's

Construct a PDA that accepts the language by empty stacks:

$$L = \{w \in \{a, b\}^* \mid N_a(w) \neq N_b(w)\}$$

where $N_a(w)$ and $N_b(w)$ are the number of a's and b's in w , respectively.

$a [Z \rightarrow PZ]$

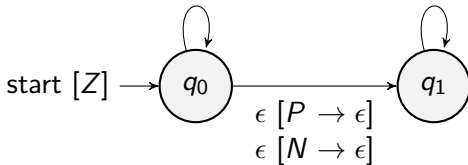
$a [P \rightarrow PP]$

$a [N \rightarrow \epsilon]$

$b [Z \rightarrow NZ] \quad \epsilon [Z \rightarrow \epsilon]$

$b [P \rightarrow \epsilon] \quad \epsilon [P \rightarrow \epsilon]$

$b [N \rightarrow NN] \quad \epsilon [N \rightarrow \epsilon]$



Example 6: Not of the Form ww

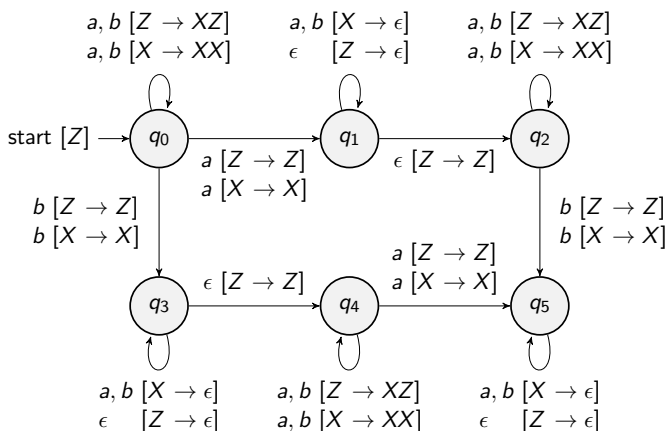
Construct a PDA that accepts the language by empty stacks:

$$L = \{x \in \{a, b\}^* \mid x \text{ is not of the form } ww\}$$

Example 6: Not of the Form ww

Construct a PDA that accepts the language by empty stacks:

$$L = \{x \in \{a, b\}^* \mid x \text{ is not of the form } ww\}$$



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- Equivalence of Pushdown Automata and Context-Free Grammars

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