The problem is on the radar in psychology; for example, Smith (2012) in a recent editorial in *Journal of Personality and Social Psychology* stated:

"if the independent variable (X) is manipulated and the mediator (M) and dependent variable (Y) are measured, the usual analysis will be biased if unobserved causes of M are correlated with unobserved causes of Y" (p. 2).

$$x \to m \to y$$

Why? There could be a third unmeasured variable, q, that predicts both m and y. Or m could be endogenous for <u>other reasons</u> (i.e., measurement errors)

With randomization there is no correlation between x and q (Antonakis, et al., 2010). Thus measuring or not q and including it in a regression model will make no difference to the effect of x on m or any other DV. That is:

$$m = \gamma_0 + \gamma_1 x + v$$

$$m = \delta_0 + \delta_1 x + \delta_2 q + w$$

Asymptotically: $\delta_1 = \gamma_1$

Although in Eq. 1, q is in v, v is orthogonal to x. Even if $\delta_2 \neq 0$, δ_1 is unaffected.

Thus, the OLS (ordinary least squares) or ML (maximum likelihood) estimate of ξ_1 in Eq. 6 will be biased because (from Eq. 5 and 10):

$$\xi_1 = \frac{cov(y,m)}{var(m)} = \eta_1 + \eta_2 \kappa_1 \neq \eta_1$$
 Eq. 11

Recall, the true model is:

$$m = \lambda_0 + \lambda_1 x + \lambda_2 q + \varepsilon$$
 Eq. 3

$$y = \eta_0 + \eta_1 m + \eta_2 q + \theta$$
 Eq. 4

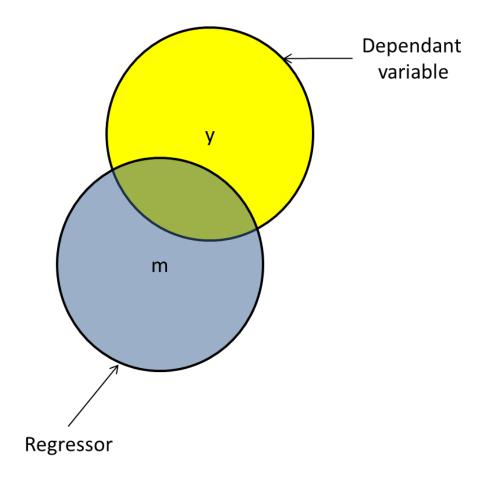
And we estimate:

$$m = \omega_0 + \omega_1 x + u$$
 Eq. 5

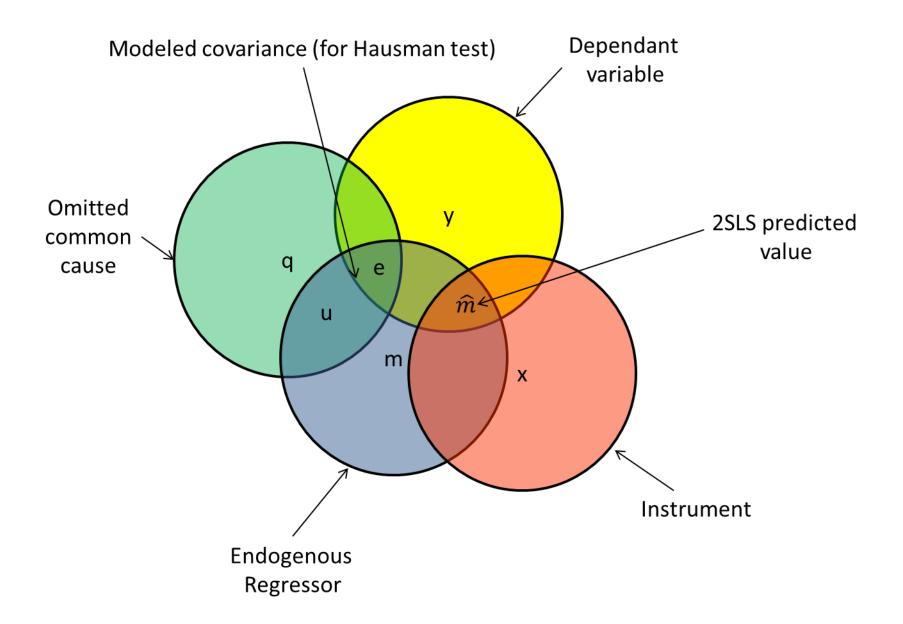
$$y = \xi_0 + \xi_1 m + o$$
 Eq. 6

 ξ_1 will only equal η_1 if either η_2 or κ_1 are zero (i.e., if q is unrelated to y or to m).

The solution to the endogeneity problem in mediation.



The overlap in ballentines is the variance that the two variables share.



Voilà: This is the general formula for an instrumental-variable estimate (e.g., see Bollen, 2012):

$$\hat{\eta}_1 = \frac{Cov(y, x)}{Cov(m, x)}$$

Thus, *x*, the <u>instrument</u>, *must* correlate with *y* and *m* if *m* is a true cause of *y* (and if *m* is endogenous); else, the estimate of Eq. 15 is either zero or undefined, which shows up another fallacy in mediation—and a widely believed one too—that *x* need not correlate with *y* to establish mediation (Shrout & Bolger, 2002; Zhao, Lynch, & Chen, 2010); this belief is only true if *m* is exogenous!