Comparison of algorithms

In [9]:
```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt
import math
import scipy.interpolate
import os
%matplotlib inline
plt.rcParams['figure.figsize'] = (15, 8)

### for Palatino and other serif fonts use:
#plt.rcParams.update({
#    "text.usetex": True,
#    "font.family": "serif",
#    "font.serif": ["Palatino"],
#})

# only with Python2.7 !!
import sys
sys.path.insert(0,'/usr/lib/python2.7/pyobs-master/')
from pyobs import *
```
executed in 303ms, finished 12:21:08 2020-11-16

In [2]:
```
def round_on_error(value, error):
    significant_digits = 1-int(math.floor(math.log(error, 10)))
    return round(value,significant_digits)

def get_2_significant(value):
    return round(value,1-int(math.floor(math.log(value, 10))))

def get_2_significant_0(value):
    return int(round(10.0/(10.0**int(math.floor(math.log(value, 10))))*value,0))

def get_position_sign(error):
    return 1-int(math.floor(math.log(error, 10)))

def printwe(value,error):
    e = get_2_significant(error)
    a = get_position_sign(e)
    e = get_2_significant_0(error)
    stri = '{:.' + str(a) + 'f}({})'
    print(stri.format(value,e))
    #parece qeu solo funciona si el error es menor que 1. Mirar la Q de doubaltw. Corregir

#def printwe2(value,error):
#    e = get_2_significant(error)
#    v = round_on_error(value,e)
#    e = get_2_significant_0(error)
#    print(v,'(',e,')', sep='')
```
executed in 23ms, finished 11:46:34 2020-11-16

In [351]:
```
-int(math.floor(math.log(0.5, 10)))
```
executed in 9ms, finished 18:25:00 2020-11-13

Out[351]:
```
1
```

1 Plaquette

In [4]:
```
P_hmc = np.loadtxt("plaq-hmc.data", skiprows=500, usecols=2);
P_altwinding = np.loadtxt("plaq-altwindinghmc.data", skiprows=500, usecols=2);
P_altinstanton = np.loadtxt("plaq-altinstantonhmc.data", skiprows=500, usecols=2);
P_doublewinding  = np.loadtxt("plaq-doublealtwinding.data", skiprows=500, usecols=2);
#P_altw2  = np.loadtxt("plaq-altw3.data", skiprows=500, usecols=2);
```
executed in 1.79s, finished 11:47:56 2020-11-16

In [8]:
```
print("Configuraciones HMC: {}".format(len(P_hmc)))
print("Configuraciones altwinding: {}".format(len(P_altwinding)))
print("Configuraciones altinstanton: {}".format(len(P_altinstanton)))
print("Configuraciones doublewinding: {}".format(len(P_doublewinding)))
```
executed in 10ms, finished 13:25:05 2020-11-13

```
Configuraciones HMC: 111738
Configuraciones altwinding: 32677
Configuraciones altinstanton: 15458
Configuraciones doublewinding: 28476
```

Algorithm	Statistics	P	τ_{int}	$\tau_{\mathrm{int}} / \tau_{\mathrm{int}}^{(\mathrm{HMC})}$
HMC	111738	0.6700214(49)	3.33(10)	0.934(59)
Alt Winding	32677	0.6700175(85)	2.77(14)	0.831(48)
Alt Instanton	15458	0.670029(12)	2.75(19)	0.826(62)
Alt Double Winding	28476	0.6700272(94)	3.11(17)	0.934(59)

In [366]:
```
a = 2.5
```
executed in 6ms, finished 23:13:38 2020-11-13

1.1 HMC

$P =$ 0.6700214(49)

$\tau_{int,P}$ = 3.33(10)

In [5]:
```
P_hmc = P_hmc[:]
MCtime_for_P_hmc = np.arange(1, len(P_hmc)+1, 1)
corr_phmc = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr_phmc.primary_observable(0,'Plaquette $P(t=0)$', [0], ['R0'], [MCtime_for_P_hmc.tolist()], [(P_hm

[Phmc, ePhmc]= corr_phmc.vwerr(errinfo=einfo)
[tauPhmc, etauPhmc] = corr_phmc.tauint()
tauPhmc = tauPhmc[0][0][0]
etauPhmc = etauPhmc[0][0][0]

print(corr_phmc.vwerr(plot=False,errinfo=einfo))

printwe(Phmc,ePhmc)
```

executed in 7.88s, finished 11:48:07 2020-11-16

```
[0.6700214468636829, 4.946715688873867e-06]
0.6700214(49)
```

1.2 Alt winding

$P =$ {{printwe(Paltwinding, ePaltwinding)}}

$\tau_{int,P}$ = {{printwe(tauPaltwinding, etauPaltwinding)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Paltwinding, etauR_Paltwinding)}}

In [6]:
```
P_altwinding = P_altwinding[:]
MCtime_for_P_altwinding = np.arange(1, len(P_altwinding)+1, 1)
corr_paltwinding = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr_paltwinding.primary_observable(0,'Plaquette $P(t=0)$', [0], ['R0'], [MCtime_for_P_altwinding.tol
[(P_altwinding).tolist()], (1,1))

[Paltwinding, ePaltwinding]= corr_paltwinding.vwerr(errinfo=einfo)
[tauPaltwinding, etauPaltwinding] = corr_paltwinding.tauint()
tauPaltwinding = tauPaltwinding[0][0][0]
etauPaltwinding = etauPaltwinding[0][0][0]

print(corr_paltwinding.vwerr(plot=False,errinfo=einfo))

#printwe(Paltwinding,ePaltwinding)

tauR_Paltwinding = tauPaltwinding/tauPhmc
etauR_Paltwinding = tauR_Paltwinding *  np.sqrt( (etauPhmc/tauPhmc)**2.0 + (etauPaltwinding/tauPaltwi
```

executed in 1.60s, finished 11:48:08 2020-11-16

```
[0.670017460679774, 8.502901030588478e-06]
```

1.3 Alt instanton

$P =$ {{printwe(Paltinstanton, ePaltinstanton)}}

$\tau_{int,P}$ = {{printwe(tauPaltinstanton, etauPaltinstanton)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Paltinstanton, etauR_Paltinstanton)}}

In [7]:
```
P_altinstanton = P_altinstanton[:]
MCtime_for_P_altinstanton = np.arange(1, len(P_altinstanton)+1, 1)
corr_paltinstanton = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr_paltinstanton.primary_observable(0,'Plaquette $P(t=0)$', [0], ['R0'], [MCtime_for_P_altinstanton
[(P_altinstanton).tolist()], (1,1))

[Paltinstanton, ePaltinstanton]= corr_paltinstanton.vwerr(errinfo=einfo)
[tauPaltinstanton, etauPaltinstanton] = corr_paltinstanton.tauint()
tauPaltinstanton = tauPaltinstanton[0][0][0]
etauPaltinstanton = etauPaltinstanton[0][0][0]

print(corr_paltinstanton.vwerr(plot=False,errinfo=einfo))

#printwe(Paltinstanton,ePaltinstanton)

tauR_Paltinstanton = tauPaltinstanton/tauPhmc
etauR_Paltinstanton = tauR_Paltinstanton *  np.sqrt( (etauPhmc/tauPhmc)**2.0 + (etauPaltinstanton/tau
```

executed in 544ms, finished 11:48:09 2020-11-16

```
[0.6700294241738907, 1.2334126716496382e-05]
```

1.4 Double winding

$P =$ {{printwe(Pdoublewinding, ePdoublewinding)}}

$\tau_{int,P}$ = {{printwe(tauPdoublewinding, etauPdoublewinding)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Pdoublewinding, etauR_Pdoublewinding)}}

In [8]:
```
P_doublewinding = P_doublewinding[:]
MCtime_for_P_doublewinding = np.arange(1, len(P_doublewinding)+1, 1)
corr_pdoublewinding = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr_pdoublewinding.primary_observable(0,'Plaquette $P(t=0)$', [0], ['R0'], [MCtime_for_P_doublewindi
[(P_doublewinding).tolist()], (1,1))

[Pdoublewinding, ePdoublewinding]= corr_pdoublewinding.vwerr(errinfo=einfo)
[tauPdoublewinding, etauPdoublewinding] = corr_pdoublewinding.tauint()
tauPdoublewinding = tauPdoublewinding[0][0][0]
etauPdoublewinding = etauPdoublewinding[0][0][0]

print(corr_pdoublewinding.vwerr(plot=False,errinfo=einfo))

#printwe(Pdoublewinding,ePdoublewinding)

tauR_Pdoublewinding = tauPdoublewinding/tauPhmc
etauR_Pdoublewinding = tauR_Pdoublewinding *  np.sqrt( (etauPhmc/tauPhmc)**2.0 +
(etauPdoublewinding/tauPdoublewinding)**2.0 )
```
executed in 1.21s, finished 11:48:10 2020-11-16

```
[0.6700272023440166, 9.399314260470143e-06]
```

2 t_0

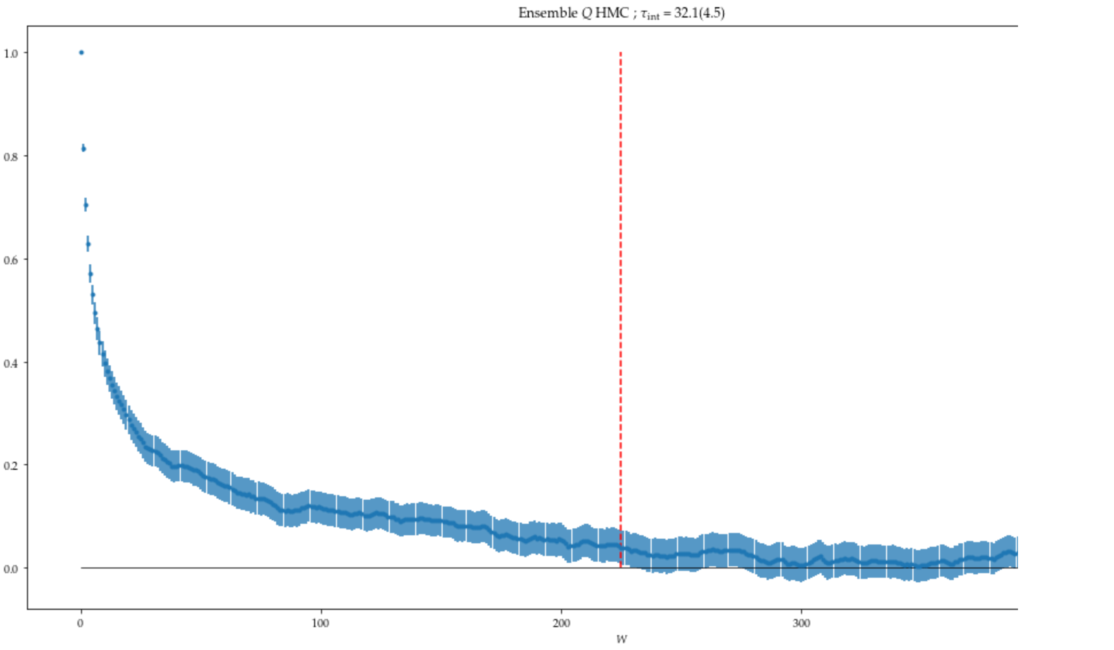
In [43]:
```
t0hmc = np.loadtxt("t0hmc.txt")
MCtimeHMC = np.arange(1, len(t0hmc)+1, 1)

t0altw = np.loadtxt("t0altw.txt")
MCtimealtw = np.arange(1, len(t0altw)+1, 1)

t0alti = np.loadtxt("t0alti.txt")
MCtimealti = np.arange(1, len(t0alti)+1, 1)

t0doubaltw = np.loadtxt("t0doubaltw.txt")
MCtimedoubaltw = np.arange(1, len(t0doubaltw)+1, 1)
```
executed in 601ms, finished 13:35:08 2020-11-10

2.1 HMC

In [42]:
```
len(t0altw)
```
executed in 16ms, finished 13:26:05 2020-11-10

Out[42]:
```
12000
```

In [35]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0, W=225)
corr.primary_observable(0,'$Q$ HMC', [0], ['R0'], [MCtimeHMC.tolist()], [(t0hmc).tolist()], (1,1))
[qhmc, eqhmc] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```
executed in 5.81s, finished 13:15:50 2020-11-10



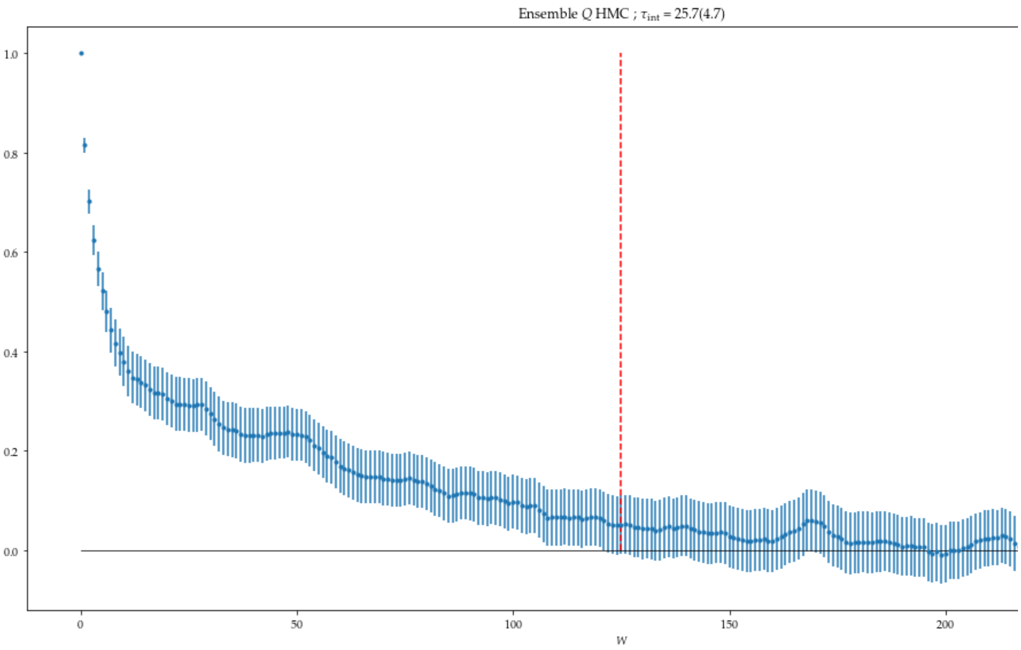
```
[0.7320578378315383, 0.0021742537553067227]
```

2.2 Alt Winding

In [40]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0, W=125)
corr.primary_observable(0,'$Q$ HMC', [0], ['R0'], [MCtimealtw.tolist()], [(t0altw).tolist()], (1,1))
[qaltw, eqaltw] = corr.vwerr(errinfo=einfo, )
```

```
print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 2.47s, finished 13:24:55 2020-11-10

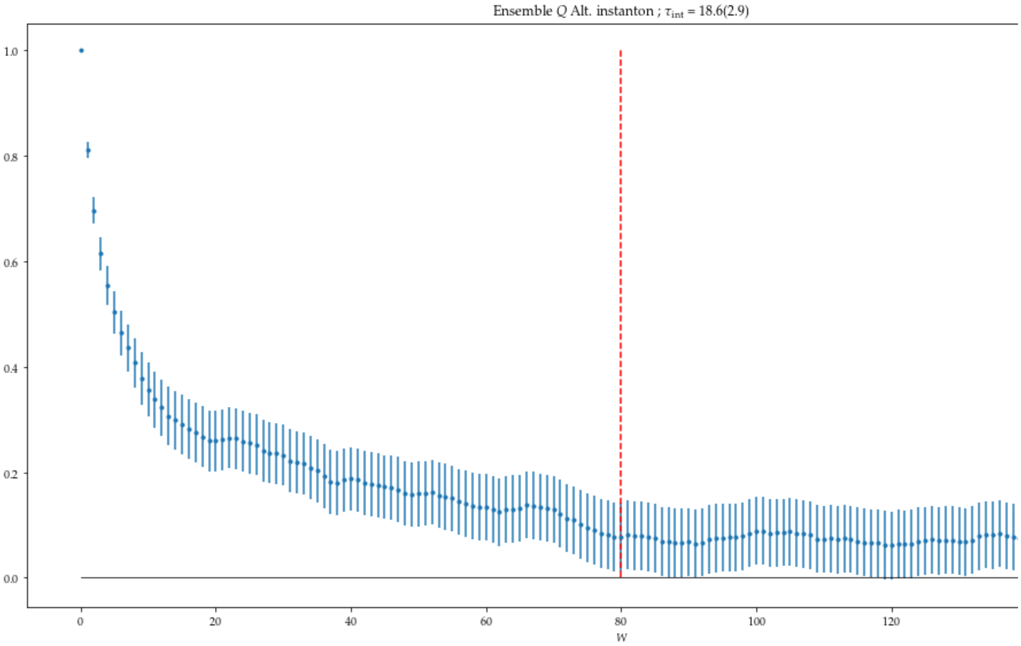


```
[0.7416314018029203, 0.003767475289450 22]
```

2.3 Alt instanton

In [53]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0, W=80)
corr.primary_observable(0,'$Q$ Alt. instanton', [0], ['R0'], [MCtimealti.tolist()], [(t0alti).tolist(
[qalti, eqalti] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 2.96s, finished 14:57:18 2020-11-10



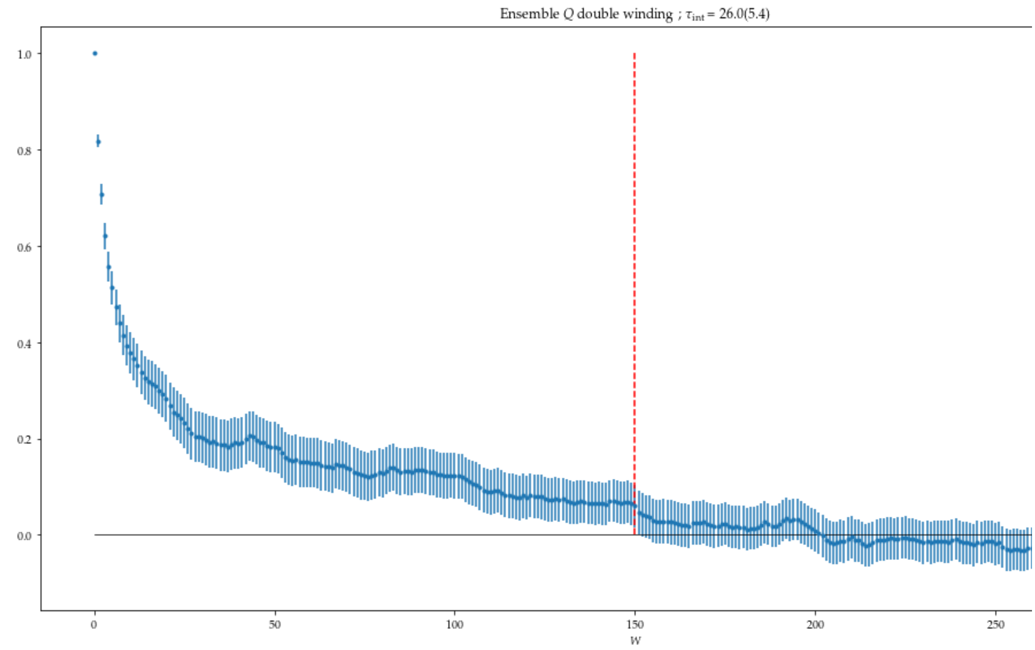
```
[0.7384171182115985, 0.00343629500885052]
```

2.4 Double winding

In [50]:

```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0, W=150)
corr.primary_observable(0,'$Q$ double winding', [0], ['R0'], [MCtimedoubaltw.tolist()], [(t0doubaltw)
[qdoubaltw, eqdoubaltw] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 2.80s, finished 14:56:03 2020-11-10



```
[0.7355934847022595, 0.003750142107685617]
```

3 t^2E at $t = 3$

In [79]:

```
t2Eat3hmc = np.loadtxt("t2Eat3hmc.txt")
t2Eat3hmc = t2Eat3hmc[:49000]
MCtimeHMC = np.arange(1, len(t2Eat6hmc)+1, 1)

t2Eat3altw = np.loadtxt("t2Eat3altw.txt")
t2Eat3altw = t2Eat3altw[:29000]
MCtimealtw = np.arange(1, len(t2Eat6altw)+1, 1)

#t2Eat6alti = np.loadtxt("t2Eat6alti.txt")
#MCtimealti = np.arange(1, len(t2Eat6alti)+1, 1)
#
#t2Eat6doubaltw = np.loadtxt("t2Eat6doubaltw.txt")
#MCtimedoubaltw = np.arange(1, len(t2Eat6doubaltw)+1, 1)

Nhmc = len(t2Eat6hmc)
Naltw = len(t2Eat6altw)
Ndoubaltw = len(t2Eat6doubaltw)
Nalti = len(t2Eat6alti)
```

executed in 604ms, finished 17:55:29 2020-11-11

In [76]:

```
Nhmc
```

executed in 8ms, finished 17:53:29 2020-11-11

Out[76]:

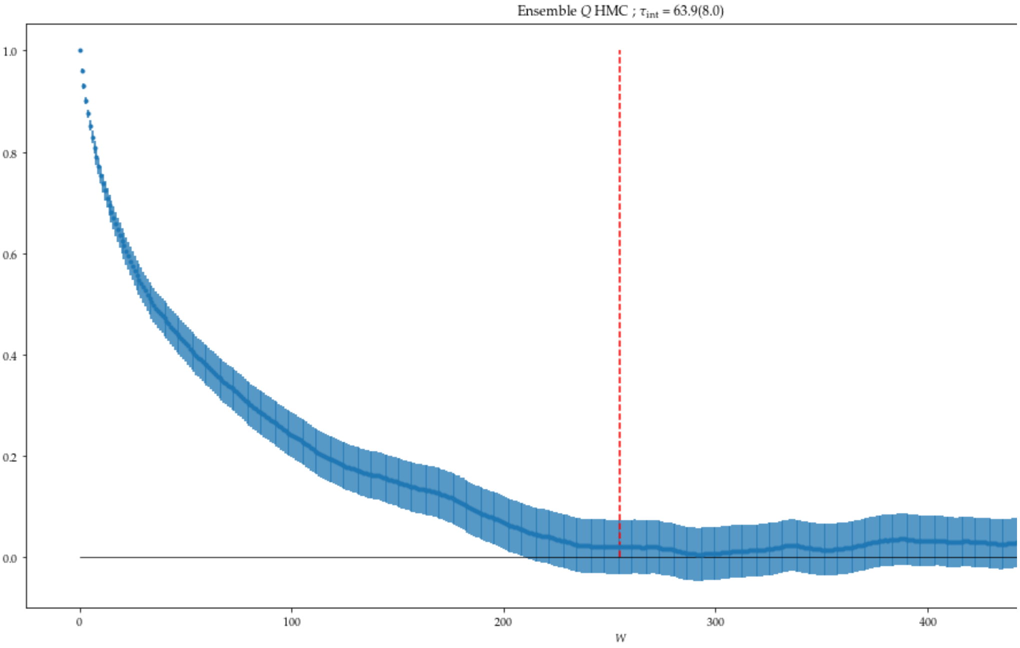
```
49000
```

3.1 HMC

In [80]:

```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr.primary_observable(0,'$Q$ HMC', [0], ['R0'], [MCtimeHMC.tolist()], [(t2Eat3hmc).tolist()], (1,1)
[qhmc, eqhmc] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 6.03s, finished 17:55:38 2020-11-11



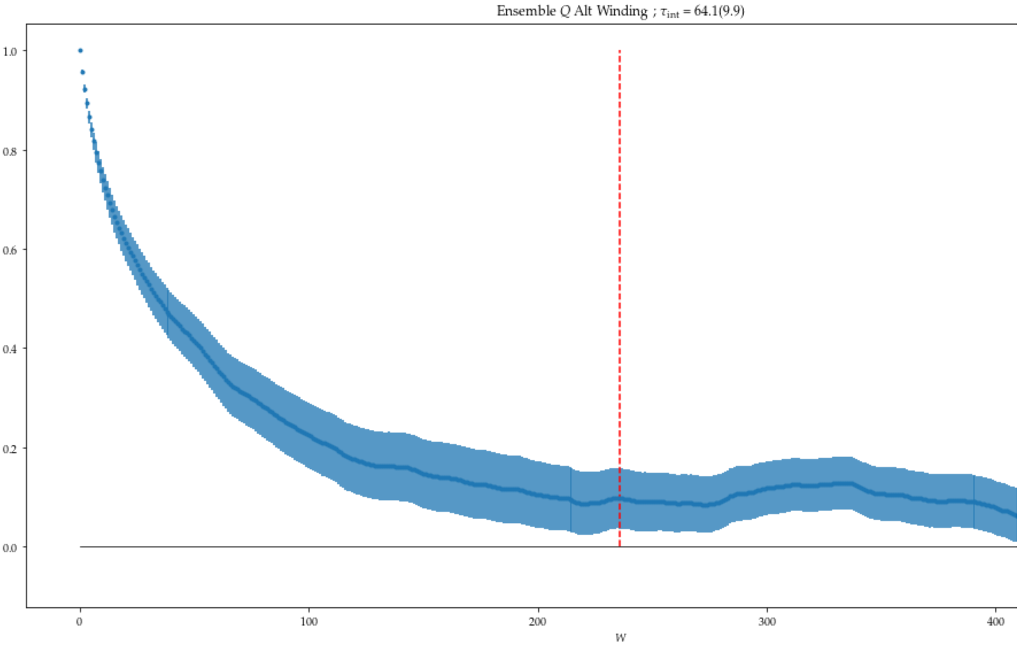
```
[0.0860729033160204, 0.0006674214615664195]
```

3.2 Alt Winding

In [81]:

```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr.primary_observable(0,'$Q$ Alt Winding', [0], ['R0'], [MCtimealtw.tolist()], [(t2Eat3altw).tolist
[qaltw, eqaltw] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 3.46s, finished 17:55:57 2020-11-11



```
[0.0832572622986207, 0.0008308000737363403]
```

4 t^2E at $t = 6$

In [20]:

```
t2Eat6hmc = np.loadtxt("t2Eat6hmc.txt")
t2Eat6hmc = t2Eat6hmc[:69000]
MCtimeHMC = np.arange(1, len(t2Eat6hmc)+1, 1)

t2Eat6altw = np.loadtxt("t2Eat6altw.txt")
t2Eat6altw = t2Eat6altw[:49000]
```

```
MCtimealtw = np.arange(1, len(t2Eat6altw)+1, 1)

t2Eat6alti = np.loadtxt("t2Eat6alti.txt")
MCtimealti = np.arange(1, len(t2Eat6alti)+1, 1)

t2Eat6doubaltw = np.loadtxt("t2Eat6doubaltw.txt")
MCtimedoubaltw = np.arange(1, len(t2Eat6doubaltw)+1, 1)

Nhmc = len(t2Eat6hmc)
Naltw = len(t2Eat6altw)
Ndoubaltw = len(t2Eat6doubaltw)
Nalti = len(t2Eat6alti)
```

executed in 1.03s, finished 12:54:36 2020-11-13

In [19]:
```
Nhmc
```

executed in 8ms, finished 12:52:26 2020-11-13

Out[19]:
```
69000
```

4.1 HMC

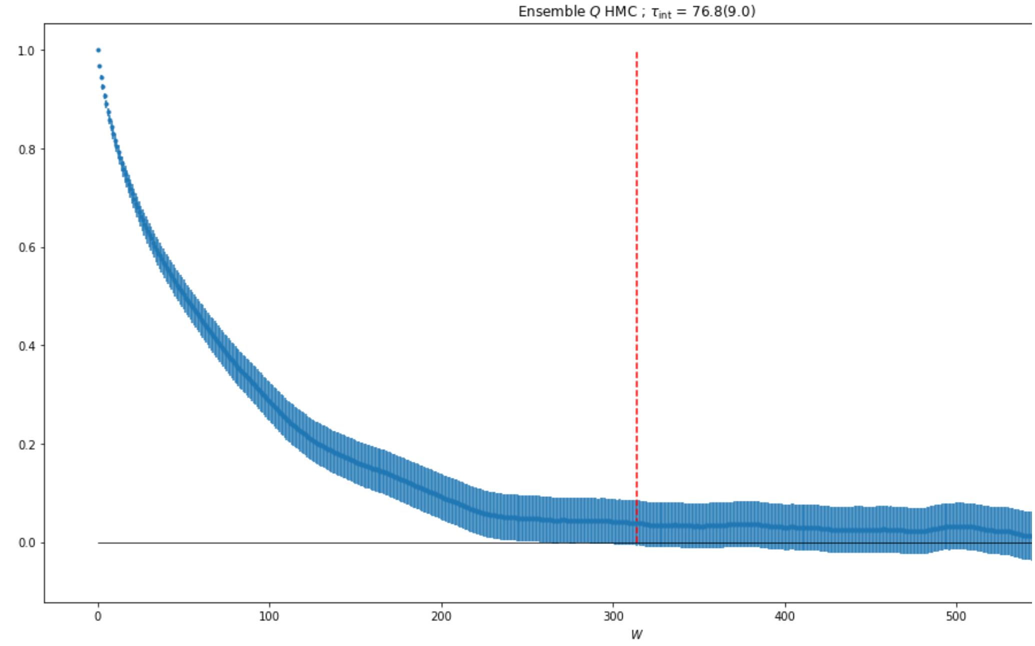
In [22]:
```
a = 2.5
```

executed in 6ms, finished 12:59:11 2020-11-13

value is {{a}}

In [16]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr.primary_observable(0,'$Q$ HMC', [0], ['R0'], [MCtimeHMC.tolist()], [(t2Eat6hmc).tolist()], (1,1)
[qhmc, eqhmc] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 8.04s, finished 12:43:49 2020-11-13



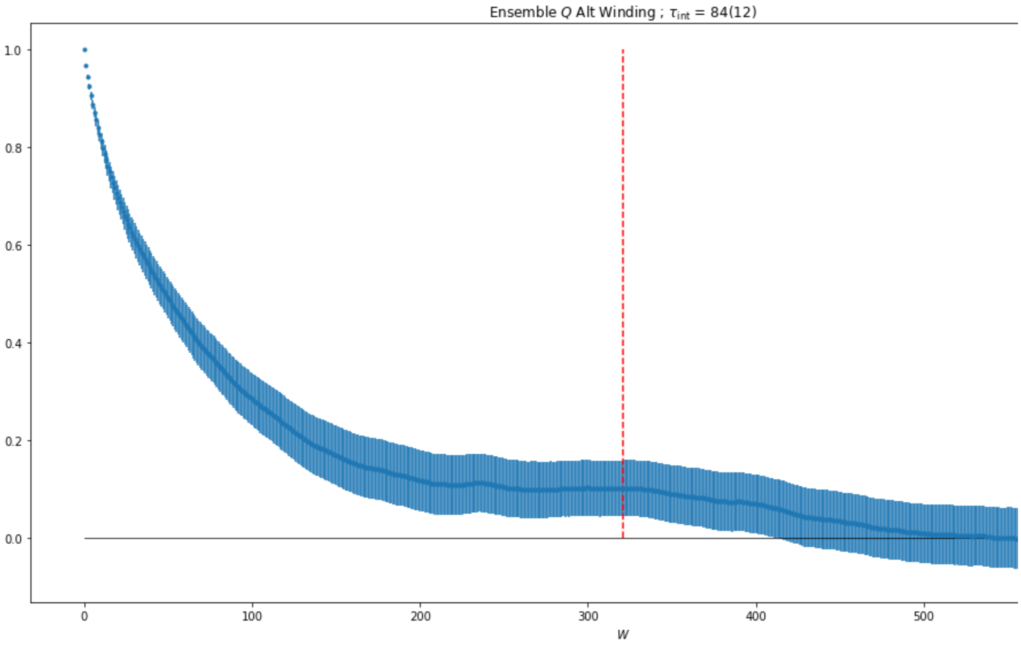
```
[0.15266109705804343, 0.002100366383712266]
```

qhmc

4.2 Alt Winding

In [21]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr.primary_observable(0,'$Q$ Alt Winding', [0], ['R0'], [MCtimealtw.tolist()], [(t2Eat6altw).tolist
[qaltw, eqaltw] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 4.00s, finished 12:54:43 2020-11-13



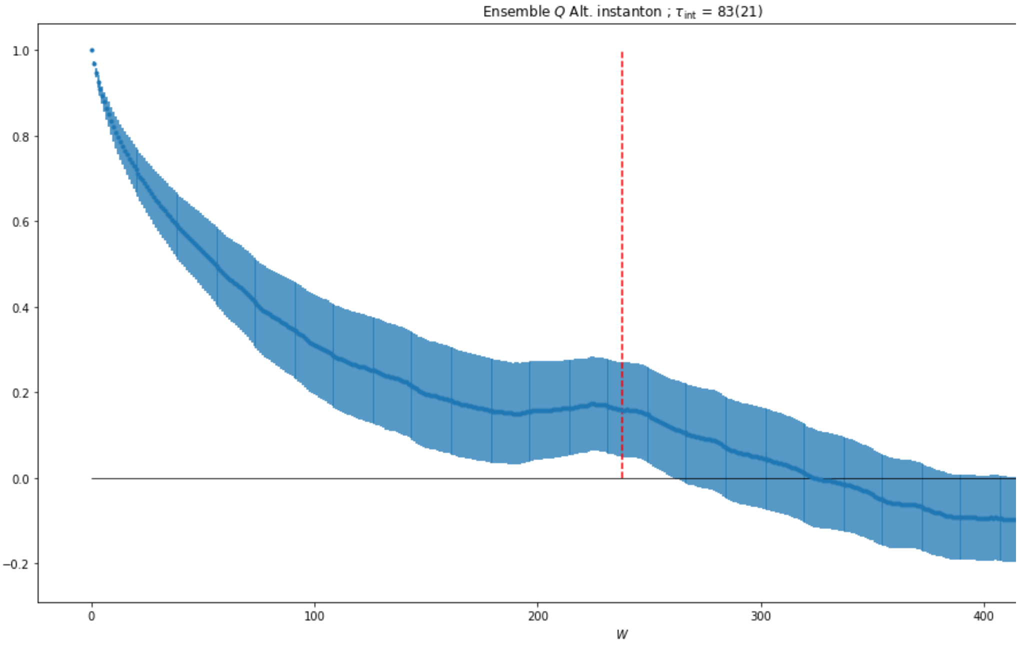
```
[0.14784941006316327, 0.002551051766441731]
```

Type *Markdown* and LaTeX: α^2

4.3 Alt instanton

In [11]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
corr.primary_observable(0,'$Q$ Alt. instanton', [0], ['R0'], [MCtimealti.tolist()], [(t2Eat6alti).tol
[qalti, eqalti] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True,errinfo=einfo))
```

executed in 828ms, finished 12:08:34 2020-11-13



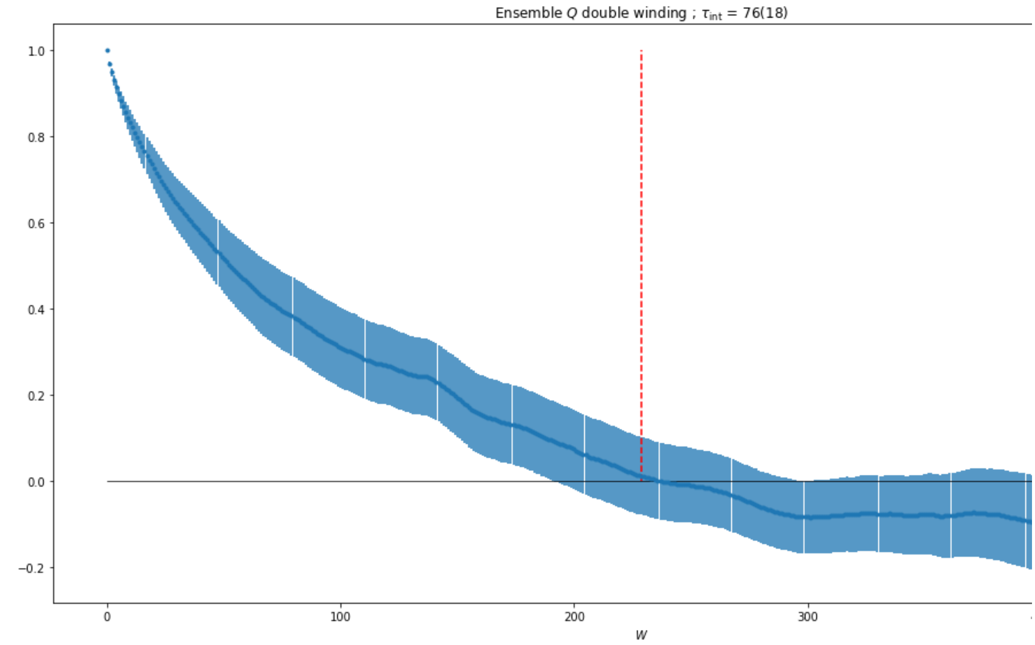
```
[0.1468619911095, 0.005760415291199244]
```

4.4 Double winding

In [12]:
```
corr = observa()
einfo = errinfo()
einfo.addEnsemble(0,Stau=1.0)
```

```
corr.primary_observable(0,'$Q$ double winding', [0], ['R0'], [MCtimedoubaltw.tolist()], [(t2Eat6douba
(1,1))
[qdoubaltw, eqdoubaltw] = corr.vwerr(errinfo=einfo, )

print(corr.vwerr(plot=True, errinfo=einfo))
```

executed in 868ms, finished 12:08:35 2020-11-13



```
[0.15304974108391306, 0.005234934776222587]
```

4.5 Binning

In [13]:

```
errors = []
ns = []

burn_in = 0 #how many initial states to discard
discard = 1 #pick 1 every discard number of states

gls = (t2Eat6hmc).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorshmc = (errors)
nshmc = (ns)


errors = []
ns = []

gls = (t2Eat6altw).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsaltw = (errors)
nsaltw = (ns)


errors = []
ns = []

gls = (t2Eat6alti).tolist()


for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsalti = (errors)
nsalti = (ns)



errors = []
ns = []

gls = (t2Eat6doubaltw).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsdoubaltw = (errors)
nsdoubaltw = (ns)
```

executed in 3.67s, finished 12:08:42 2020-11-13

```
250.0% completed!!
```

In [88]:

```
(eqaltw)**2.0/(eqhmc)**2.0*12.0/39.5
```

executed in 6ms, finished 16:44:08 2020-11-10

Out[88]:

```
0.641056655804379
```

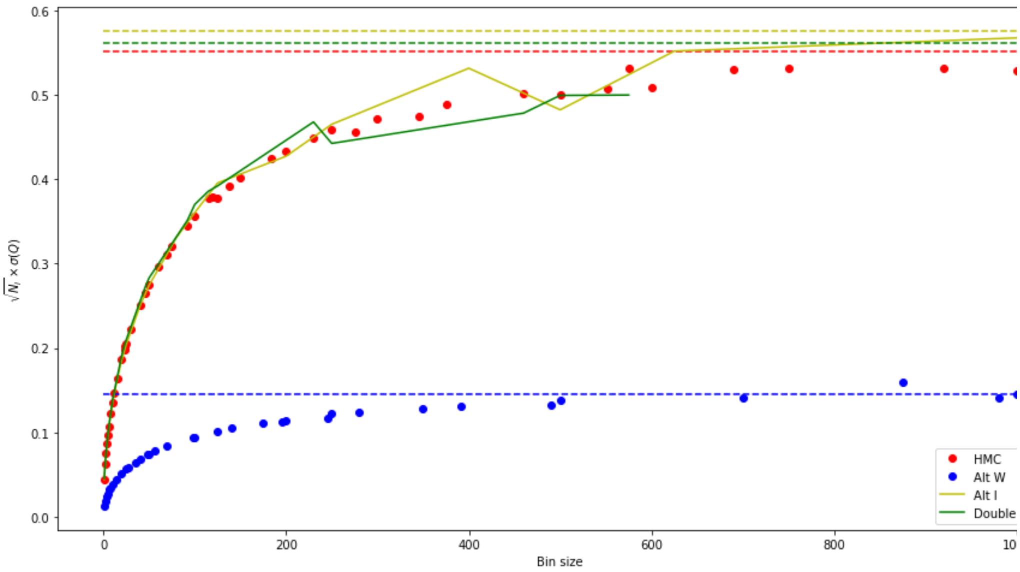
In [14]:

```
plt.plot(np.array(nshmc), np.sqrt(Nhmc)*np.array(errorshmc),'ro', label="HMC")
plt.plot([0,1000],[np.sqrt(Nhmc)*eqhmc,np.sqrt(Nhmc)*eqhmc], 'r--')

plt.plot(np.array(nsaltw), np.sqrt(Naltw)*np.array(errorsaltw),'bo', label="Alt W")
plt.plot([0,1000],[np.sqrt(Naltw)*eqaltw,np.sqrt(Naltw)*eqaltw], 'b--')

plt.plot(np.array(nsalti), np.sqrt(Nalti)*np.array(errorsalti),'y-', label="Alt I")
plt.plot([0,1000],[np.sqrt(Nalti)*eqalti,np.sqrt(Nalti)*eqalti], 'y--')
plt.plot(np.array(nsdoubaltw), np.sqrt(Ndoubaltw)*np.array(errorsdoubaltw),'g-', label="Double Alt W"
plt.plot([0,1000],[np.sqrt(Ndoubaltw)*eqdoubaltw,np.sqrt(Ndoubaltw)*eqdoubaltw], 'g--')

plt.legend()

plt.ylabel("$\sqrt{N_i} \\times \sigma (Q)$")
plt.xlabel("Bin size")
#plt.xlim(0,0.0.1)
```

executed in 673ms, finished 12:08:44 2020-11-13

Out[14]:

```
Text(0.5,0,'Bin size')
```



5 Q

In [359]:

```
topchargehmc = np.loadtxt("topchargeHMC.txt")
topchargehmc = topchargehmc[:69000]
topchargealtw = np.loadtxt("topchargealtw.txt")
topchargealtw = topchargealtw[:49000]
topchargedoubaltw = np.loadtxt("topchargedoubaltw.txt")
topchargedoubaltw = topchargedoubaltw[:11000]
topchargealti = np.loadtxt("topchargealti.txt")

topchargealtw2 = np.loadtxt("topchargealtw2.txt")
topchargealtw2 = topchargealtw2[:22000]
topchargealtw3 = np.loadtxt("topchargealtw3.txt")
topchargealtw3 = topchargealtw3[:16000]

MCtimehmc = np.arange(1, len(topchargehmc)+1, 1)
MCtimealtw = np.arange(1, len(topchargealtw)+1, 1)
MCtimedoubaltw = np.arange(1, len(topchargedoubaltw)+1, 1)
MCtimealti = np.arange(1, len(topchargealti)+1, 1)
MCtimealtw2 = np.arange(1, len(topchargealtw2)+1, 1)
MCtimealtw3 = np.arange(1, len(topchargealtw3)+1, 1)

Nhmc = len(topchargehmc)
Naltw = len(topchargealtw)
Ndoubaltw = len(topchargedoubaltw)
Nalti = len(topchargealti)
Naltw2 = len(topchargealtw2)
Naltw3 = len(topchargealtw3)
```

executed in 1.40s, finished 18:46:20 2020-11-13

In [360]:

```
print("Configuraciones HMC: {}".format(len(topchargeHMC)))
print("Configuraciones altwinding: {}".format(len(topchargealtw)))
print("Configuraciones double winding: {}".format(len(topchargedoubaltw)))
print("Configuraciones alt instanton: {}".format(len(topchargealti)))
print("Configuraciones alt instanton: {}".format(len(topchargealtw2)))
print("Configuraciones alt instanton: {}".format(len(topchargealtw3)))
```

executed in 13ms, finished 18:46:26 2020-11-13

```
Configuraciones HMC: 69000
Configuraciones altwinding: 49000
Configuraciones double winding: 11000
Configuraciones alt instanton: 10000
Configuraciones alt instanton: 22000
Configuraciones alt instanton: 16000
```

Algorithm	Statistics	Q	τ_{int}	
HMC	{{Nhmc}}	{{printwe(Qhmc,eQhmc)}}	{{printwe(tauQhmc, etauQhmc)}}	
Alt Winding	{{Naltw}}	{{printwe(Qaltw,eQaltw)}}	{{printwe(tauQaltw, etauQaltw)}}	{{printwe(tauR_Qa
Alt Instanton	{{Nalti}}	{{printwe(Qalti,eQalti)}}	{{printwe(tauQalti, etauQalti)}}	{{printwe(tauR_(
Alt Double Winding	{{Ndoubaltw}}	{{printwe(Qdoubaltw,eQdoubaltw)}}	{{printwe(tauQdoubaltw, etauQdoubaltw)}}	{{printwe(tauR_Qdoubaltw, e
Alt Winding 2	{{Naltw2}}	{{printwe(Qaltw2,eQaltw2)}}	{{printwe(tauQaltw2, etauQaltw2)}}	{{printwe(tauR_Qaltw
Alt Winding 3	{{Naltw3}}	{{printwe(Qaltw3,eQaltw3)}}	{{printwe(tauQaltw3, etauQaltw3)}}	{{printwe(tauR_Qaltw

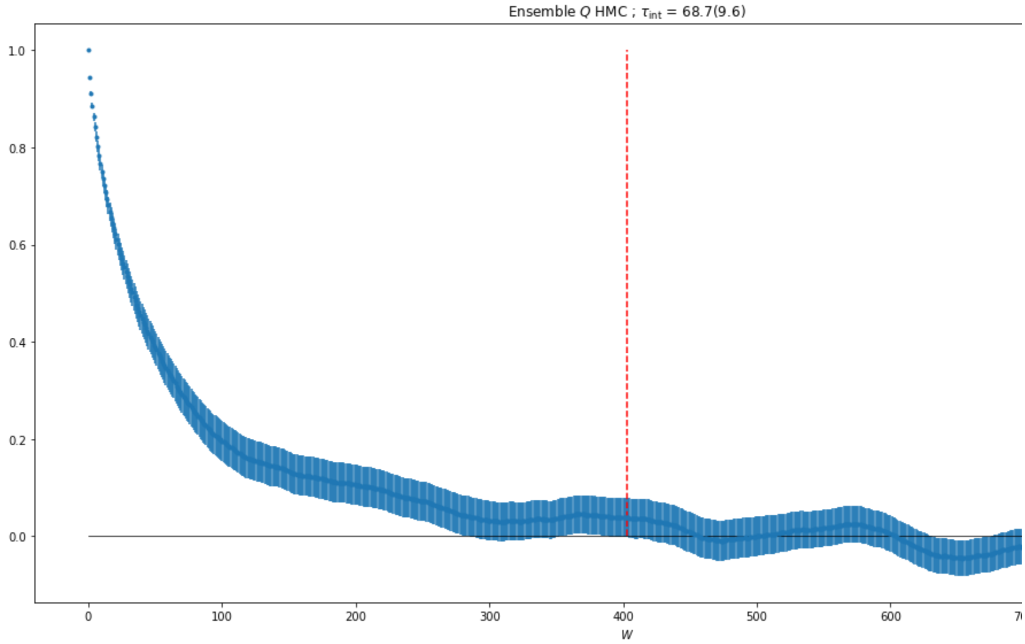
5.1 HMC

P = {{printwe(Qhmc, eQhmc)}}

$\tau_{int,Q}$ = {{printwe(tauQhmc, etauQhmc)}}

In [240]:
```
corr_qhmc = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_qhmc.primary_observable(0,'$Q$ HMC', [0], ['R0'], [MCtimehmc.tolist()], [(topchargehmc).tolist()

[Qhmc, eQhmc]= corr_qhmc.vwerr(errinfo=einfo)
[tauQhmc, etauQhmc] = corr_qhmc.tauint()
tauQhmc = tauQhmc[0][0][0]
etauQhmc = etauQhmc[0][0][0]

print(corr_qhmc.vwerr(plot=True,errinfo=einfo))

printwe(Qhmc, eQhmc)
```

executed in 7.61s, finished 17:11:30 2020-11-13



```
[-0.0029484408451943356, 0.0404168404662309]
-0.003(40)
```

5.2 Alternating Winding

$Q$ = {{printwe(Qaltw, eQaltw)}}

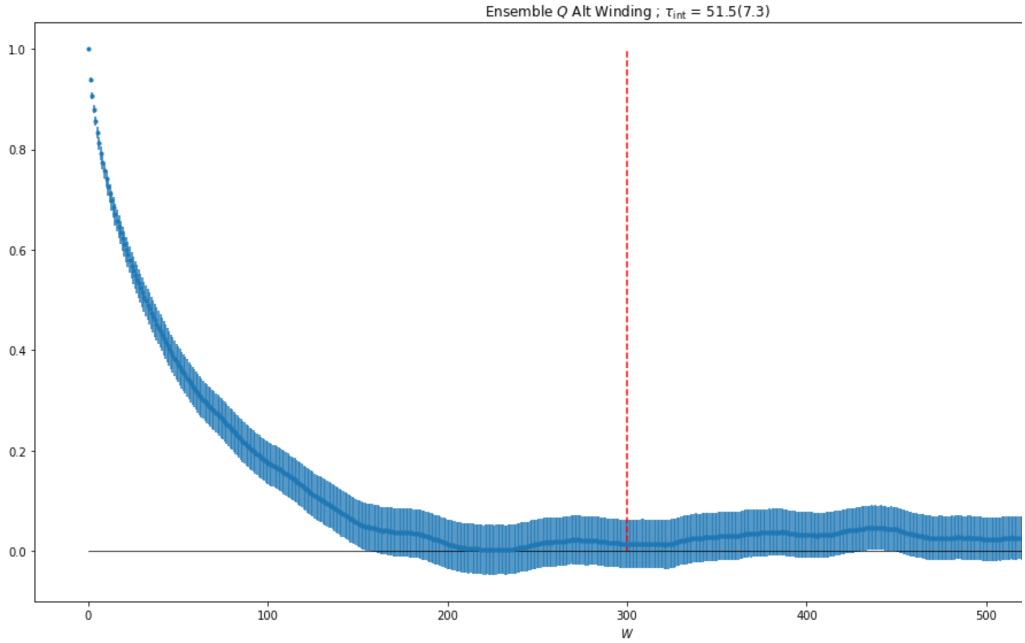
$\tau_{int,Q}$ = {{printwe(tauQaltw, etauQaltw)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Qaltw, etauR_Qaltw)}}

In [251]:

```
corr_qaltw = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_qaltw.primary_observable(0,'$Q$ Alt Winding', [0], ['R0'], [MCtimealtw.tolist()], [(topchargealt

[Qaltw, eQaltw]= corr_qaltw.vwerr(errinfo=einfo)
[tauQaltw, etauQaltw] = corr_qaltw.tauint()
tauQaltw = tauQaltw[0][0][0]
etauQaltw = etauQaltw[0][0][0]

print(corr_qaltw.vwerr(plot=True,errinfo=einfo))

printwe(Qaltw,eQaltw)

tauR_Qaltw = tauQaltw/tauQhmc
etauR_Qaltw = tauR_Qaltw *  np.sqrt( (etauQhmc/tauQhmc)**2.0 + (etauQaltw/tauQaltw)**2.0 )
```

executed in 5.26s, finished 17:20:51 2020-11-13



```
[-0.0700155782083696, 0.0399013919622557]
-0.07(40)
```

5.3 Alternating instanton

$Q$ = {{printwe(Qalti, eQalti)}}

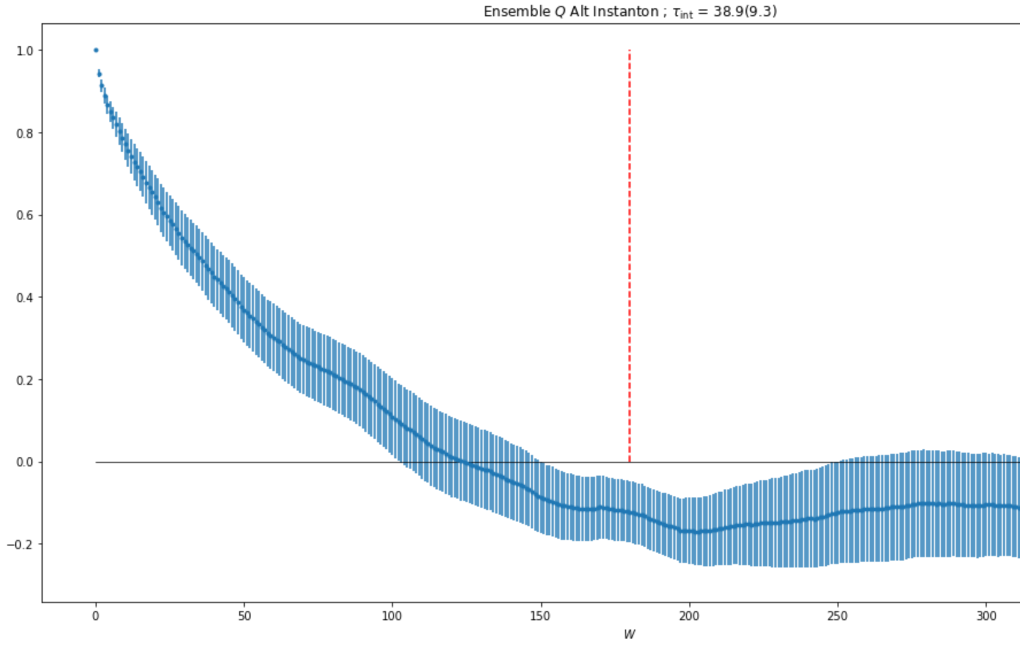
$\tau_{int,Q}$ = {{printwe(tauQalti, etauQalti)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Qalti, etauR_Qalti)}}

In [252]:

```
corr_qalti = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_qalti.primary_observable(0,'$Q$ Alt Instanton', [0], ['R0'], [MCtimealti.tolist()], [(topchargea
(1,1))

[Qalti, eQalti]= corr_qalti.vwerr(errinfo=einfo)
[tauQalti, etauQalti] = corr_qalti.tauint()
tauQalti = tauQalti[0][0][0]
etauQalti = etauQalti[0][0][0]

print(corr_qalti.vwerr(plot=True,errinfo=einfo))

printwe(Qalti,eQalti)

tauR_Qalti = tauQalti/tauQhmc
etauR_Qalti = tauR_Qalti * np.sqrt( (etauQhmc/tauQhmc)**2.0 + (etauQalti/tauQalti)**2.0 )
```

executed in 932ms, finished 17:20:52 2020-11-13



```
[0.14106735732349657, 0.07680352873332864]
0.141(77)
```

5.4 Double Winding

$Q$ = {{printwe(Qdoubaltw, eQdoubaltw)}}

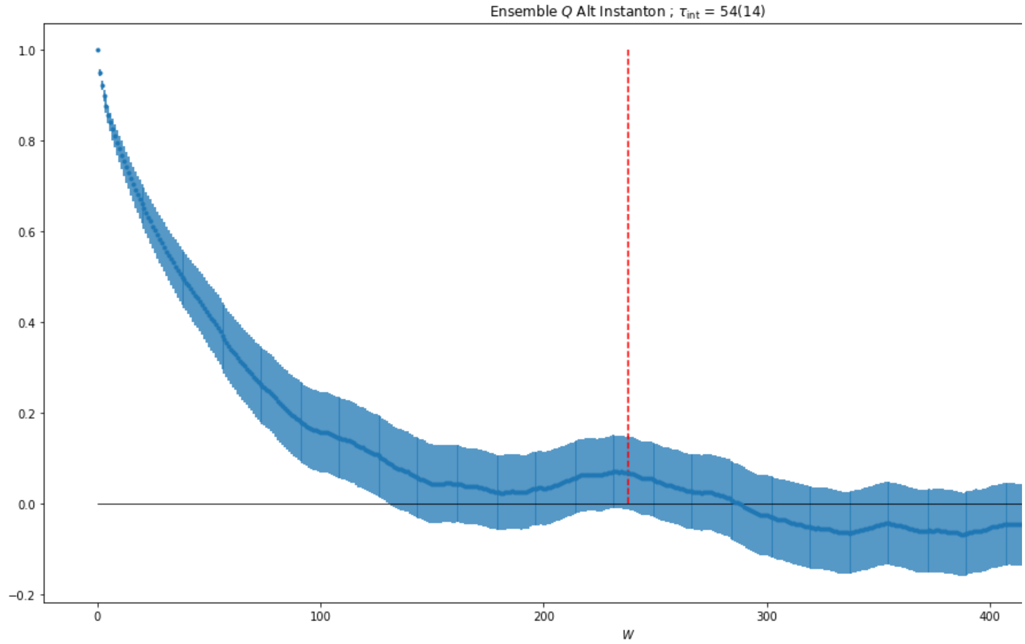
$\tau_{int,Q}$ = {{printwe(tauQdoubaltw, etauQdoubaltw)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Qdoubaltw, etauR_Qdoubaltw)}}

In [255]:

```
corr_qdoubaltw = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_qdoubaltw.primary_observable(0,'$Q$ Alt Instanton', [0], ['R0'], [MCtimedoubaltw.tolist()],
[(topchargedoubaltw).tolist()], (1,1))

[Qdoubaltw, eQdoubaltw]= corr_qdoubaltw.vwerr(errinfo=einfo)
[tauQdoubaltw, etauQdoubaltw] = corr_qdoubaltw.tauint()
tauQdoubaltw = tauQdoubaltw[0][0][0]
etauQdoubaltw = etauQdoubaltw[0][0][0]

print(corr_qdoubaltw.vwerr(plot=True,errinfo=einfo))

printwe(Qdoubaltw,eQdoubaltw)

tauR_Qdoubaltw = tauQdoubaltw/tauQhmc
etauR_Qdoubaltw = tauR_Qdoubaltw *  np.sqrt( (etauQhmc/tauQhmc)**2.0 + (etauQdoubaltw/tauQdoubaltw)**
```

executed in 1.12s, finished 17:21:30 2020-11-13



```
[-0.14008025448150208, 0.09506703292075573]
-0.14(95)
```

5.5 Alternating Winding half side 2

$Q$ = {{printwe(Qaltw2, eQaltw2)}}

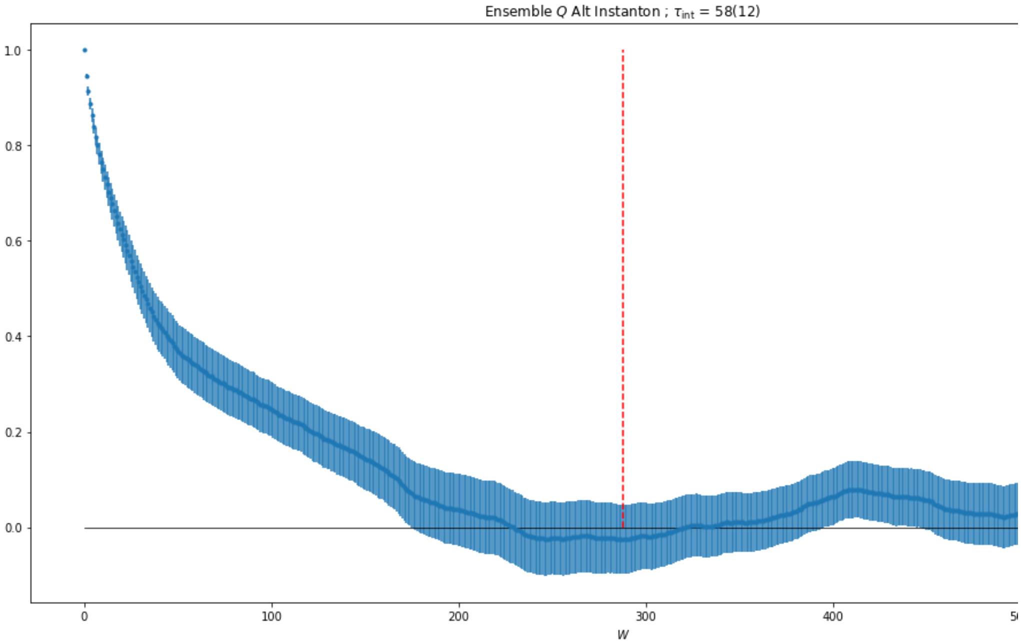
$\tau_{int,Q}$ = {{printwe(tauQaltw2, etauQaltw2)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Qaltw2, etauR_Qaltw2)}}

In [357]:

```
corr_qaltw2 = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_qaltw2.primary_observable(0,'$Q$ Alt Instanton', [0], ['R0'], [MCtimealtw2.tolist()], [(topcharg
(1,1))

[Qaltw2, eQaltw2]= corr_qaltw2.vwerr(errinfo=einfo)
[tauQaltw2, etauQaltw2] = corr_qaltw2.tauint()
tauQaltw2 = tauQaltw2[0][0][0]
etauQaltw2 = etauQaltw2[0][0][0]

print(corr_qaltw2.vwerr(plot=True,errinfo=einfo))

printwe(Qaltw2,eQaltw2)

tauR_Qaltw2 = tauQaltw2/tauQhmc
etauR_Qaltw2 = tauR_Qaltw2 *  np.sqrt( (etauQhmc/tauQhmc)**2.0 + (etauQaltw2/tauQaltw2)**2.0 )
```

executed in 1.67s, finished 18:40:46 2020-11-13



```
[-0.04345579221622017, 0.0653456523523522]
-0.043(65)
```

5.6 Alternating Winding half side 3

$Q$ = {{printwe(Qaltw3, eQaltw3)}}

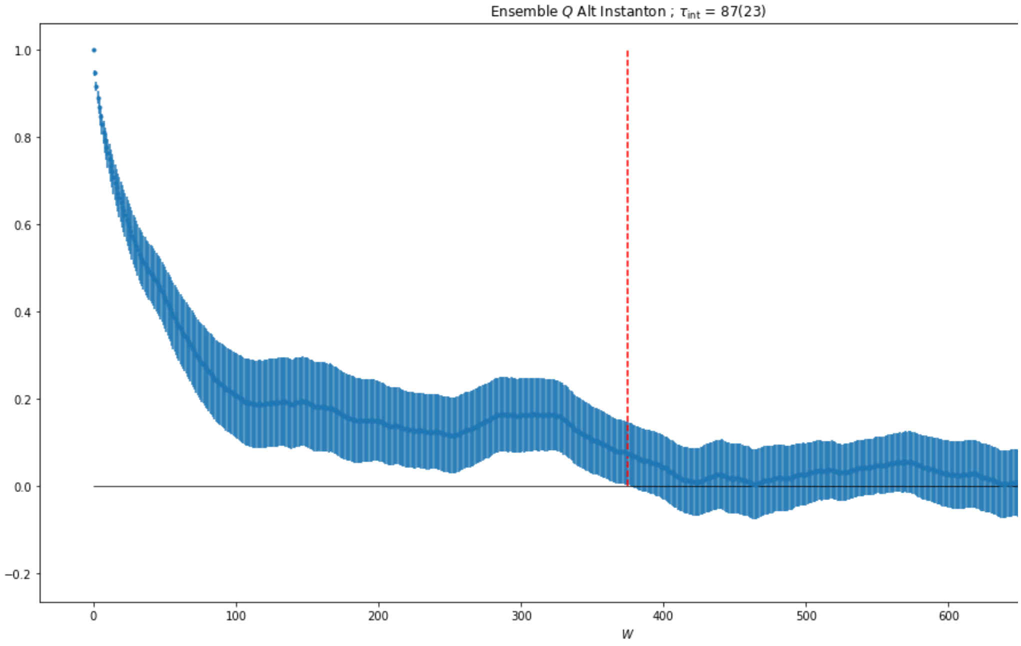
$\tau_{int,Q}$ = {{printwe(tauQaltw3, etauQaltw3)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Qaltw3, etauR_Qaltw3)}}

Contents

In [361]:
```
corr_qaltw3 = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_qaltw3.primary_observable(0,'$Q$ Alt Instanton', [0], ['R0'], [MCtimealtw3.tolist()], [(topcharg
(1,1))

[Qaltw3, eQaltw3]= corr_qaltw3.vwerr(errinfo=einfo)
[tauQaltw3, etauQaltw3] = corr_qaltw3.tauint()
tauQaltw3 = tauQaltw3[0][0][0]
etauQaltw3 = etauQaltw3[0][0][0]

print(corr_qaltw3.vwerr(plot=True,errinfo=einfo))

printwe(Qaltw3,eQaltw3)

tauR_Qaltw3 = tauQaltw3/tauQhmc
etauR_Qaltw3 = tauR_Qaltw3 *  np.sqrt( (etauQhmc/tauQhmc)**2.0 + (etauQaltw3/tauQaltw3)**2.0 )
```

executed in 1.33s, finished 18:46:48 2020-11-13



```
[0.07214080833018939, 0.09838212376404576]
0.072(98)
```

5.7 Binning

In [27]:
```
errors = []
ns = []

burn_in = 0 #how many initial states to discard
discard = 1 #pick 1 every discard number of states

gls = (topchargeHMC).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorshmc = (errors)
nshmc = (ns)


errors = []
ns = []

gls = (topchargealtw).tolist()
```

```
for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsaltw = (errors)
nsaltw = (ns)


errors = []
ns = []

gls = (topchargealti).tolist()


for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsalti = (errors)
nsalti = (ns)



errors = []
ns = []

gls = (topchargedoubaltw).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsdoubaltw = (errors)
nsdoubaltw = (ns)
```

executed in 2.53s, finished 17:17:45 2020-11-11

```
250.0% completed!!
```

In [29]:

```
Nhmc
```

executed in 8ms, finished 17:17:50 2020-11-11

Out[29]:

```
39000
```

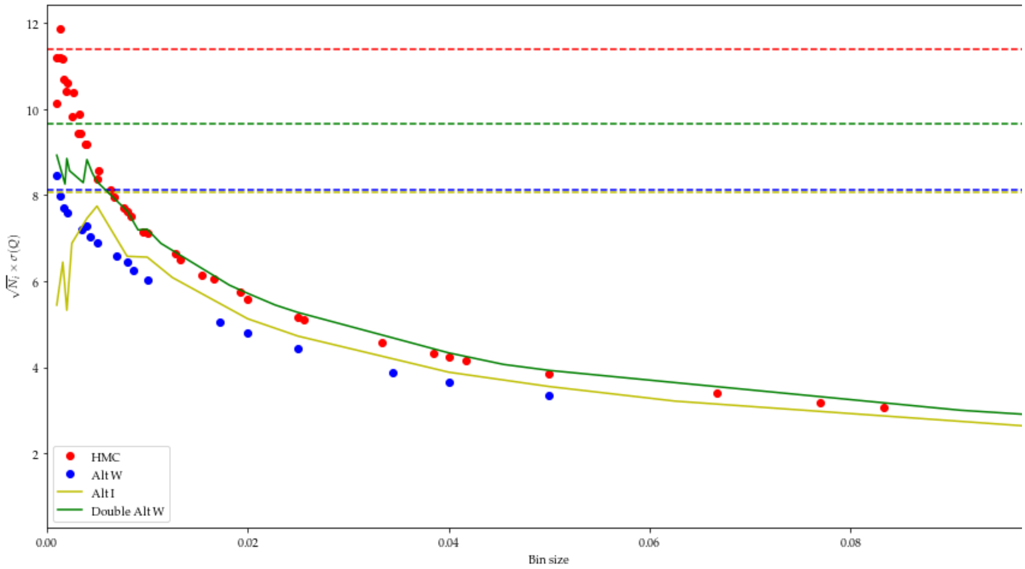
In [31]:

```
plt.plot(1.0/np.array(nshmc), np.sqrt(Nhmc)*np.array(errorshmc),'ro', label="HMC")
plt.plot([0,1],[np.sqrt(Nhmc)*eqhmc,np.sqrt(Nhmc)*eqhmc], 'r--')

plt.plot(1.0/np.array(nsaltw), np.sqrt(Naltw)*np.array(errorsaltw),'bo', label="Alt W")
plt.plot([0,1],[np.sqrt(Naltw)*eqaltw,np.sqrt(Naltw)*eqaltw], 'b--')

plt.plot(1.0/np.array(nsalti), np.sqrt(Nalti)*np.array(errorsalti),'y-', label="Alt I")
plt.plot([0,1],[np.sqrt(Nalti)*eqalti,np.sqrt(Nalti)*eqalti], 'y--')

plt.plot(1.0/np.array(nsdoubaltw), np.sqrt(Ndoubaltw)*np.array(errorsdoubaltw),'g-', label="Double Al
plt.plot([0,1],[np.sqrt(Ndoubaltw)*eqdoubaltw,np.sqrt(Ndoubaltw)*eqdoubaltw], 'g--')

plt.legend()

plt.ylabel("$\sqrt{N_i} \\times \sigma (Q)$")
plt.xlabel("Bin size")
plt.xlim(0.0,0.1)
```

executed in 2.44s, finished 17:18:21 2020-11-11

Out[31]:

```
(0.0, 0.1)
```



5.7.1 Binning errors

This computes the errors of the error with a Jackknife method. You can compare the results with automatic windowing procedure: the relat error should be similar to the relative error of the autocorrelation time.

In [42]:

```
import numpy as np
import glob
import numpy.ma as ma
import time

def iden(x):
    return x

def stdev(x):
    return np.std(x)/np.sqrt(len(x))

def removeelement (array, index):
    return np.delete(array, (index), axis=0)


def jackknife(x, func):
    """Jackknife estimate of the estimator func"""
    n = len(x)
    idx = np.arange(n)
    avg = 0.
    for i in idx:
        xaux = np.apply_along_axis(stdev,0,removeelement(x,i))
        #       avg = avg + np.mean(np.apply_along_axis(func, 1, removeelement(x,i)))
        avg = avg + func(xaux)
    return (avg/n)

def jackknifevar(x, xmean, func):
    n = len(x)
    idx = np.arange(n)
    var = 0.
    for i in idx:
        #  print (np.apply_along_axis(func, 1, removeelement(x,i)))
        #  print np.mean(np.apply_along_axis(func, 1, removeelement(x,i)))
        xi = func(np.apply_along_axis(stdev,0,removeelement(x,i)))
        var = var + (xi-xmean)**2
    return np.sqrt(var*(n-1)/n)
```

executed in 29ms, finished 09:06:32 2020-11-13

```
In [8]: err = jackknife(gls2,iden)
```

executed in 22ms, finished 08:19:03 2020-11-13

```
In [9]: jackknifevar(gls2,err,iden)
```

executed in 20ms, finished 08:19:04 2020-11-13

```
Out[9]: 0.0042627183143538775
```

```
In [10]: err
```

executed in 8ms, finished 08:19:05 2020-11-13

```
Out[10]: 0.03809522962034062
```

```
In [11]: evalue
```

executed in 9ms, finished 08:23:51 2020-11-13

```
Out[11]: 0.037825741661202214
```

In [43]:
```
errors = []
errorsbar = []
ns = []

burn_in = 0 #how many initial states to discard
discard = 1 #pick 1 every discard number of states

gls = (topchargeHMC).tolist()

for n in range(50,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        #evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        evalue = jackknife(gls2,iden)
        evalueerr = jackknifevar(gls2,evalue,iden)
        errors.append(evalue)
        errorsbar.append(evalueerr)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorshmc = (errors)
errorsbarhmc = errorsbar
nshmc = (ns)


errors = []
errorsbar = []
ns = []

gls = (topchargealtw).tolist()

for n in range(50,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        #evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        evalue = jackknife(gls2,iden)
        evalueerr = jackknifevar(gls2,evalue,iden)
        errors.append(evalue)
        errorsbar.append(evalueerr)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsaltw = (errors)
errorsbaraltw = errorsbar
nsaltw = (ns)
```

executed in 7.71s, finished 09:06:43 2020-11-13

```
250.0% completed!!
```

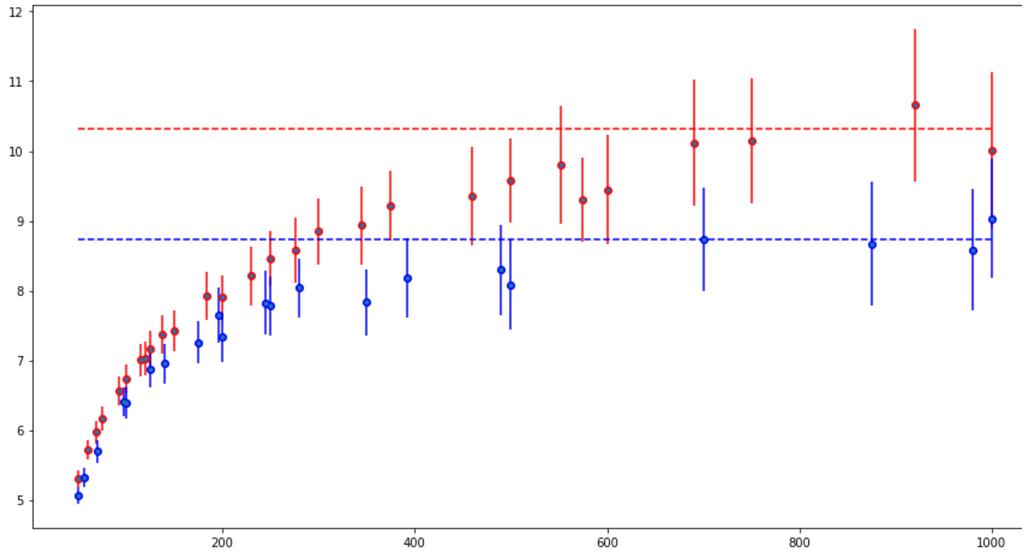
In [44]:

```
plt.plot(np.array(nshmc), np.sqrt(Nhmc)*np.array(errorshmc),'ro', label="HMC")
plt.errorbar(np.array(nshmc), np.sqrt(Nhmc)*np.array(errorshmc), yerr=np.sqrt(Nhmc)*np.array(errorsba
fmt='.')
plt.plot([50,1000],[np.sqrt(Nhmc)*eqhmc,np.sqrt(Nhmc)*eqhmc], 'r--')

plt.plot(np.array(nsaltw), np.sqrt(Naltw)*np.array(errorsaltw),'bo', label="Alt W")
plt.errorbar(np.array(nsaltw), np.sqrt(Naltw)*np.array(errorsaltw),np.sqrt(Naltw)*np.array(errorsbara
fmt='.')
plt.plot([50,1000],[np.sqrt(Naltw)*eqaltw,np.sqrt(Naltw)*eqaltw], 'b--')
```

executed in 365ms, finished 09:06:45 2020-11-13

Out[44]:

```
[<matplotlib.lines.Line2D at 0x7fcbc4d75090>]
```



6 Q^2

Algorithm	Statistics	Q^2	τ_{int}	
HMC	{{Nhmc}}	{{printwe(Q2hmc,eQ2hmc)}}	{{printwe(tauQ2hmc, etauQ2hmc)}}	
Alt Winding	{{Naltw}}	{{printwe(Q2altw,eQ2altw)}}	{{printwe(tauQ2altw, etauQ2altw)}}	{{printwe(tauR_C
Alt Instanton	{{Nalti}}	{{printwe(Q2alti,eQ2alti)}}	{{printwe(tauQ2alti, etauQ2alti)}}	{{printwe(tauR_
Alt Double Winding	{{Ndoubaltw}}	{{printwe(Q2doubaltw,eQ2doubaltw)}}	{{printwe(tauQ2doubaltw, etauQ2doubaltw)}}	{{printwe(tauR_Q2doubaltw
Alt Instanton	{{Naltw2}}	{{printwe(Q2altw2,eQ2altw2)}}	{{printwe(tauQ2altw2, etauQ2altw2)}}	{{printwe(tauR_Q2a
Alt Instanton	{{Naltw3}}	{{printwe(Q2altw3,eQ2altw3)}}	{{printwe(tauQ2altw3, etauQ2altw3)}}	{{printwe(tauR_Q2a

6.1 HMC

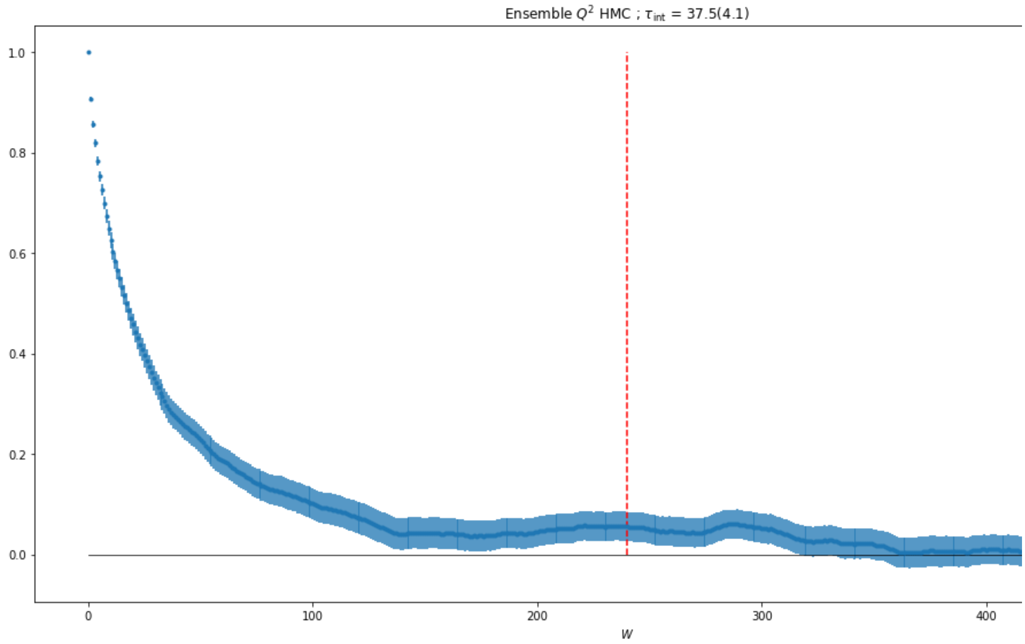
$P =$ {{printwe(Q2hmc, eQ2hmc)}}

$\tau_{int,Q^2} =$ {{printwe(tauQ2hmc, etauQ2hmc)}}

In [258]:

```
corr_q2hmc = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q2hmc.primary_observable(0,'$Q^2$ HMC', [0], ['R0'], [MCtimehmc.tolist()], [(topchargehmc**2).to

[Q2hmc, eQ2hmc]= corr_q2hmc.vwerr(errinfo=einfo)
[tauQ2hmc, etauQ2hmc] = corr_q2hmc.tauint()
tauQ2hmc = tauQ2hmc[0][0][0]
etauQ2hmc = etauQ2hmc[0][0][0]

print(corr_q2hmc.vwerr(plot=True,errinfo=einfo))

printwe(Q2hmc,eQ2hmc)
```

executed in 7.81s, finished 17:29:45 2020-11-13



```
[0.8191819850747745, 0.04432421659296179]
0.819(44)
```

6.2 Alternating Winding

$Q^2$ = {{printwe(Q2altw, eQ2altw)}}

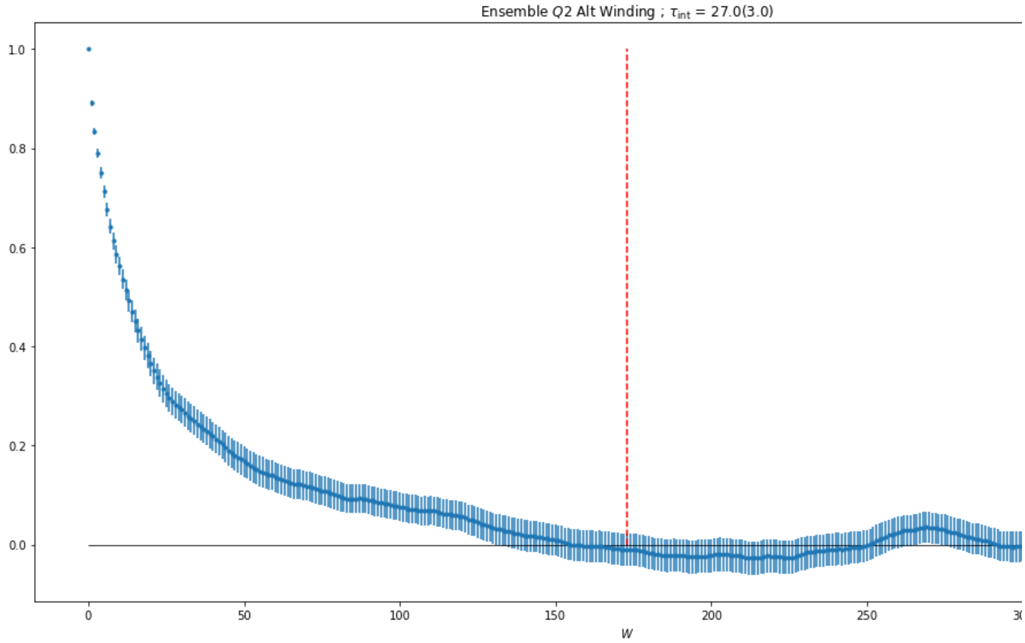
τ_{int,Q^2} = {{printwe(tauQ2altw, etauQ2altw)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q2altw, etauR_Q2altw)}}

In [261]:

```
corr_q2altw = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q2altw.primary_observable(0,'$Q2$ Alt Winding', [0], ['R0'], [MCtimealtw.tolist()], [(topchargea
(1,1))

[Q2altw, eQ2altw]= corr_q2altw.vwerr(errinfo=einfo)
[tauQ2altw, etauQ2altw] = corr_q2altw.tauint()
tauQ2altw = tauQ2altw[0][0][0]
etauQ2altw = etauQ2altw[0][0][0]

print(corr_q2altw.vwerr(plot=True,errinfo=einfo))

printwe(Q2altw,eQ2altw)

tauR_Q2altw = tauQ2altw/tauQ2hmc
etauR_Q2altw = tauR_Q2altw * np.sqrt( (etauQ2hmc/tauQ2hmc)**2.0 + (etauQ2altw/tauQ2altw)**2.0 )
```

executed in 4.63s, finished 17:31:49 2020-11-13



```
[0.7607011398656379, 0.04046302417028977]
0.761(40)
```

6.3 Alternating instanton

Q^2 = {{printwe(Q2alti, eQ2alti)}}

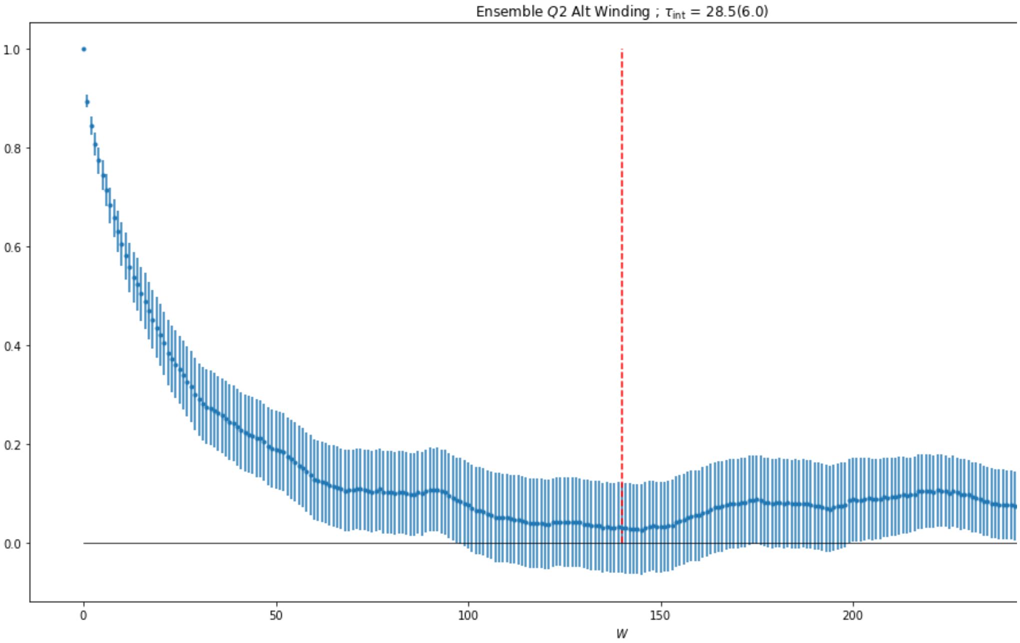
τ_{int,Q^2} = {{printwe(tauQ2alti, etauQ2alti)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q2alti, etauR_Q2alti)}}

In [263]:

```
corr_q2alti = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q2alti.primary_observable(0,'$Q2$ Alt Winding', [0], ['R0'], [MCtimealti.tolist()], [(topchargea
(1,1))

[Q2alti, eQ2alti]= corr_q2alti.vwerr(errinfo=einfo)
[tauQ2alti, etauQ2alti] = corr_q2alti.tauint()
tauQ2alti = tauQ2alti[0][0][0]
etauQ2alti = etauQ2alti[0][0][0]

print(corr_q2alti.vwerr(plot=True,errinfo=einfo))

printwe(Q2alti,eQ2alti)

tauR_Q2alti = tauQ2alti/tauQ2hmc
etauR_Q2alti = tauR_Q2alti * np.sqrt( (etauQ2hmc/tauQ2hmc)**2.0 + (etauQ2alti/tauQ2alti)**2.0 )
```

executed in 674ms, finished 17:33:44 2020-11-13



```
[0.7714815003631615, 0.09498620447061464]
0.771(95)
```

6.4 Double Winding

Q^2 = {{printwe(Q2doubaltw, eQ2doubaltw)}}

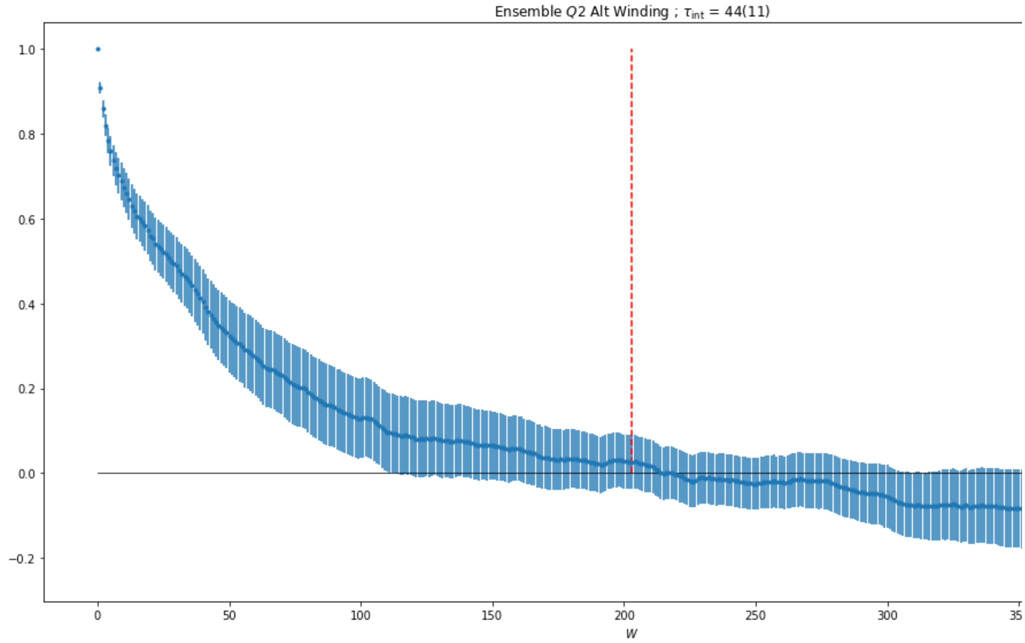
τ_{int,Q^2} = {{printwe(tauQ2doubaltw, etauQ2doubaltw)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q2doubaltw, etauR_Q2doubaltw)}}

In [265]:

```
corr_q2doubaltw = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q2doubaltw.primary_observable(0,'$Q2$ Alt Winding', [0], ['R0'], [MCtimedoubaltw.tolist()],
[(topchargedoubaltw**2).tolist()], (1,1))

[Q2doubaltw, eQ2doubaltw]= corr_q2doubaltw.vwerr(errinfo=einfo)
[tauQ2doubaltw, etauQ2doubaltw] = corr_q2doubaltw.tauint()
tauQ2doubaltw = tauQ2doubaltw[0][0][0]
etauQ2doubaltw = etauQ2doubaltw[0][0][0]

print(corr_q2doubaltw.vwerr(plot=True,errinfo=einfo))

printwe(Q2doubaltw,eQ2doubaltw)

tauR_Q2doubaltw = tauQ2doubaltw/tauQ2hmc
etauR_Q2doubaltw = tauR_Q2doubaltw * np.sqrt( (etauQ2hmc/tauQ2hmc)**2.0 + (etauQ2doubaltw/tauQ2douba
```

executed in 1.14s, finished 17:34:39 2020-11-13



```
[0.9304809269445213, 0.1285291691817063]
0.93(13)
```

6.5 Alternating Winding 2

Q^2 = {{printwe(Q2altw2, eQ2altw2)}}

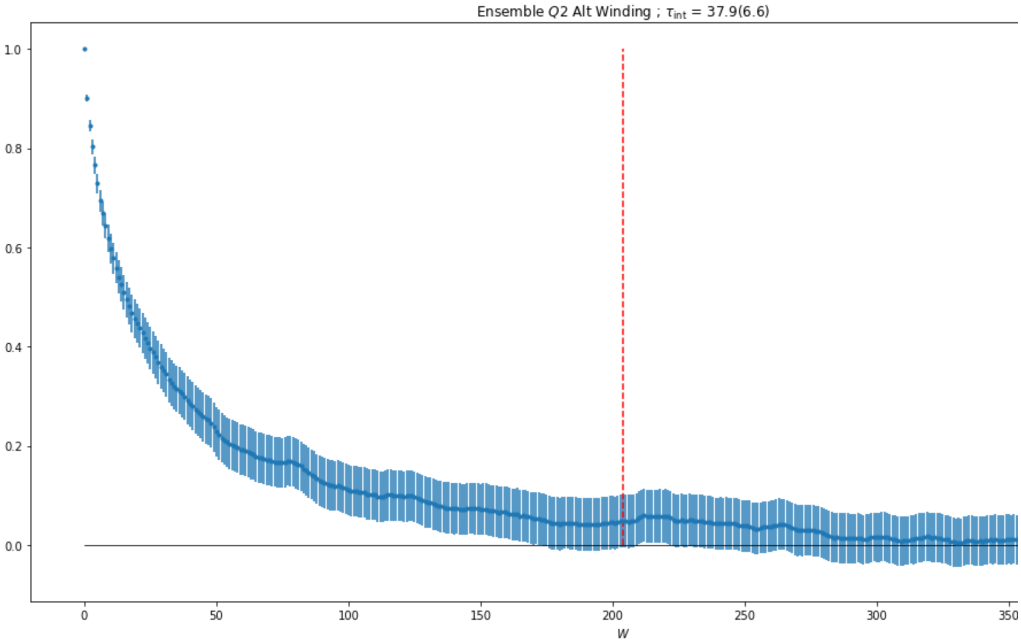
τ_{int,Q^2} = {{printwe(tauQ2altw2, etauQ2altw2)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q2altw2, etauR_Q2altw2)}}

In [362]:

```
corr_q2altw2 = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q2altw2.primary_observable(0,'$Q2$ Alt Winding', [0], ['R0'], [MCtimealtw2.tolist()], [(topcharg
(1,1))

[Q2altw2, eQ2altw2]= corr_q2altw2.vwerr(errinfo=einfo)
[tauQ2altw2, etauQ2altw2] = corr_q2altw2.tauint()
tauQ2altw2 = tauQ2altw2[0][0][0]
etauQ2altw2 = etauQ2altw2[0][0][0]

print(corr_q2altw2.vwerr(plot=True,errinfo=einfo))

printwe(Q2altw2,eQ2altw2)

tauR_Q2altw2 = tauQ2altw2/tauQ2hmc
etauR_Q2altw2 = tauR_Q2altw2 *  np.sqrt( (etauQ2hmc/tauQ2hmc)**2.0 + (etauQ2altw2/tauQ2altw2)**2.0 )
```

executed in 1.13s, finished 18:51:24 2020-11-13



```
[0.8114269888738852, 0.07800976752252282]
0.811(78)
```

6.6 Alternating Winding 3

Q^2 = {{printwe(Q2altw3, eQ2altw3)}}

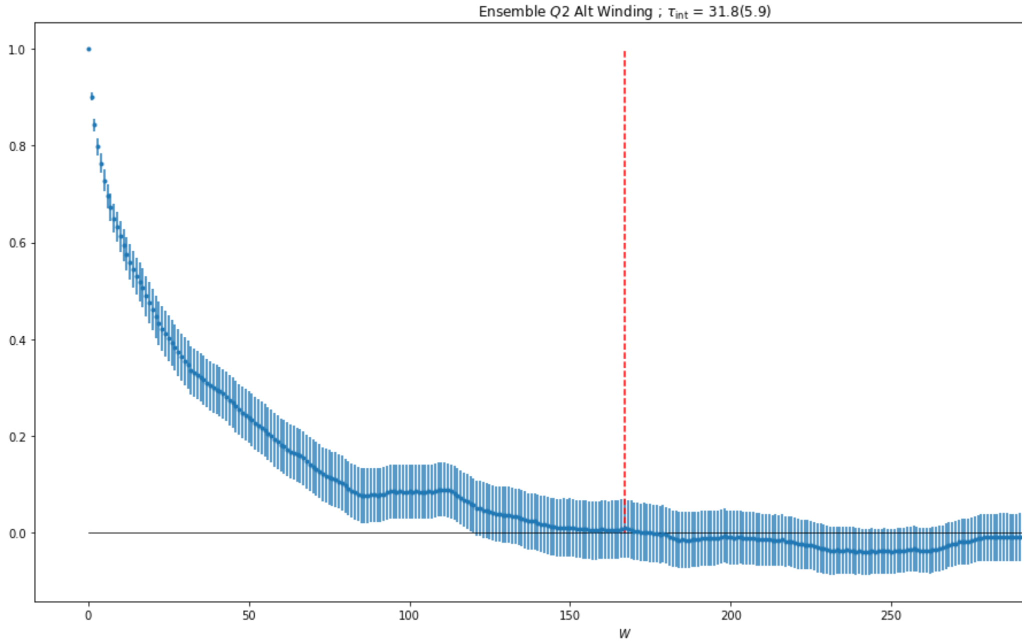
τ_{int,Q^2} = {{printwe(tauQ2altw3, etauQ2altw3)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q2altw3, etauR_Q2altw3)}}

In [363]:

```
corr_q2altw3 = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q2altw3.primary_observable(0,'$Q2$ Alt Winding', [0], ['R0'], [MCtimealtw3.tolist()], [(topcharg
(1,1))

[Q2altw3, eQ2altw3]= corr_q2altw3.vwerr(errinfo=einfo)
[tauQ2altw3, etauQ2altw3] = corr_q2altw3.tauint()
tauQ2altw3 = tauQ2altw3[0][0][0]
etauQ2altw3 = etauQ2altw3[0][0][0]

print(corr_q2altw3.vwerr(plot=True,errinfo=einfo))

printwe(Q2altw3,eQ2altw3)

tauR_Q2altw3 = tauQ2altw3/tauQ2hmc
etauR_Q2altw3 = tauR_Q2altw3 *  np.sqrt( (etauQ2hmc/tauQ2hmc)**2.0 + (etauQ2altw3/tauQ2altw3)**2.0 )
```

executed in 1.13s, finished 18:51:24 2020-11-13



```
[0.8830498775128622, 0.08261842332889216]
0.883(83)
```

6.7 Binning

In [143]:

```
errors = []
ns = []

burn_in = 0 #how many initial states to discard
discard = 1 #pick 1 every discard number of states

gls = (topchargeHMC**2).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorshmc = (errors)
nshmc = (ns)


errors = []
ns = []

gls = (topchargealtw**2).tolist()
```

```
for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsaltw = (errors)
nsaltw = (ns)


errors = []
ns = []

gls = (topchargealti**2).tolist()


for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsalti = (errors)
nsalti = (ns)



errors = []
ns = []

gls = (topchargedoubaltw**2).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsdoubaltw = (errors)
nsdoubaltw = (ns)
```

executed in 2.21s, finished 18:07:15 2020-11-09

```
250.0% completed!!
```

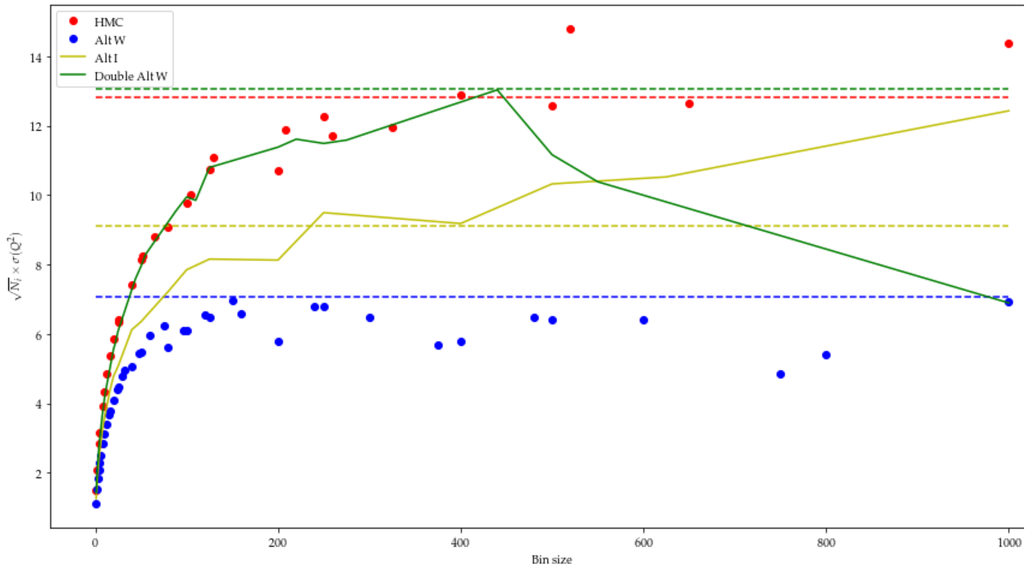
In [144]:

```
plt.plot(nshmc, np.sqrt(Nhmc)*np.array(errorshmc),'ro', label="HMC")
plt.plot([0,1000],[np.sqrt(Nhmc)*eqhmc,np.sqrt(Nhmc)*eqhmc], 'r--')

plt.plot(nsaltw, np.sqrt(Naltw)*np.array(errorsaltw),'bo', label="Alt W")
plt.plot([0,1000],[np.sqrt(Naltw)*eqaltw,np.sqrt(Naltw)*eqaltw], 'b--')

plt.plot(nsalti, np.sqrt(Nalti)*np.array(errorsalti),'y-', label="Alt I")
plt.plot([0,1000],[np.sqrt(Nalti)*eqalti,np.sqrt(Nalti)*eqalti], 'y--')

plt.plot(nsdoubaltw, np.sqrt(Ndoubaltw)*np.array(errorsdoubaltw),'g-', label="Double Alt W")
plt.plot([0,1000],[np.sqrt(Ndoubaltw)*eqdoubaltw,np.sqrt(Ndoubaltw)*eqdoubaltw], 'g--')

plt.legend()

plt.ylabel("$\sqrt{N_i} \\times \sigma (Q^2)$")
plt.xlabel("Bin size")
```

executed in 3.13s, finished 18:07:18 2020-11-09

Out[144]:

```
Text(0.5,0,'Bin size')
```



7 Q^4

Algorithm	Statistics	Q^4	τ_{int}	
HMC	{{Nhmc}}	{{printwe(Q4hmc,eQ4hmc)}}	{{printwe(tauQ4hmc, etauQ4hmc)}}	
Alt Winding	{{Naltw}}	{{printwe(Q4altw,eQ4altw)}}	{{printwe(tauQ4altw, etauQ4altw)}}	{{printwe(tauR_C
Alt Instanton	{{Nalti}}	{{printwe(Q4alti,eQ4alti)}}	{{printwe(tauQ4alti, etauQ4alti)}}	{{printwe(tauR_
Alt Double Winding	{{Ndoubaltw}}	{{printwe(Q4doubaltw,eQ4doubaltw)}}	{{printwe(tauQ4doubaltw, etauQ4doubaltw)}}	{{printwe(tauR_Q4doubaltw
Alt Winding 2	{{Naltw2}}	{{printwe(Q4altw2,eQ4altw2)}}	{{printwe(tauQ4altw2, etauQ4altw2)}}	{{printwe(tauR_Q4a
Alt Winding 3	{{Naltw3}}	{{printwe(Q4altw3,eQ4altw3)}}	{{printwe(tauQ4altw3, etauQ4altw3)}}	{{printwe(tauR_Q4a

7.1 HMC

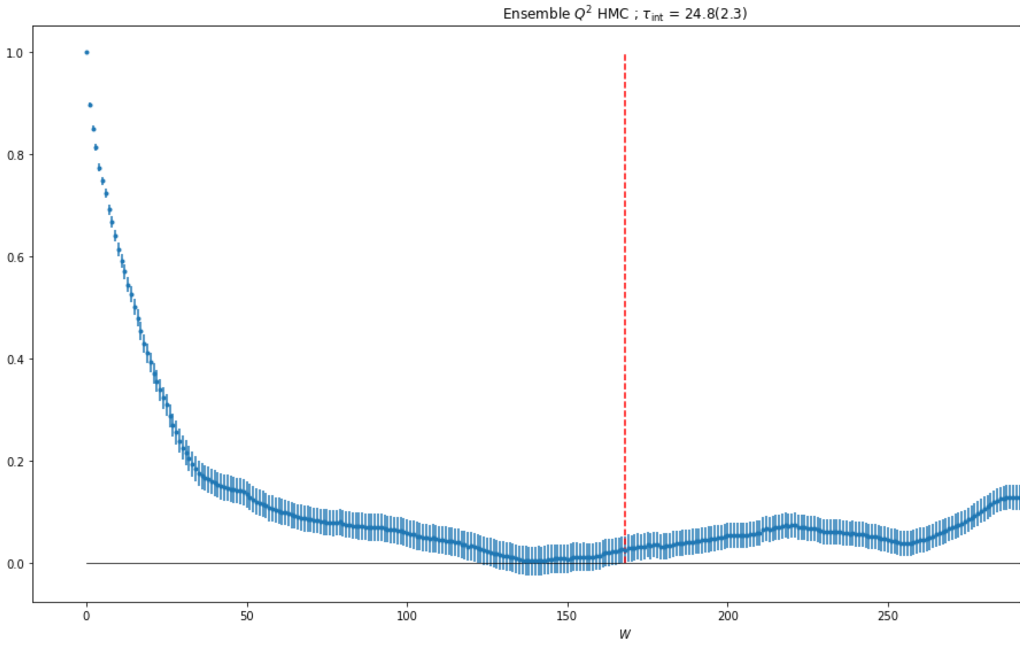
$P =$ {{printwe(Q4hmc, eQ4hmc)}}

$\tau_{int,Q^4} =$ {{printwe(tauQ4hmc, etauQ4hmc)}}

In [269]:

```
corr_q4hmc = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q4hmc.primary_observable(0,'$Q^2$ HMC', [0], ['R0'], [MCtimehmc.tolist()], [(topchargehmc**4).to

[Q4hmc, eQ4hmc]= corr_q4hmc.vwerr(errinfo=einfo)
[tauQ4hmc, etauQ4hmc] = corr_q4hmc.tauint()
tauQ4hmc = tauQ4hmc[0][0][0]
etauQ4hmc = etauQ4hmc[0][0][0]

print(corr_q4hmc.vwerr(plot=True,errinfo=einfo))

printwe(Q4hmc,eQ4hmc)
```

executed in 7.66s, finished 17:36:44 2020-11-13



```
[2.4769506093459195, 0.2868759866872537]
2.48(29)
```

7.2 Alternating Winding

Q^4 = {{printwe(Q4altw, eQ4altw)}}

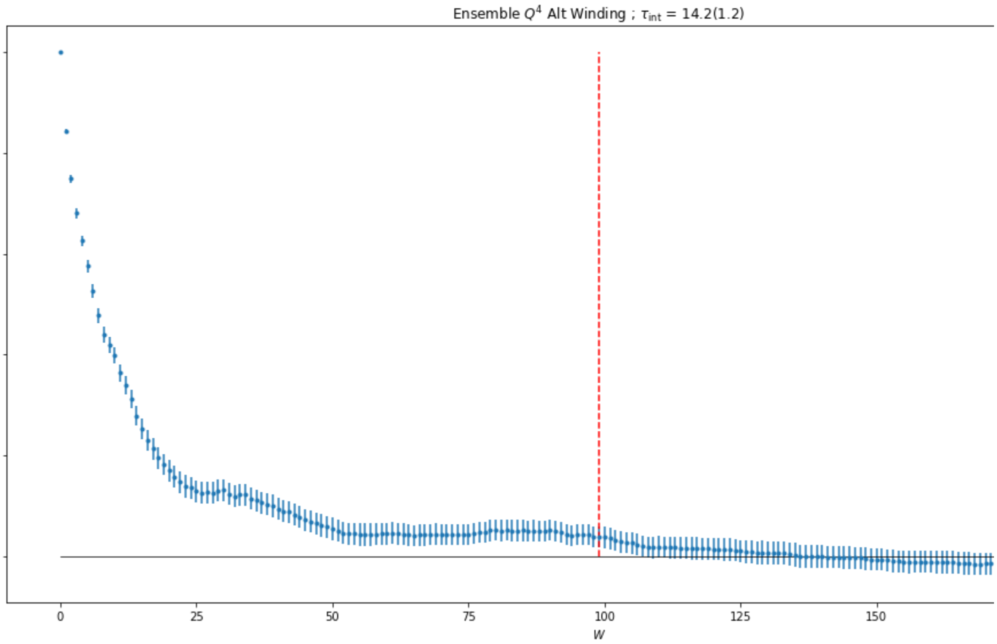
τ_{int,Q^4} = {{printwe(tauQ4altw, etauQ4altw)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q4altw, etauR_Q4altw)}}

In [272]:

```
corr_q4altw = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q4altw.primary_observable(0,'$Q^4$ Alt Winding', [0], ['R0'], [MCtimealtw.tolist()], [(topcharge
(1,1))

[Q4altw, eQ4altw]= corr_q4altw.vwerr(errinfo=einfo)
[tauQ4altw, etauQ4altw] = corr_q4altw.tauint()
tauQ4altw = tauQ4altw[0][0][0]
etauQ4altw = etauQ4altw[0][0][0]

print(corr_q4altw.vwerr(plot=True,errinfo=einfo))

printwe(Q4altw,eQ4altw)

tauR_Q4altw = tauQ4altw/tauQ4hmc
etauR_Q4altw = tauR_Q4altw *  np.sqrt( (etauQ4hmc/tauQ4hmc)**2.0 + (etauQ4altw/tauQ4altw)**2.0 )
```

executed in 4.34s, finished 17:40:20 2020-11-13



```
[2.0625923840122407, 0.21076036159963832]
2.06(21)
```

7.3 Alternating instanton

$Q^4$ = {{printwe(Q4alti, eQ4alti)}}

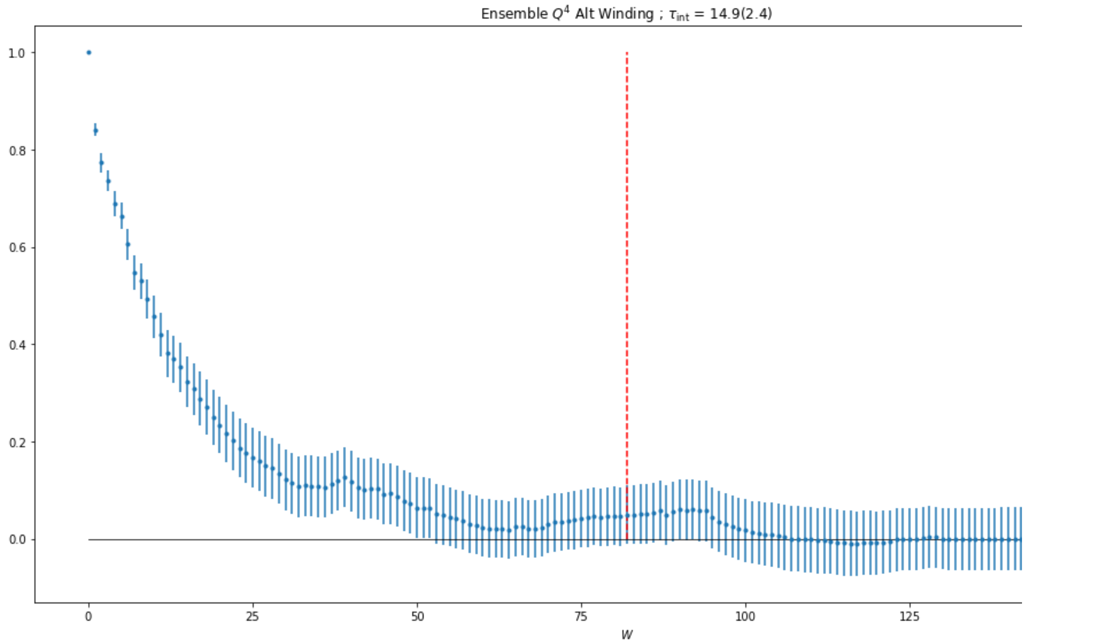
τ_{int,Q^4} = {{printwe(tauQ4alti, etauQ4alti)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q4alti, etauR_Q4alti)}}

In [274]:

```
corr_q4alti = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q4alti.primary_observable(0,'$Q^4$ Alt Winding', [0], ['R0'], [MCtimealti.tolist()], [(topcharge
(1,1))

[Q4alti, eQ4alti]= corr_q4alti.vwerr(errinfo=einfo)
[tauQ4alti, etauQ4alti] = corr_q4alti.tauint()
tauQ4alti = tauQ4alti[0][0][0]
etauQ4alti = etauQ4alti[0][0][0]

print(corr_q4alti.vwerr(plot=True,errinfo=einfo))

printwe(Q4alti,eQ4alti)

tauR_Q4alti = tauQ4alti/tauQ4hmc
etauR_Q4alti = tauR_Q4alti * np.sqrt( (etauQ4hmc/tauQ4hmc)**2.0 + (etauQ4alti/tauQ4alti)**2.0 )
```

executed in 740ms, finished 17:41:05 2020-11-13



```
[2.1666092092307054, 0.44543838673549631
2.17(45)
```

7.4 Double Winding

$Q^4$ = {{printwe(Q4doubaltw, eQ4doubaltw)}}

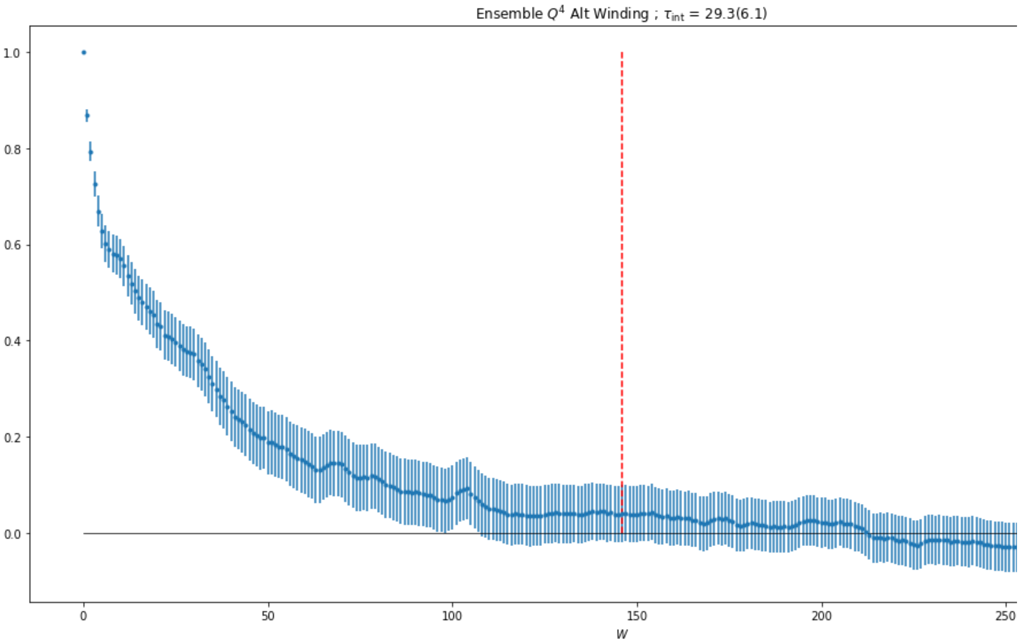
τ_{int,Q^4} = {{printwe(tauQ4doubaltw, etauQ4doubaltw)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q4doubaltw, etauR_Q4doubaltw)}}

In [343]:

```
corr_q4doubaltw = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q4doubaltw.primary_observable(0,'$Q^4$ Alt Winding', [0], ['R0'], [MCtimedoubaltw.tolist()],
[(topchargedoubaltw**4).tolist()], (1,1))

[Q4doubaltw, eQ4doubaltw]= corr_q4doubaltw.vwerr(errinfo=einfo)
[tauQ4doubaltw, etauQ4doubaltw] = corr_q4doubaltw.tauint()
tauQ4doubaltw = tauQ4doubaltw[0][0][0]
etauQ4doubaltw = etauQ4doubaltw[0][0][0]

print(corr_q4doubaltw.vwerr(plot=True,errinfo=einfo))

printwe(Q4doubaltw,eQ4doubaltw)

tauR_Q4doubaltw = tauQ4doubaltw/tauQ4hmc
etauR_Q4doubaltw = tauR_Q4doubaltw * np.sqrt( (etauQ4hmc/tauQ4hmc)**2.0 + (etauQ4doubaltw/tauQ4douba
```

executed in 972ms, finished 18:15:47 2020-11-13



```
[2.9015676860650816, 0.6608553493941995]
2.90(66)
```

7.5 Alternating Winding 2

$Q^4$ = {{printwe(Q4altw2, eQ4altw2)}}

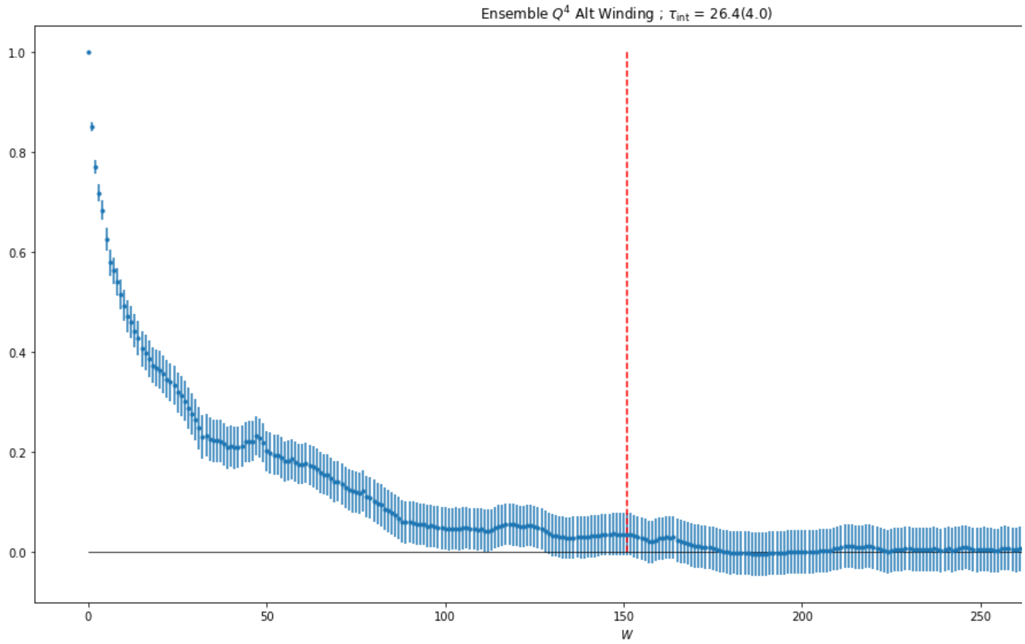
τ_{int,Q^4} = {{printwe(tauQ4altw2, etauQ4altw2)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q4altw2, etauR_Q4altw2)}}

In [364]:

```
corr_q4altw2 = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q4altw2.primary_observable(0,'$Q^4$ Alt Winding', [0], ['R0'], [MCtimealtw2.tolist()],
[(topchargealtw2**4).tolist()], (1,1))

[Q4altw2, eQ4altw2]= corr_q4altw2.vwerr(errinfo=einfo)
[tauQ4altw2, etauQ4altw2] = corr_q4altw2.tauint()
tauQ4altw2 = tauQ4altw2[0][0][0]
etauQ4altw2 = etauQ4altw2[0][0][0]

print(corr_q4altw2.vwerr(plot=True,errinfo=einfo))

printwe(Q4altw2,eQ4altw2)

tauR_Q4altw2 = tauQ4altw2/tauQ4hmc
etauR_Q4altw2 = tauR_Q4altw2 *  np.sqrt( (etauQ4hmc/tauQ4hmc)**2.0 + (etauQ4altw2/tauQ4altw2)**2.0 )
```

executed in 1.32s, finished 18:53:27 2020-11-13



```
[2.419397741376706, 0.46031498784845537]
2.42(46)
```

7.6 Alternating Winding 3

$Q^4$ = {{printwe(Q4altw3, eQ4altw3)}}

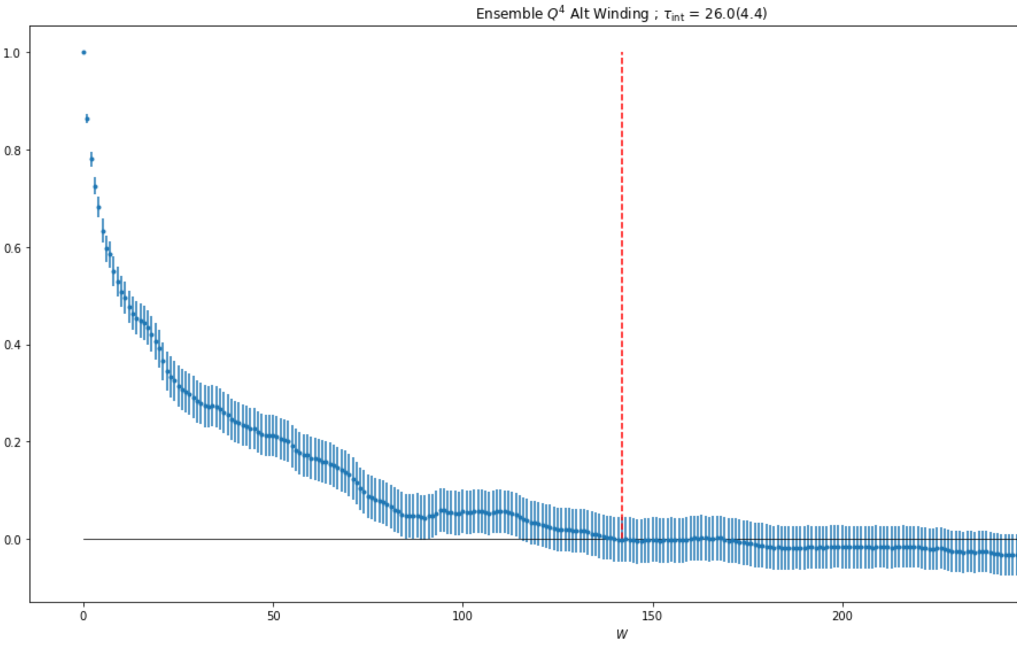
τ_{int,Q^4} = {{printwe(tauQ4altw3, etauQ4altw3)}}

$\tau_{int}/\tau_{int}^{(HMC)}$ = {{printwe(tauR_Q4altw3, etauR_Q4altw3)}}

In [365]:

```
corr_q4altw3 = observa()
einfo = errinfo()
einfo.addEnsemble(0, Stau=1.5)
corr_q4altw3.primary_observable(0,'$Q^4$ Alt Winding', [0], ['R0'], [MCtimealtw3.tolist()],
[(topchargealtw3**4).tolist()], (1,1))

[Q4altw3, eQ4altw3]= corr_q4altw3.vwerr(errinfo=einfo)
[tauQ4altw3, etauQ4altw3] = corr_q4altw3.tauint()
tauQ4altw3 = tauQ4altw3[0][0][0]
etauQ4altw3 = etauQ4altw3[0][0][0]

print(corr_q4altw3.vwerr(plot=True,errinfo=einfo))

printwe(Q4altw3,eQ4altw3)

tauR_Q4altw3 = tauQ4altw3/tauQ4hmc
etauR_Q4altw3 = tauR_Q4altw3 *  np.sqrt( (etauQ4hmc/tauQ4hmc)**2.0 + (etauQ4altw3/tauQ4altw3)**2.0 )
```

executed in 1.32s, finished 18:53:27 2020-11-13



```
[2.488788196015555, 0.495612964165533]
2.49(50)
```

7.7 Binning

In [149]:

```
errors = []
ns = []

burn_in = 0 #how many initial states to discard
discard = 1 #pick 1 every discard number of states

gls = (topchargeHMC**4).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorshmc = (errors)
nshmc = (ns)


errors = []
ns = []

gls = (topchargealtw**4).tolist()
```

```
for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsaltw = (errors)
nsaltw = (ns)


errors = []
ns = []

gls = (topchargealti**4).tolist()


for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsalti = (errors)
nsalti = (ns)



errors = []
ns = []

gls = (topchargedoubaltw**4).tolist()

for n in range(1,1001):

    gls2=[]


    if( (np.double(len(gls))/np.double(n)).is_integer() == True  ):
        ns.append(n);
        for i in range(int(len(gls)/n)):

            med = 0.0
            for j in range(0,n):
                med += gls[n*i+j]

            gls2.append(med/n)
            #gls2 = np.double(med)/np.double(n)
        #gls3.append(gls2)


        value = np.mean(np.asarray(gls2[burn_in::discard]))
        evalue = np.std(np.asarray(gls2[burn_in::discard]))/np.sqrt(len(gls2[burn_in::discard]))
        errors.append(evalue)
        #disc.append(abs(mom3.evalf()-value)/evalue)

    print("{}% completed!".format(100.0*n/400.0), end='\r')

errorsdoubaltw = (errors)
nsdoubaltw = (ns)
```

executed in 2.35s, finished 18:07:50 2020-11-09

```
250.0% completed!!
```

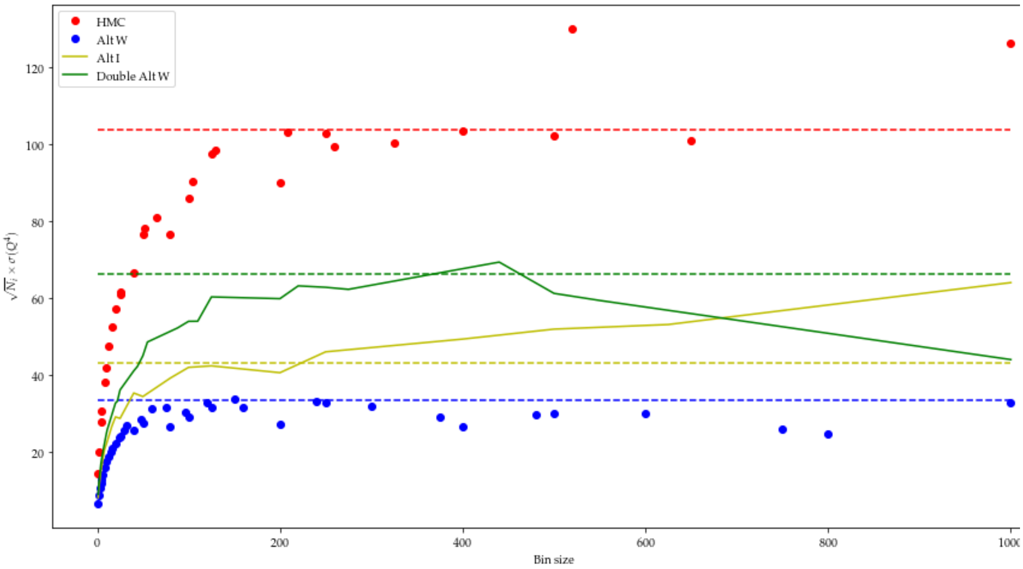
In [150]:

```
plt.plot(nshmc, np.sqrt(Nhmc)*np.array(errorshmc),'ro', label="HMC")
plt.plot([0,1000],[np.sqrt(Nhmc)*eqhmc,np.sqrt(Nhmc)*eqhmc], 'r--')

plt.plot(nsaltw, np.sqrt(Naltw)*np.array(errorsaltw),'bo', label="Alt W")
plt.plot([0,1000],[np.sqrt(Naltw)*eqaltw,np.sqrt(Naltw)*eqaltw], 'b--')

plt.plot(nsalti, np.sqrt(Nalti)*np.array(errorsalti),'y-', label="Alt I")
plt.plot([0,1000],[np.sqrt(Nalti)*eqalti,np.sqrt(Nalti)*eqalti], 'y--')

plt.plot(nsdoubaltw, np.sqrt(Ndoubaltw)*np.array(errorsdoubaltw),'g-', label="Double Alt W")
plt.plot([0,1000],[np.sqrt(Ndoubaltw)*eqdoubaltw,np.sqrt(Ndoubaltw)*eqdoubaltw], 'g--')

plt.legend()

plt.ylabel("$\sqrt{N_i} \\times \sigma (Q^4)$")
plt.xlabel("Bin size")
```

executed in 2.94s, finished 18:07:53 2020-11-09

Out[150]:

```
Text(0.5,0,'Bin size')
```



In [ ]: