

**CSC 226 SPRING 2017**  
**ALGORITHMS AND DATA STRUCTURES II**  
**ASSIGNMENT 3**  
**UNIVERSITY OF VICTORIA**

1. Consider a union-find implementation that uses the same basic strategy as weighted quick-union but keeps track of tree height and always links the shorter tree to the taller one. Prove a logarithmic upper bound on the height of trees for  $N$  sites for this scheme.
2. Give a proof of correctness for Algorithm 4.10, for computing shortest paths in edge-weighted Directed Acyclic Graphs (DAGS). Use proof by contradiction technique.
3. If the PQ is implemented as an unsorted sequence, show that Dijkstra's algorithm runs in  $O(n^2)$  time. For what type of graphs is this implementation preferred?
4. If at the end of the execution of Bellman-Ford algorithm, there is an edge  $(u, z)$  that can be potentially relaxed (that is,  $D[u] + w(u, z) < D[z]$ , then show that the input digraph  $G$  contains a negative-weight cycle.