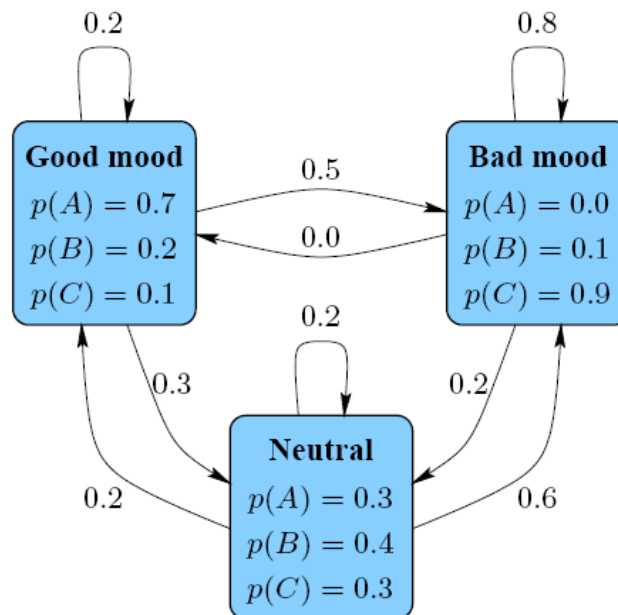


A school teacher gives one assignment daily. He has three different types of assignments: A, B & C. Assignment of type “A” takes about 5 minutes to complete, assignment of type “B” takes about 1 hour to complete, and assignment of type “C” takes about 3 hours to complete.

He has three moods: Good, Neutral and Bad. His assignments are usually dependant on his moods and he usually gives assignments of type “C” if he is in a bad mood. His moods only change overnight and he remains in the same mood throughout the day. His moods are not openly revealed however they can guess his mood by the type of assignments which he gives on a particular day.

Given below is a Markov model of the whole process.



1. Which are the hidden states and which are the visible states for this problem. Make matrices of transition probabilities and of visible signal emission probabilities.
2. One week, the teacher gave the following homework assignments:  
Mon – Type A, Tues – Type C, Wed – Type B, Thurs – Type A, Fri – Type C.
  - a. What is the probability of this sequence
  - b. What was, most likely, his mood curve during that week

**Solution:**

1. The hidden states are moods: Good Mood, Bad Mood & Neutral

The visible states are types of assignments: Type A, B & C

The hidden state transition probability matrix is

	Good Mood	Bad Mood	Neutral
Good Mood	0.2	0.5	0.3
Bad Mood	0	0.8	0.2
Neutral	0.2	0.6	0.2

The visible signal emission probability matrix is

	Type A	Type B	Type C
Good Mood	0.7	0.2	0.1
Bad Mood	0	0.1	0.9
Neutral	0.3	0.4	0.3

2. Suppose the teacher was in a good mood on the last day of the previous week (i.e. assume initial state to be Good Mood). The probability of the sequence can be calculated as:

		A	C	B	A	C
Good	1	x11	x12	x13	x14	x15
Bad	0	x21	x22	x23	x24	x25
Neutral	0	x31	x32	x33	x34	x35

Where

$$x11 = 0.2 \times 0.7 = 0.14$$

$$x21 = 0.5 \times 0 = 0$$

$$x31 = 0.3 \times 0.3 = 0.09$$

$$x12 = 0.14 \times 0.2 \times 0.1 + 0 \times 0 \times 0.1 + 0.09 \times 0.2 \times 0.1 = 0.0046$$

$$x22 = 0.14 \times 0.5 \times 0.9 + 0 \times 0.8 \times 0.9 + 0.09 \times 0.6 \times 0.9 = 0.1116$$

$$x32 = 0.14 \times 0.3 \times 0.3 + 0 \times 0.2 \times 0.3 + 0.09 \times 0.2 \times 0.3 = 0.018$$

$$x13 = 0.0046 \times 0.2 \times 0.2 + 0.1116 \times 0 \times 0.2 + 0.018 \times 0.2 \times 0.2 \\ = 0.000184 + 0 + 0.00072 = 0.000904$$

$$x23 = 0.0046 \times 0.5 \times 0.1 + 0.1116 \times 0.8 \times 0.1 + 0.018 \times 0.6 \times 0.1 \\ = 0.00023 + 0.008928 + 0.00144 = 0.010598$$

$$x33 = 0.0046 \times 0.3 \times 0.4 + 0.1116 \times 0.2 \times 0.4 + 0.018 \times 0.2 \times 0.4 \\ = 0.000552 + 0.008928 + 0.00144 = 0.01092$$

$$x14 = 0.000904 \times 0.2 \times 0.7 + 0.010598 \times 0 \times 0.7 + 0.01092 \times 0.2 \times 0.7 \\ = 0.0001265 + 0 + 0.0015288 = 0.0016553$$

$$x24 = 0.000904 \times 0.5 \times 0 + 0.010598 \times 0.8 \times 0 + 0.01092 \times 0.6 \times 0 = 0$$

$$x34 = 0.000904 \times 0.3 \times 0.3 + 0.010598 \times 0.2 \times 0.3 + 0.01092 \times 0.2 \times 0.3$$

$$= 0.0000813 + 0.0006358 + 0.0006552 = 0.0013723$$

$$\begin{aligned} x_{15} &= 0.0016553 \times 0.2 \times 0.1 + 0 \times 0 \times 0.1 + 0.0013723 \times 0.2 \times 0.1 = \\ &= 0.0000331 + 0 + 0.0000274 = 0.0000605 \end{aligned}$$

$$\begin{aligned} x_{25} &= 0.0016553 \times 0.5 \times 0.9 + 0 \times 0.8 \times 0.9 + 0.0013723 \times 0.6 \times 0.9 = \\ &= 0.0007448 + 0 + 0.0007409 = 0.0014857 \end{aligned}$$

$$\begin{aligned} x_{35} &= 0.0016553 \times 0.3 \times 0.3 + 0 \times 0.2 \times 0.3 + 0.0013723 \times 0.2 \times 0.3 = \\ &= 0.0001489 + 0 + 0.0000823 = 0.0002312 \end{aligned}$$

Hence the probability of this sequence is: 0.0014857

The following sequence has the highest probability for teacher's mood:

Good – Bad – Neutral – Good – Bad