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(All dimensions taken in mm)

Homework-04

• **Updated DH Table** :—

Link, i	a	Alpha α	Theta θ	d
1	0	θ_1^*	$\pi/2$	d_1
2	0	θ_2^*	$-\pi/2$	0
3	a_3	$\theta_3^\dagger$	$-\pi/2$	d_3
4	$-a_3$	θ_4^*	$\pi/2$	0
5	0	θ_5^*	$\pi/2$	d_5
6	a_3	θ_6^*	$-\pi/2$	0
n (or 7)	0	θ_7^*	0	$-(d_7+100)$

* — variable

Changes to DH Table :— • $^\dagger\theta_3 = 0$; (Since the joint will be locked)

• For n, d is mentioned as $-d_7-100$ to include end-effector i.e., the pen length.

• Remaining table remains the same

Used the following equation to find Homogeneous transformation between Links 'i' & 'i-1'

Homogeneous Transformation Matrix between Links 'i' and 'i-1' :

$$T_{i-1}^{i} = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the above matrix, we can use method 2 for finding the Jacobian

Refer Jupyter terminal for the Jacobian matrix

For method 2 the partial derivatives need to be taken for the fourth row of 'n' frame matrix wrt base frame.

The matrix required can be calculated as below :—

$${}^{0}_{1}T = {}^{0}_{1}T$$

$${}^{0}_{2}T = {}^{0}_{1}T\,{}^{1}_{2}T$$

$${}^{0}_{3}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T$$

$${}^{0}_{4}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T$$

$${}^{0}_{5}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T$$

$${}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T$$

$${}^{0}_{7}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T\,{}^{6}_{7}T$$

${}^{0}_{1}T, {}^{1}_{2}T, {}^{2}_{3}T, {}^{3}_{4}T, {}^{4}_{5}T, {}^{5}_{6}T, {}^{6}_{7}T$ are obtained from D-H table using the formula given earlier.

The 4th row of ${}^{0}_{7}T$ will be equalto column vector ${}^{0}X_{p}$

The 3rd row of ${}^0_1T$, ${}^0_2T$, ${}^0_3T$, ${}^0_4T$, ${}^0_5T$, ${}^0_6T$, ${}^0_7T$ will be
${}^0Z_1$, ${}^0Z_2$, ${}^0Z_3$, ${}^0Z_4$, ${}^0Z_5$, ${}^0Z_6$, ${}^0Z_7$ column vectors

Note :- Padding of '1' is removed above column vectors

The manipulator Jacobian will be

$$ {}^0J = \begin{bmatrix} \frac{\partial {}^0X_p}{\partial \theta_1} & \frac{\partial {}^0X_p}{\partial \theta_2} & \frac{\partial {}^0X_p}{\partial \theta_3} & \frac{\partial {}^0X_p}{\partial \theta_4} & \frac{\partial {}^0X_p}{\partial \theta_5} & \frac{\partial {}^0X_p}{\partial \theta_6} & \frac{\partial {}^0X_p}{\partial \theta_7} \\ {}^0Z_1 & {}^0Z_2 & {}^0Z_3 & {}^0Z_4 & {}^0Z_5 & {}^0Z_6 & {}^0Z_7 \end{bmatrix} $$

This is a 6 x 7 matrix

Since Joint 3 is locked we will remove the 3rd row from above Jacobian to make it square

$$ {}^0J_{\text{Joint3 locked}} = \begin{bmatrix} \frac{\partial {}^0X_p}{\partial \theta_1} & \frac{\partial {}^0X_p}{\partial \theta_2} & \frac{\partial {}^0X_p}{\partial \theta_4} & \frac{\partial {}^0X_p}{\partial \theta_5} & \frac{\partial {}^0X_p}{\partial \theta_6} & \frac{\partial {}^0X_p}{\partial \theta_7} \\ {}^0Z_1 & {}^0Z_2 & {}^0Z_4 & {}^0Z_5 & {}^0Z_6 & {}^0Z_7 \end{bmatrix} $$

The element's mentioned in above matrix are as below :-

$$^0Z_1 = \begin{bmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{bmatrix}$$

$$^0Z_2 = \begin{bmatrix} -\sin(\theta_2)\cos(\theta_1) \\ -\sin(\theta_1)\sin(\theta_2) \\ \cos(\theta_2) \end{bmatrix}$$

$$^0Z_4 = \begin{bmatrix} -\sin(\theta_2)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2) \\ -\sin(\theta_1)\sin(\theta_2)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_4)\cos(\theta_2) \\ \sin(\theta_2)\sin(\theta_4) + \cos(\theta_2)\cos(\theta_4) \end{bmatrix}$$

$$^0Z_5 = \begin{bmatrix} (\sin(\theta_2)\sin(\theta_4)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_4))\sin(\theta_5) + \sin(\theta_1)\cos(\theta_5) \\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_4) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_4))\sin(\theta_5) - \cos(\theta_1)\cos(\theta_5) \\ (\sin(\theta_2)\cos(\theta_4) - \sin(\theta_4)\cos(\theta_2))\sin(\theta_5) \end{bmatrix}$$

$$^0Z_6 = \begin{bmatrix} -((\sin(\theta_2)\sin(\theta_4)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_4))\cos(\theta_5) - \sin(\theta_1)\sin(\theta_5))\sin(\theta_6) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2))\cos(\theta_6) \\ -((\sin(\theta_1)\sin(\theta_2)\sin(\theta_4) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_4))\cos(\theta_5) + \sin(\theta_5)\cos(\theta_1))\sin(\theta_6) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_4)\cos(\theta_2))\cos(\theta_6) \\ (\sin(\theta_2)\sin(\theta_4) + \cos(\theta_2)\cos(\theta_4))\cos(\theta_6) - (\sin(\theta_2)\cos(\theta_4) - \sin(\theta_4)\cos(\theta_2))\sin(\theta_6)\cos(\theta_5) \end{bmatrix}$$

$$ {}^{0}Z_7 = $$

$$\begin{bmatrix} -\left(\left(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)-\sin(\theta_1)\sin(\theta_5)\right)\sin(\theta_6)+\left(-\sin(\theta_2)\cos(\theta_1)\cos(\theta_4)+\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\right)\cos(\theta_6) \\ -\left(\left(\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)+\sin(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)+\sin(\theta_5)\cos(\theta_1)\right)\sin(\theta_6)+\left(-\sin(\theta_1)\sin(\theta_2)\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\right)\cos(\theta_6) \\ \left(\sin(\theta_2)\sin(\theta_4)+\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_6)-\left(\sin(\theta_2)\cos(\theta_4)-\sin(\theta_4)\cos(\theta_2)\right)\sin(\theta_6)\cos(\theta_5) \end{bmatrix}$$

Recommended to check in code since the matrix is too large to paste a screenshot here

$$ {}^{0}X_p = $$

```
[a3·((sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ4))·cos(θ5) - sin(θ1)·sin
|
|a3·((sin(θ1)·sin(θ2)·sin(θ4) + sin(θ1)·cos(θ2)·cos(θ4))·cos(θ5) + sin(θ5)·cos
|
[                                                                           a3

(θ5))·cos(θ6) + a3·(-sin(θ2)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·cos(θ2))·sin(θ6

(θ1))·cos(θ6) + a3·(-sin(θ1)·sin(θ2)·cos(θ4) + sin(θ1)·sin(θ4)·cos(θ2))·sin(θ6

·(sin(θ2)·sin(θ4) + cos(θ2)·cos(θ4))·sin(θ6) + a3·(sin(θ2)·cos(θ4) - sin(θ4)·c

) - a3·sin(θ2)·sin(θ4)·cos(θ1) - a3·cos(θ1)·cos(θ2)·cos(θ4) + a3·cos(θ1)·cos(θ

) - a3·sin(θ1)·sin(θ2)·sin(θ4) - a3·sin(θ1)·cos(θ2)·cos(θ4) + a3·sin(θ1)·cos(θ

os(θ2))·cos(θ5)·cos(θ6) - a3·sin(θ2)·cos(θ4) + a3·sin(θ2) + a3·sin(θ4)·cos(θ2)

2) - d3·sin(θ2)·cos(θ1) + d5·(-sin(θ2)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·cos(θ

2) - d3·sin(θ1)·sin(θ2) + d5·(-sin(θ1)·sin(θ2)·cos(θ4) + sin(θ1)·sin(θ4)·cos(θ

 + d1 + d3·cos(θ2) + d5·(sin(θ2)·sin(θ4) + cos(θ2)·cos(θ4)) + (-d7 - 100)·((si

2)) + (-d7 - 100)·(-((sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ4))·cos(θ

2)) + (-d7 - 100)·(-((sin(θ1)·sin(θ2)·sin(θ4) + sin(θ1)·cos(θ2)·cos(θ4))·cos(θ

n(θ2)·sin(θ4) + cos(θ2)·cos(θ4))·cos(θ6) - (sin(θ2)·cos(θ4) - sin(θ4)·cos(θ2))

5) - sin(θ1)·sin(θ5))·sin(θ6) + (-sin(θ2)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·co

5) + sin(θ5)·cos(θ1))·sin(θ6) + (-sin(θ1)·sin(θ2)·cos(θ4) + sin(θ1)·sin(θ4)·co

·sin(θ6)·cos(θ5))

s(θ2))·cos(θ6))]
               |
s(θ2))·cos(θ6))|
               |
               ]
```

$$ \frac{\partial\, {}^{0}X_p}{\partial \theta_1} = $$

```
[a3·((-sin(θ1)·sin(θ2)·sin(θ4) - sin(θ1)·cos(θ2)·cos(θ4))·cos(θ5) - sin(θ5)·co
|
|a3·((sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ4))·cos(θ5) - sin(θ1)·sin
|
[

s(θ1))·cos(θ6) + a3·(sin(θ1)·sin(θ2)·cos(θ4) - sin(θ1)·sin(θ4)·cos(θ2))·sin(θ6

(θ5))·cos(θ6) + a3·(-sin(θ2)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·cos(θ2))·sin(θ6


) + a3·sin(θ1)·sin(θ2)·sin(θ4) + a3·sin(θ1)·cos(θ2)·cos(θ4) - a3·sin(θ1)·cos(θ

) - a3·sin(θ2)·sin(θ4)·cos(θ1) - a3·cos(θ1)·cos(θ2)·cos(θ4) + a3·cos(θ1)·cos(θ


2) + d3·sin(θ1)·sin(θ2) + d5·(sin(θ1)·sin(θ2)·cos(θ4) - sin(θ1)·sin(θ4)·cos(θ2

2) - d3·sin(θ2)·cos(θ1) + d5·(-sin(θ2)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·cos(θ

      0

)) + (-d7 - 100)·(-((-sin(θ1)·sin(θ2)·sin(θ4) - sin(θ1)·cos(θ2)·cos(θ4))·cos(θ

2)) + (-d7 - 100)·(-((sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ4))·cos(θ


5) - sin(θ5)·cos(θ1))·sin(θ6) + (sin(θ1)·sin(θ2)·cos(θ4) - sin(θ1)·sin(θ4)·cos

5) - sin(θ1)·sin(θ5))·sin(θ6) + (-sin(θ2)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·co


(θ2))·cos(θ6)) ]
               |
s(θ2))·cos(θ6))|
               |
               ]
```

$$ \frac{\partial\, {}^{0}X_p}{\partial \theta_2} = $$

Recommended to check in code since the matrix is too large to paste a screenshot here

```
[a3·(-sin(θ2)·sin(θ4)·cos(θ1) - cos(θ1)·cos(θ2)·cos(θ4))·sin(θ6) + a3·(-sin(θ2
|
|a3·(-sin(θ1)·sin(θ2)·sin(θ4) - sin(θ1)·cos(θ2)·cos(θ4))·sin(θ6) + a3·(-sin(θ1
|
[                                                        a3·(sin(θ2)·sin(θ4) +

)·cos(θ1)·cos(θ4) + sin(θ4)·cos(θ1)·cos(θ2))·cos(θ5)·cos(θ6) + a3·sin(θ2)·cos(

)·sin(θ2)·cos(θ4) + sin(θ1)·sin(θ4)·cos(θ2))·cos(θ5)·cos(θ6) + a3·sin(θ1)·sin(

 cos(θ2)·cos(θ4))·cos(θ5)·cos(θ6) + a3·(-sin(θ2)·cos(θ4) + sin(θ4)·cos(θ2))·si

θ1)·cos(θ4) - a3·sin(θ2)·cos(θ1) - a3·sin(θ4)·cos(θ1)·cos(θ2) - d3·cos(θ1)·cos

θ2)·cos(θ4) - a3·sin(θ1)·sin(θ2) - a3·sin(θ1)·sin(θ4)·cos(θ2) - d3·sin(θ1)·cos

n(θ6) - a3·sin(θ2)·sin(θ4) - a3·cos(θ2)·cos(θ4) + a3·cos(θ2) - d3·sin(θ2) + d5

(θ2) + d5·(-sin(θ2)·sin(θ4)·cos(θ1) - cos(θ1)·cos(θ2)·cos(θ4)) + (-d7 - 100)·(

(θ2) + d5·(-sin(θ1)·sin(θ2)·sin(θ4) - sin(θ1)·cos(θ2)·cos(θ4)) + (-d7 - 100)·(

·(-sin(θ2)·cos(θ4) + sin(θ4)·cos(θ2)) + (-d7 - 100)·(-(sin(θ2)·sin(θ4) + cos(θ

(-sin(θ2)·sin(θ4)·cos(θ1) - cos(θ1)·cos(θ2)·cos(θ4))·cos(θ6) - (-sin(θ2)·cos(θ

(-sin(θ1)·sin(θ2)·sin(θ4) - sin(θ1)·cos(θ2)·cos(θ4))·cos(θ6) - (-sin(θ1)·sin(θ

2)·cos(θ4))·sin(θ6)·cos(θ5) + (-sin(θ2)·cos(θ4) + sin(θ4)·cos(θ2))·cos(θ6))

1)·cos(θ4) + sin(θ4)·cos(θ1)·cos(θ2))·sin(θ6)·cos(θ5))]
                                                      |
2)·cos(θ4) + sin(θ1)·sin(θ4)·cos(θ2))·sin(θ6)·cos(θ5))|
                                                      |
                                                      ]
```

$$\frac{\partial X_p}{\partial \theta_4} =$$

Recommended to check in code since the matrix is too large to paste a screenshot here

```
⎡a₃·(sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·sin(θ₆) + a₃·(sin(θ₂)·
⎢
⎢a₃·(sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)·cos(θ₂)·cos(θ₄))·sin(θ₆) + a₃·(sin(θ₁)·
⎢
⎣                                                   a₃·(-sin(θ₂)·sin(θ₄) - cos(θ₂)·

cos(θ₁)·cos(θ₄) - sin(θ₄)·cos(θ₁)·cos(θ₂))·cos(θ₅)·cos(θ₆) - a₃·sin(θ₂)·cos(θ₁

sin(θ₂)·cos(θ₄) - sin(θ₁)·sin(θ₄)·cos(θ₂))·cos(θ₅)·cos(θ₆) - a₃·sin(θ₁)·sin(θ₂

cos(θ₄))·cos(θ₅)·cos(θ₆) + a₃·(sin(θ₂)·cos(θ₄) - sin(θ₄)·cos(θ₂))·sin(θ₆) + a₃

)·cos(θ₄) + a₃·sin(θ₄)·cos(θ₁)·cos(θ₂) + d₅·(sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)

)·cos(θ₄) + a₃·sin(θ₁)·sin(θ₄)·cos(θ₂) + d₅·(sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)

·sin(θ₂)·sin(θ₄) + a₃·cos(θ₂)·cos(θ₄) + d₅·(sin(θ₂)·cos(θ₄) - sin(θ₄)·cos(θ₂))

·cos(θ₂)·cos(θ₄)) + (-d₇ - 100)·((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·co

·cos(θ₂)·cos(θ₄)) + (-d₇ - 100)·((sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)·cos(θ₂)·co

 + (-d₇ - 100)·(-(-sin(θ₂)·sin(θ₄) - cos(θ₂)·cos(θ₄))·sin(θ₆)·cos(θ₅) + (sin(θ

s(θ₄))·cos(θ₆) - (sin(θ₂)·cos(θ₁)·cos(θ₄) - sin(θ₄)·cos(θ₁)·cos(θ₂))·sin(θ₆)·c

s(θ₄))·cos(θ₆) - (sin(θ₁)·sin(θ₂)·cos(θ₄) - sin(θ₁)·sin(θ₄)·cos(θ₂))·sin(θ₆)·c

₂)·cos(θ₄) - sin(θ₄)·cos(θ₂))·cos(θ₆))

os(θ₅))⎤
       ⎥
os(θ₅))⎥
       ⎥
       ⎦
```

$$\frac{\partial X_p}{\partial \theta_5} =$$

Recommended to check in code since the matrix is too large to paste a screenshot here

$$\begin{bmatrix} a_3\left(-\left(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\sin(\theta_5)-\sin(\theta_1)\cos(\theta_5)\right)\cos(\theta_6)-(-d_7-100)\left(-\left(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\sin(\theta_5)-\sin(\theta_1)\cos(\theta_5)\right)\sin(\theta_6) \\ a_3\left(-\left(\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)+\sin(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\sin(\theta_5)+\cos(\theta_1)\cos(\theta_5)\right)\cos(\theta_6)-(-d_7-100)\left(-\left(\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)+\sin(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\sin(\theta_5)+\cos(\theta_1)\cos(\theta_5)\right)\sin(\theta_6) \\ -a_3\left(\sin(\theta_2)\cos(\theta_4)-\sin(\theta_4)\cos(\theta_2)\right)\sin(\theta_5)\cos(\theta_6)+(-d_7-100)\left(\sin(\theta_2)\cos(\theta_4)-\sin(\theta_4)\cos(\theta_2)\right)\sin(\theta_5)\sin(\theta_6) \end{bmatrix}$$

$$\frac{\partial x_p}{\partial \theta_6} =$$

Recommended to check in code since the matrix is too large to paste a screenshot here

```
⎡-a₃·((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅) - sin(θ₁)·si
⎢
⎢-a₃·((sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅) + sin(θ₅)·co
⎢
⎣                                                      a₃·(sin(θ₂)·sin(θ₄) + co

n(θ₅))·sin(θ₆) + a₃·(-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ₂))·cos(θ

s(θ₁))·sin(θ₆) + a₃·(-sin(θ₁)·sin(θ₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·cos(θ₂))·cos(θ

s(θ₂)·cos(θ₄))·cos(θ₆) - a₃·(sin(θ₂)·cos(θ₄) - sin(θ₄)·cos(θ₂))·sin(θ₆)·cos(θ₅

₆) + (-d₇ - 100)·(-((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅

₆) + (-d₇ - 100)·(-((sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅

) + (-d₇ - 100)·(-(sin(θ₂)·sin(θ₄) + cos(θ₂)·cos(θ₄))·sin(θ₆) - (sin(θ₂)·cos(θ

) - sin(θ₁)·sin(θ₅))·cos(θ₆) - (-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos

) + sin(θ₅)·cos(θ₁))·cos(θ₆) - (-sin(θ₁)·sin(θ₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·cos

₄) - sin(θ₄)·cos(θ₂))·cos(θ₅)·cos(θ₆))

(θ₂))·sin(θ₆))⎤
              ⎥
(θ₂))·sin(θ₆))⎥
              ⎥
              ⎦
```

$$\frac{\partial x_p}{\partial \theta_7} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Points S of circle in parametric form can be written as; wrt base frame

$x = 0.679 \times 1000 = 679\ mm$

$y = 0.1 \sin\theta \times 1000 = 100 \sin\theta$

$z = (0.725 + 0.1 \cos\theta) \times 1000$

$= 725 + 100 \cos\theta$

$\dot{x} = 0$

$\dot{y} = 0.1 \dot{\theta} \cos\theta$

$\dot{z} = 0 - 0.1 \dot{\theta} \sin\theta = -0.1 \dot{\theta} \sin\theta$

$\dot{\theta} = \frac{2\pi}{T}$; T = Total time to complete one 360° revolution

Given T = 5 seconds

$\dot{\theta} = \frac{2\pi}{5}$

$\dot{x} = 0$

$\dot{y} = \frac{0.1 \times 2\pi}{5} \cos\theta = 0.04\pi \cos\theta \times 1000$

$\dot{y} = 40\pi \cos\theta$

$\dot{z} = \frac{-0.1 \times 2\pi}{5} \sin\theta = -0.04\pi \sin\theta \times 1000$

$\dot{z} = -40\pi \sin\theta$

x dot column matrix can be written as

$$x\ dot = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} & \dot{\psi} & \dot{\theta} & \dot{\phi} \end{bmatrix}$$

$\dot{\psi} = \dot{\theta} = \dot{\phi} = 0$ (Given)

$$x\ dot = \begin{bmatrix} 0 \\ 0.04\pi\cos\theta \times 1000 \\ -0.04\pi\sin\theta \times 1000 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 40\pi\cos\theta \\ -40\pi\sin\theta \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

qdot, qinitial column vector is given

$$q_{initial} = \begin{bmatrix} 0 \\ 0 \\ \pi/2 \\ 0 \\ \pi \\ 0 \end{bmatrix}$$

$$qdot = \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \\ \dot{q}_7 \end{bmatrix}$$

For running the integration loop

Given:-

$$\dot{q} = J^{-1} \cdot (xdot)$$

$$q_{next} = q_{current} + \dot{q}_{next} \Delta t$$

We know the $\dot{x}$ & $\dot{y}$ & $\dot{z}$ at various points of time throughout the circle.

So I will divide the rotation of the ballpoint θ/theta in 1000 parts.

So

$$\dot{x} = 0$$

$$\dot{y} = 40\pi \cos\left(\frac{2\pi}{1000} * i\right)$$

$$\dot{z} = -40\pi \sin\left(\frac{2\pi}{1000} * i\right)$$

i will be varied from 0 to 1000 for 0 to 2π rotation

$$\theta = \frac{2\pi * i}{1000}$$

For updating the values of q or Joint angles

$$q_{next} = q_{current} + \dot{q}_{next} \Delta t$$

$$\Delta t = \frac{T}{N} = \frac{5}{N}$$

N is the number of iterations

$$\Delta t = \frac{5}{1000} = 0.005$$

Integrator loop