This Test has 14 attempts. For information on editing questions, click **More Help** below.

Test Canvas: Midterm Examination

The Test Canvas lets you add, edit, and reorder questions, as well as review a test. More Help

Question Settings

You can edit, delete, or change the point values of test questions on this page. If necessary, test attempts will be regraded after you submit your changes.

Description	
Instructions	
Total Questions	7
Total Points	110
Number of Attempts	14

Select: All None Select by Type: - Question Type -

Delete and Regrade Points Update and Regrade Hide Question Details

1. File Response: Question 1:

Points: 20

Question

(20 points) Suppose that the percentage return of a stock $(r_n)_{n\geq 0}$ is modeled by an auto regressive process

$$r_{n+1} = 1 + r_n + \varepsilon_n$$

where the noise ε_n's are independent and have standard normal distribution $\mathcal{N}(0,1)$.

Given that $r_0 = 2, r_1 = 1.5, r_2 = 1.8$.

(a) Compute the probability that the return at time $n = 3$ is greater than 3.

(b) Find the conditional distribution of the return at time $n = 4$.

2. File Response: Question 2:

Points: 10

Question

(10 points) Suppose that the random variables X_i are independent and have the same distribution with zero mean $E(X_i) = 0$ and finite variance $\sigma^2 = Var(X_i)$.
Let

$$S_n = X_1 + \cdots + X_n.$$

and

$$M_n = S_n^2 - n\sigma^2$$

Prove that $(M_n)_{0 \le n \le N}$ is a martingale with respect to the filtration $(\mathscr{F}_n)_{n \ge 0}$ where $\mathscr{F}_n = \sigma(X_1, \ldots, X_n)$ is the σ - algebra generated by $X_1, \ldots, X_n$.

3. File Response: Question 3:

Points: 20

Question

(20 points) Suppose that $(N_t)_{t \ge 0}$ is a Poisson process with rate $\lambda = 2$. Compute

(a) $P(N_2 = 7 | N_1 = 3)$.

(b) $P(N_1 = 3 | N_2 = 7)$.

4. File Response: Question 4:

Points: 20

Question

(20 points) Customers arrive at a bakery according to a Poisson process with rate 5 per hour. Given that two customers arrived in the first 30 minutes, find the probability that

(a) both arrived in the first 15 minutes.

(b) at least one arrived in the first 15 minutes.

5. File Response: Question 5:

Points: 10

Question

(10 points) Consider a Poisson process $(N_t)_{t \ge 0}$ with rate $\lambda = 1$. Let $S_1, S_2, \ldots$ be arrival times of the Poisson process. Compute

$$E\left(\sum_{i=1}^{N_t} S_i^2.\right)$$

6. File Response: Question 6:

Points: 20

Question

(20 points) Consider a particular utility stock whose daily price is recorded as increased, decreased or unchanged . The probability that it increases or decreases in price depends only on the result of the preceding day's trading. The sequence of stock price $(X_n)_{n\geq 0}$ forms a Markov chain with the following transition probability

$$P = \begin{array}{c} \\ \textit{increase} \\ \textit{decrease} \\ \textit{unchange} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{increase} & \textit{decrease} & \textit{unchange} \end{array} \\ \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.3 & 0.5 & 0.2 \\ 0.3 & 0.1 & 0.6 \end{bmatrix} \end{array}$$

The initial distribution of stock price is

$$\pi_0 = \begin{pmatrix} 0.4 & 0.4 & 0.2 \end{pmatrix}.$$

(a) Find the conditional probability that the stock price will increase at day 5 given that the stock price decreased at day 2.

(b) Find the probability that the stock price at day 4 will increase.

7. File Response: Question 7:

Points: 10

Question

(Optional - 10 points) Suppose you are playing a game. At each round, you toss a a fair coin. If the coin turns head, you wins then you bet \$1 on the next round but if the coin turn tail then you lose \$1 and you bet twice the previous amount. The amount that you win or lose is equal to the amount of betting. For example, if the sequence tossing result is TTTH then

Tossing result	T	T	T	H
Play result	Lose	Lose	Lose	Win
Bet amount	1	2	4	8
Net profit	-1	-3	-7	1

Let M_n be your net profit after round n and $\mathscr{F}_n = \sigma(M_k, k \leq n)$ be the σ- algebra which contains information about your net profit up to time n.
Is the net profit process $(M_n)_{n\geq 1}$ be a martingale with respect to the filtration $\mathscr{F} = (\mathscr{F}_n)_{n\geq 1}$?

Select: All None Select by Type: - Question Type -

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