THE INTERNATIONAL UNIVERSITY - HO CHI MINH CITY

FINAL EXAMINATION

Semester 2, 2019-20 • Date: 22/06/2020 • Duration: 90 minutes

<table>
<tr><td colspan="2">SUBJECT: RANDOM PROCESSES</td></tr>
<tr><td>Department of Mathematics</td><td>Lecturer

Dr. Tran Vinh Linh</td></tr>
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INSTRUCTIONS: Each student is allowed a scientific calculator. All other documents and electronic devices are forbidden.

Throughout this exam, let $(B_t)_{t\geq 0}$ be a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathcal{P})$.

1. (15 points) Prove that $(B_t^2 - t)$ is a martingale.
2. Let τ_m to be the first passage time at level m of (B_t)

$$\tau_m = \min\{t \leq 0 | B_t = m\}$$

   (a) (5 points) Use Reflection Principle to show that

$$P(\tau_m \leq t) = 2P(B_t \geq m)$$

   (b) (10 points) Use part (a) to calculate $P(\tau_1 \leq 2)$

   (c) (5 points) A particle moves according to Brownian motion start at 2. At $t = 3$ hours, the particle is at level 1.5. Find the probability that the particle reaches level 3.5 before $t = 5$ hours.

3. (a) (10 points) Use Ito's lemma to evaluate $I_t = \int_0^t B_s dB_s$.

   (b) (10 points) Calculate $E(I_t)$.

   (c) (5 points) Show that I_t is a martingale.

4. (20 points) Suppose that the process $(X_t)_{t\geq 0}$ is governed by the arithmetic Brownian motion:

$$dX_t = -2dt + 3dB_t.$$

   Find the distribution, the mean and variance of the random variable $(X_4 | X_2 = 2)$.

5. (20 points) Suppose that the process $(X_t)_{t\geq 0}$ is governed by the geometric Brownian motion:

$$dX_t = 2X_t dt + 5X_t dB_t.$$

Find the distribution, the mean and variance of the random variable $(X_5|X_1 = 1)$.

—END—

Z-value table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998