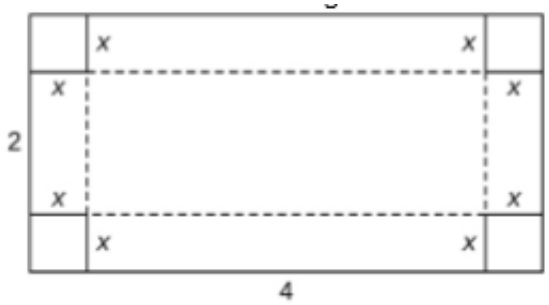

Figure 1:

HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION, Academic year 2019-2020, Semester 3

Duration: 90 minutes

Instructions: *Each student is allowed a scientific calculator and a maximum of two double-sided sheets ofreference material (size A4 or similar), stapled together and marked with their name and ID.All other documents and electronic devices are forbidden.*

Question 1. (20 marks) Find $\lim_{x\to 0}\left(\dfrac{e^x - x - 1}{\cos x - 1}\right)$.

Question 2. (20 marks) The curve implicitly defined by $x\sin y + y\sin x = \pi$, passes through the point $P = P(\frac{\pi}{2}, \frac{\pi}{2})$. (a) Find the slope of the tangent line through P. (b) Write the tangent line through P.

Question 3. (20 marks) Let a_0, a_1, a_2 are given real numbers. Consider the function

$$f(x) := a_2\frac{(\ln x)^3}{3} + a_1\frac{(\ln x)^2}{2} + a_0\ln x.$$

(i) Evaluate $f'(x)$.

(ii) Assume that $a_2\frac{2^2}{3} + a_1 + a_0 = 0$. Show that the equation $a_2(\ln x)^2 + a_1\ln x + a_0 = 0$, has at least one real root in $[1, \infty)$.

Question 4. (20 marks) You are constructing a cardboard box with the dimensions 2 m by 4 m. You then cut equal-size squares from each corner so you may fold the edges. What are the dimensions of the box with the largest volume? See Figure 1.

Question 5. (20 marks) Find the maximum and minimum values of $f(x) := \sqrt{x} - \sqrt{x^3}, \quad x \in [0, 4]$.

End.