

## FINAL EXAMINATION-EXAM A

Semester 1, 2021-22 • Date: January 20, 2022 • Total duration: 120 minutes

SUBJECT: PROBABILITY (MAFE206IU)	
Department of Mathematics	Lecturer
Chair:	Nguyen Minh Quan, PhD

## INSTRUCTIONS:

- **Exam format: 7 questions. Total pages: 2.**
- Explain the solutions in detail. No scores will be given for the answer alone.
- Student submits the scanned hand-writing solutions in a single PDF file on Blackboard/Assignments. Final exam weight: 50% of the total score

1. The daily profit of a certain restaurant is a normal random variable with mean \$200 and standard deviation \$50.

(a) (10 points) What is the probability that tomorrow profit is between \$150 and \$250?

(b) (5 points) What is the cutoff for the highest 5% of tomorrow profit?

(c) (10 points) What is the probability that daily profit exceed \$250 in at least 2 of the next 5 days?

2. (10 points) The Capital Asset Pricing Model (CAPM) is a financial model that assumes returns on a portfolio are normally distributed. Suppose a portfolio has an average annual return of 15% (i.e. an average gain of 15%) with a standard deviation of 20%. A return of 0% means the value of the portfolio doesn't change and a positive return means that the portfolio gains money. What percent of years does this portfolio gain money?

3. The joint probability density function of  $X$  and  $Y$  is given by

$$f(x, y) = \begin{cases} 2e^{-x-2y} & \text{if } 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) (10 points) Evaluate  $\text{Cov}(X, Y)$  and  $E(X^2Y)$ .

(b) (10 points) Evaluate  $P(3X > Y)$ .

4. (10 points) Each item produced by a certain manufacturer is, independently, of acceptable quality with probability 0.98. Approximate the probability that at most 19 of the next 1000 items produced are unacceptable.

5. (10 points) The time  $T$  (in hours) required to repair a device is an exponentially distributed random variable with the expected value of  $E(T) = 3$ . What is the probability that a repair time exceeds 2.5 hours?

6. (10 points) The joint probability density function of  $X$  and  $Y$  is given by

$$f(x, y) = \begin{cases} x^2 e^{-x(y+1)} & \text{if } 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Are  $X$  and  $Y$  independent? Explain your answer in detail.

7. (15 points) Let  $Z$  be a standard normal random variable, that is,  $Z \sim N(0, 1)$ . Find  $E(Z^2)$ ,  $E(Z^4)$ , and  $E(\frac{Z}{Z^2 + 1})$ .

—THE END. GOOD LUCK!—