

## FINAL EXAMINATION SAMPLE, Academic year 2019-2020, Semester 3

Duration: 120 minutes

**Instructions:** Send files of solution to the email address hiletr1985@gmail.com**Question 1.** (15 marks) Find

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}.$$

**Question 2.** (15 marks) Let

$$f(x) := 2 \arctan \sqrt{\frac{1+x}{1-x}} - \arcsin x, \quad -1 \leq x < 1.$$

Evaluate  $f'(x)$ . What is the function  $f$ ?

**Question 3.** (20 marks) In a simple video game, a rocket travels in an elliptical orbit whose path is described by the equation  $4x^2 + 25y^2 = 100$ . The rocket can fire missiles along lines tangent to its path. The object of the game is to destroy an incoming asteroid traveling along the positive x-axis toward  $(0, 0)$ . If the rocket fires a missile when it is located at  $(3, \frac{8}{5})$ , where will it intersect the x-axis?

**Question 4.** (20 marks) Assume that  $a, b, c, d$  are real numbers such that

$$\frac{a}{4} + \frac{b}{3} + \frac{c}{2} + \frac{d}{1} = 0.$$

Show that the equation  $ax^3 + bx^2 + cx + d = 0$  has at least one real root in  $(0, 1)$ .

**Question 5.** (15 marks) Find the maximum and minimum values of  $f(x) := (x - 2020)e^{-x}$ ,  $x \in [0, \infty)$ .

**Question 6.** (15 marks) Two poles are connected by a wire that is also connected to the ground. The first pole is 20 ft tall and the second pole is 10 ft tall. There is a distance of 30 ft between the two poles. Where should the wire be anchored to the ground to minimize the amount of wire needed? See the figure bellow.

End.

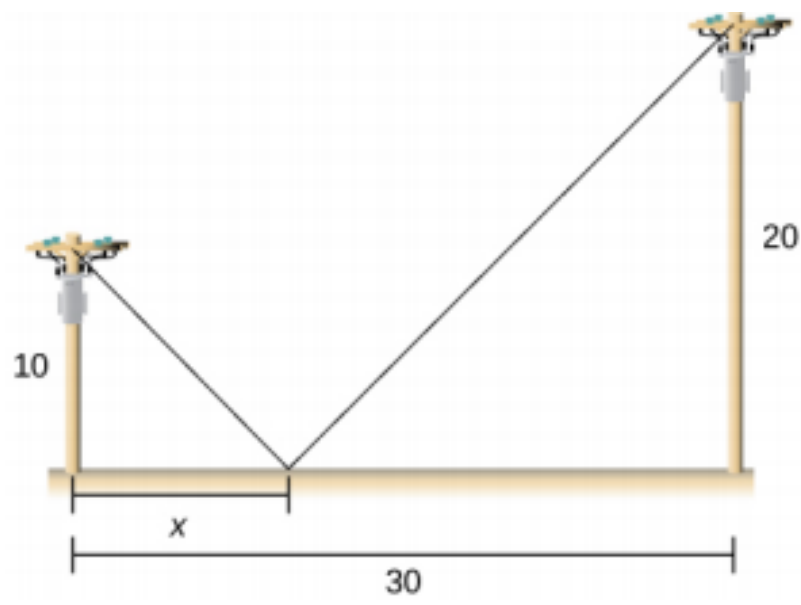


Figure 1: