1) $dX_t + X_t dt = dB_t$

$$a'(t) = 1 \Rightarrow a(t) = t$$

$Y_t = e^{a(t)} X_t = e^t X_t = f(t, X_t)$ where $f(t,x) = e^t x$

$dY_t = e^t dX_t + e^t X_t dt = e^t dB_t$, so

$$e^T X_T - 2 = Y_T - Y_0 = \int_0^T dY_t = \int_0^T e^t dB_t$$

$$X_T \sim \mathcal{N}(2/e^T, (e^{2T} - 1)/(2e^{2T}))$$

2/ $dX_t - X_t dt = 3dB_t + dt \Rightarrow a(t) = -t$

$Y_t = e^{-t} X_t = f(t, X_t)$ where $f(t,x) = e^{-t}x$

$$dY_t = e^{-t} dX_t - e^{-t} X_t dt = e^{-t}(3dB_t + dt)$$

$$e^{-T} X_T - X_0 = \int_0^T 3e^{-t} dB_t + 1 - e^{-T}$$

$$X_T \sim \mathcal{N}(e^T(X_0 + 1) - 1, 9(e^{2T} - 1)/2)$$

3/ $dR_t + \beta R_t dt = \sigma dB_t + \alpha dt \Rightarrow a(t) = \beta t$

$Y_t = e^{\beta t} R_t = f(t, R_t)$ where $f(t, x) = e^{\beta t} x$

$$dY_t = e^{\beta t} dR_t + \beta e^{\beta t} R_t dt = e^{\beta t}(\sigma dB_t + \alpha dt)$$

$$e^{\beta T} R_T - R_0 = \int_0^T \sigma e^{\beta t} dB_t + \alpha(e^{\beta T} - 1)/T$$

$$R_T \sim \mathcal{N}\left(\frac{\alpha(e^{\beta T} - 1)}{Te^{\beta T}} + R_0, \frac{\sigma^2(e^{2\beta T} - 1)}{2\beta e^{2\beta T}}\right)$$