LINEAR ALGEBRA – FINAL EXAMINATION

Semester 2, 2021-22 • Duration: 90 minutes

Student's name: Student ID:	Proctor's signature

Vice chair of Dept. of Mathematics	Lecturer	Score and Examiner
Dr. Nguyen Minh Quan		

INSTRUCTIONS:

- Use of calculator is allowed.
- Each student is allowed two double-sided sheet of reference material (size A4 or similar).
- All other documents and electronic devices are forbidden.
- You must explain your answers in detail; no marks will be given for the answer alone.
- There are a total of 6 questions in this final examination.
- GOOD LUCK!

Question 1. (20 points)
Find a basis for the column space, row space and null space of A.

$$A = \begin{bmatrix} -4 & 3 & -27 & 16 \\ -8 & 6 & -42 & 24 \\ -16 & 12 & -92 & 53 \end{bmatrix}$$

Question 2. (20 points)

Let $x_1 = \begin{bmatrix} -2 \\ -2 \\ 1 \\ -2 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 5 \end{bmatrix}$ and $x_3 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, and write H=Span$\{x_1, x_2, x_3\}$.

Use the Gram-Schmidt process to find an orthogonal basis for H.

----- please turn over -----

Question 3. (20 points)
Find the eigenvalues and eigenvectors of B, i.e. the λ which satisfies $det|B-\lambda I_3|=0$

$$B=\begin{bmatrix} 2 & 0 & 4 \\ 3 & -4 & 5 \\ 1 & 0 & 5 \end{bmatrix}$$

Question 4. (20 points)
Define the below linear transformation L: $R^2 \to R^4$. Find basis for range and kernel of L.

$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)=\begin{bmatrix} 19x_2 \\ -3x_1 \\ 7x_1-14x_2 \\ 4x_1+12x_2 \end{bmatrix}$$

Question 5. (20 points)
Let $S=\{u_1,u_2,u_3\}$, where

$$u_1=\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}, u_2=\begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, u_3=\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

a) Show that S is an orthonormal basis for R^3.

b) Express

$$w=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

as a linear combination of the vectors in S.

c) Let $V=span\{u_1,u_2\}$. Determine the orthogonal projection of

$$w=\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

onto V.

—END—