CHAPTER 6: HYPOTHESIS TESTING

STATISTICS (FERM)

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CONTENTS

Introduction to Hypothesis Testing

Two most important types of statistical inference:

- Confidence intervals: to estimate a parameter with confidence level.
- **Significance tests**: to assess if the data provide sufficient evidence against or in favor of some claim about the population.

Introduction to Hypothesis Testing

We start by considering an example.

Introduction Example

A public health official claims that the mean home water use is 350 gallons a day. To verify this claim, a study of 20 randomly selected homes was instigated with the result that the average daily water uses of these 20 homes were as follows:

340 344 362 375 356 386 354 364 332 402 340 355 362 322 372 324 318 360 338 370

Do the data contradict the official's claim?

- The statement being tested (called the **null hypothesis**): $\mu = 350$
- The hypothesis $H_a : \mu \neq 350$ is called the alternative hypothes

Null and Alternative Hypotheses

- The statement being tested in a significance test is called the **null hypothesis** (denoted as $\mathbf{H_0}$). Usually it is a statement of "no effect" or "no difference".
- The statement we hope or suspect is true instead of H_0 is called the **alternative hypothesis** (denoted as $\mathbf{H_a}$ or $\mathbf{H_1}$).
- For the previous example,

$$H_0 : \mu = 350.$$

$$H_a : \mu \neq 350.$$

Types of error

We have "Type I Error" when we reject the null when the null is in fact true. We have "Type II Error" when we fail to reject the null when the null is in fact false.

	H_0 is true	H_0 is false
Reject H_0	Type I Error	Correct decision
Accept H_0	Correct decision	Type II Error

P(Type I error) $= \alpha$ [usually 0.05 or 0.01]
P(Type II error) $= \beta$ [usually 0.05 or 0.01]

Purpose

Purpose: Test if

- H_0: $\mu = \mu_0$ vs. H_a: $\mu \neq \mu_0$.
- H_0: $\mu \leq \mu_0$ vs. H_a: $\mu > \mu_0$.
- H_0: $\mu \geq \mu_0$ vs. H_a: $\mu < \mu_0$.

Significance Level

- A significance level $\alpha \in (0, 1)$.

- If the event we observe has a probability **as small or smaller than** α then we say that **the result is significant**.

- If the result is significant then we reject the null hypothesis H_0 and adopt the alternative hypothesis H_a.

Case of Known Variance

Suppose that the standard deviation σ of the population is known and a significance level α is given.

1. Select a simple random sample of size n from the population and calculate the sample mean $\bar{x}$.
2. Calculate the **test statistic** which is defined by
$$TS = t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$
3. Depending on the alternative hypothesis, calculate the **p-Value** (p). Let Z be $N(0, 1)$.
    - If H_a: $\mu \neq \mu_0$ then define the p-Value
    $$p = 2P(Z \geq |TS|).$$
    - If H_a: $\mu > \mu_0$ then define the p-Value
    $$p = P(Z \geq TS).$$
    - If H_a: $\mu < \mu_0$ then define the p-Value
    $$p = P(Z \leq TS).$$

Case of Known Variance

4. Finally, compare the p-Value (p) and the significance level α

- If $p < \alpha$ then we say that the result is significant at level α. We hence reject H_0 and adopt H_a.

- If $p \geq \alpha$ then we say that the result is not significant at level α. Thus the data do not provide sufficient evidence against H_0.

Case of Known Variance

Alternatively, we can reject H_0 by comparing the value of TS and $z_{\alpha/2}$.

TABLE 8.1 *$X_1, \dots, X_n$ Is a Sample from a $\mathcal{N}(\mu, \sigma^2)$*
Population σ^2 Is Known, $\overline{X} = \sum_{i=1}^{n} X_i/n$

H_0	H_1	Test Statistic TS	Significance Level α Test	p-Value if $TS = t$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/\sigma$	Reject if $\lvert TS \rvert > z_{\alpha/2}$	$2P\{Z \geq \lvert t \rvert\}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/\sigma$	Reject if $TS > z_\alpha$	$P\{Z \geq t\}$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/\sigma$	Reject if $TS < -z_\alpha$	$P\{Z \leq t\}$

Z is a standard normal random variable.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Example 1

Given a data set with $n = 5$, $\bar{x} = 9.5$. Also, it is known that the population standard deviation is $\sigma = 2$.
Test the hypothesis $H_0 : \mu = 8$ vs. H_a: $\mu \neq 8$.

Solution 1 (use the p-value) Test $H_0 : \mu = \mu_0$ vs. H_a: $\mu \neq \mu_0$.

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{9.5 - 8}{2/\sqrt{5}} = 1.68$$

$$p = 2P\left(Z \geqslant 1.68\right) = 0.093 > \alpha = 0.05$$

Therefore, H_0 is "accepted" (not rejected)!

Note that if $\alpha = 0.1$ then the null hypothesis would have been rejected.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Solution 2 (compare the TS with $z_{\alpha/2}$)

Test $H_0 : \mu = \mu_0$ vs. H_a: $\mu \neq \mu_0$.

We compare

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{9.5 - 8}{2/\sqrt{5}} = 1.68 < z_{\alpha/2} = 1.96 \rightarrow H_0 \text{ is "accepted"!}$$

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{9.5 - 8}{2/\sqrt{5}} = 1.68$$

Note that if $\alpha = 0.1$ then the null hypothesis would have been rejected since $z_{\alpha/2} = 1.645 < 1.68$ with $\alpha = 0.1$.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Example 2

In Example 1, suppose that the average of the 5 values received is $\bar{x} = 8.5$. Calculate the p-Value and test with $\alpha = 0.05$.

Test $H_0 : \mu = \mu_0$ and H_a: $\mu \neq \mu_0$.

Solution 1 (use the p-Value)

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{8.5 - 8}{2/\sqrt{5}} = 0.559$$

$$p = 2P\left(Z \geqslant 0.559\right) = 0.0576 > \alpha = 0.05 \rightarrow H_0 \text{ is accepted!}$$

Note that if $\bar{x} = 11.5$ then the null hypothesis would have been rejected.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Denote

$$\beta(\mu) = P_\mu(acceptance\ of\, H_0) = P(Type\ \ II\ \ error)$$

Preposition: The probability of a type II error

$$\beta(\mu) = \Phi\left(\frac{\mu_0 - \mu}{\sigma/\sqrt{n}} + z_{\alpha/2}\right) - \Phi\left(\frac{\mu_0 - \mu}{\sigma/\sqrt{n}} - z_{\alpha/2}\right)$$

The function $\beta(\mu)$ is called the operating characteristic (or OC) curve and represents the probability that H_0 will be accepted when the true mean is μ.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

The operating characteristic (or OC) curve:

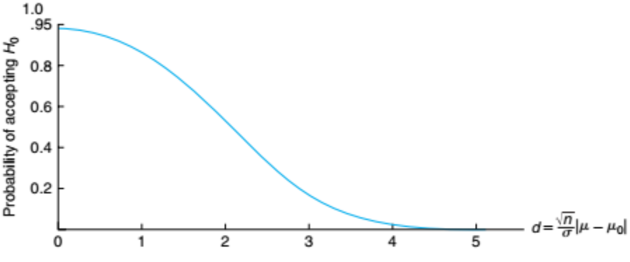

FIGURE 8.2 *The OC curve for the two-sided normal test for significance level $\alpha = .05$.*

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Example 3

Back to Example 1. Determine the probability of accepting the null hypothesis that $\mu = 8$ when the actual value sent is 10.

Solution

$$\frac{\mu_0 - \mu}{\sigma/\sqrt{n}} = -\frac{\sqrt{5}}{2} \times 2 = -\sqrt{5}$$

$$z_{\alpha/2} = z_{0.025} = 1.96$$

$$\beta(\mu) = \Phi\left(-\sqrt{5} + 1.96\right) - \Phi\left(-\sqrt{5} - 1.96\right) = 0.392$$

Remark: The function $1 - \beta(\mu) = P_\mu(\textit{rejection of } H_0)$ is called the power-function of the test.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Sample size n such that $\beta(\mu_1) = \beta$

$$n \approx \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \sigma^2}{(\mu_1 - \mu_0)^2}$$

Example 4: Sample size n such that $\beta(\mu_1) = \beta$

For the problem of Example 1, how many signals need be sent so that the 0.05 level test of $H_0 : \mu = 8$ has at least a 75 percent probability of rejection when $\mu = 9.2$?

Since $z_{0.025} = 1.96$, $z_{0.25} = 0.67$, we have

$$n \approx \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \sigma^2}{(\mu_1 - \mu_0)^2} = \frac{(1.96 + 0.67)^2 4}{1.2^2} = 19.21$$

Hence a sample of size 20 is needed.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Example 5

Do middle-aged male executives have different average blood pressure than the general population? The National Center for Health Statistics reports that the blood pressure for males aged 35 to 44 has the mean 128. Suppose that the blood pressure for male executives aged 35-44 has the standard deviation 15. A simple random sample of 72 male executives aged 35-42 was selected. The mean blood pressure for this sample was $\bar{x} = 126.07$. At the significance level $\alpha = 0.05$, is this evidence that executive blood pressures **differ from the national average?**

Solution

Let μ be the mean blood pressure of male executives aged 35-42.

$$H_0 : \mu = 128 \quad H_a : \mu \neq 128$$

$\bar{x} = 126.07$, $n = 72$, $\alpha = 0.05$, $\mu_0 = 128$ and $\sigma = 15$. The test statistic

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{126.07 - 128}{(15/\sqrt{72})} = -1.09$$

The P-value

$$p = 2P(Z \geq |z|) = p = 2P(Z \geq 1.09) = 0.2758$$

Since $p > \alpha$, there is no sufficient evidence that the mean blood pressure of executives differs from that of other men.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Example 6

Suppose in Example 1 that we know in advance that the signal value is **at least** as large as 8. What can be concluded in this case?

Solution

To see if the data are consistent with the hypothesis that the mean is 8, we test $H_0 : \mu = 8$ against the one-sided alternative $H_a : \mu > 8$.
The value of the test statistic is

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{9.5 - 8}{2/\sqrt{5}} = 1.68$$

and the p-Value is

$$p = P(Z \geq TS) = 1 - \Phi(1.68) = 0.0465$$

Thus, we can reject the null hypothesis at the $\alpha = 0.05$ level of significance.

Case of Known Variance. Test $H_0 : \mu = \mu_0$

Example 7

In a discussion of SAT scores, someone comments that if all high school seniors in California took the test, then the mean SAT math score would be no more than 450. A test was given to a simple random sample of 500 seniors from California. The mean SAT math score for the sample is $\bar{x} = 461$. Assume that the SAT math score for all California seniors has the standard deviation $\sigma = 100$. At the significance level $\alpha = 0.05$, is this a good evidence against the claim that the mean of all California seniors is **no more than** 450**?**

Solution

Let μ be the mean SAT math score of all California seniors.

$$H_0 : \mu \leq 450 \quad H_a : \mu > 450$$

$n = 500, \bar{x} = 461, \sigma = 100, \alpha = 0.05, \mu_0 = 450.$
The test statistic

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{461 - 450}{(100/\sqrt{500})} = 2.46$$

The p-Value

$$p = 0.0069$$

Since $p < \alpha$, the result is significant. We reject H_0 and adopt H_a. In other words, the data provide good evidence that the mean of all California seniors is more than 450.

Exercise

Wall Street securities firms paid out record year-end bonuses of \$125,500 per employee for 2005. Suppose we would like to take a sample of employees at the Jones & Ryan securities firm to see whether the mean year-end bonus is different from the reported mean of \$125,500 for the population.

(a) Suppose a sample of 40 Jones & Ryan employees showed a sample mean year-end bonus of \$118,000. Assume a population standard deviation of $\sigma =$\$30,000 and compute the p-value.

(b) With $\alpha = 0.05$ as the level of significance, what is your conclusion?

Case of Unknown Variance: The t-test (two-sided)

Two-sided test

Consider the test

$$H_0 : \mu = \mu_0 \quad \text{vs. } H_a : \mu \neq \mu_0$$

and the variance σ^2 is unknown.

Key idea: Replace σ by s and use $\frac{\overline{X}-\mu_0}{S/\sqrt{n}} \sim T_{n-1}$ (a t-random variable with $n-1$ degree of freedom).

Let $TS := \frac{\sqrt{n}(\bar{x}-\mu_0)}{S}$ be the value of the test statistic.

- Accept H_0 if $|TS| \leqslant t_{\alpha/2,n-1}$.
- Reject H_0 if $|TS| > t_{\alpha/2,n-1}$.

Case of Unknown Variance: The t-test (two-sided)

The two-sided t-test.

Alternatively, we can reject/accept H_0 via the p-Value of the test, which is defined by

$$p = 2P\left(T_{n-1} \geqslant |TS|\right)$$

- If $p < \alpha$ then reject H_0 and adopt H_a.
- If $p \geq \alpha$ then we say that the data do not provide sufficient evidence against H_0 (accept H_0).

Case of Unknown Variance: The t-test

Example 8

A public health official claims that the mean home water use is 350 gallons a day. To verify this claim, a study of 20 randomly selected homes was instigated with the result that the average daily water uses of these 20 homes were as follows:

340 344 362 375 356 386 354 364 332 402 340 355 362 322 372 324 318 360 338 370

Do the data contradict the official's claim?

Solution

We need to test $H_0 : \mu = 350$ versus $H_a : \mu = 350$. We compare the value of the test statistic and $t_{\alpha/2, n-1}$:

$$|TS| = |\frac{\sqrt{20}\,(353.8 - 350)}{21.8478}| = 0.7778 < t_{0.05,19} = 1.73$$

$\rightarrow H_0$ is accepted.

Note: $p = 2P\,(T_{19} > 0.7778) = 0.4462 > \alpha$.

Case of Unknown Variance. Summarize: Use the t-test

TABLE 8.2 $X_1, \ldots, X_n$ *Is a Sample from a* $\mathcal{N}(\mu, \sigma^2)$
Population σ^2 *Is Unknown,* $\overline{X} = \sum_{i=1}^{n} X_i / n S^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2 / (n-1)$

H_0	H_1	Test Statistic TS	Significance Level α Test	p-Value if $TS = t$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/S$	Reject if $\lvert TS \rvert > t_{\alpha/2,\, n-1}$	$2P\{T_{n-1} \geq \lvert t \rvert\}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/S$	Reject if $TS > t_{\alpha,\, n-1}$	$P\{T_{n-1} \geq t\}$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/S$	Reject if $TS < -t_{\alpha,\, n-1}$	$P\{T_{n-1} \leq t\}$

T_{n-1} is a t-random variable with $n-1$ degrees of freedom: $P\{T_{n-1} > t_{\alpha, n-1}\} = \alpha$.

Case of Unknown Variance: The t-test

Example 9

The manufacturer of a new fiberglass tire claims that its average life will be at least 40,000 miles. To verify this claim a sample of 12 tires is tested, with their lifetimes (in 1,000s of miles) being as follows:

Tire	1	2	3	4	5	6	7	8	9	10	11	12
Life	36.1	40.2	33.8	38.5	42	35.8	37	41	36.8	37.2	33	36

Solution

To determine whether the foregoing data are consistent with the hypothesis that the mean life is at least 40, 000 miles, we will test: $H_0 : \mu \geq 40,000$ versus $H_a : \mu < 40,000$.

$$\overline{X} = 37.2833, S = 2.7319$$

Case of Unknown Variance: The t-test

Solution (cont.)

We calculate

$$\overline{x} = 37.2833, s = 2.7319$$

$$TS = \frac{\sqrt{12}\,(37.2833 - 40)}{2.7319} = -3.4448 < -t_{0.05,11} = -1.796$$

The null hypothesis is rejected at the 5 percent level of significance.

Remark: The p-Value of the test data is

$$p = P\,(T_{11} < -3.4448) = P\,(T_{11} > 3.4448) = 0.0028$$

indicating that the manufacturer's claim would be rejected at any significance level greater than 0.003.

Case of Unknown Variance: The t-test

Exercises

A shareholders' group, in lodging a protest, claimed that the mean tenure for a chief exective office (CEO) was at least nine years. A survey of companies reported in The Wall Street Journal found a sample mean tenure of $\bar{x} = 7.27$ years for CEOs with a standard deviation of $s =$6.38 years (The Wall Street Journal, January 2, 2007).

(a) Formulate hypotheses that can be used to challenge the validity of the claim made by the shareholders' group.

(b) Assume 85 companies were included in the sample. What is the p-value for your hypothesis test?

(c) At $\alpha = 0.01$, what is your conclusion?

Testing a proportion

- Consider the following two-tailed test:
  $\mathbf{H}_0$: $p = p_0$ **vs.** $\mathbf{H}_a : p \neq p_0$

- The p-value is a way of quantifying the strength of the evidence against the null hypothesis and in favor of the alternative hypothesis:
  $p-$value$=2P(Z \geq \dfrac{|\hat{p} - p_0|}{SE})$.

- The test statistic (the z-score of $\hat{p}$) is $t := \dfrac{\hat{p} - p_0}{SE_{\hat{p}}}$, where
  $SE_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$.

Testing a proportion

Example

In the Pew Research poll about solar energy in 2018, they also inquired about other forms of energy, and 84.8% of the 1000 respondents supported expanding the use of wind turbines.

(a) A person believes in the fact that 85% of American adults support the expansion of solar power in 2018. Test the hypothesis:

$\mathbf{H}_0$**:** $p = 0.85$ **vs.** $\mathbf{H}_a : p \neq 0.85$

(b) A person claims that 80% of American adults support the expansion of solar power in 2018. Test the hypothesis:

$\mathbf{H}_0$**:** $p = 0.80$ **vs.** $\mathbf{H}_a : p \neq 0.80$

Testing a proportion

Example

It was claimed that the proportion p of respondents who pick the answer, that *80% of 1 year olds have been vaccinated against some disease*, is about 33.3%. Assume for a survey of $n = 5000$ college-educated adults, it is computed that $\hat{p}$ =24% of respondents got the question correct that 80% of 1 year olds have been vaccinated against some disease. Construct a test to reject/not reject the null hypothesis $\mathrm{H}_0 : p = 0.333$.

Testing the Equality of Means of Two Normal Populations

Case 1 (known variance):

Suppose that $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_n$ are in dependent samples from normal populations having unknown means μ_x and μ_y but known variances σ_x^2 and σ_y^2. Let us consider the problem of testing the hypothesis

$$H_0 : \mu_x = \mu_y \text{ versus the alternative } H_a : \mu_x \neq \mu_y$$

Note that $\overline{X} - \overline{Y} \sim N\left(\mu_x - \mu_y, \frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}\right)$.

$$\rightarrow \frac{\overline{X} - \overline{Y} - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} \sim N(0, 1)$$

Testing the Equality of Means of Two Normal Populations. Case 1.

Therefore,

$$P_{H_0}\left(-z_{\alpha/2} \leqslant \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} \leqslant z_{\alpha/2}\right) = 1 - \alpha$$

The value of the test statistic is defined by

$$TS := \frac{\overline{x} - \overline{y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}}$$

- Accept H_0 if $|TS| \leqslant z_{\alpha/2}$.
- Reject H_0 if $|TS| > z_{\alpha/2}$.

Testing the Equality of Means of Two Normal Populations. Case 1

Example

Two new methods for producing a tire have been proposed. To ascertain which is superior, a tire manufacturer produces a sample of 10 tires using the first method and a sample of 8 using the second. The first set is to be road tested at location A and the second at location B. It is known from past experience that the lifetime of a tire that is road tested at one of these locations is normally distributed with a mean life due to the tire but with a variance due (for the most part) to the location. Specifically, it is known that the lifetimes of tires tested at location A are normal with standard deviation equal to 4,000 kilometers, whereas those tested at location B are normal with $\sigma = 6{,}000$ kilometers. If the manufacturer is interested in testing the hypothesis that there is no appreciable difference in the mean life of tires produced by either method, what conclusion should be drawn at the 5 percent level of significance if the resulting data are as given in following Table?

Testing the Equality of Means of Two Normal Populations. Case 1

TABLE 8.3 *Tire Lives in Units of 100 Kilometers*

Tires Tested at A	Tires Tested at B
61.1	62.2
58.2	56.6
62.3	66.4
64	56.2
59.7	57.4
66.2	58.4
57.8	57.6
61.4	65.4
62.2	
63.6	

Solution:

The value of the test statistic is $TS = \frac{\overline{x}-\overline{y}}{\sqrt{\frac{\sigma_x^2}{n}+\frac{\sigma_y^2}{m}}} = 0.066$. For such a small value of the test statistic (which has a standard normal distribution when H_0 is true), it is clear that the null hypothesis is accepted.

Testing the Equality of Means of Two Normal Populations

Case 2 (unknown variance):

Suppose that $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_n$ are in dependent samples from normal populations having unknown means μ_x and μ_y and unknown variances σ_x^2 and σ_y^2. Let us consider the problem of testing the hypothesis

$$H_0 : \mu_x = \mu_y \text{ versus the alternative } H_a : \mu_x \neq \mu_y$$

with assumption $\sigma^2 = \sigma_x^2 = \sigma_y^2$.

Theorem

$$\frac{\overline{X} - \overline{Y} - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n} + \frac{1}{m}\right)}} \sim t_{n+m-2}$$

where

$$S_p^2 = \frac{(n-1)\,S_x^2 + (m-1)\,S_y^2}{n+m-2}$$

Testing the Equality of Means of Two Normal Populations

Case 2 (unknown variance) The test statistic

$$TS := \frac{\overline{x} - \overline{y}}{\sqrt{S_p^2 \left(\frac{1}{n} + \frac{1}{m}\right)}}$$

- Accept H_0 if $|TS| \leqslant t_{\alpha/2,n+m-2}$.
- Reject H_0 if $|TS| > t_{\alpha/2,n+m-2}$.

Testing the Equality of Means of Two Normal Populations. Case 2

Example 8.4b

Twenty-two volunteers at a cold research institute caught a cold after having been exposed to various cold viruses. A random selection of 10 of these volunteers was given tablets containing 1 gram of vitamin C. These tablets were taken four times a day. The control group consisting of the other 12 volunteers was given placebo tablets that looked and tasted exactly the same as the vitamin C tablets. This was continued for each volunteer until a doctor, who did not know if the volunteer was receiving the vitamin C or the placebo tablets, decided that the volunteer was no longer suffering from the cold. The length of time the cold lasted was then recorded. At the end of this experiment, the following data resulted.

Testing the Equality of Means of Two Normal Populations. Case 2

Treated with Vitamin C	Treated with Placebo
5.5	6.5
6.0	6.0
7.0	8.5
6.0	7.0
7.5	6.5
6.0	8.0
7.5	7.5
5.5	6.5
7.0	7.5
6.5	6.0
	8.5
	7.0

Hint: $\bar{x} = 6.450, \bar{y} = 7.125, S_x^2 = 0.581, S_y^2 = 0.778$. $S_p^2 = \frac{9}{20}S_x^2 + \frac{11}{20}S_y^2 = 0.689$; $TS = \frac{-0.675}{\sqrt{0.689(1/10+1/12)}} = -1.90$; $t_{0.025,20} = 2.086$.

–End of Chapter 6–