Continuous random variables

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1 Joint discrete random variables

1. A product is classified according to the number of defects it contains and the factory that produces it. Let X1 and X2 be the random variables that represent the number of defects per unit (taking on possible values of 0, 1, 2, or 3) and the factory number (taking on possible values 1 or 2), respectively. The entries in the table represent the joint possibility mass function of a randomly chosen product.

	x_1	
x_2	1	2
0	1/8	1/16
1	1/16	1/16
2	3/16	1/8
3	1/8	1/4

   (a) Find the marginal probability distributions of X_1 and X_2.

   (b) Are X_1 and X_2 are independent?

   (c) Find $E[(X_1)]$,$E[(X_2)]$, $Var(X_1)$, $Var(X_2)$.

2. The joint probability mass function of X and Y is

x	-1	0	0	1
y	0	-1	1	0
p(x,y)	1/4	1/4	1/4	1/4

   Show that X and Y are not independent.

3. Consider four independent rolls of a fair 6-sided die. Let X be the number of 1s and let Y be the number of 2s obtained. What is the joint PMF of X and Y?

4. Let X and Y be independent random variables with non-negative integer values. Show that:

   (a) $P(X+Y=n) = \sum_{k=0}^{n} P(X=k)P(Y=n-k)$.

   (b) Find the probability that the sum of numbers on four dice is 8, by taking X to be the sum on two of the dice, Y the sum on the other two.

5. A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B. respectively. over a certain time period. and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-2 \leq x \leq 4, -1 \leq y - x \leq 1.$$

   (a) Find the marginal PMFs and the means of X and Y.

   (b) Find the mean of the trader's profit.

6. Consider four independent rolls of a 6-sided die. Let X be the numberof 1s and let Y be the number of 2s obtained. What is the joint PMF of X and Y?

7. Joint probability mass function $p(x,y)$ of (X,Y) is given by

$$\begin{array}{ccc} p(1,1)=\frac{1}{9} & p(1,2)=\frac{1}{9} & p(1,3)=0 \\ p(2,1)=\frac{1}{3} & p(2,2)=0 & p(2,3)=\frac{1}{6} \\ p(3,1)=\frac{1}{9} & p(3,2)=\frac{1}{18} & p(3,3)=\frac{1}{9} \end{array}$$

   (a) Compute $P(X=k|Y=i)$ for $k,i=1,2,3$.

   (b) Are X and Y are independent?

8. Suppose that random variables X and Y, each with a finite number of possible values, have joint probabilities of the form $P(X=x,Y=y)=f(x)g(y)$ for some functions f and g, for all (x,y).

   (a) Find formulae for $P(X=x)$ and $P(Y=y)$ in terms of f and g.

   (b) Use your formulae to show that X and Y are independent.

9. Consider a ferry that can carry both buses and cars across a waterway. Each trip costs the owner approximately \$10. The fee for cars is \$3 and the fee for buses is \$8. Let X and Y denote the number of buses and cars, respectively, carried on a given trip. The joint distribution of X and Y is given by

		x		
		0	1	2
	0	0.01	0.01	0.03
	1	0.03	0.08	0.07
	2	0.03	0.06	0.06
y	3	0.07	0.07	0.13
	4	0.12	0.04	0.03
	5	0.08	0.06	0.02

   Compute the expected profit for the ferry trip

2 Joint continuous random variables

1. Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pounds per square inch (psi). Let X denote the actual air pressure for the right tire and Y denote the actual air pressure for the left tire. Suppose that X and Y are random variables with the joint density function

$$f(x,y)=\begin{cases} k(x^2+y^2), & 30\le x<50, 30\le y<50 \\ 0 & elsewhere, \end{cases}$$

   (a) Find k.

   (b) Find $P(30\le X\le 40$ and $40\le Y<50)$.

   (c) Find the probability that both tires are underfilled.

2. A fast-food restaurant operates both a drivethrough facility and a walk-in facility. On a randomly selected day, let X and Y , respectively, be the proportions of the time that the drive-through and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y)=\begin{cases} \frac{2}{3}(x+2y), & 0\le x\le 1, 0\le y\le 1, \\ 0, & elsewhere \end{cases}$$

   (a) Find the marginal density of X.

   (b) Find the marginal density of Y .

   (c) Find the probability that the drive-through facility is busy less than one-half of the time

3. There are two service lines. The random variables X and Y are the proportions of time that line 1 and line 2 are in use, respectively. The joint probability density function for (X, Y) is given by

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2), & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & elsewhere \end{cases}.$$

   (a) Are X and Y independent?
   (b) Find $Var(X)$, $Var(Y)$.

4. The joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} 6x, & 0 < x < 1, 1 < y < 1-x \\ 0, & elsewhere \end{cases}$$

   Show that X and Y are not independent.

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