Discrete Random Variables

March 24, 2021

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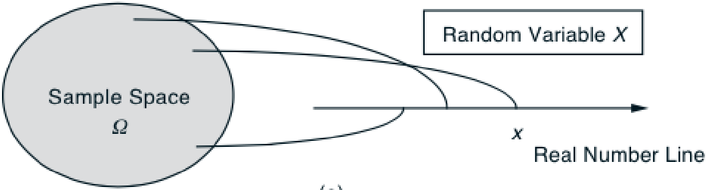
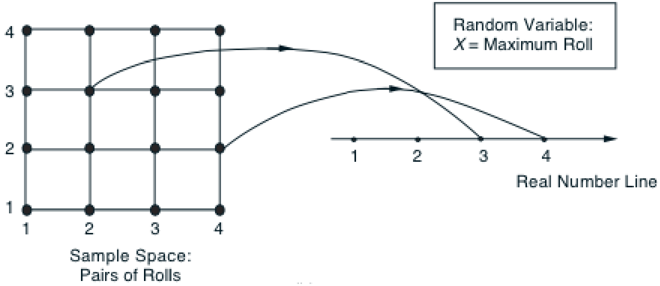
Random variable



(a)


Random Variable:
X = Maximum Roll
4
3
2
1
1
2
3
4
Sample Space:
Pairs of Rolls
1
2
3
4
Real Number Line

Example

Tossing 3 dice

- X=times that number 1 shown up
- Y=largest number shown
- Z=sum of 3 numbers

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Three balls are to be randomly chosen without replacement from a box containing 20 balls numbered 1 through 20. If we bet that at least one of the balls that are drawn has a number at least 17, what is the probability that we win the bet?

- X = largest number
- X takes value $3, 4, \ldots, 20$

$$P(X = i) = \frac{\binom{i-1}{2}}{\binom{20}{3}}$$

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$$\begin{aligned} P(X \geq 17) =& P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20) \\ =& 0.508 \end{aligned}$$

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- Random variable

$$X : (\Omega, \mathcal{F}, P) \longrightarrow (\mathbb{R}, \mathcal{B}(\mathbb{R}))$$

$\mathcal{B}(\mathbb{R})$ is generated by $(-\infty, x]$ for $X \in \mathbb{R}$

- *Cumulative Distribution Function* (cdf) of a random variable X is

$$F(x) = P(X \leq x) \qquad -\infty < x < \infty$$

- $\{\omega \in \Omega : X(\omega) \leq x\} \in \mathcal{F}$ for all x then X is $\mathcal{F}$ - measurable

Properties

- cdf is non decreasing

$$\lim_{b \to \infty} F(b) = 1 \qquad \lim_{b \to -\infty} F(b) = 0$$

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- Example:

$$P(a < X \leq b) = F(b) - F(a)$$

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- **Discrete RV**: Range of RV is countable
- **Continuous RV**: Range of RV is not countable

- Dicrete RV is a RV can take at most countable number of possible values
- the *probablity mass function* (pmf) of a discrete RV X is

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If X takes on one of the values $x_1, x_2, \ldots$ then

- $$p(x_i) \geq 0 \quad \text{for } i = 1, 2, \ldots$$
- $$\sum_{i=1}^{\infty} p(x_i) = 1$$

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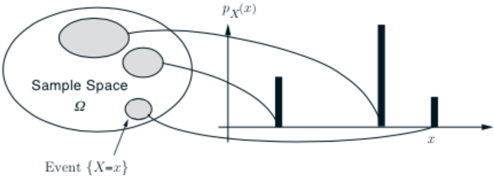
Calculate pmf of a discrete RV

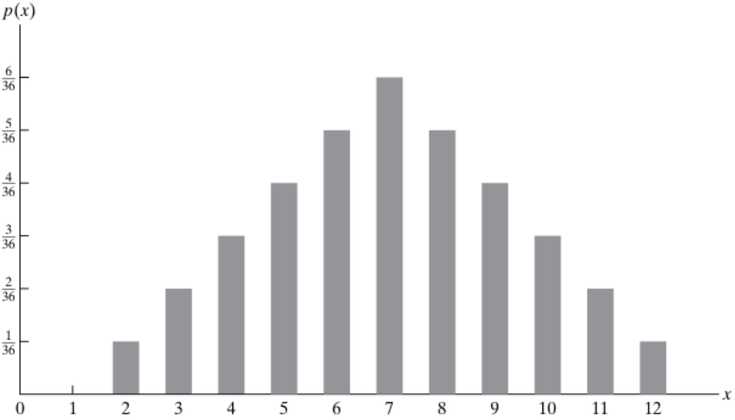


- Collect all the possible outcomes that give rise to the event $\{X = x\}$.
- Add their probabilities to obtain $p_X(x)$.

Let the experiment consist of two independent tosses of a fair coin, and let X be the number of heads obtained. Then the pmf of X is

x	0	1	2
outcome	$\{TT\}$	$\{TH, HT\}$	$\{HH\}$
$p_X(x)$	$1/4$	$2/4$	$1/4$

Example: X= sum of two dice



Can write cdf in terms of pmf

$$F(a) = \sum_{x \leq a} p(x)$$

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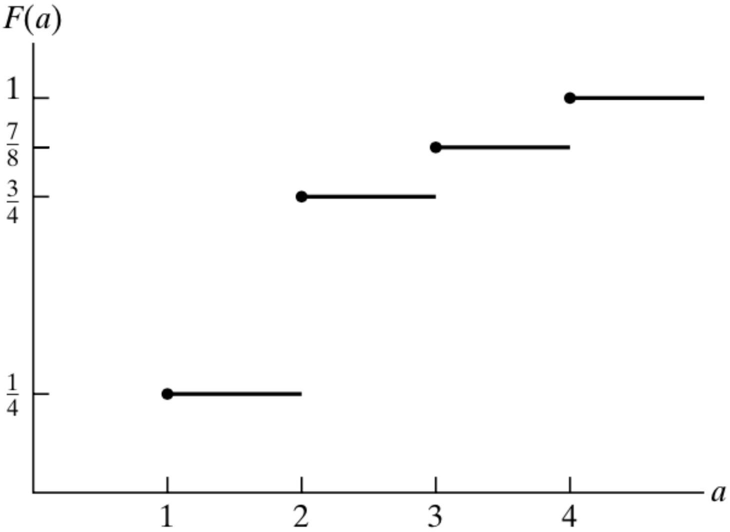
Suppose X has pmf given by

$$p(1) = \frac{1}{4}, \quad p(2) = \frac{1}{2}$$

$$p(3) = \frac{1}{8}, \quad p(4) = \frac{1}{8}$$

then the cdf is

$$F(a) = \begin{cases} 0 & a < 1 \\ \frac{1}{4} & 1 \leq a < 2 \\ \frac{3}{4} & 2 \leq a < 3 \\ \frac{7}{8} & 3 \leq a < 4 \\ 1 & 4 \leq a \end{cases}$$


$F(a)$
1
$\frac{7}{8}$
$\frac{3}{4}$
$\frac{1}{4}$
1
2
3
4
a