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Probability, Homework #1.

I/ Sample space - Events:

1/ Denote the color red, blue, green by the letter R, B, G respectively.

Case 1: Replace the 1st marble in the box after taking it out.
Then each marble can be either R, B or G.
⇒ Sample space: $\Omega_1 = \{(RR), (RB), (RG), (BR), (BB), (BG), (GR), (GB), (GG)\}$.

Case 2: No replacement.
Then the 1st marble can be either R, B or G while the 2nd is different from the 1st.
⇒ Sample space: $\Omega_2 = \{(RB), (RG), (BR), (BG), (GR), (GB)\}$.

2/
a) Each student can either be M or F, here the order of the group is not important.
Sample space: $\Omega_1 = \{(M,M,M,M), (M,M,M,F), (M,M,F,F), (M,F,F,F), (F,F,F,F)\}$.

b) The number of females selected is an integer between 0 and 4.
Sample space: $\Omega_2 = \{0,1,2,3,4\}$.

5/ $\Omega = \{x \in \mathbb{R} : x > 0\} = (0, \infty)$.

a) $A^C = \{x \in \Omega : x \geq 72.5\} = [72.5, \infty)$

b) $B^C = \{x \in \Omega : x \leq 52.5\} = (0, 52.5]$

c) $AB = \{x \in \Omega : 52.5 < x < 72.5\} = (52.5, 72.5)$.

d) $A \cup B = \{x \in \Omega : x > 52.5 \text{ or } x < 72.5\} = \Omega$.

6/ Let $O = \{1,3,5\}$ and $E = \{2,4,6\}$. Then the sample space is:

$$\Omega = \left\{ \{x_k\}_{k=1}^{n} \subset \mathbb{N}, n \in \mathbb{N} : x_k \in O \ \forall k = \overline{1, n-1} \text{ and } x_n \in E \right\}$$

7/
a) $S = \{1, 01, 001, 0001, \ldots, 0000\ldots\}$.
Here 1 represents head and 0 represents tail.
Each element 000...01 (n-1 zeroes) of S represents a game that ends after n flips.
The 0000... element represents a game where all flips land on tail (thus no one wins).

b) $A = \{x \in S : x \text{ has } k \text{ zeroes and one } 1,\ k \geq 0,\ k \bmod 3 = 0\} = \{1, 0001, 000001, \ldots\}$

$B = \{x \in S : x \text{ has } k \text{ zeroes and one } 1,\ k \geq 0,\ k \bmod 3 = 1\} = \{01, 00001, 0000001, \ldots\}$

$(A \cup B)^C = \{x \in S : x \text{ has } k \text{ zeroes and one } 1, k \geq 0, k \bmod 3 = 2\} \cup \{x \in S : x \text{ has no } 1\}$

$= \{001, 000001, 000000001, \ldots, 0000\ldots\}$.

II/ Axiom of Probability:

5/ Consider the following events:

U = a U.S driver is using a seat belt.

N = a Northeastern driver is using a seat belt.

M = a Midwestern driver is using a seat belt.

S = a Southern driver is using a seat belt.

W = a Western driver is using a seat belt.

a) $P(U) = \frac{858}{858+228} \approx 0.7901$.

b) He would have been pleased with the survey results, since the probability is higher than the expectation, i.e. $P(U) \approx 0.7901 > 0.78$.

c) For Northeast: $P(N) = \frac{148}{148+52} = 0.74$.

For Midwest: $P(M) = \frac{162}{162+54} = 0.75$.

For Southern: $P(S) = \frac{296}{296+74} = 0.8$

For Western: $P(W) = \frac{252}{252+48} = 0.84$.

$\Rightarrow$ The West region has the highest seat belt usage.

d) Northeast proportion: $\frac{148+52}{858+228} \times 100\% \approx 18.4162\%$

Midwest proportion: $\frac{162+54}{858+228} \times 100\% \approx 19.8895\%$

Southern proportion: $\frac{296+74}{858+228} \times 100\% \approx 34.07\%$

Western proportion: $\frac{252+48}{858+228} \times 100\% \approx 27.6243\%$

$\Rightarrow$ South and West region had the 1st and 2nd most drivers selected, respectively.