Chapter 7

Multivariate models

1. Simultaneous Equations Models

- All the models we have looked at thus far have been single equations models of the form $\boldsymbol{y = X\beta + u}$
- All of the variables contained in the X matrix are assumed to be **EXOGENOUS**.
- y is an **ENDOGENOUS** variable.

An example from economics to illustrate - the demand and supply of a good:

$$Q_{dt} = \alpha + \beta P_t + \gamma S_t + u_t \qquad (1)$$

$$Q_{st} = \lambda + \mu P_t + \kappa T_t + v_t \qquad (2)$$

$$Q_{dt} = Q_{st} \qquad (3)$$

where Q_{dt} = quantity of the good demanded

Q_{st} = quantity of the good supplied

S_t = price of a substitute good

T_t = some variable embodying the state of technology

Simultaneous Equations Models: The Structural Form

- Assuming that the market always clears (i.e. equilibrium), and dropping the time subscripts for simplicity:

$$Q = \alpha + \beta P + \gamma S + u \qquad (4)$$

$$Q = \lambda + \mu P + \kappa T + v \qquad (5)$$

This is a **simultaneous STRUCTURAL FORM** of the model.

- The point is that price and quantity are determined simultaneously (price affects quantity and quantity affects price).

- P and Q are endogenous variables, while S and T are exogenous.

- We can obtain **REDUCED FORM** equations corresponding to (4) and (5) by solving equations (4) and (5) for P and for Q (separately).

Obtaining the Reduced Form

- Solving for Q,

$$\alpha + \beta P + \gamma S + u = \lambda + \mu P + \kappa T + v \tag{6}$$

- Solving for P,

$$\frac{Q}{\beta} - \frac{\alpha}{\beta} - \frac{\gamma S}{\beta} - \frac{u}{\beta} = \frac{Q}{\mu} - \frac{\lambda}{\mu} - \frac{\kappa T}{\mu} - \frac{v}{\mu} \tag{7}$$

- Rearranging (6),

$$\beta P - \mu P = \lambda - \alpha + \kappa T - \gamma S + v - u$$

$$(\beta - \mu) P = (\lambda - \alpha) + \kappa T - \gamma S + (v - u)$$

$$P = \frac{\lambda - \alpha}{\beta - \mu} + \frac{\kappa}{\beta - \mu} T - \frac{\gamma}{\beta - \mu} S - \frac{v - u}{\beta - \mu} \tag{8}$$

2. Obtaining the Reduced Form (cont'd)

- Multiplying (7) through by $\beta\mu$,

$$\mu Q - \mu\alpha - \mu\gamma S - \mu u = \beta Q - \beta\lambda - \beta\kappa T - \beta v$$

$$\mu Q - \beta Q = \mu\alpha - \beta\lambda - \beta\kappa T + \mu\gamma S + \mu u - \beta v$$

$$(\mu - \beta)Q = (\mu\alpha - \beta\lambda) - \beta\kappa T + \mu\gamma S + (\mu u - \beta v)$$

$$Q = \frac{\mu\alpha - \beta\lambda}{\mu - \beta} - \frac{\beta\kappa}{\mu - \beta}T + \frac{\mu\gamma}{\mu - \beta}S + \frac{\mu u - \beta v}{\mu - \beta} \qquad (9)$$

- (8) and (9) are the reduced form equations for P and Q.

Simultaneous Equations Bias

- **But what would happen if we had estimated equations (4) and (5), i.e. the structural form equations, separately using OLS**?

- Both equations depend on P. One of the CLRM assumptions was that $E(X'u) = 0$, where X is a matrix containing all the variables on the RHS of the equation.

- It is clear from (8) that ***P* is related to the errors in (4) and (5) - i.e. it is stochastic.**

- What would be the consequences for the OLS estimator, $\hat{\beta}$, if we ignored the simultaneity?

Simultaneous Equations Bias (cont'd)

- Recall that $\hat{\beta} = (X'X)^{-1}X'y$ and $y = X\beta + u$
- So that $\hat{\beta} = (X'X)^{-1}X'(X\beta + u)$
  $= (X'X)^{-1}X'X\beta + (X'X)^{-1}X'u$
  $= \beta + (X'X)^{-1}X'u$
- Taking expectations, $E(\hat{\beta}) = E(\beta) + E((X'X)^{-1}X'u)$
  $= \beta + (X'X)^{-1}E(X'u)$
- If the X's are non-stochastic, E($X'u$) = 0, which would be the case in a single equation system, so that $E(\hat{\beta}) = \beta$ which is the condition for unbiasedness.
- But .... if the equation is part of a system, then E($X'u$) $\neq$ 0, in general.

Simultaneous Equations Bias (cont'd)

- Conclusion: Application of OLS to structural equations which are part of a simultaneous system will lead to **biased coefficient estimates**.
- **<u>Question</u>**: Is the OLS estimator still consistent, even though it is biased?
- No - In fact the estimator is **inconsistent** as well.
- Hence it would **not be possible to estimate equations (4) and (5) validly using OLS**.

3. Estimating Simultaneous Equations

So What Can We Do?

- Taking equations (8) and (9), we can rewrite them as

$$P = \pi_{10} + \pi_{11}T + \pi_{12}S + \varepsilon_1 \qquad (10)$$

$$Q = \pi_{20} + \pi_{21}T + \pi_{22}S + \varepsilon_2 \qquad (11)$$

- **We CAN estimate equations (10) & (11) using OLS since all the RHS variables are exogenous.**

- But… what we wanted were the original parameters in the structuralequations - α, β, γ, λ, μ, κ.

Identification of Simultaneous Equations

Can We Retrieve the Original Coefficients from the π's?

Short answer: sometimes.

- As well as simultaneity, we sometimes encounter another problem: identification.
- Consider the following demand and supply equations

Supply equation $Q = \alpha + \beta P$ (12)

Demand equation $Q = \lambda + \mu P$ (13)

We cannot tell which is which!

- Both equations are **NOT IDENTIFIED**
- The problem is that we do not have enough information from the equations to estimate 4 parameters. Notice that we would not have had this problem with equations (4) and (5) since they have different exogenous variables.

What Determines whether an Equation is Identified or not?

- We could have three possible situations:

1. An equation is **unidentified**
- like (12) or (13)
- we cannot get the structural coefficients from the reduced form estimates

2. An equation is **exactly identified**
- e.g. (4) or (5)
- can get unique structural form coefficient estimates

3. An equation is **over-identified**
- Example given later
- More than one set of structural coefficients could be obtained from the reduced form.

What Determines whether an Equation is Identified or not? (cont'd)

- How can we tell if an equation is identified or not?
- There are two conditions we could look at:

- The **order condition** - is a **necessary** but not sufficient condition for an equation to be identified.

- The **rank condition** - is a **necessary and sufficient condition** for identification. We specify the structural equations in a matrix form and consider the rank of a coefficient matrix.

Simultaneous Equations Bias (cont'd)

Statement of the **Order Condition** (from Ramanathan 1995, pp.666)

- Let ***G* denote the number of structural equations**.
- An equation is **just identified** if the number of variables excluded from an equation is *G*-1.
- If more than *G*-1 variables are absent, it is **over-identified**. (common)
- If less than *G*-1 variables are absent, it is **not identified**.

Example

- In the following system of equations, the *Y*'s are endogenous, while the *X*'s are exogenous. Determine whether each equation is over-, under-, or just-identified.

$$Y_1 = \alpha_0 + \alpha_1 Y_2 + \alpha_3 Y_3 + \alpha_4 X_1 + \alpha_5 X_2 + u_1$$
$$Y_2 = \beta_0 + \beta_1 Y_3 + \beta_2 X_1 + u_2$$
$$Y_3 = \gamma_0 + \gamma_1 Y_2 + u_3 \qquad (14)\text{-}(16)$$

Simultaneous Equations Bias (cont'd)

Solution

$G = 3$;

If # excluded variables = 2, the eq^n is just identified

If # excluded variables > 2, the eq^n is over-identified

If # excluded variables < 2, the eq^n is not identified

Equation 14: Not identified

Equation 15: Just identified

Equation 16: Over-identified

Tests for Exogeneity

- How do we tell whether variables really need to be treated as endogenous or not?
- Consider again equations (14)-(16). Equation (14) contains **Y_2 and Y_3** - but do we really need equations for them?
- We can formally test this using a **Hausman test**, which is calculated as follows:

1. Obtain the **reduced form** equations corresponding to (14)-(16). The reduced forms turn out to be:

$$\begin{aligned}
Y_1 &= \pi_{10} + \pi_{11}X_1 + \pi_{12}X_2 + v_1 \\
Y_2 &= \pi_{20} + \pi_{21}X_1 + v_2 \\
Y_3 &= \pi_{30} + \pi_{31}X_1 + v_3
\end{aligned} \qquad (17)\text{-}(19)$$

Estimate the reduced form equations (17)-(19) using **OLS**, and obtain the fitted values, $\hat{Y}_1, \hat{Y}_2, \hat{Y}_3$

Tests for Exogeneity (cont'd)

2. Run the **regression** corresponding to equation (14), ignoring simultaneity

3. Run the **regression (14) again, but now also including the fitted values** $\hat{Y}_2, \hat{Y}_3$ as additional regressors:

$$Y_1 = \alpha_0 + \alpha_1 Y_2 + \alpha_3 Y_3 + \alpha_4 X_1 + \alpha_5 X_2 + \lambda_2 \hat{Y}_2^{\,1} + \lambda_3 \hat{Y}_3^{\,1} + u_1 \qquad (20)$$

4. Use an **F-test** to test the joint restriction that $\boldsymbol{\lambda_2 = 0}$**, and** $\boldsymbol{\lambda_3 = 0}$. If the null hypothesis is rejected, Y_2 and Y_3 should be treated as endogenous.

7. Triangular Systems

- Consider the following system of equations:

$$
\begin{aligned}
Y_1 &= \beta_{10} & &+ \gamma_{11}X_1 + \gamma_{12}X_2 + u_1 \\
Y_2 &= \beta_{20} + \beta_{21}Y_1 & &+ \gamma_{21}X_1 + \gamma_{22}X_2 + u_2 \\
Y_3 &= \beta_{30} + \beta_{31}Y_1 + \beta_{32}Y_2 & &+ \gamma_{31}X_1 + \gamma_{32}X_2 + u_3
\end{aligned}
\qquad (21\text{-}23)
$$

- **Assume** that the 3 error terms are not correlated with each other. Can we estimate the equations individually using OLS?

- **Equation 21**: Contains no endogenous variables, so X_1 and X_2 are not correlated with u_1. So we can use OLS on (21).

- **Equation 22**: Contains endogenous Y_1 together with exogenous X_1 and X_2. We can use OLS on (22) if all the RHS variables in (22) are uncorrelated with that equation's error term. In fact, Y_1 is not correlated with u_2 because there is no Y_2 term in equation (21). So we can use OLS on (22).

- **Equation 23**: Contains both Y_1 and Y_2; we require these to be uncorrelated with u_3. By similar arguments to the above, equations (21) and (22) do not contain Y_3, so we can use OLS on (23).

- This is known as a **TRIANGULAR system**. We do not have a simultaneity problem here.

- **But in practice not many systems of equations will be recursive...**

8. Estimation Procedures for Simultaneous Equations

1st. Indirect Least Squares (ILS)

- Cannot use **OLS** on structural equations, but we can validly apply it to the **reduced form** equations.

- **If the system is just identified**, ILS involves estimating the reduced form equations using OLS, and then using them to substitute back to obtain the structural parameters.

- However, ILS is not used much because

  1. Solving back to get the structural parameters can be tedious.

  2. **Most simultaneous equations systems are over-identified.**

2^{nd}. Estimation of Systems Using Two-Stage Least Squares (2SLS)

- In fact, we can use this technique **for just-identified and over-identified systems.**

- Two stage least squares (2SLS or TSLS) is done in two stages:

Stage 1:

- Obtain and estimate the reduced form equations using OLS. Save the fitted values for the dependent variables.

Stage 2:

- Estimate the structural equations, but replace any RHS endogenous variables with their stage 1 fitted values.

Estimation of Systems
Using Two-Stage Least Squares (cont'd)

Example: Say equations (14)-(16) are required.

Stage 1:

- Estimate the reduced form equations (17)-(19) individually by OLS and obtain the fitted values, $\hat{Y}_1, \hat{Y}_2, \hat{Y}_3$.

Stage 2:

- Replace the RHS endogenous variables with their stage 1 estimated values:

$$\begin{aligned} Y_1 &= \alpha_0 + \alpha_1 \hat{Y}_2 + \alpha_3 \hat{Y}_3 + \alpha_4 X_1 + \alpha_5 X_2 + u_1 \\ Y_2 &= \beta_0 + \beta_1 \hat{Y}_3 + \beta_2 X_1 + u_2 \\ Y_3 &= \gamma_0 + \gamma_1 \hat{Y}_2 + u_3 \end{aligned} \qquad (24)\text{-}(26)$$

- Now $\hat{Y}_2$ and $\hat{Y}_3$ will not be correlated with u_1, $\hat{Y}_3$ will not be correlated with u_2 , and $\hat{Y}_2$ will not be correlated with u_3 .

3rd. Instrumental Variables

- Recall that the reason we cannot use OLS directly on the structural equations is that the endogenous variables are correlated with the errors.

- One solution to this would be not to use Y_2 or Y_3 , but rather to use some other variables instead.

- We want these other variables to be (highly) correlated with Y_2 and Y_3, but not correlated with the errors - they are called **INSTRUMENTS**.

- Say we found suitable instruments for Y_2 and Y_3, z_2 and z_3 respectively. We do not use the instruments directly, but run regressions of the form

$$Y_2 = \lambda_1 + \lambda_2 z_2 + \varepsilon_1$$
$$Y_3 = \lambda_3 + \lambda_4 z_3 + \varepsilon_2 \qquad (27)\ \&\ (28)$$

Instrumental Variables (cont'd)

- Obtain the fitted values from (27) & (28), $\hat{Y}_2$ and $\hat{Y}_3$, and replace Y_2 and Y_3 with these in the structural equation.

- If the instruments are the variables in the reduced form equations, then IV is equivalent to 2SLS.

<u>Other Estimation Techniques</u>

1. 3SLS - allows for non-zero covariances between the error terms.
2. LIML - estimating reduced form equations by maximum likelihood
3. FIML - estimating all the equations simultaneously using maximum likelihood

11. Vector Autoregressive Models (VAR)

- A natural generalisation of autoregressive models popularised by Sims
- A **VAR** is in a sense a systems regression model i.e. there is more than one dependent variable.
- Simplest case is a **bivariate VAR(k)**

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \ldots + \beta_{1k} y_{1t-k} + \alpha_{11} y_{2t-1} + \ldots + \alpha_{1k} y_{2t-k} + u_{1t}$$

$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \ldots + \beta_{2k} y_{2t-k} + \alpha_{21} y_{1t-1} + \ldots + \alpha_{2k} y_{1t-k} + u_{2t}$$

where u_{it} is an iid disturbance term with $E(u_{it})=0$, $i=1,2$; $E(u_{1t}\, u_{2t})=0$.

- The analysis could be extended to a **VAR(k)** model with g variables and g equations.

Vector Autoregressive Models: Notation and Concepts

- One important feature of VARs is the compactness with which we can write the notation. For example, consider the case from above where k=1.
- We can write this as
$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t}$$
$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}$$

or
$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \beta_{10} \\ \beta_{20} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \alpha_{11} \\ \alpha_{21} & \beta_{21} \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

or even more compactly as

$$\underset{g\times 1}{y_t} = \underset{g\times 1}{\beta_0} + \underset{g\times g}{\beta_1}\ \underset{g\times 1}{y_{t\text{-}1}} + \underset{g\times 1}{u_t}$$

Vector Autoregressive Models: Notation and Concepts (cont'd)

- This model can be extended to the case where there are *k* lags of each variable in each of *g* equation:

$$
\underset{g\times 1}{y_t} = \underset{g\times 1}{\beta_0} + \underset{g\times g}{\beta_1}\,\underset{g\times 1}{y_{t-1}} + \underset{g\times g}{\beta_2}\,\underset{g\times 1}{y_{t-2}} + \ldots + \underset{g\times g}{\beta_k}\,\underset{g\times 1}{y_{t-k}} + \underset{g\times 1}{u_t}
$$

Vector Autoregressive Models Compared with Structural Equations Models

- **Advantages of VAR Modelling**

  - Do not need to specify which variables are endogenous or exogenous - all are endogenous

  - Allows the value of a variable to depend on more than just its own lags or combinations of white noise terms, so more general than ARMA modelling

  - Provided that there are no contemporaneous terms on the right hand side of the equations, can simply use OLS separately on each equation

  - Forecasts are often better than "traditional structural" models.

- **Problems with VAR's**

  - VAR's are a-theoretical (as are ARMA models)

  - How do you decide the appropriate lag length?

  - So many parameters! If we have g equations for g variables and we have k lags of each of the variables in each equation, we have to estimate $(g+kg^2)$ parameters. e.g. $g=3$, $k=3$, parameters = 30

  - Do we need to ensure all components of the VAR are stationary? (if not using 1st diff.)

  - How do we interpret the coefficients?

Choosing the Optimal Lag Length for a VAR

- 2 possible approaches: cross-equation restrictions and information criteria

1. Cross-Equation Restrictions

- In the spirit of (unrestricted) VAR modelling, each equation should have the same lag length
- Suppose that a bivariate VAR(8) estimated using quarterly data has 8 lags of the two variables in each equation, and we want to examine a restriction that the coefficients on lags 5 through 8 are jointly zero. This can be done using a likelihood ratio test
- Denote the variance-covariance matrix of residuals (given by $\hat{u}\hat{u}'/T$), as $\hat{\Sigma}$. The **likelihood ratio test** for this joint hypothesis is given by

$$LR = T\left[\log\left|\hat{\Sigma}_r\right| - \log\left|\hat{\Sigma}_u\right|\right]$$

Choosing the Optimal Lag Length for a VAR (cont'd)

where $\hat{\Sigma}_r$ is the variance-covariance matrix of the residuals for the restricted model (with 4 lags), $\hat{\Sigma}_u$ is the variance-covariance matrix of residuals for the unrestricted VAR (with 8 lags), and T is the sample size.

- The test statistic is asymptotically distributed as a χ^2 with degrees of freedom equal to the total number of restrictions. In the VAR case above, we are restricting 4 lags of two variables in each of the two equations = a total of 4 * 2 * 2 = 16 restrictions.
- In the general case where we have a **VAR with *p* equations**, and we want to impose the restriction that the last ***q* lags** have zero coefficients, there would be **p^2q restrictions** altogether
- **Disadvantages**: Conducting the LR test is cumbersome and requires a normality assumption for the disturbances.

Information Criteria for VAR Lag Length Selection

• 2. Multivariate versions of the **information criteria** are required. These can be defined as:

$$MAIC = \ln\left|\hat{\Sigma}\right| + 2k'/T$$

$$MSBIC = \ln\left|\hat{\Sigma}\right| + \frac{k'}{T}\ln(T)$$

$$MHQIC = \ln\left|\hat{\Sigma}\right| + \frac{2k'}{T}\ln(\ln(T))$$

where all notation is as above and ***k′* is** the total number of regressors in all equations, which will be equal to $g^2k + g$ for g equations, each with k lags of the g variables, plus a constant term in each equation. The values of the information criteria are constructed for 0, 1, … lags (up to some pre-specified maximum $\bar{k}$).

Does the VAR Include Contemporaneous Terms?

- So far, we have assumed the VAR is of the form

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t}$$
$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}$$

- But what if the equations had a contemporaneous feedback term?

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + \alpha_{12} y_{2t} + u_{1t}$$
$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + \alpha_{22} y_{1t} + u_{2t}$$

- We can write this as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \beta_{10} \\ \beta_{20} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \alpha_{11} \\ \alpha_{21} & \beta_{21} \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \alpha_{12} & 0 \\ 0 & \alpha_{22} \end{pmatrix} \begin{pmatrix} y_{2t} \\ y_{1t} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

- This VAR is in **primitive form**.

Primitive versus Standard Form VARs

- We can take the contemporaneous terms over to the LHS and write

$$\begin{pmatrix} 1 & -\alpha_{12} \\ -\alpha_{22} & 1 \end{pmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \beta_{10} \\ \beta_{20} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \alpha_{11} \\ \alpha_{21} & \beta_{21} \end{pmatrix}\begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

or

$$\mathrm{B}\, y_t = \beta_0 + \beta_1 y_{t-1} + u_t$$

- We can then pre-multiply both sides by B^{-1} to give

$$y_t = \mathrm{B}^{-1}\beta_0 + \mathrm{B}^{-1}\beta_1 y_{t-1} + \mathrm{B}^{-1}u_t$$

or

$$y_t = \mathrm{A}_0 + \mathrm{A}_1 y_{t-1} + e_t$$

- This is known as a **standard form** VAR, which we can estimate using OLS if $\alpha_{12}=0$ or $\alpha_{21}=0$

13. Block Significance and Causality Tests

- It is likely that, when a VAR includes many lags of variables, it will be difficult to see which sets of variables have significant effects on each dependent variable and which do not. For illustration, consider the following bivariate VAR(3):

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{10} \\ \alpha_{20} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \begin{pmatrix} y_{1t-3} \\ y_{2t-3} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

- This VAR could be written out to express the individual equations as

$$y_{1t} = \alpha_{10} + \beta_{11} y_{1t-1} + \beta_{12} y_{2t-1} + \gamma_{11} y_{1t-2} + \gamma_{12} y_{2t-2} + \delta_{11} y_{1t-3} + \delta_{12} y_{2t-3} + u_{1t}$$

$$y_{2t} = \alpha_{20} + \beta_{21} y_{1t-1} + \beta_{22} y_{2t-1} + \gamma_{21} y_{1t-2} + \gamma_{22} y_{2t-2} + \delta_{21} y_{1t-3} + \delta_{22} y_{2t-3} + u_{2t}$$

- We might be interested in testing the following hypotheses, and their implied restrictions on the parameter matrices:

Block Significance and Causality Tests (cont'd)

Hypothesis	Implied Restriction
1. Lags of y_{1t} do not explain current y_{2t}	$\beta_{21} = 0$ and $\gamma_{21} = 0$ and $\delta_{21} = 0$
2. Lags of y_{1t} do not explain current y_{1t}	$\beta_{11} = 0$ and $\gamma_{11} = 0$ and $\delta_{11} = 0$
3. Lags of y_{2t} do not explain current y_{1t}	$\beta_{12} = 0$ and $\gamma_{12} = 0$ and $\delta_{12} = 0$
4. Lags of y_{2t} do not explain current y_{2t}	$\beta_{22} = 0$ and $\gamma_{22} = 0$ and $\delta_{22} = 0$

- Each of these four joint hypotheses can be tested within the F-test framework, since each set of restrictions contains only parameters drawn from one equation.
- These tests could also be referred to as **Granger causality tests**.
- Granger causality tests seek to answer questions such as "Do changes in y_1 cause changes in y_2?" If y_1 causes y_2, lags of y_1 should be significant in the equation for y_2. If this is the case, we say that **y_1 "Granger-causes" y_2**.
- If y_2 causes y_1, lags of y_2 should be significant in the equation for y_1.
- If both sets of lags are significant, there is "**bi-directional causality**"

Impulse Responses

- VAR models are often difficult to interpret: one solution is to construct the impulse responses and variance decompositions.
- **Impulse responses** trace out the **responsiveness of the dependent variables in the VAR to shocks to the error term**. A unit shock is applied to each variable and its effects are noted.
- Consider for example a simple bivariate VAR(1):

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t}$$
$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}$$

- A change in u_{1t} will immediately change y_1. It will change y_2 and also y_1 during the next period.
- We can examine how long and to what degree a shock to a given equation has on all of the variables in the system.

Variance Decompositions

- **Variance decompositions** offer a slightly different method of examining VAR dynamics. They give the **proportion of the movements in the dependent variables that are due to their "own" shocks**, versus shocks to the other variables.

- **Note:** for calculating impulse responses and variance decompositions, the **ordering of the variables is important**. The main reason for this is that above, we assumed that the VAR **error terms were statistically independent** of one another, which is generally not true, however. The error terms will typically be correlated to some degree.

16. An Example of the use of VAR Models: The Interaction between Property Returns and the Macroeconomy.

- Brooks and Tsolacos (1999) employ a VAR methodology for investigating the interaction between the UK property market and various macroeconomic variables.
- Monthly data are used for the period December 1985 to January 1998.
- It is assumed that stock returns are related to macroeconomic and business conditions.
- The **variables included in the VAR are**
  - FTSE Property Total Return **Index**
  - The rate of **unemployment**
  - **Nominal interest rates**
  - The **spread between long and short term interest rates**
  - **Unanticipated inflation**
  - The **dividend yield**.

The property index and unemployment are I(1) and hence are differenced.

Marginal Significance Levels associated with Joint *F*-tests that all 14 Lags have not Explanatory Power for that particular Equation in the VAR

- Multivariate AIC selected 14 lags of each variable in the VAR
- Marginal significance levels associated with joint F-tests:

	Lags of Variable					
Dependent variable	SIR	DIVY	SPREAD	UNEM	UNINFL	PROPRES
SIR	0.0000	0.0091	0.0242	0.0327	0.2126	0.0000
DIVY	0.5025	0.0000	0.6212	0.4217	0.5654	0.4033
SPREAD	0.2779	0.1328	0.0000	0.4372	0.6563	0.0007
UNEM	0.3410	0.3026	0.1151	0.0000	0.0758	0.2765
UNINFL	0.3057	0.5146	0.3420	0.4793	0.0004	0.3885
PROPRES	0.5537	0.1614	0.5537	0.8922	0.7222	0.0000

- Conclusion: all 14 lags have no explanatory power for this equation in VAR. Variation on property returns cannot be explain by any economic variables