Numerical Analysis

by

Assoc.Prof. Mai Duc Thanh

1. Course Syllabus

- Lecturer: Assoc. Prof. Mai Duc Thanh Office: Room A2.610
  Phone: 091 323 899 E-Mail: mdthanh@hcmiu.edu.vn
- Contents
  1. Roots of Nonlinear Equations
  2. Solutions of Linear Systems of Equations
  3. Curve Fitting and Interpolation
  4. Numerical Differentiation and Integration
  5. Numerical Solutions of Differential Equations
  6. Numerical Methods for Partial Differential Equations
- Grading
  1. Assignments (Homework, Classwork): 20%
  2. Midterm Test: 30%
  3. Final Exam: 50%

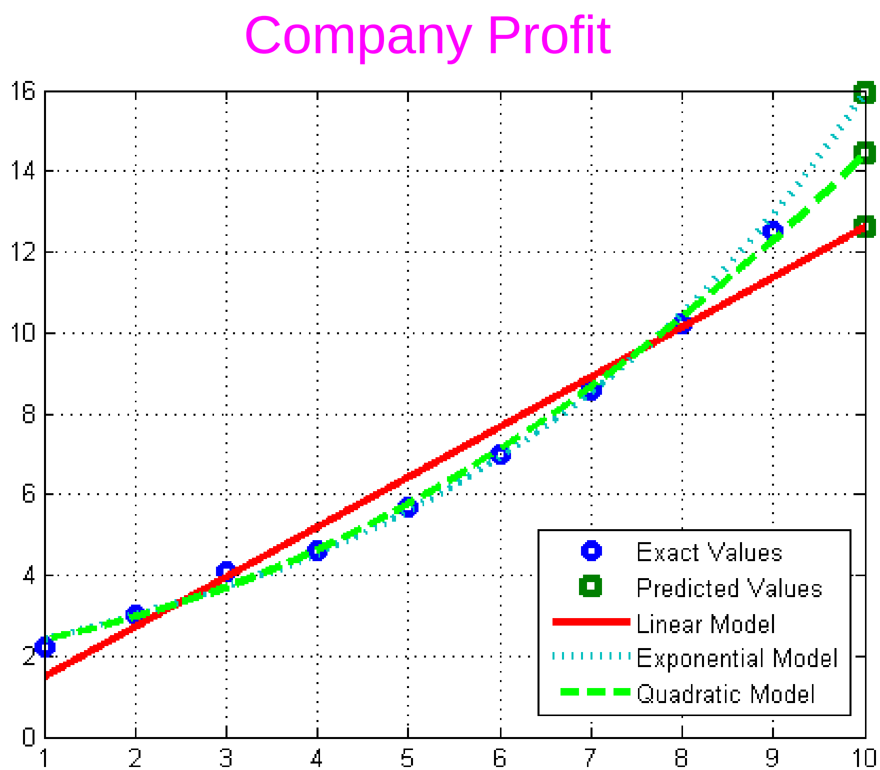
Textbooks

- Burden and Faires, Numerical Analysis, Brooks Cole; 7 edition, 2000
- **S. Chapra & R.P. Canale, Numerical Methods for Engineers: with software and Programming Appl, McGraw-Hill, 7th ed., 2015**
- S.S. Rao, Applied Numerical methods for Engineers and Scientists, Prentice Hall, 2001

Homework and Classwork

- **HW assignments** will be given for each chapter
- **Quizes** are given in the class, not informed in advance.
- Late submission of HWs will lose 10% of the total points/day, not accepted after one week.
- **Extra marks** E from 0-20 are given for those who solve exercises on the board. Rule: ***Averaged HW/Quiz+ E = Assignment Score***

Motivations



- **Numerical methods: powerful problem-solving tools.**

  Usable solutions of a math model: solution should be computational, can be programmed, be used in a computer

  Course enhances your problem-solving skills. Example: use data to forecast future developments, estimate quantities needed, etc

Example

Facebook's ad revenue:

2020: $84.5 billion*
2019: $69.4 billion*
2018: $55.0 billion
2017: $39.9 billion
2016: $26.9 billion
2015: $17.1 billion
2014: $11.5 billion
2013: $6.9 billion
2012: $4.3 billion
2011: $3.2 billion
2010: $1.9 billion
2009: $764 million

Source: Jon Erlichman Twitter

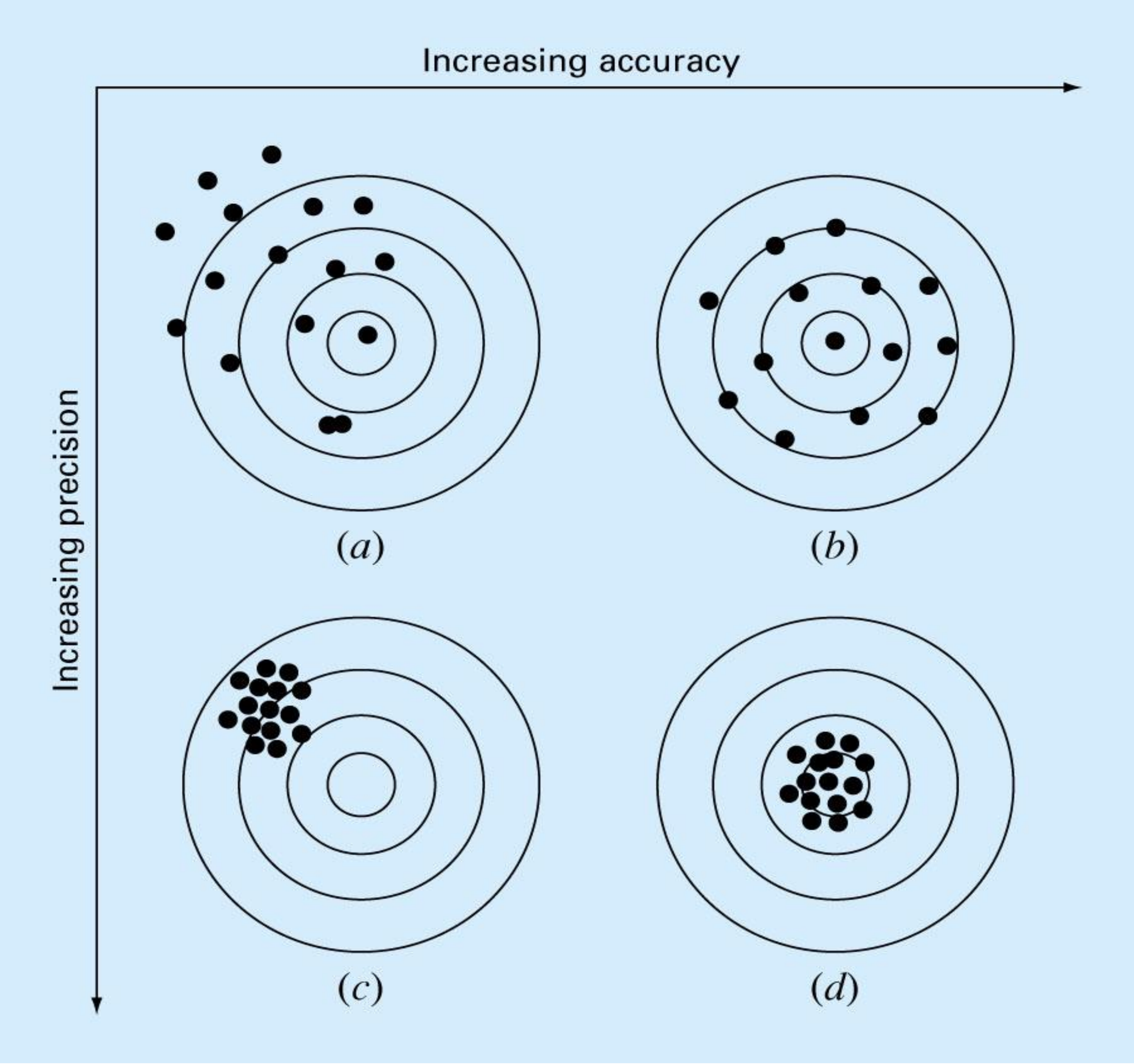
Question: Estimate the Giant's revenue next year

https://thanhnien.vn/cong-nghe/facebook-kiem-tien-nhu-the-nao-1174596.html

Lecture 1: Accuracy and Errors

Accuracy and Precision

- Errors associated with both calculations and measurements can be characterized with regard to their accuracy and precision
- **Accuracy** refers to how closely a computed or measured value agrees with the true value
- **Precision** refers to how closely individual computed or measured values agree with each other


Increasing accuracy
Increasing precision
(a)
(b)
(c)
(d)

Error Definitions E_T

Error, E_T, or true error, is defined as the difference between the true value in a calculation and the approximate value found using a numerical method etc.

$$\boxed{\text{True Error} = \text{True Value} - \text{Approximate Value}}$$

Example 1

The derivative, $f'(x)$ of a function $f(x)$ can be approximated by the equation,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

If $f(x) = 7e^{0.5x}$ and $h = 0.3$

a) Find the approximate value of $f'(2)$

b) True value of $f'(2)$

c) Error for part (a)

Solution

a) For $x=2$ and $h=0.3$

$$f'(2) \approx \frac{f(2+0.3)-f(2)}{0.3}$$

$$= \frac{f(2.3)-f(2)}{0.3}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5(2)}}{0.3}$$

$$= \frac{22.107-19.028}{0.3} = 10.263$$

Example 1…

b) The exact value of $f'(2)$ can be found by using our knowledge of differential calculus.

$$f(x) = 7e^{0.5x}$$

$$f'(x) = 7 \times 0.5 \times e^{0.5x} \quad = 3.5e^{0.5x}$$

So the true value of $f'(2)$ is

$$f'(2) = 3.5e^{0.5(2)} \quad = 9.5140$$

Error is given by

$$E_t = \text{ True Value – Approximate Value}$$

$$= 9.5140 - 10.263 = -0.722$$

Relative Error ε_t

Defined as the ratio between the true error, and the true value.

$$\text{Relative True Error } \varepsilon_t = \frac{\text{True error}}{\text{True Value}}$$

Example 2

Following from the previous example for true error, find the relative true error for $f(x) = 7e^{0.5x}$ at $f'(2)$ with $h = 0.3$

From the previous example,

$$E_t = -0.722$$

Relative True Error is defined as

$$\varepsilon_t = \frac{\text{True Error}}{\text{True Value}}$$

$$= \frac{-0.722}{9.5140} = -0.075888$$

as a percentage,

$$\varepsilon_t = -0.075888 \times 100\% = -7.5888\%$$

Approximate Error E_a

What can be done if true values are not known or are very difficult to obtain?

Approximate error is defined as the difference between the present approximation and the previous approximation

$$\text{Approximate Error } E_a = \text{Present Approximation} - \text{Previous Approximation}$$

Example 3

Use the approximation formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \qquad \text{for} \qquad f(x) = 7e^{0.5x}$$

to estimate $f'(2)$ using a) h=0.3, b) h=0.15.
c) Find the approximate error for b)

Solution:

a) $$f'(2) \approx \frac{f(2+0.3) - f(2)}{0.3}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5(2)}}{0.3} = 10.263$$

Example 3…

b) $$f'(2) \approx \frac{f(2+0.15) - f(2)}{0.15}$$

$$= \frac{7e^{0.5(2.15)} - 7e^{0.5(2)}}{0.15} \quad = 9.8800$$

c) So the approximate error is

$$E_a = \text{Present Approximation} - \text{Previous Approximation}$$

$$= 9.8800 - 10.263$$

$$= -0.38300$$

Relative Approximate Error ε_a

Defined as the ratio between the approximate error and the present approximation

$$\text{Relative Approximate Error } \varepsilon_a = \frac{\text{Approximate error}}{\text{Present Approximation}}$$

Example 4

Find relative approximate error when using

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \qquad \text{for} \qquad f(x) = 7e^{0.5x}$$

to estimate $f'(2)$ with h=0.3 and h=0.15

Solution:

From Example 3, $E_a = -0.3830$

$$\varepsilon_a = \frac{\text{Approximate Error}}{\text{Present Approximation}} = \frac{-0.38300}{9.8800} = -0.038765$$

as a percentage,

$$\varepsilon_a = -0.038765 \times 100\% = -3.8765\%$$

Absolute relative approximate errors may also need to be calculated,

$$|\varepsilon_a| = |-0.038765| = 0.038765 \text{ or } 3.8765\%$$

Round-off Error

Since only a finite number of digits are stored in a computer, the actual number may undergo chopping or rounding of the last digits

For example, let a decimal number x be given by

$$x = 0.b_1 b_2 \ldots b_i b_{i+1} b_{i+2} \ldots, \quad \text{where} \quad 0 \le b_k \le 9, k \ge 1$$

Round-off Error

Let *i* be the maximum number of decimal digits used in the computation. Consider:

$$x = 0.b_1 b_2 \ldots b_i b_{i+1} b_{i+2} \ldots, \quad \text{where} \quad 0 \le b_k \le 9, k \ge 1$$

Chopped representation of x is $x_{chop} = 0.b_1 b_2 \ldots b_i$

Rounded representation:

-If $b_{i+1} \geq 5$, we add 1 to b_i

-If $b_{i+1} < 5$, we merely chop off all but the first *i* digits

Exercise 1

Calculate errors, approximate errors for approximating

$f\,'(0)$, using $f\,'(x) \approx \dfrac{f(x+h) - f(x)}{h}$ for $h = 0.1$ and $h = 0.05$

where

$$f(x) = \sqrt{x^2 + 1}$$

Exercise 2

Calculate errors, relative errors, approximate errors, and relative approximate errors for approximating

$$f'(1), \text{ using } f'(x) \approx \frac{f(x+h) - f(x)}{h} \text{ for } h = 0.1 \text{ and } h = 0.05$$

where

$$f(x) = \sqrt{2x^2 + 1}$$

Exercise 3

Calculate errors, relative errors, approximate errors, and relative approximate errors for approximating

$$f'(2),\ \text{using}\ f'(x) \approx \frac{f(x+h)-f(x)}{h}\ \text{for}\ h = 0.1\ \text{and}\ h = 0.05$$

where

$$f(x) = 8\ln\left(x^2+1\right)$$