Analysis 2
Final Exam Cheat Sheet

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Contents

1. Introduction

The final exam focuses on the following sections:

- Applications of Integrals: Area Between Curves, Volumes (Disk - Shell Method), Length of Curves;
- Convergence of Sequences;
- Series Test of Convergence: Root Test, Ratio Test, Integral Test,...;
- Power Series: Radius & Interval of Convergence, Power Series Representation;
- Taylor & Maclaurin Series.

This Cheat Sheet will cover the formulas and calculation steps for each section. Several useful online websites are included in the last slide.

2. Applications of Integrals

Area Between Curves:

- Area between $f(x)$ and $g(x)$ on $[a, b]$: $A = \int_a^b |f(x) - g(x)| dx$;
- Area between $f(y)$ and $g(y)$ on $[a, b]$: $A = \int_a^b |f(y) - g(y)| dy$;
- Special cases when both formulas can be applied:
  Choose the simpler one.

Disk Method Steps:

- Sketch the graph and a particular cross-section (disk);
- Determine the formula for the outer radius R and inner radius r;
- Determine the disk area: $D = \pi(R^2 - r^2)$;
- Volume: $V = \int D$. Integrating with respect to the specifed variable.

If stuck, try applying **Shell Method**.

Shell Method Steps:

- Sketch the graph and a particular cross-section (shell);
- Determine the formula for the radius R and height h;
- Determine the shell area: $S = 2\pi Rh$;
- Volume: $V = \int S$. Integrating with respect to the specifed variable.

If stuck, try applying **Disk Method**.

2. Applications of Integrals

Parametric Curves: $(x, y) = (x(t), y(t)), t \in [a, b]$.

- Formula of **Tangent Line** at $t = t_0$: $\frac{x - x(t_0)}{x'(t_0)} = \frac{y - y(t_0)}{y'(t_0)}$;
- **Length**: $L = \displaystyle\int_a^b \sqrt{x'(t)^2 + y'(t)^2}dt$. Special cases:

$$y = f(x), x \in [a, b] : L = \int_a^b \sqrt{1 + f'(x)^2}dx;$$

$$x = f(y), y \in [a, b] : L = \int_a^b \sqrt{1 + f'(y)^2}dy.$$

To verify the **convergence** of a given sequence $\{x_n\}$:

- Show that $\{x_n\}$ is bounded;
- Show that $\{x_n\}$ is monotone (increasing/decreasing), or:
- Show that $y_n \leq x_n \leq z_n$, where $\lim_{n\to\infty} y_n = \lim_{n\to\infty} z_n = a \in \mathbb{R}$.

Otherwise, the sequence diverges.

4. Series

To verify the **convergence** of a given series $\sum a_n$:

- Check for special form: geometric series ($\sum ar^n$) or p-series ($\sum \frac{1}{n^p}$).
- Apply **Series Convergence Flowchart**, available at: `https://www.shorturl.at/apKOY`

4. Series

Given a **power series** $\sum c_n(x-a)^n$:

- Find the **Radius of Convergence** R, by Root Test or Ratio Test:

  If $\left|\frac{c_{n+1}}{c_n}\right| \to L$ or $\sqrt[n]{|c_n|} \to L$, then $R = \frac{1}{L}$.

- Find the **Interval of Convergence** I from R:

  - If $R = 0$, then $I = \{a\}$.
  - If $R = \infty$, then $I = \mathbb{R}$.
  - Otherwise, $(a-R, a+R) \subset I \subset [a-R, a+R]$.

    Check if $a \pm R \in I$ by Series Tests at $x = a \pm R$.

4. Series

To find the **series representation** for a given function f:

- **Power Series**: differentiate/integrate/separate f into functions representable by geometric series: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$;
- **Maclaurin/Taylor Series**: look at the **Maclaurin Series List**.

  `https://www.shorturl.at/fq459`

5. Online Websites

- Online Derivative Calculator;
  https://www.derivative-calculator.net
- Online Integral Calculator;
  https://www.integral-calculator.com
- Geogebra - online graphing tool;
  https://www.geogebra.org/graphing
- Desmos - online graphing tool;
  https://www.desmos.com/calculator
- Symbolab - multi-functional online tool for calculus.
  https://www.symbolab.com