Chapter 1: Vector Functions

Lecture 1:
Vector Functions and Space Curves

Vector Functions

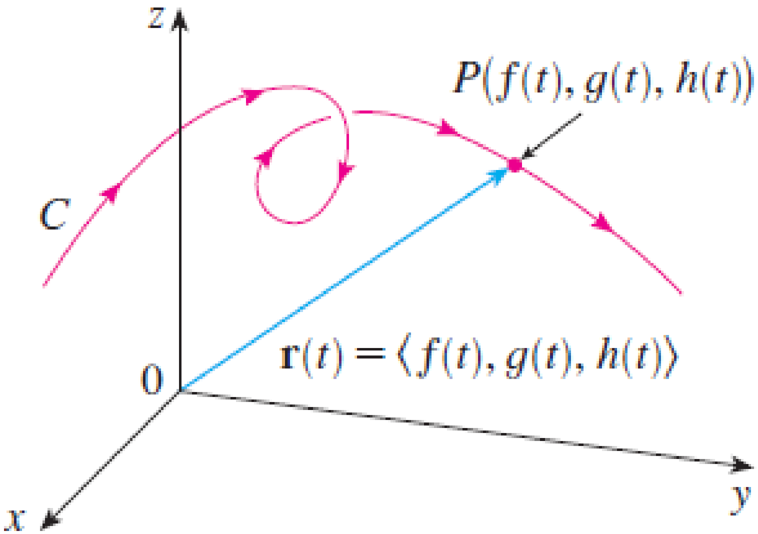


FIGURE 1
C is traced out by the tip of a moving position vector $\mathbf{r}(t)$.

A function is a **rule** that assigns to each element in the domain an element in the range

A **vector-valued function**, or **vector function**, is simply a function whose domain is a set of real numbers and whose range is a set of vectors.

Vector-Valued Function

$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ $\qquad$ t: real variable

Space Curves

A ***space curve*** is a set C of points (x, y, z) satisfying

$$x = x(t), \quad y = y(t), \quad z = z(t), \; t\text{: parameter}$$

are continuous functions of the real variable t

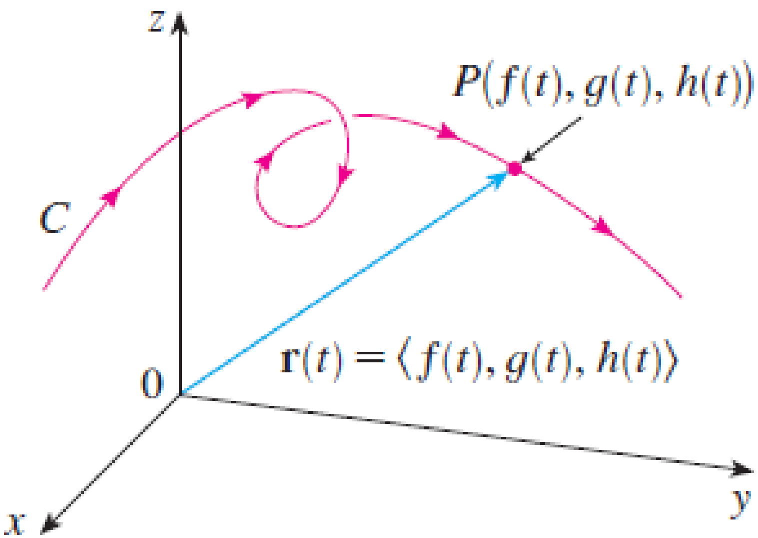


FIGURE 1
C is traced out by the tip of a moving position vector $\mathbf{r}(t)$.

Example 1: Circular Helix

Graph the curve by

$$\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}, \quad t \geq 0$$

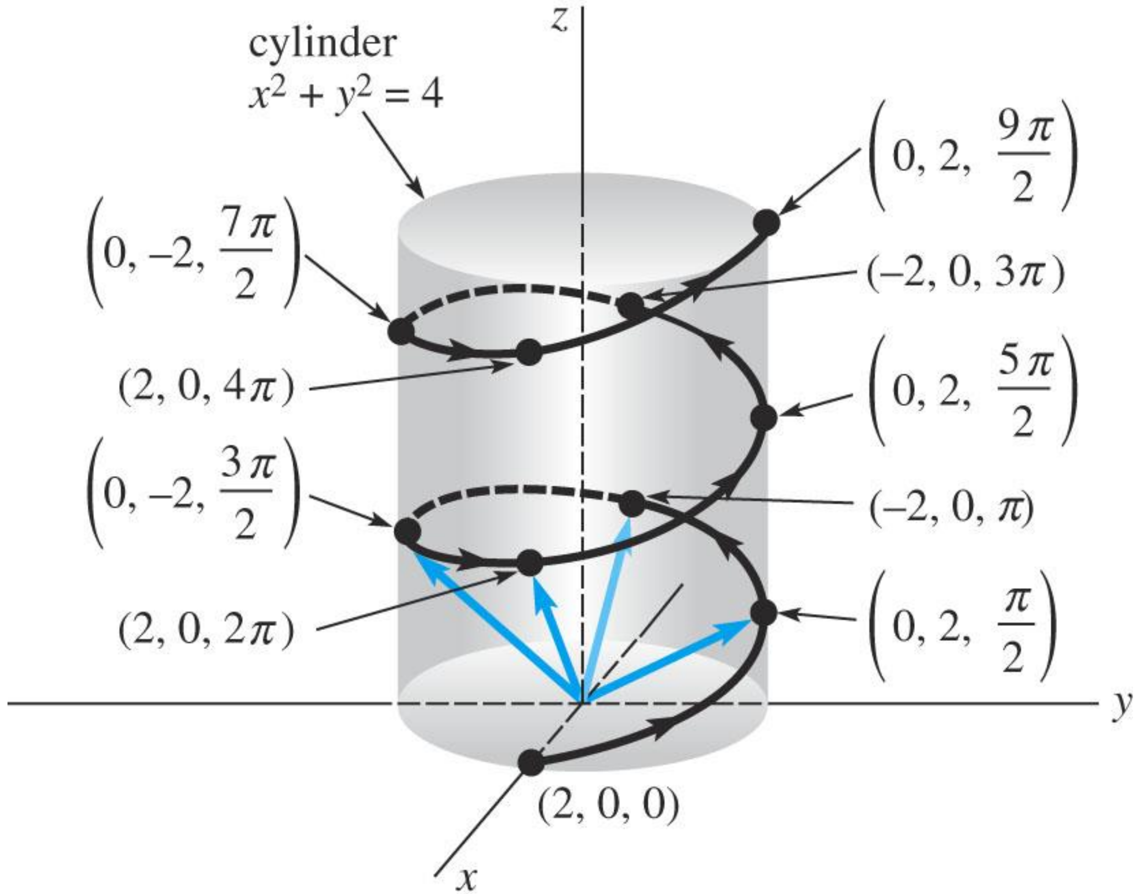
$\mathbf{i} = \langle 1, 0, 0 \rangle$
$\mathbf{j} = \langle 0, 1, 0 \rangle$
$\mathbf{k} = \langle 0, 0, 1 \rangle$

Solution

$$x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2 = 2^2$$

See Fig 2. The curve winds upward in spiral or circular helix.

Fig 2



$$\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k},$$

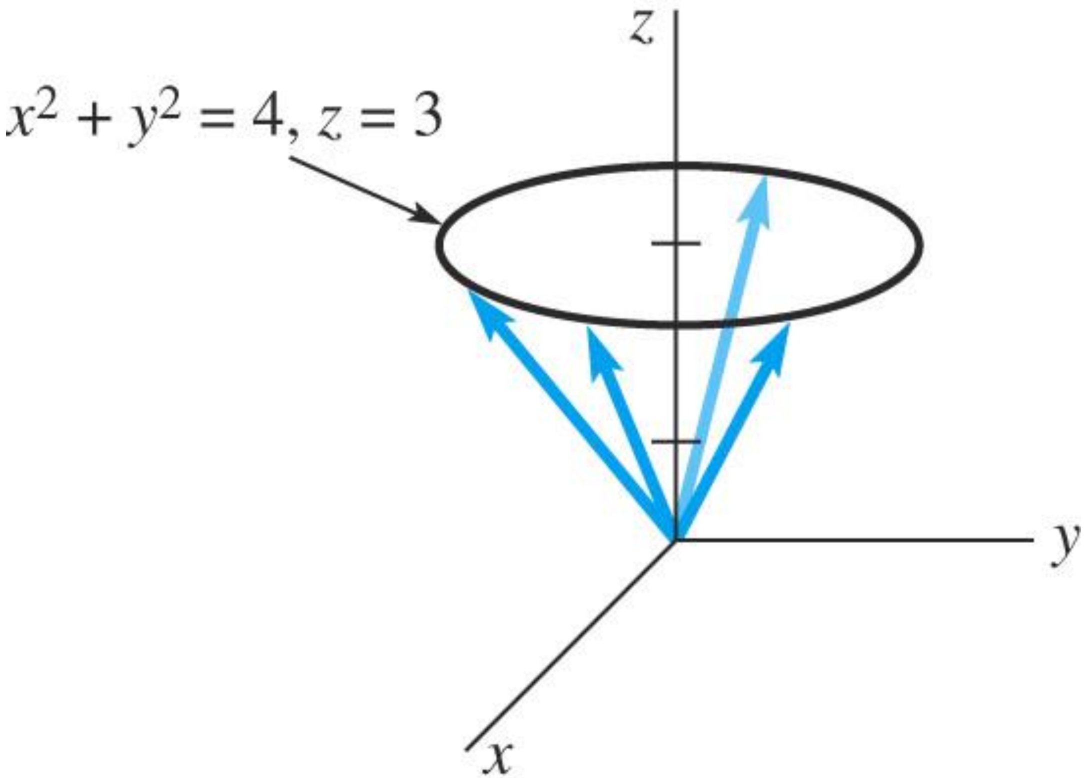
Example 2

Graph the curve by

$$\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 3\mathbf{k}$$

Solution

$$x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2 = 4,\ \ z = 3$$



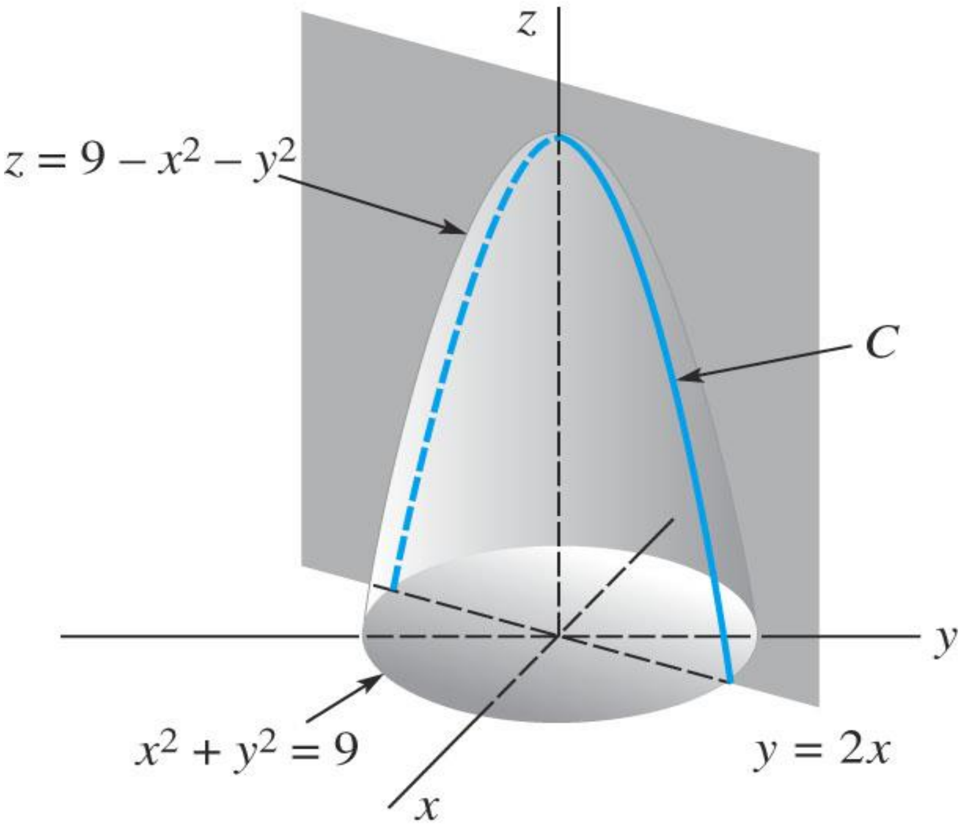
Example 3

Find the vector function that describes the curve C of the intersection of $y = 2x$ and $z = 9 - x^2 - y^2$.

Solution

Let $x = t$, then $y = 2t$, $z = 9 - t^2 - 4t^2 = 9 - 5t^2$

Thus, $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + (9 - 5t^2)\mathbf{k}$.



Limit of a Vector Function

Vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

DEFINITION 1

If $\lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)$ exist, then

$$\lim_{t\to a} \mathbf{r}(t) = \left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \right\rangle$$

Properties of Limits

THEOREM 1

If $\lim_{t\to a}\mathbf{r}_1(t)=\mathbf{L}_1$, $\lim_{t\to a}\mathbf{r}_2(t)=\mathbf{L}_2$, then

(i) $\lim_{t\to a} c\mathbf{r}_1(t)=c\mathbf{L}_1$, c a scalar

(ii) $\lim_{t\to a}[\mathbf{r}_1(t)+\mathbf{r}_2(t)]=\mathbf{L}_1+\mathbf{L}_2$

(iii) $\lim_{t\to a}\mathbf{r}_1(t)\cdot\mathbf{r}_2(t)=\mathbf{L}_1\cdot\mathbf{L}_2$

DEFINITION 2

Continuity

A vector function $\mathbf{r}$ is said to be ***continuous*** at $t = a$ if

(i) $\boldsymbol{r}(a)$ is defined, (ii) $\lim_{t \to a} \mathbf{r}(t)$ exists, and

(iii) $\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$.

Homework Chapter 1

- Section 13.1: 2, 3, 6, 8, 31, 32, 40, 42, 49
- Section 13.2 : 3, 6, 10, 12, 18, 20, 24, 26, 35, 36, 38
- Section 13.3: 1, 2, 4, 6, 9, 12, 14