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Probability, Homework 8

I/ Special Continuous Random Variable:

2/ Let Z be the standard normal random variable.

By the Normal Probability Table, $P(\{Z>1.96\}) = 1 - P(\{Z \leq 1.96\}) = 1 - 0.975 = 0.025$.

3/ Let Z be the standard normal random variable.

a) $P(\{Z>z\}) = 0.3622 \Rightarrow P(\{Z \leq z\}) = 1 - P(\{Z>z\}) = 1 - 0.3622 = 0.6378$.

By the Normal Probability Table, $z = 0.3526$.

b) By the Normal Probability Table, $P(\{Z<z\}) = 0.1131 \Rightarrow z = -1.21$.

c) $P(\{0<Z<z\}) = 0.4838$

$\Rightarrow P(\{Z<z\}) = P(\{Z<0\}) + P(\{0<Z<z\}) = 0.5 + 0.4838 = 0.9838$.

By the Normal Probability Table, $z = 2.14$.

d) $P(\{-z<Z<z\}) = 0.95$. By the symmetry of Z by the y-axis,

$P(\{-z<Z<z\}) = 1 - P(\{Z \leq -z\}) - P(\{Z \geq z\}) = 1 - 2P(\{Z \geq z\})$

$\Rightarrow P(\{Z \geq z\}) = \dfrac{1 - P(\{-z<Z<z\})}{2} = \dfrac{1 - 0.95}{2} = 0.025$

$\Rightarrow P(\{Z<z\}) = 1 - P(\{Z \geq z\}) = 1 - 0.025 = 0.975$.

By the Normal Probability Table, $z = 1.96$.

4/

a) $P(\{Z \leq k\}) = 1 - P(\{Z>k\}) = 1 - 0.2946 = 0.7054$.

By the Normal Probability Table, $k = 0.54$.

b) By the Normal Probability Table, $k = -1.72$.

c) By the Normal Probability Table, $P(\{Z \leq -0.93\}) = 0.1762$

$\Rightarrow P(\{Z<k\}) = P(\{Z \leq -0.93\}) + P(\{-0.93<Z<k\}) = 0.1762 + 0.7235 = 0.8997$.

By the Normal Probability Table, $k = 1.28$.

5/ Let X be the random variable representing the amount of drink distributed.

Then X is a normal random variable with parameters $\mu = 200$, $\sigma = 15$.

$\Rightarrow Z = \dfrac{X-200}{15} \sim \mathcal{N}(0,1)$.

a) By the Normal Probability Table,

$P(\{X > 224\}) = P(\{Z > 1.6\}) = 1 - P(\{Z \le 1.6\}) = 1 - 0.9452 = 0.0548$.

b) By the Normal Probability Table,

$P(\{191 \le X \le 209\}) = P(\{-0.6 \le Z \le 0.6\}) = P(\{Z \le 0.6\}) - P(\{Z < -0.6\})$

$= 0.7257 - 0.2743 = 0.4514$.

c) By the Normal Probability Table,

$P(\{X > 230\}) = P(\{Z > 2\}) = 1 - P(\{Z \le 2\}) = 1 - 0.9772 = 0.0228$.

$\Rightarrow$ # of overflow cups $\approx 0.0228 \times 1000 \approx 23$.

d) $P(\{X < x\}) = 0.25 \Leftrightarrow P\left(\left\{Z < \dfrac{x-200}{15}\right\}\right) = 0.25$.

By the Normal Probability Table, $\dfrac{x-200}{15} = -0.68 \Rightarrow x = 189.95$.

6/ Let X be the random variable representing the piston ring diameter.

Then $X \sim \mathcal{N}(10, 0.03^2) \Rightarrow Z = \dfrac{X-10}{0.03} \sim \mathcal{N}(0,1)$.

a) By the Normal Probability Table,

$P(\{X > 10.075\}) = P(\{Z > 2.5\}) = 1 - P(\{Z \le 2.5\}) = 1 - 0.9938 = 0.0062 = 0.62\%$.

b) By the Normal Probability Table,

$P(\{9.97 < X < 10.03\}) = P(\{-1 < Z < 1\}) = P(\{Z < 1\}) - P(\{Z \le -1\})$

$= 0.8413 - 0.1587 = 0.6826$.

c) $P(\{X < x\}) = 15\% \Leftrightarrow P\left(\left\{Z < \dfrac{x-10}{0.03}\right\}\right) = 0.15$.

By the Normal Probability Table, $\dfrac{x-10}{0.03} = -1.036 \Rightarrow x = 9.9689$.

7/ Let Y be the random variable represnting # of defective chips.

Then $Y \sim \text{Bino}(1000, 0.02)$. Here $n = 1000$ and $p = 2\% = 0.02$

Thus, Y can be approximated by $X \sim \mathcal{N}(20, 19.6)$.

$$\Rightarrow Z = \frac{X - 20}{\sqrt{19.6}} \sim \mathcal{N}(0, 1).$$

a) By the Normal Probability Table,

$$P(\{Y > 25\}) \approx P(\{X > 25.5\}) = P(\{Z > 1.24\}) = 1 - P(\{Z \leq 1.24\})$$

$$= 1 - 0.8925 = 0.1075.$$

b) By the Normal Probability Table,

$$P(\{20 < Y < 30\}) \approx P(\{20.5 < X < 29.5\}) = P(\{-0.11 < Z < 2.37\})$$

$$= P(\{Z < 2.37\}) - P(\{Z \leq -0.11\}) = 0.9911 - 0.4562 = 0.5349.$$

II/ Extra Exercises:

1/ $X \sim \mathcal{N}(102, 100)$, $W = 5X + 525$.

a) Since normality is preserved by linear transformation,

$W \sim \mathcal{N}(5 \times 102 + 525, 5^2 \times 100)$, i.e. $W \sim \mathcal{N}(1035, 2500)$.

The cumulative distribution function of W is given by

$$F(x) = P(\{W \leq x\}) = \int_{-\infty}^{x} \frac{e^{-(t-1035)^2/5000}}{50\sqrt{2\pi}} \, dt., \forall x \in \mathbb{R}.$$

b) $P(\{W > 1100\}) = 1 - P(\{W \leq 1100\}) = 1 - 0.9032 = 0.0968.$

c) $Y = \dfrac{W - 1035}{50} \sim \mathcal{N}(0, 1)$.

$$P(\{W < z\}) = 0.05 \Leftrightarrow P\left(\left\{Y < \frac{z - 1035}{50}\right\}\right) = 0.05.$$

By the Normal Probability Table, $\dfrac{z - 1035}{50} = -1.64 \Rightarrow z = 953.$

Interpretation: There is a 5% chance that your wealth after 1 year < \$953.