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Probability, Homework 6.

1/ Let Y be the random variable representing the spending on new word processors.

Then $Y = 1200X - 50X^2$ and thus, the probability distribution of Y is:

y	0	1150	2200	3150
$p(y) = P(\{Y = y\})$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{5}$

$\Rightarrow E(Y) = 0p(0) + 1150p(1150) + 2200p(2200) + 3150p(3150) = 1855.$

Thus the firm expects to spend \$1855 on new word processors this year.

2/ Let X_1, X_2 be the random variables representing the commission on appointment 1, 2.

Then the probability distribution of X_1, X_2 are:

x	0	1000
$p_1(x) = P(\{X_1 = x\})$	0.3	0.7

x	0	1500
$p_2(x) = P(\{X_2 = x\})$	0.6	0.4

$\Rightarrow E(X_1) = 0p_1(0) + 1000p_1(1000) = 700, \quad E(X_2) = 0p_2(0) + 1500p_2(1500) = 600.$

$\Rightarrow$ Total expected commission $= E(X_1) + E(X_2) =$ \$1300.

3/ Let X be the random variable representing the profit of the insurance company.

Assume that the charge of customer is C, then the probability distribution of X is:

x	$C - A$	C
$p(x) = P(\{X = x\})$	p	$1 - p$

$\Rightarrow E(X) = (C - A)p(C - A) + Cp(C) = (C - A)p + C(1 - p) = C - Ap = 0.1A$

$\Rightarrow C = Ap + 0.1A$. Thus the charge should be $Ap + 0.1A$.

4/

a) $E(X)$ is larger, since the selected student is more likely to be carried on the bus with more students, whereas the chance each bus driver is selected is equally likely.

b) The probability distribution for X and Y:

x	25	33	40	50
$P_X(x) = P(\{X = x\})$	$\frac{25}{148}$	$\frac{33}{148}$	$\frac{40}{148}$	$\frac{50}{148}$
$P_Y(x) = P(\{Y = x\})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$$\Rightarrow E(X) = \frac{25^2 + 33^2 + 40^2 + 50^2}{148} = 39.28, \quad E(Y) = \frac{25+33+40+50}{4} = 37.$$

5/ $$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} \frac{x e^{-x/4}}{4}\,dx = \lim_{a\to+\infty} \int_0^a \frac{x e^{-x/4}}{4}\,dx = \lim_{a\to+\infty} \left(-(x+4)\, e^{-x/4} \Big|_0^a \right) = 4.$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\infty} \frac{x^2 e^{-x/4}}{4}\,dx = \lim_{a\to+\infty} \int_0^a \frac{x^2 e^{-x/4}}{4}\,dx = \lim_{a\to+\infty} \left(-(x^2+8x+32)\, e^{-x/4} \Big|_0^a \right) = 32.$$

$$\Rightarrow \mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 32 - 16 = 16.$$

By the linearity of expectation and variance,

$$\begin{cases} E(Y) = E(3X-2) = 3E(X) - 2 = 10. \\ \mathrm{Var}(Y) = \mathrm{Var}(3X-2) = 9\,\mathrm{Var}(X) = 144. \end{cases}$$