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Probability, Homework 4.

I/ Discrete random variables:

1/ - Discrete random variables: X, N, P.

- Continuous random variables: Y, M, Q.

5/ The probability mass function of X, $p(x)$, is given by the following table:

x	$-500{,}000$	$5{,}000{,}000$	$15{,}000{,}000$
$p(x) = P(\{X=x\})$	0.1	0.3	0.6

6/ Denote the given random variable by X.

Then the probability mass function of X, $p(x)$, is given by the following table:

x	1	4	7
$p(x) = P(\{X=x\})$	0.7	0.2	0.1

The above data can be calculated based on the given cumulative distribution function.

For example. $p(1) = P(\{X=1\}) = P(\{X \le 1\}) - P(\{X<1\}) = F(1) - P(\{X<1\})$

$= 0.7 - \lim_{n\to\infty} P(\{X \le 1 - \frac{1}{n}\}) = 0.7 - \lim_{n\to\infty} F(1-\frac{1}{n}) = 0.7 - \lim_{n\to\infty} 0 = 0.7 - 0 = 0.7$.

7/ Consider the following events:

F_i: Fischer wins the match by winning game i, $i = \overline{1,10}$.

S_i: Spassky wins the match by winning game i, $i = \overline{1,10}$.

D_i: The first i games are draw, $i = \overline{1,10}$. (D_{10} means the match is draw).

a) For any $i = \overline{1,10}$, F_i happens if and only if the first $i-1$ games are draw and Fischer wins game i.

Since all games are independent, $P(D_i) = (0.3)^i$, $\forall i = \overline{1,10}$ and hence

$P(F_i) = P(D_{i-1}) \cdot 0.4 = (0.3)^{i-1} \cdot 0.4$, $\forall i = \overline{1,10}$.

Therefore, the probability that Fischer wins the match is:

$P(\bigcup_{i=1}^{10} F_i) = \sum_{i=1}^{10} P(F_i) = \sum_{i=1}^{10} (0.3)^{i-1} \cdot 0.4 \approx 0.5714$

b) Let X be the random variable representing the duration of the match.

Then X can takes on the values from 1 (no draw game) to 10 (draw match).

For any $i = \overline{1, 9}$, the duration of the match is i games if and only if the first $i-1$ games are draw while game i is not draw. Hence,

$$P(\{X = i\}) = P(D_{i-1}) \cdot 0.7 = (0.3)^{i-1} \cdot 0.7, \forall i = \overline{1, 9}.$$

Thus the probability mass function of X, $p(x)$, is given by the following table:

x	1	2	3	4	5	6	7
$p(x) = P(\{X = x\})$	0.7	0.21	0.063	0.0189	0.00567	0.001701	0.0005103

x	8	9	10
$p(x) = P(\{X = x\})$	0.00015309	0.000045927	0.000019683

II / Extra exercises:

1/ The probability mass function of X, $p(x)$, is given by the following table:

x	0	1	2	3
$p(x) = P(\{X = x\})$	$4c$	$5c$	$8c$	$13c$

Then $\sum_{x=0}^{3} p(x) = 1 \Leftrightarrow 30c = 1 \Leftrightarrow c = \frac{1}{30}$.

2/ X can takes on the values between -3 (3 whites selected) and 3 (3 reds selected).

- $X = -3 \Leftrightarrow$ 3 whites are selected.

$$P(\{X = -3\}) = P(3 \text{ whites}) = \frac{\binom{3}{3}}{\binom{11}{3}} = \frac{1}{165}.$$

- $X = -2 \Leftrightarrow$ 2 whites & 1 blue are selected.

$$P(\{X = -2\}) = P(2 \text{ whites}, 1 \text{ blue}) = \frac{\binom{3}{2} \cdot \binom{5}{1}}{\binom{11}{3}} = \frac{1}{11}.$$

- $X = -1 \Leftrightarrow$ 2 whites & 1 red are selected, or 1 white & 2 blues are selected.

$$P(\{X = -1\}) = P(2 \text{ whites}, 1 \text{ red}) + P(1 \text{ white}, 2 \text{ blues}) = \frac{\binom{3}{2} \cdot \binom{3}{1}}{\binom{11}{3}} + \frac{\binom{3}{1} \cdot \binom{5}{2}}{\binom{11}{3}} = \frac{13}{55}.$$

- $X = 0 \Leftrightarrow$ 3 blues are selected, or 1 white - 1 red - 1 blue are selected.

$$P(\{X = 0\}) = P(3 \text{ blues}) + P(1 \text{ white}, 1 \text{ red}, 1 \text{ blue}) = \frac{C_5^3}{\binom{11}{3}} + \frac{\binom{3}{1} \cdot \binom{3}{1} \cdot \binom{5}{1}}{\binom{11}{3}} = \frac{1}{3}.$$

- $X = 1 \Leftrightarrow$ 2 reds & 1 white are selected, or 1 red & 2 blues are selected.

$$P(\{X=1\}) = P(2\text{ reds}, 1\text{ white}) + P(1\text{ red}, 2\text{ blues}) = \frac{\binom{3}{2}\cdot\binom{3}{1}}{\binom{11}{3}} + \frac{\binom{3}{1}\cdot\binom{5}{2}}{\binom{11}{3}} = \frac{13}{55}.$$

- $X = 2 \Leftrightarrow$ 2 reds & 1 blue are selected.

$$P(\{X=2\}) = P(2\text{ reds}, 1\text{ blue}) = \frac{\binom{3}{2}\cdot\binom{5}{1}}{\binom{11}{3}} = \frac{1}{11}.$$

- $X = 3 \Leftrightarrow$ 3 reds are selected.

$$P(\{X=3\}) = P(3\text{ reds}) = \frac{\binom{3}{3}}{\binom{11}{3}} = \frac{1}{165}.$$

Thus the probability mass function of X, $p(x)$, is given by the following table:

x	-3	-2	-1	0	1	2	3
$p(x) = P(\{X=x\})$	$\frac{1}{165}$	$\frac{1}{11}$	$\frac{13}{55}$	$\frac{1}{3}$	$\frac{13}{55}$	$\frac{1}{11}$	$\frac{1}{165}$