

FINAL EXAMINATION

Semester 1, 2017-2018 • Date: January 15, 2018 • Duration: 120 minutes

ANALYSIS I	
Department of Mathematics	Lecturer:
Chair:	Assoc. Prof. Pham Huu Anh Ngoc
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INSTRUCTIONS: Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.

1. [20 points] Find the following limit

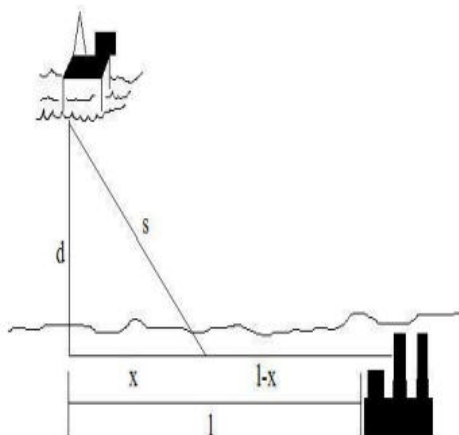
$$\lim_{x \rightarrow 0} \frac{(1 + 2018x)^{2017} - (1 + 2017x)^{2018}}{x^2}.$$

2. [20 points] Suppose that the amount of money in a bank account after t years is given by,

$$A(t) = 2000 - 10te^{5 - \frac{t^2}{8}}.$$

Determine the minimum and maximum amount of money in the account during the first 10 years that it is open.

3. [15 points] An oil rig is 10 miles out to sea and the refinery that the oil needs to go to is 20 miles up the shore line from the oil rig (see the figure given below). If it costs \$25 million per mile to place a pipeline in the water and \$10 million per mile on land where should the pipeline make landfall in order to minimize cost.



($d := 10, l := 20$).

4. [15 points] Let f be continuous on $[1, 3]$ and differentiable on $(1, 3)$. Suppose that, for all x in $(1, 3)$, $1 \leq f'(x) \leq 2$. Prove that

$$2 + f(1) \leq f(3) \leq f(1) + 4.$$

5. [20 points] Evaluate the following indefinite integral

$$\int (\sin x)^{2018} (\cos x)^3 dx.$$

6. [10 points] Find all differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f'(x) = 2018f(x) + 2018, \quad \forall x \in \mathbb{R}.$$

End.