Continuous random variables

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1 Special discrete random variables

1.1 Geometric distribution

1. Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.
   (a) What is the probability that your first call that connects is your tenth call?
   (b) What is the probability that it requires more than five calls for you to connect?
   (c) What is the mean number of calls needed to connect?
2. A player of a video game is confronted with a series of opponents and has an 80% probability of defeating each one. Success with any opponent is independent of previous encounters. The player continues to contest opponents until defeated.
   (a) What is the probability mass function of the number of opponents contested in a game?
   (b) What is the probability that a player defeats at least two opponents in a game?
   (c) What is the expected number of opponents contested in a game?
   (d) What is the probability that a player contests four or more opponents in a game?
   (e) What is the expected number of game plays until a player contests four or more opponents?

1.2 Binomial distribution

1. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, exactly 2 contain the pollutant.
2. A particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.
   (a) Over five mornings, what is the probability that the light is green on exactly one day?
   (b) Over 20 mornings, what is the probability that the light is green on exactly four days?
   (c) Over 20 mornings, what is the probability that the light is green on more than four days?
3. Heart failure is due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances or foreign objects. Natural occurrences are caused by arterial blockage, disease, and infection. Suppose that 20 patients will visit an emergency room with heart failure. Assume that causes of heart failure between individuals are independent.
   (a) What is the probability that three individuals have conditions caused by outside factors?
   (b) What is the probability that three or more individuals have conditions caused by outside factors?
   (c) What are the mean and standard deviation of the number of individuals with conditions caused by outside factors?

4. **A statistical process control chart example.** Samples of 20 parts from a metal punching process are selected every hour. Typically, 1% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework. A process problem is suspected if X exceeds its mean by more than three standard deviations.

   (a) If the percentage of parts that require rework remains at 1%, what is the probability that X exceeds its mean by more than three standard deviations?

   (b) If the rework percentage increases to 4%, what is the probability that X exceeds 1?

   (c) If the rework percentage increases to 4%, what is the probability that X exceeds 1 in at least one of the next five hours of samples?

1.3 Poisson distribution

1. The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.

   (a) What is the probability that there are exactly five calls in one hour?

   (b) What is the probability that there are three or fewer calls in one hour?

   (c) What is the probability that there are exactly 15 calls in two hours?

2. The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.

   (a) What is the probability that there are two flaws in 1 square meter of cloth?

   (b) What is the probability that there is one flaw in 10 square meters of cloth?

   (c) What is the probability that there are no flaws in 20 square meters of cloth?

   (d) What is the probability that there are at least two flaws in 10 square meters of cloth?

2 Special continuous random variables

1. Two points are chosen randomly and independently from the interval [0.1] according to a uniform distribution. Show that the expected distance between the two points is $\frac{1}{3}$

2. Given a standard normal distribution, find the area under the curve that lies

   (a) to the right of $z = 1.96$;

3. Find the value of z if the area under a standard normal curve

   (a) to the right of z is 0.3622;

   (b) to the left of z is 0.1131;

   (c) between 0 and z, with $z > 0$, is 0.4838;

   (d) between $-z$ and z, with $z > 0$, is 0.9500

4. Given a standard normal distribution, find the value of k such that

   (a) $P(Z > k) = 0.2946$;

   (b) $P(Z < k) = 0.0427$;

   (c) $P(-.93 < Z < k) = 0.7235$

5. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

   (a) what fraction of the cups will contain more than 224 milliliters?

   (b) what is the probability that a cup contains between 191 and 209 milliliters?

   (c) how many cups will probably overflow if 230 - milliliter cups are used for the next 1000 drinks?

(d) below what value do we get the smallest 25% of the drinks?

6. The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.03 centimeter.

   (a) What proportion of rings will have inside diameters exceeding 10.075 centimeters?

   (b) What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimeters?

   (c) Below what value of inside diameter will 15% of the piston rings fall?

7. The manufacturing of semiconductor chips produces 2% defective chips. Assume the chips are independent and that a lot contains 1000 chips.

   (a) Approximate the probability that more than 25 chips are defective.

   (b) Approximate the probability that between 20 and 30 chips are defective.