ANALYSIS 2 - FINAL EXAMINATION

Semester 1, 2020-21 • Duration: 90 minutes

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Instructions: Each student is allowed to bring a maximum of two A4 sheets of reference material, stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.

1. (20 points) Let R be the region in the first quadrant of the plane, bounded by the curve $x = -y^2+2y+3$ and coordinates axes. Let V be the solid obtained by rotating R about the x-axis.

a/ (10 points) Find the formulas to compute the volume of V using the disc method and the shell method.

b/ (10 points) Compute the volume of V using one of the formulas obtained in part a/.

2. (20 points) The sequence $\{a_n\}$ is defined as follows:

$$a_1 = 2, \quad a_{n+1} = 4 - \frac{3}{a_n} \text{ for } n \geq 1$$

a/ (10 points) Show that $\{a_n\}$ is an increasing sequence and is bounded from above.

b/ (10 points) Show that $\{a_n\}$ converges and find the limit.

3. a/ (10 points) Determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges or diverges.

b/ (10 points) Calculate the sum $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}-1}{3^{2n}}$.

4. (20 points) Find the series representation of $\dfrac{1-x}{2x^2+3x+1}$ in powers of x. Find the interval of convergence of the series.

5. (20 points) By computing the derivatives of $f(x) = (\ln x)^2$, find the Taylor polynomial $T_3(x)$ of $f(x)$ at at $c = 1$.

END

SOLUTION - ANALYSIS 2 - FINAL EXAMINATION

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1. a/ Disc method:

$$\pi \int_0^3 (1+\sqrt{4-x})^2 dx + \pi \int_3^4 4\sqrt{4-x} dx.$$

Shell method:

$$2\pi \int_0^3 y(-y^2+2y+3) dy.$$

b/ It is easier to use shell method. The volume is $\frac{45}{2}\pi$.

2. a/ Use induction: First, $a_2 = 2.5 > a_1$. Then since the function $f(x) = 4 - \frac{3}{x}$ is increasing, if $a_n > a_{n-1}$ then $a_{n+1} = f(a_n) > f(a_{n-1}) = a_n$.

Clearly $a_n < 4$ because a_n's are positive.

b/ By MCT, the sequence converges. From $L = 4 - \frac{3}{L}$ we get $L = 3$. (Ignore $L = 1$ as the limit has to be bigger than a_1).

3. a/ Converges, by integral test.

b/ 25/56.

4. Since

$$\frac{1-x}{2x^2+3x+1} = -\frac{2}{1+x} + \frac{3}{1+2x},$$

using the geometric series, one gets

$$\sum_{n=0}^{\infty} (-1)^n [3 \cdot 2^n - 2] x^n.$$

By the ratio test, the radius of convergence is $R = 1/2$. At the two endpoints, the series diverges since the terms do not go to 0. Thus, the interval of convergence is $(-\frac{1}{2}, \frac{1}{2})$.

5.

$$f'(x) = \frac{2\ln x}{x},$$

$$f''(x) = \frac{2}{x^2} - \frac{2\ln x}{x^2},$$

$$f'''(x) = -\frac{6}{x^3} + \frac{4\ln x}{x^3}.$$

At $c = 1$,

$$T_3(x) = (x-1)^2 - (x-1)^3.$$