

PROBABILITY-MIDTERM EXAM

Semester 1, 2021-22 • Duration: 80 minutes

INSTRUCTIONS:

- Assignment format: 5 questions. Midterm weight: 30% of the total score.
- The grader can contact students to clarify their writing/solutions when grading.
- Student submits the scanned hand-writing solutions in a single PDF file on Blackboard/Assignments.

1. Let (Ω, \mathcal{F}, P) be a probability space and $A, B, C \in \mathcal{F}$.

(a) (10 points) Show that $P(A|B) \geq P(AB|(A \cup B))$.

(b) (5 points) Assume A , B , and C are independent events. Show that A and $B \cup C$ are independent events.

2. (20 points) A city has an experiment to test for a kind of Covid-19 vaccine from participants. The participants are of two groups: 80% of participants are vaccinated (group A) and 20% of participants did *not* get the vaccine (group B). After testing for both groups, the probability of “a positive result” for group A is 1% and is 6% for group B. If a person gets “a positive result”, what is a probability that she/he did not take the vaccine?

3. In a city, 40% of workers take public transportation daily. Consider a sample of 20 workers.

(a) (10 points) Find the probability that exactly four workers take public transportation daily.

(b) (10 points) Find the probability that at least ten workers take public transportation daily.

4. A box contains seventeen balls, four of which are marked \$3, six \$5, and seven \$10. A set of three balls is randomly selected.

(a) (10 points) Find the probability that each of the balls will be of the same value.

(b) (10 points) Find the probability that the total value of the three balls is \$18.

5. A coffee shop serves an average of 64 customers per hour during the morning rush.

(a) (10 points) Find the probability that this coffee shop serves 64 customers in one hour during this time of day.

(b) (10 points) Find the probability that the number of customers served in one hour is between 60 and 70 during this time of day.

(c) (5 points) Would it be considered unusually low if only 55 customers showed up to this coffee shop in one hour during this time of day? Note that if the number of customers is within 2 standard deviations of the mean, it would *not* be considered unusual.

—GOOD LUCK!—