ANALYSIS 2 - FINAL EXAMINATION

Semester 2, 2020-21 • Duration: 90 minutes

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Instructions: Each student is allowed to bring a calculator and a maximum of two A4 sheets of reference material, stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.

1. (20 points) Let R be the region bounded by the curves $x^2 + (y-1)^2 = 1$ and $y = \frac{2}{x+1}$. (The intersection points are $(0, 2)$ and $(1, 1)$.) Let V be the solid obtained by rotating R about the y-axis. Find the area of R and the volume of V.

2. (20 points) The sequence $\{a_n\}$ is defined as follows:

$$a_1 = 1, \quad a_{n+1} = a_n + \frac{1}{a_n^2} \text{ for } n \geq 1.$$

Determine whether $\{a_n\}$ converges. If it does, find the limit.

3. (20 points) Calculate the sum

$$\sum_{n=1}^{\infty} \big((-1)^{n-1} - \frac{1}{n+1}\big)\frac{1}{n}.$$

4. (20 points) Find the Taylor series of $\ln(2x^2 + 3x + 1)$ centered at $a = 1$. Find the interval of convergence of the series.

5. (20 points) Find the first three terms of the Maclaurin series of $\sqrt{9+x}$.

END