HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION

Semester 2, Academic Year 2020-2021

Duration: 90 minutes (online)

SUBJECT: **Numerical Analysis**	
Chair of Department of Mathematics	Lecturer:
Full name: Prof. Pham Huu Anh Ngoc	Full name: Assoc.Prof. Mai Duc Thanh

<u>Instructions</u>: **Students have to follow the IU regulations for online exams**

Problem 1. (*20 marks*) Use central difference approximations to estimate $y'(1), y"(1)$ for $y(x) = e^x, h = 0.3$ by both $O(h^2)$ and $O(h^4)$ formulas, and then compute the errors.

Problem 2. (*30 marks*) A population $P = P(t)$ (in thousands) of a certain animals is given as a function of the time t (in years) by the following logistic model

$$\begin{aligned} &\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right), \quad t > 0, \\ &P(0) = P_0, \end{aligned} \tag{1}$$

where P_0 is the initial population, M is the carrying capacity. Let $k = 0.6, P_0 = 300, M = 1200$. Estimate the population after 4 years with step-size $\Delta t = 2$ using a) Heun's method, and b) the 4th-order Runge-Kutta method. Compute the errors, given the solution of the problem (1):

$$P(t) = \frac{M}{1 + Ae^{-kt}}, \quad A = \frac{M - P_0}{P_0}.$$

Problem 3. (*20 marks*) Using Midpoint method with step-size $h = 0.1$, compute the approximate solution of the following initial-value problem for 2nd-order differential equations

$$\begin{aligned} &y'' + 6y' + 9y = 0, \quad 0 \le x \le 0.2, \\ &y(0) = 1, \quad y'(0) = 2. \end{aligned} \tag{2}$$

Then, compute the errors, given the exact solution: $y(x) = (5x + 1)e^{-3x}$.

Problem 4. (*30 marks*) Use the implicit Crank-Nicholson method with $\Delta x = 0.2, \Delta t = 0.1$ to approximate the values of solution of the following problem

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{1}{9}\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \\ &u(0,t) = u(1,t) = 0, \quad t > 0, \\ &u(x,0) = \sin(3\pi x), \quad 0 \le x \le 1. \end{aligned} \tag{3}$$

at the times $t_1 = 0.1$ and $t_2 = 0.2$. Then, find the errors, given the exact solution: $u(x,t) = e^{-\pi^2 t}\sin(3\pi x)$.

*********** END OF QUESTIONS ***********