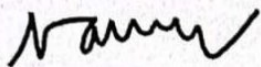
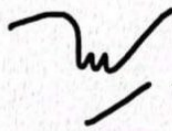


**LINEAR ALGEBRA – FINAL EXAMINATION**

Semester 2, 2021-22 • Duration: 90 minutes

Student's name:		Proctor's signature
Student ID:		
Vice chair of Dept. of Mathematics	Lecturer	Score and Examiner
 Dr. Nguyen Minh Quan		

**INSTRUCTIONS:**

- Use of calculator is allowed.
- Each student is allowed two double-sided sheet of reference material (size A4 or similar).
- All other documents and electronic devices are forbidden.
- You must explain your answers in detail; no marks will be given for the answer alone.
- There are a total of 6 questions in this final examination.
- **GOOD LUCK!**

**Question 1. (20 points)**

Find a basis for the column space, row space and null space of A.

$$A = \begin{bmatrix} -4 & 3 & -27 & 16 \\ -8 & 6 & -42 & 24 \\ -16 & 12 & -92 & 53 \end{bmatrix}$$

**Question 2. (20 points)**

Let  $x_1 = \begin{bmatrix} -2 \\ -2 \\ 1 \\ -2 \end{bmatrix}$  and  $x_2 = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 5 \end{bmatrix}$  and  $x_3 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ , and write  $H = \text{Span}\{x_1, x_2, x_3\}$ .

Use the Gram-Schmidt process to find an orthogonal basis for H.

----- please turn over -----



**Question 3. (20 points)**

Find the eigenvalues and eigenvectors of  $B$ , i.e. the  $\lambda$  which satisfies  $\det|B - \lambda I_3| = 0$

$$B = \begin{bmatrix} 2 & 0 & 4 \\ 3 & -4 & 5 \\ 1 & 0 & 5 \end{bmatrix}$$

**Question 4. (20 points)**

Define the below linear transformation  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^4$ . Find basis for range and kernel of  $L$ .

$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 19x_2 \\ -3x_1 \\ 7x_1 - 14x_2 \\ 4x_1 + 12x_2 \end{bmatrix}$$

**Question 5. (20 points)**

Let  $S = \{u_1, u_2, u_3\}$ , where

$$u_1 = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}, u_2 = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, u_3 = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

a) Show that  $S$  is an orthonormal basis for  $\mathbb{R}^3$ .

b) Express

$$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

as a linear combination of the vectors in  $S$ .

c) Let  $V = \text{span}\{u_1, u_2\}$ . Determine the orthogonal projection of

$$w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

onto  $V$ .