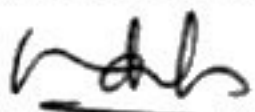



FINAL EXAMINATION

Semester 1, Academic year 2014-2015

Duration: 120 minutes

SUBJECT: Real Analysis	
Chair of Department of Mathematics	Lecturer:
Signature: 	Signature: 
Full name: Assoc. Prof. Nguyen Dinh	Full name: Assoc. Prof. Mai Duc Thanh

Instructions:

- All documents and electronic devices, except scientific calculators and dictionaries, are not allowed.
- Marks are shown in italics.

~~Question 1.~~ (20 marks) State the definition and basic properties of measures ✓

Question 2. (20 marks)

~~(a)~~ State the definition of outer measures.

(b) Prove that any set of outer measure zero is measurable.

~~Question 3.~~ (20 marks) Let (X, \mathcal{M}) be a measurable space. Prove that a function $f : X \rightarrow \mathbb{R} = [-\infty, \infty]$ is measurable if and only if for each open set $O \subseteq \mathbb{R}$, the set $f^{-1}(O)$ is measurable.

~~Question 4.~~ (20 marks) Let (X, \mathcal{M}, μ) be a measure space, and let $f \geq 0$ be a measurable function on X and $\int_X f d\mu < \infty$. Prove that f is finite almost everywhere on X and the set $\{x \in X \mid f(x) > 0\}$ is σ -finite.

~~Question 5.~~ (20 marks) Prove Hölder's inequality for integrals: For $1 \leq p < \infty$, $1/p + 1/q = 1$, if $f \in L^p(X, \mu)$ and $g \in L^q(X, \mu)$ then

$$\int_X |f \cdot g| d\mu = \|f \cdot g\|_1 \leq \|f\|_p \cdot \|g\|_q. \quad (1)$$

Moreover, for $f \neq 0$, show that the function

$$\underline{f^* = \|f\|_p^{1-p} \cdot \text{sign}(f) \cdot |f|^{p-1} \in L^q(X, \mu),}$$

and prove that

$$\left| \int_X f \cdot f^* d\mu = \|f\|_p \quad \text{and} \quad \|f^*\|_q = 1. \right|$$

*** END OF QUESTIONS ***

