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Probability, Homework 5

1/ Let X be the given random variable

a) $P(\{x<0.2\}) = P(\{x\leq 0.2\}) = F(0.2) \approx 0.798$.

b) Let f be the probability density function of X, then

$$f(x) = F'(x) = \begin{cases} 0, & x<0 \\ 8e^{-8x}, & x\geq 0 \end{cases}$$

Thus $P(\{x<0.2\}) = P(\{x\leq 0.2\}) = \int_{-\infty}^{0.2} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{0.2} f(x)dx$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{0.2} 8e^{-8x}dx = -e^{-8x}\Big|_0^{0.2} \approx 0.798$$

2/ Let X be the given random variable.

a) $P(\{X>3000\}) = P(\{X\geq 3000\}) = \int_{3000}^{\infty} f(x)dx = \int_{3000}^{\infty} \frac{e^{\frac{-x}{1000}}}{1000}dx$

$$= -e^{\frac{-x}{1000}}\Big|_{3000}^{\infty} \approx 0.05.$$

b) $P(\{1000\leq X\leq 2000\}) = \int_{1000}^{2000} f(x)dx = \int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{1000}^{2000} \approx 0.233.$

c) $P(\{x<1000\}) = P(\{X\leq 1000\}) = \int_{-\infty}^{1000} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1000} f(x)dx$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{1000} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_0^{1000} \approx 0.632.$$

d) Let t be the number of hours at which 10% of all components have failed, then

$$10\% = 0.1 = P(\{X\leq t\}) = \int_{-\infty}^{t} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{t} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{t} \frac{e^{\frac{-x}{1000}}}{1000}dx$$

$$= -e^{\frac{-x}{1000}}\Big|_0^t = 1 - e^{\frac{-t}{1000}} \Rightarrow t \approx 105.36 \text{ (hrs)}.$$

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a) $P(\{X > 50\}) = P(\{X \geq 50\}) = \int_{50}^{\infty} f(x)dx = \int_{50}^{50.25} f(x)dx + \int_{50.25}^{\infty} f(x)dx$

$$\int_{50}^{50.25} 2dx + \int_{50.25}^{\infty} 0dx = 2x\Big|_{50}^{50.25} = 0.5.$$

b) Assume that 90% of all packages contain at least t pounds of chemical, then

$$90\% = 0.9 = P(\{X \geq t\}) = \int_{t}^{\infty} f(x)dx = \int_{t}^{50.25} f(x)dx + \int_{50.25}^{\infty} f(x)dx$$

$$= \int_{t}^{50.25} 2dx + \int_{50.25}^{\infty} 0dx = 2x\Big|_{t}^{50.25} = 100.5 - 2t \Rightarrow t = 49.8 \text{ (lbs)}.$$