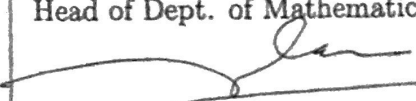



## MIDTERM EXAMINATION

November 2020

Duration: 90 minutes

SUBJECT: REAL ANALYSIS	
Head of Dept. of Mathematics:	Lecturer:
	
Prof. Pham Huu Anh Ngoc	Assoc. Prof. Nguyen Ngoc Hai

**INSTRUCTIONS:** Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.

**Question 1** (25 marks) Let  $X = (0, \infty)$ . Show that

$$\rho(x, y) = |\ln x - \ln y|, \quad x, y \in X,$$

is a distance on  $X$ . Determine the closed unit ball  $\overline{B}(1, 1)$  in the metric space  $(X, \rho)$ .

**Question 2** (a) (15 marks) Let  $a, b, c$  be constants such that  $a^2 + b^2 > 0$ . Let

$$A = \{(x, y) \in \mathbb{R}^2 : ax + by \leq c\} \quad \text{and} \quad B = \{(x, y) \in \mathbb{R}^2 : ax + by < c\}.$$

Show that  $A$  is closed and  $B$  is open in the Euclidean space  $\mathbb{R}^2$ .

(b) (10 marks) Let  $S$  be a nonempty and closed subset of  $\mathbb{R}$ . Show that if  $a := \sup S < \infty$ , then  $a \in S$ .

**Question 3** (20 marks) Let  $(X, d)$  be a metric space and  $f : X \rightarrow \mathbb{R}$ . Suppose that the sets  $\{x \in X : f(x) > \alpha\}$  and  $\{x \in X : f(x) < \alpha\}$  are open for every  $\alpha \in \mathbb{R}$ .

(a) Show that for all  $a, b \in \mathbb{R}$  with  $a < b$ , the sets  $\{x \in X : a < f(x) < b\}$  are open.

(b) Show that  $f$  is continuous.

**Question 4** (a) (15 marks) Let  $(X, d)$  and  $(Y, \rho)$  be metric spaces and  $f : X \rightarrow Y$ . Show that  $f$  is continuous if and only if

$$\overline{f^{-1}(E)} \subset f^{-1}(\overline{E}) \quad \text{for each } E \subset Y.$$

(b) (15 marks) Let  $a, b, c$  be positive numbers. Show that the set

$$A = \left\{ (x, y, z) \in \mathbb{R}^3 : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right\}$$

is compact.

\*\*\* END \*\*\*