Random variables

1 Discrete random variables

1. Classify the following random variables as discrete or continuous:
   (a) X: the number of automobile accidents per year in Virginia.
   (b) Y : the length of time to play 18 holes of golf.
   (c) M: the amount of milk produced yearly by a par- ticular cow.
   (d) N: the number of eggs laid each month by a hen.
   (e) P: the number of building permits issued each month in a certain city.
   (f) Q: the weight of grain produced per acre.

2. The sample space of a random experiment is $\{a, b, c, d, e, f\}$, and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

   Determine the probability mass function of X. Use the probability mass function to determine the following probabilities:

   $a.\, P(X = 1.5)$ $\quad b.\, P(0.5 < X < 2.7)$ $\quad c.\, P(0 \leq X < 2)$ $\quad d.\, P(X > 3)$

3. The data from 200 endothermic reactions involving sodium bicarbonate are summarized as follows:

Final Temperature Conditional	Number of Reactions
$266K$	48
$271K$	60
$274K$	92

   Calculate the probability mass function of final temperature.

4. An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are 0.95 and 0.98. Assume that the components are independent. Determine the probability mass function of the number of components in the assembly that meet specifications.

5. The distributor of a machine for cytogenics has developed a new model. The company estimates that when it is introduced into the market, it will be very successful with a probability 0.6, moderately successful with a probability 0.3, and not successful with probability 0.1. The estimated yearly profit associated with the model being very successful is \$15 million and with it being moderately successful is \$5 million; not successful would result in a loss of \$500,000. Let X be the yearly profit of the new model. Determine the probability mass function of X.

6. Errors in an experimental transmission channel are found when the transmission is checked by a certifier that detects missing pulses. The number of errors found in an eight - bit byte is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.7 & \text{if } 1 \leq x < 4 \\ 0.9 & \text{if } 4 \leq x < 7 \\ 1 & \text{if } x \geq 7 \end{cases}$$

   Determine the probability mass function.

7. Fischer and Spassky play a chess match in which the first player to win a game wins the match. After 10 successive draws. the match is declared drawn. Each game is won by Fischer with probability 0.4. is won by Spassky with probability 0.3. and is a draw with probability 0.3. independent of previous games.
   (a) What is the probability that Fischer wins the match?
   (b) What is the PMF of the duration of the match?

2 Continuous random variables

1. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}.$$

   Find the probability of waiting less than 12 minutes between successive speeders

   (a) using the cumulative distribution function of X;

   (b) using the probability density function of X.

2. The probability density function of the time to failure of an electronic component in a copier (in hours) is

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{1000} e^{-x/1000}, & x \geq 0 \end{cases}.$$

   Determine the probability that

   (a) A component lasts more than 3000 hours before failure.

   (b) A component fails in the interval from 1000 to 2000 hours.

   (c) A component fails before 1000 hours.

   (d) Determine the number of hours at which 10% of all components have failed.

3. The probability density function of the net weight in pounds of a packaged chemical herbicide is

$$f(x) = \begin{cases} 2, & 49.75 \leq x \leq 50.25 \\ 0 & \text{elsewhere} \end{cases}.$$

   (a) Determine the probability that a package weighs more than 50 pounds.

   (b) How much chemical is contained in 90% of all packages?

4. **Simulating a continuous random variable.** A computer has asubroutine that can generate values of a random variable U that is uniformly distributedin the interval $[0, 1]$. Such a subroutine can be used to generate values of a continuousrandom variable with given CDF $F(x)$ as follows. If U takes a value u, we let the value of X be a number x that satisfies $F(x) = u$. For simplicity, we assume that the given CDF is strictly increasing over the range S of values of interest where $S = \{x : 0 < F(x) < 1\}$. This condition guarantees that for any $u \in (0, 1)$. there is a unique x that satisfies $F(x) = u$.

   (a) Show that the CDF of the random variable X thus generated is indeed equal to the given CDF.

   (b) Describe how this procedure can be used to simulate an exponential random variable with parameter λ.

   (c) How can this procedure be generalized to simulate a discrete integer-valued random variable?