HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION, Academic year 2020-2021, Semester 1

Duration: 120 minutes

SUBJECT: Analysis I	**(For FERM program)**
Head of Department of Mathematics Signature: Professor Pham Huu Anh Ngoc	Lecturer: Signature: Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.*

Question 1. (15 marks) Evaluate

$$\lim_{x\to 1^+}\left((x-1)\tan\left(\frac{\pi}{2}x\right)\right) =$$

$$\lim_{x\to 1^+}\frac{(x-1)\sin(\frac{\pi}{2}x)}{\cos(\frac{\pi}{2}x)} \quad \left(\frac{0}{0}\right) = \lim_{x\to 1^+}\frac{\sin(\frac{\pi}{2}x)+(x-1)\frac{\pi}{2}\cos(\frac{\pi}{2}x)}{-\frac{\pi}{2}\sin(\frac{\pi}{2}x)} = \frac{1+0}{-\frac{\pi}{2}} = -\frac{2}{\pi}.$$

Question 2. (20 marks) Find an equation of the line tangent to the graph of $x^2+(y-x)^3=9$ at $x=1$.

Question 3. (15 marks) Find the maximum and minimum values of the function

$$f(x)=\frac{1}{x(1-x)}+2021, \quad x\in[2,3].$$

Question 4. A rectangular box has a square base with edge length x of at least 1 unit. The total surface area of its six sides is 150 square units.

(i) (10 marks) Express the volume V of the box as a function of x.

(ii) (15 marks) Find the dimensions of the box in part (i) with the greatest possible volume. What is this greatest possible volume?

Question 5. (15 marks) Find all functions $f(x)$ such that $f(x)$ is continuous on $[0,1]$, differentiable on $(0,1)$, $f(0)=f(1)=\dfrac{2021}{2020}$ and $2019f'(x)+2020f(x)\geq 2021$, for any $x\in(0,1)$.

Question 6. (10 marks) Suppose f is differentiable on $[a, b]$ and $f'_+(a).f'_-(b) < 0$. Show that there exists $c \in (a, b)$ such that $f'(c) = 0$.

The end.