Covariance - Sum of random variable

1 Covariance

1. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

$p(x,y)$		x		
		1	2	3
	1	.05	.05	.1
y	3	.05	.1	.35
	5	0	.2	.1

   Find the covariance and the correlation coefficient of X and Y.

2. The fraction X of male runners and the fraction Y of female runners who compete in marathon races are described by the joint density function
$$f(x,y) = \begin{cases} 8xy, & 0 \le y \le x \le 1 \\ 0, & otherwise \end{cases}.$$
   Find the covariance and the correlation coefficient of X and Y.

3. Random variables X and Y follow a joint distribution
$$f(x,y) = \begin{cases} 2, & 0 \le x \le y \le 1 \\ 0, & otherwise \end{cases}.$$
   Find the covariance and the correlation coefficient of X and Y .

4. Show that $Cov(aX, bY) = abCov(X,Y)$.

5. Suppose that X and Y are random variables with the same variance. Show that $X - Y$ and $X + Y$ are uncorrelated.

6. Suppose that a random variable X satisfies
$$E(X) = 0,\ E(X^2) = 1,\ E(X^3) = 0,\ E(X^4) = 3$$
   and
$$Y = a + bX + cX^2$$
   Find $Cov(X,Y)$.

2 Sum of RVs

1. Let T_n be the sum of numbers from n fair 6 -sided dice. Find $E(T_n)$.

2. Suppose a system has n components, and that at a particular time the jth component is working with probability P_j, $j = 1, ..., n$. Let X be the number of components working at that time. Find $E(X)$.

3. Suppose $E(X^2) = 3$, $E(Y^2) = 4$, $E(XY) = 2$. Find $E[(X+Y)^2]$.

4. Let X and Y be two independent Bernoulli random variables with parameters p and r respective ly. Find $E(X - Y)^2$.

5. Suppose $E(X^2) = 3$, $E(Y^2) = 4$, $E(XY) = 2$. Find $E[(X+Y)^2]$.