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Probability, Homework 7

I/ Geometric distribution:

1/ Let X be the random variable representing the number of calls needed to connect.

X is a geometric random variable with parameter $p = 0.02$.

Thus, the probability mass function of X is given by:

$$p(x) = P(\{X = x\}) = 0.98^{x-1} \times 0.02, \quad \forall x \in \mathbb{N}.$$

a) $P(\{X = 10\}) = p(10) = 0.0167$

b) $P(\{X > 5\}) = 1 - \left[\sum_{i=1}^{5} P(\{X = i\})\right] = 0.9039.$

c) $E(X) = \frac{1}{p} = 50.$

2/ Let X be the random variable representing the number of opponents contested in a game.

X is a geometric random variable with parameter $p = 1 - 80\% = 0.2$.

a) The probability mass function of X is given by:

$$p(x) = P(\{X = x\}) = 0.8^{x-1} \times 0.2, \quad \forall x \in \mathbb{N}.$$

b) $P(\{X \geq 3\}) = 1 - P(\{X = 1\}) - P(\{X = 2\}) = 1 - 0.2 - 0.16 = 0.64.$

c) $E(X) = \frac{1}{p} = 5.$

d) $P(\{X \geq 4\}) = P(\{X \geq 3\}) - P(\{X = 3\}) = 0.64 - 0.128 = 0.512.$

e) Let Y be the random variable representing the number of games played until four or more opponents are contested in a game.

Then Y is a geometric random variable with parameter $p' = 0.512$.

Thus $E(Y) = \frac{1}{p'} = 1.9531 \approx 2.$

II/ Binomial distribution:

1/ Let X be the random variable representing the number of polluted samples in the next 18 samples.

Then X is a binomial random variable with parameters $n=18$, $p=0.1$.

Thus the probability mass function of X is given by:

$$p(x) = P(\{X=x\}) = \binom{18}{x} 0.1^{x} \times 0.9^{18-x}, \quad \forall x = \overline{0,18}.$$

Hence $P(\{X=2\}) = p(2) = 0.2835$.

2/ Let X be the random variable representing the number of days when the green light is observed during the next n days.

Then X is a binomial random variable with parameters n, $p=0.2$.

Thus the probability mass function of X is given by:

$$p(x) = P(\{X=x\}) = \binom{n}{x} 0.2^{x} \times 0.8^{n-x}, \quad \forall x = \overline{0,n}.$$

a) For $n=5$: $p(x) = \binom{5}{x} 0.2^{x} \times 0.8^{5-x}$

$\Rightarrow P(\{X=1\}) = p(1) = 0.4096$.

b) For $n=20$: $p(x) = \binom{20}{x} 0.2^{x} \times 0.8^{20-x}$

$\Rightarrow P(\{X=4\}) = p(4) = 0.2182$.

c) $P(\{X>4\}) = 1 - \left[\sum_{i=0}^{4} P(\{X=i\})\right] = 0.3704$.

3/ Let X be the random variable representing the number of heart failures caused by outside factors among the 20 patients.

Then X is a binomial random variable with parameters $n = 20$, $p = 0.13$.

Thus the probability mass function of X is given by:

$$p(x) = P(\{X = x\}) = \binom{20}{x} 0.13^x \, 0.87^{20-x}, \quad \forall x = \overline{0,20}.$$

a) $P(\{X = 3\}) = p(3) = 0.2347.$

b) $P(\{X \geq 3\}) = 1 - P(\{X = 0\}) - P(\{X = 1\}) - P(\{X = 2\})$

$= 1 - 0.0617 - 0.1844 - 0.2618 = 0.4921.$

c) $E(X) = np = 2.6.$

$\sigma_X = \sqrt{Var(X)} = \sqrt{np(1-p)} = 1.5040.$

4/ X is a binomial random variable with parameters $n = 20$, p.

Thus the probability mass function of X is given by:

$$p(x) = P(\{X = x\}) = \binom{20}{x} p^x (1-p)^{20-x}, \quad \forall x = \overline{0,20}.$$

a) For $p = 0.01$: $p(x) = \binom{20}{x} 0.01^x \times 0.99^{20-x}, \quad \forall x = \overline{0,20}.$

$E(X) = np = 0.2$

$\sigma_X = \sqrt{Var(X)} = \sqrt{np(1-p)} = 0.445.$

$\Rightarrow P(\{X > E(X) + 3\sigma_X\}) = P(\{X > 1.535\}) = P(\{X \geq 2\})$

$= 1 - P(\{X = 0\}) - P(\{X = 1\}) = 0.0169.$

b) For $p = 0.04$: $p(x) = \binom{20}{x} 0.04^x \times 0.96^{20-x}, \quad \forall x = \overline{0,20}.$

$P(\{X > 1\}) = 1 - P(\{X = 0\}) - P(\{X = 1\}) = 0.1897$

c) Let Y be the random variable representing the number of hours that X exceeds 1 in the next 5 hours.

Then Y is a binomial random variable with parameters $n = 5$, $p' = 0.1897$

Thus the probability mass function of Y is given by:

$$p'(x) = P(\{Y = x\}) = \binom{5}{x} 0.1897^x \, 0.8103^{5-x}, \quad \forall x = \overline{0,5}$$

$\Rightarrow P(\{Y \geq 1\}) = 1 - P(\{Y = 0\}) = 1 - 0.3493 = 0.6507.$

III/ Poisson distribution:

1/ Let X be the given Poisson random variable.

a) For $\lambda = 10$ (10 calls / hour):

$$p(x) = P(\{X = x\}) = e^{-10} \frac{10^i}{i!}, \forall i \in \mathbb{N} \cup \{0\}.$$

$\Rightarrow P(\{X = 5\}) = p(5) = 0.0378$

b) $P(\{X \leq 3\}) = \sum_{x=0}^{3} P(\{X = x\}) = 0.0103.$

c) For $\lambda = 20$ (20 calls / 2 hours):

$$p'(x) = P(\{X = x\}) = e^{-20} \frac{20^i}{i!}, \forall i \in \mathbb{N} \cup \{0\}.$$

$\Rightarrow P(\{X = 15\}) = p'(15) = 0.0516.$

2/ Let X be the given Poisson random variable.

a) For $\lambda = 0.1$ (0.1 flaw / m^2):

$$p(x) = P(\{X = x\}) = e^{-0.1} \frac{0.1^i}{i!}, \forall i \in \mathbb{N} \cup \{0\}$$

$\Rightarrow P(\{X = 2\}) = p(2) = 0.0045.$

b) For $\lambda = 1$ (1 flaw / 10 m^2):

$$p_1(x) = P(\{X = x\}) = \frac{e^{-1}}{i!}, \forall i \in \mathbb{N} \cup \{0\}$$

$\Rightarrow P(\{X = 1\}) = p_1(1) = 0.3679.$

c) For $\lambda = 2$ (2 flaw / 20 m^2):

$$p_2(x) = P(\{X = x\}) = e^{-2} \cdot \frac{2^i}{i!}, \forall i \in \mathbb{N} \cup \{0\}$$

$\Rightarrow P(\{X = 0\}) = p_2(0) = 0.1353.$

d) For $\lambda = 1$: $P(\{X \geq 2\}) = 1 - P(\{X = 0\}) - P(\{X = 1\})$

$$= 1 - p_1(0) - p_1(1) = 0.2642.$$