Pre-final EXAMINATION (just A TEST)

Academic year 2010-2011, Semester 3
Duration: 90 minutes

SUBJECT: **Differential Equations**	
Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name: Prof. Phan Quoc Khanh	Full name: Dr. Pham Huu Anh Ngoc

Instructions:

- *Open-book examination. Laptops are NOT allowed.*

Question 1. *Solve the following differential equation*

$$y''' + y'' - 2y' = x - e^x.$$

Question 2. *Find the general solution of the differential equation:*

$$y'' - \frac{1}{x}y' = x.$$

Question 3. *Find the general solution of the system of linear differential equations*

$$\frac{dx}{dt} = x - 4y; \quad \frac{dy}{dt} = x + y$$

Question 4. *(***Population Growth***) The rate of change of the population of a city is proportional to the population P at any time t. In 1998, the population was 400,000, and the constant of proportionality was 0.015. Estimate the population of the city in the year 2005.*

SOLUTIONS:

Question 1. *Note that*

$$0 = (e^x \ln y + 2x)dx + (\frac{e^x}{y} + 2y)dy = (\ln y de^x + e^x d\ln y) + dx^2 + dy^2 = d(e^x \ln y + x^2 + y^2).$$

Thus the general solution is given by

$$e^x \ln y + x^2 + y^2 = C.$$

Question 2.

$$xy' - y = x^2 \cos x, \qquad y(\pi) = 1.$$

The given equation can be rewritten as

$$y' - \frac{1}{x}y = x\cos x, \qquad x \neq 0.$$

The integrating factor is given by $I(x) = \frac{1}{x}$. *Thus, we get*

$$\frac{1}{x}y' - \frac{1}{x^2}y = \cos x,$$

or equivalently,

$$\frac{d}{dx}(\frac{y}{x}) = \cos x.$$

Therefore, the general solution is

$$y(x) = x(\sin x + C).$$

Since $y(\pi) = 1$, *the particular solution is given by* $y(x) = x(\sin x + \frac{1}{\pi})$.

Question 3.

The general solution of $y'' - 3y' + 2y = 0$ *is given by*

$$y(x) = c_1 e^x + c_2 e^{2x}.$$

A particular solution of $y'' - 3y' + 2y = e^x(3 - 4x)$ *is* $y_p(x) = xe^x(2x + 1)$. *Thus, the general solution of the equation*

$$y'' - 3y' + 2y = e^x(3 - 4x)$$

is given by

$$y(x) = c_1 e^x + c_2 e^{2x} + xe^x(2x + 1).$$

Question 4. $y_1(x) = e^{\sin x}$ *and* $y_2(x) = e^{-\sin x}$ *are linearly independent solutions of the linear second order differential equation*

$$y'' + (\tan x)y' - (\cos^2 x)y = 0.$$

The general solution is given by

$$y(x) = c_1 e^{\sin x} + c_2 e^{-\sin x}.$$