Abstract

This report corresponds to the evaluation of the obtained results in the first laboratory of Applied Harmonic Analysis. We shall divide the report in two parts, one for each exercise.

1 Exercise 1

On the first exercise, we have worked with COVID-19 data. First, we have applied a five-day moving average filter of the data to reduce the noise. The reader can observe in Figure 1 , the reduction of the noise that we have just talked about. Let us explain why the noise is reduced. Calling h our filter and assuming that we have that $y = x * h$, where x the original signal, we know that $Y(\omega) = X(\omega)H(\omega)$, where Y and X are the Fourier transform of the signals y and x respectively, and H is the Fourier transform of the filter h. Hence, in the frequency space, the convolution becomes a multiplication. Since we are interested on removing high frequencies, should be interesting to obtain a function $H(\omega)$, such that H takes the value 1 near ω and 0 otherwise. That is exactly what our function h is performing, see Figure 2, to observe how is the absolute value of the Fourier transform of h, $|H|$.

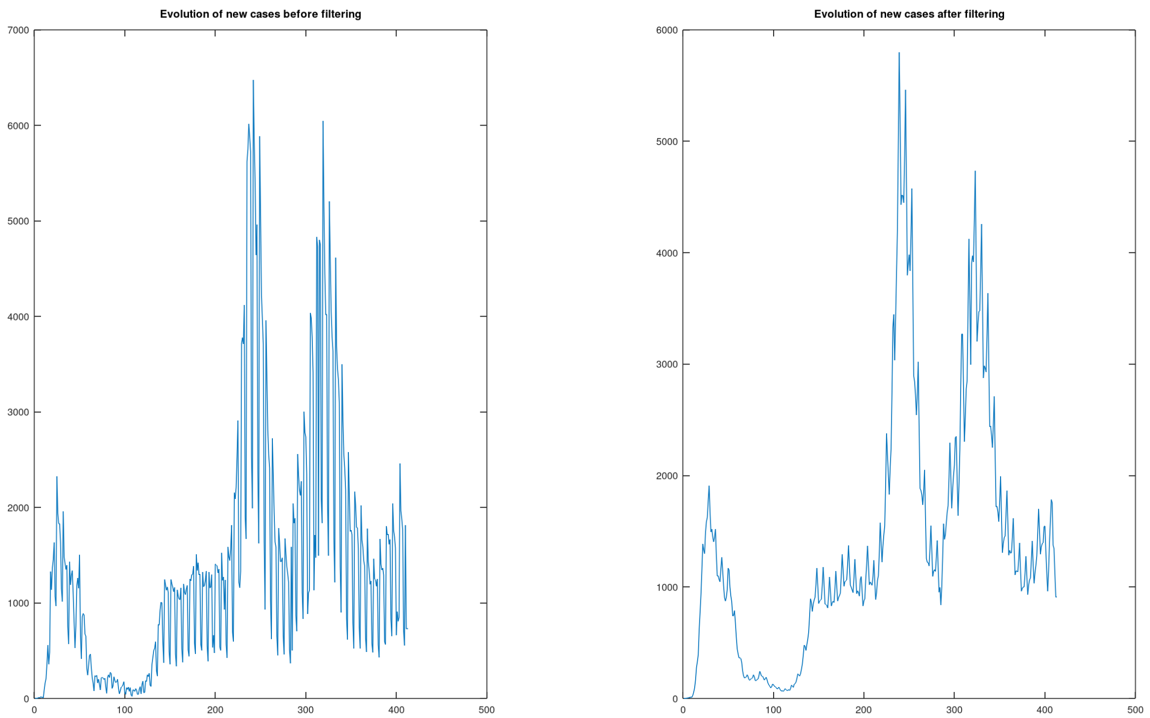

Figure 1: Data before filtering on the left and data after filtering on the right.

To compute the Fourier transform, we can either use the *freqz* function of Octave, or compute it by hand. I my case, I have done both, the plot generated by *freqz* can be found in the code. To compute it by hand, notice that

$$H(w) = \frac{1}{5}\sum_{k=-2}^{k=2} e^{ikw} = \frac{1}{5}(2cos(2w) + 2cos(2) + 1),$$

where the last equality can be easily obtained using Euler's formula. Moreover, we are asked to propose alternative filters h. We have proposed to use a seven-day moving average filter of

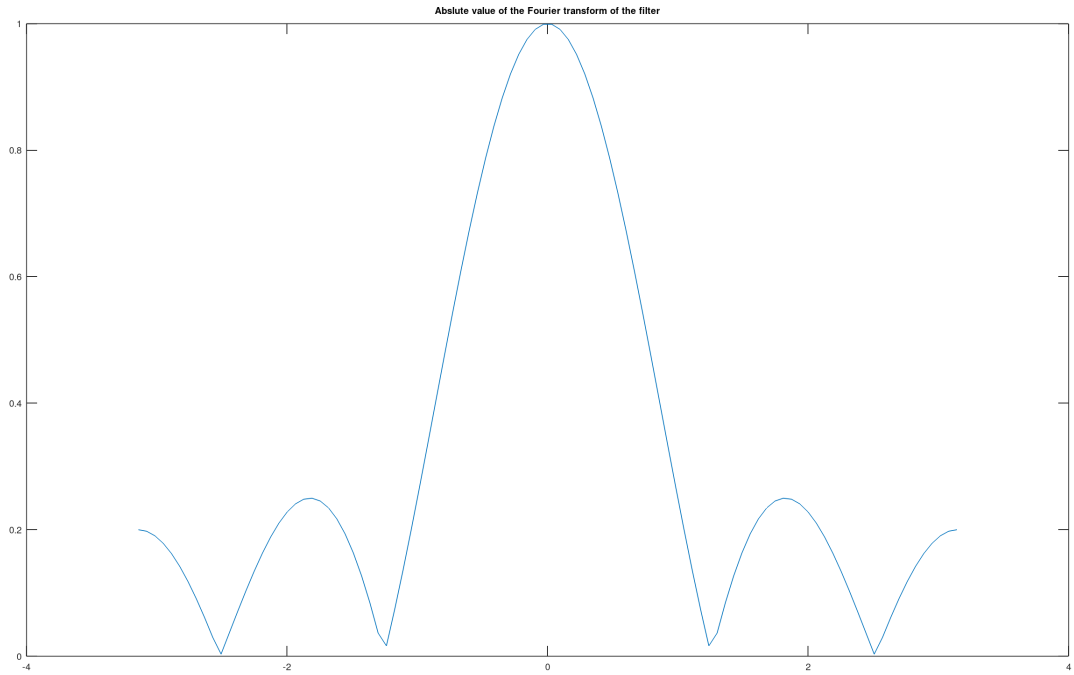

Figure 2: Fourier transform of the filter h.

data instead of a seven-day, since there is always a spike in cases every 7 days (Mondays) as there is no work at weekends. The improvements can be visualized on Figure 3.

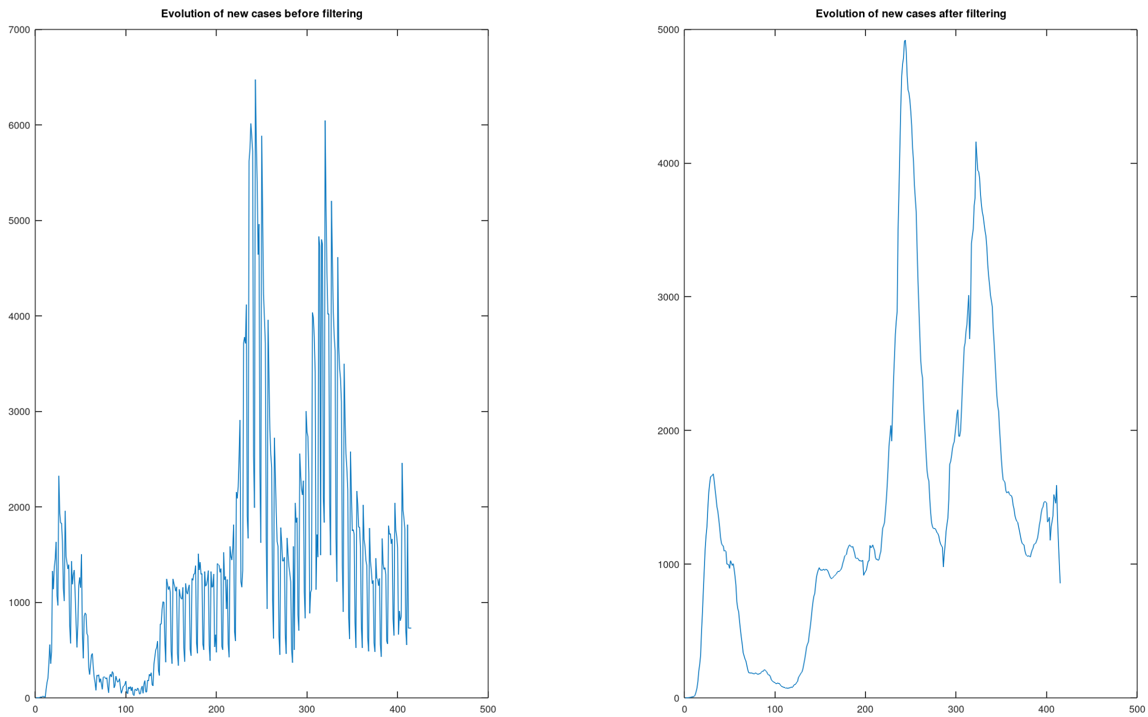

Figure 3: Data before filtering on the left and data after filtering on the right.

2 Exercise 2

On the second exercise, we have worked with data from a GPS. We are asked to eliminate the noise of the data using some filter as the previous exercise. Using the same filter as in the previous exercise, one can observe a small improvement, see Figure 4.

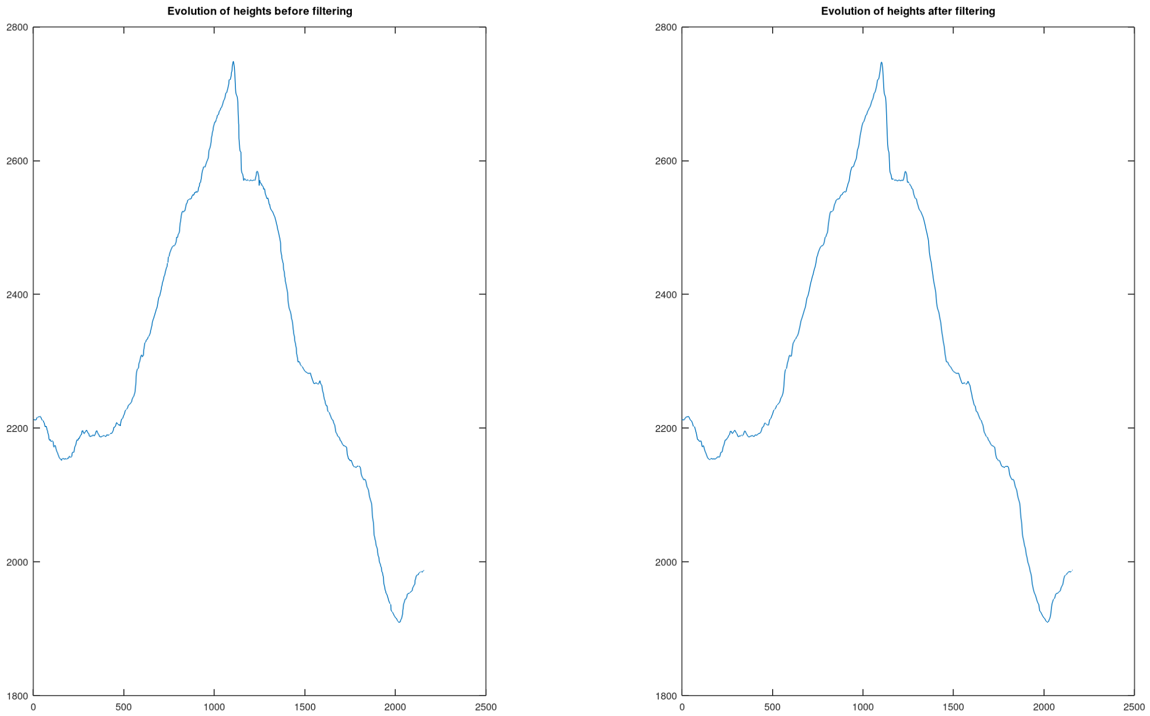

Figure 4: Data before filtering on the left and data after filtering on the right.

We are asked to calculate the total height climbed and distance, before and after applying the filter. The following table summarizes the results.

Type	Climbed	Descended
Without filtering	796.18	1021.01
Filtered	747.59	972.59

The difference in results comes from the elimination of high frequencies on the original data after passing the filter of the previous exercise. Notice that, another possible filters could be $h \in \mathbb{R}^n$ such that $h = (1/n, \ldots, 1/n)$, where n is big. In our case, we have selected $n = 5$, but increasing n, we could get a filter that removes even more high frequencies.