k-means algorithm implementation on Hadoop

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March 31, 2017

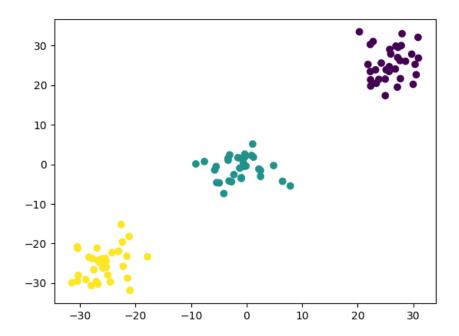
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1 Data points generation

1.1 createDataPoints.py

The initial task of the project is to generate a set of more than one million data points to be used later as input for the k-means clustering algorithm. Using this python script three isotropic Gaussian blobs for clustering are generated. More specifically, the centers are the following data points [25, 25], [-1, -1], [-25, -25]. Additionally, the data points are presented visually with the use of a scatter plot.



```
1 """createDataPoints.py: Generate data points for clustering."""
2
3 import argparse
4 import matplotlib.pyplot as plt
5 import os
6 import pandas as pd
7 from sklearn.datasets.samples_generator import make_blobs
9 __author__ = "Stratos Gounidellis, Lamprini Koutsokera"
10 __copyright__ = "Copyright 2017, BDSMasters"
11
12
13 class DataGenerator():
14
15
      def generateData(self, points, dataFile):
16
           """Generate the input data points.
17
18
           :param self: An instance of the class DataGenerator.
19
           :param points: The number of data points to be generated.
```

```
20
           :param dataFile: The file to save the data points.
21
22
           centers = [[25, 25], [-1, -1], [-25, -25]]
23
           X, labels_true = make_blobs(n_samples=long(points),
24
                                        centers=centers, cluster_std=3.5,
25
                                        n features=2)
26
27
           df = pd.DataFrame(X)
28
           df.to_csv(dataFile, header=False, index=False, sep=" ")
29
30
           plt.scatter(X[:, 0], X[:, 1], c=labels_true)
31
           directory = "../images"
32
           if not os.path.isdir(directory):
33
               os.makedirs(directory)
34
           plt.savefig("../images/data_points.png")
35
36
37 if __name__ == "__main__":
38
      parser = argparse
39
      parser = argparse.ArgumentParser()
40
      parser.add_argument("dataFile", type=str,
41
                           help="File to save the generated data points.")
42
43
      parser.add_argument("points", type=int,
44
                           help="Number of data points to create.")
45
      args = parser.parse_args()
46
       instanceDataGenerator = DataGenerator()
47
      instanceDataGenerator.generateData(args.points, args.dataFile)
```

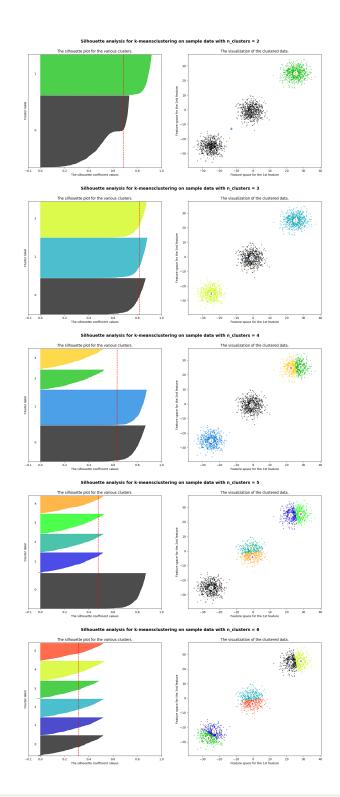
2 Number of clusters

2.1 plotSilhouetteScore.py

The silhouette score constitutes a useful criterion for determining the proper number of clusters and it was firstly suggested by Peter J. Rousseeuw. The silhouette shows which objects lie well within their cluster, and which ones are merely somewhere in between clusters. A silhouette close to 1 implies the datum is in an appropriate cluster, while a silhouette close to -1 implies the datum is in the wrong cluster.

The following python script calculates the silhouette score for different numbers of clusters ranging from 2 to 6. With this script not only the average silhouette score of each cluster is visualized but also the thickness (i.e. the number of data points) of each cluster. The number of clusters which leads to clusters of more or less similar thickness and silhouette score above the average could be the optimal number of clusters for the k-means algorithm.

As expected creating three clusters is the optimal solution in this case.



```
provides
 8 a way to assess parameters like number of clusters visually. This measure has
 9 range of [-1, 1].
10
11 Silhouette coefficients (as these values are referred to as) near +1 indicate
12 that the sample is far away from the neighboring clusters. A value of 0
13 indicates that the sample is on or very close to the decision boundary between
14 two neighboring clusters and negative values indicate that those samples might
15 have been assigned to the wrong cluster.
16
17 Source:
18 http://scikit-learn.org/stable/auto_examples/cluster/
      plot_kmeans_silhouette_analysis.html
19
20 """
21
22 import argparse
23 from kmeans import KmeansRunner
24 import matplotlib.cm as cm
25 import matplotlib.pyplot as plt
26 import numpy as np
27 import os
28 import PIL
29 from sklearn.cluster import KMeans
30 from sklearn.metrics import silhouette_samples, silhouette_score
32 __author__ = "Scikit-Learn"
33
34
35 class SilhouetteScore():
36
37
       def calculateSilhouetteScore(self, dataFile):
38
           """Calculate the silhouette score for different numbers of clusters.
39
40
           :param self: An instance of the class SilhouetteScore.
41
           :param dataFile: An array with the input data points.
42
           :return: A list with the names of the image files created.
43
           . . . . .
44
           instanceKmeans = KmeansRunner()
45
           X = instanceKmeans.retrieveData(dataFile)
46
           if (X.shape[0] > 10000):
47
               size = round(X.shape[0] * 0.001)
48
               idx = np.random.randint(X.shape[0], size=size)
49
               subset = X[idx, :]
50
               X = subset
51
           range_n_clusters = [2, 3, 4, 5, 6]
52
           list_images = []
53
54
           for n_clusters in range_n_clusters:
55
56
               fig, (ax1, ax2) = plt.subplots(1, 2)
57
               fig.set_size_inches(18, 7)
58
```

```
59
               ax1.set_xlim([-0.1, 1])
60
61
               ax1.set_ylim([0, len(X) + (n_clusters + 1) * 10])
62
63
               clusterer = KMeans(n_clusters=n_clusters, random_state=10)
64
               cluster_labels = clusterer.fit_predict(np.array(X))
65
66
                silhouette_avg = silhouette_score(X, cluster_labels)
67
               print("For n_clusters =", n_clusters,
68
                      "The average silhouette_score is :", silhouette_avg)
69
70
               sample_silhouette_values = silhouette_samples(X, cluster_labels)
71
72
               y_lower = 10
73
                for i in range(n_clusters):
74
75
                    ith_cluster_silhouette_values = \
76
                        sample_silhouette_values[cluster_labels == i]
77
78
                    ith_cluster_silhouette_values.sort()
79
80
                    size_cluster_i = ith_cluster_silhouette_values.shape[0]
81
                    y_upper = y_lower + size_cluster_i
82
83
                    color = cm.spectral(float(i) / n_clusters)
84
                    ax1.fill_betweenx(np.arange(y_lower, y_upper),
85
                                      0, ith_cluster_silhouette_values,
                                      facecolor=color, edgecolor=color, alpha=0.7)
86
87
88
                    ax1.text(-0.05, y_lower + 0.5 * size_cluster_i, str(i))
89
90
                    y_{lower} = y_{upper} + 10
91
92
               ax1.set_title("The silhouette plot for the various clusters.")
93
               ax1.set_xlabel("The silhouette coefficient values")
94
               ax1.set_ylabel("Cluster label")
95
96
               ax1.axvline(x=silhouette_avg, color="red", linestyle="--")
97
98
               ax1.set vticks([])
99
               ax1.set_xticks([-0.1, 0, 0.2, 0.4, 0.6, 0.8, 1])
100
101
               colors = cm.spectral(cluster_labels.astype(float) / n_clusters)
102
               ax2.scatter(X[:, 0], X[:, 1], marker=".", s=30, lw=0, alpha=0.7,
103
                            c=colors)
104
105
               centers = clusterer.cluster centers
106
               ax2.scatter(centers[:, 0], centers[:, 1],
107
                            marker="o", c="white", alpha=1, s=200)
108
109
               for i, c in enumerate(centers):
110
                    ax2.scatter(c[0], c[1], marker="$%d$" % i, alpha=1, s=50)
111
112
               ax2.set_title("The visualization of the clustered data.")
```

```
ax2.set_xlabel("Feature space for the 1st feature")
113
114
               ax2.set_ylabel("Feature space for the 2nd feature")
115
116
               plt.suptitle(("Silhouette analysis for k-means"
117
                              "clustering on sample data "
                              "with n_clusters = %d" % n_clusters),
118
119
                             fontsize=14, fontweight="bold")
120
                fig.savefig("cluster_" + str(n_clusters) + ".png")
121
               list_images.append("cluster_" + str(n_clusters) + ".png")
122
           return list_images
123
124
       def silhouetteScoretoPNG(self, list images):
125
           """Save the results of the plots in asingle image file.
126
127
            :param self: An instance of the class SilhouetteScore.
128
            :param list_images: A list with the name of the image files created.
129
130
           clusterImages = [PIL.Image.open(i) for i in list_images]
131
           minSize = sorted([(np.sum(i.size), i.size)
132
                              for i in clusterImages])[0][1]
133
134
           imagesCombination = np.vstack((np.asarray(i.resize(minSize)))
135
                                            for i in clusterImages))
136
           imagesCombination = PIL.Image.fromarray(imagesCombination)
137
           directory = "../images"
138
           if not os.path.isdir(directory):
139
               os.makedirs(directory)
140
           imagesCombination.save("../images/clustersScore.png")
141
           for image in list images:
142
               os.remove(image)
143
           print ("The silhouette score for the number of"
144
                   " clusters ranging from 2 "
145
                   "to 6 has been saved in the file clustersScore.png!")
146
147
148 if __name__ == "__main__":
149
150
       parser = argparse
151
       parser = argparse.ArgumentParser()
152
       parser.add_argument("dataFile", type=str,
153
                            help="File to retrieve the generated data points.")
154
       args = parser.parse args()
155
       instanceSilhouetteScore = SilhouetteScore()
156
       images = instanceSilhouetteScore.calculateSilhouetteScore(args.dataFile)
157
       instanceSilhouetteScore.silhouetteScoretoPNG(images)
```

3 k-means clustering algorithm

3.1 kmeans.py

This python script calls the k-means algorithm implemented on hadoop. However, before implementing k-means the initial centroids are computed using the k-means++ algorithm proposed in

Algorithm 1 k-means++ algorithm

- 1. Take one center c_1 , chosen uniformly at random from X.
- 2. Take a new center c_1 , choosing $x \in X$ with probability $\frac{D(x')^2}{\sum_{x \in X} D(x')^2}$.
- 3. Repeat Step 2. until we have taken k centers altogether.
- 4. Proceed as with the standard k-means algorithm.

After determining the initial centroids, k-means algorithm is called in order to detetermine the new centroids of the clusters and the results are saved as an image file.

```
1 """kmeans.py: Run the k-means algorithm."""
3 import argparse
4 import matplotlib.pyplot as plt
5 import numpy as np
6 import pandas as pd
7 import os
8 import random
9 import re
10 import sys
11 sys.tracebacklimit = 0
12
13 __author__ = "Stratos Gounidellis, Lamprini Koutsokera"
14 __copyright__ = "Copyright 2017, BDSMasters"
15
16
17 class KmeansRunner():
18
19
      def retrieveData(self, file):
20
           """Retrieve the data points from the input file.
21
22
           :param self: An instance of the class KmeansRunner.
23
           :param file: A file with the input data.
24
           :return: An array with the input data points.
25
26
          df_points = pd.read_csv(file, header=None, names=["x", "y"], sep=" ")
27
          if (len(df_points.index) < 1):</pre>
28
               raise Exception("The input file is empty!")
29
          data = [tuple(row) for row in df_points.values]
30
          points = np.array([data_point for data_point in data])
31
          return points
32
33
      def initialCentroids(self, file, nclusters):
34
           """Calculate the initial centroids to be used by the k-means
35
               clustering algorithm.
36
```

```
37
           :param self: An instance of the class KmeansRunner.
38
           :param file: A file with the input data.
39
           :param nclusters: The number of clusters.
40
           :return: A list with the initial centroids.
41
42
           points = self.retrieveData(file)
43
           initial_centroids = [list(random.choice(points))]
44
           dist = []
45
           if nclusters < 2:</pre>
46
               raise Exception("Error the number of clusters should be" +
47
                                " greater than or equal to 2!")
48
           for i in range(2, nclusters + 1):
49
               dist.append([np.linalg.norm(np.array(point) -
50
                            initial_centroids[i - 2])**2 for point in points])
51
               min dist = dist[0]
52
               if (len(dist) > 1):
53
                   min_dist = np.minimum(
54
                       min_dist, (dist[index] for index in range(1, len(dist))))
55
56
               sumValues = sum(min_dist)
57
               probabilities = [float(value) / sumValues for value in min_dist]
58
               cumulative = np.cumsum(probabilities)
59
               random_index = random.random()
60
61
               index = np.where(cumulative >= random index)[0][0]
62
               initial_centroids.append(list(points[index]))
63
64
           return initial centroids
65
66
       def retrieveCentroids(self, file):
67
           """Retrieve the centroids coordinated from the centroids file.
68
69
           :param self: An instance of the class KmeansRunner.
70
           :param file: A file with the centroids.
71
           :return: A list with the centroids.
72
73
           with open(file, "r") as inputFile:
74
               output_data = inputFile.readlines()
75
76
           centroids = []
77
           for point in output_data:
78
               p = re.search("\setminus [(.*?)\setminus ]", point).group()
79
               p = p.replace("[", "").replace("]", "")
80
               p.strip()
81
               axisx, axisy = p.split(",")
82
               axisx = float(axisx)
83
               axisy = float(axisy)
84
               point_list = [axisx, axisy]
85
               centroids.append(point_list)
86
           return centroids
87
88
       def retrieveLabels(self, dataFile, centroidsFile):
89
           """Retrieve the labels of the imput data points.
90
```

```
91
            :param self: An instance of the class KmeansRunner.
92
            :param dataFile: A file with the input data points.
93
            :param centroidsFile: A file with the centroids.
94
            :return: A list with the labels.
95
96
           data points = self.retrieveData(dataFile)
97
           centroids = self.retrieveCentroids(centroidsFile)
98
           labels = []
99
           for data_point in data_points:
100
                distances = [np.linalg.norm(data_point - centroid)
101
                             for centroid in centroids]
102
                cluster = np.argmin(distances)
103
                labels.append(int(cluster))
104
            return labels
105
106
       def writeCentroids(self, centroids, file):
107
            """Write centroids to a file.
108
109
            :param self: An instance of the class KmeansRunner.
110
            :param centroids: A list with the centroids.
111
            :param file: A file to write the centroids.
            . . . .
112
113
            f = open (CENTROIDS FILE, "w+")
114
           for item in centroids:
115
                f.write("%s\n" % str(item))
116
           f.close()
117
118
       def plotClusters(self, data points, centroids, labels):
119
           """Plot the clusters with the centroids and save the plot as an image.
120
121
            :param self: An instance of the class KmeansRunner.
122
            :param data_points: An array with the input data points.
123
            :param centroids: A list with the centroids.
124
            :param labels: The labels of the input data points.
125
126
           plt.scatter(data_points[:, 0], data_points[:, 1], c=labels)
127
           for i in range(len(centroids)):
128
                label = "Centroid " + str(i)
129
                colors = ["red", "green", "blue"]
130
                plt.scatter(centroids[i][0], centroids[i][1], s=50,
131
                            c=colors[i], label=label)
132
           plt.legend(loc="best", fancybox=True)
133
           fig = plt.gcf()
134
           plt.show()
135
           directory = "../images"
136
           if not os.path.isdir(directory):
137
                os.makedirs(directory)
138
           fig.savefig("../images/clusters.png")
139
140
141 CENTROIDS_FILE = "centroids.txt"
142 OUTPUT_FILE = "output.txt"
143
144 if __name__ == "__main__":
```

```
145
146
       parser = argparse
147
       parser = argparse.ArgumentParser(description="k-means algorithm"
148
                                          " implementation on Hadoop",
149
                                          epilog="Go ahead and try it!")
150
       parser.add_argument("inputFile", type=str,
                            help="Input data points for the clustering algorithm."
151
152
       parser.add_argument("centroids", type=int,
153
                            help="Number of clusters.")
154
       args = parser.parse_args()
155
156
       data = args.inputFile
157
       k = args.centroids
158
       instanceKmeans = KmeansRunner()
159
       centroids = instanceKmeans.initialCentroids(data, int(k))
160
       instanceKmeans.writeCentroids(centroids, CENTROIDS_FILE)
161
162
       outputFile = open(OUTPUT_FILE, "w+")
163
       outputFile.close()
164
165
       i = 1
166
       while True:
167
           print "k-means iteration #%i" % i
168
169
           command = "python kmeansAlgorithm.py < " \</pre>
170
                      + data + " --k=" \
171
                      + str(k) + " --centroids=" \
172
                      + CENTROIDS FILE + " > " + OUTPUT FILE \
173
                      + " -r hadoop"
174
           os.popen(command)
175
176
           new_centroids = instanceKmeans.retrieveCentroids(OUTPUT_FILE)
177
178
           if sorted(centroids) != sorted(new_centroids):
179
               centroids = new_centroids
180
                instanceKmeans.writeCentroids(centroids, CENTROIDS_FILE)
181
           else:
182
               break
183
           i += 1
184
185
       os.remove(OUTPUT FILE)
186
       labels = instanceKmeans.retrieveLabels(data, CENTROIDS_FILE)
187
       labelsFile = open("labels.txt", "w+")
188
       for label in labels:
189
            labelsFile.write("%s\n" % str(label))
190
       labelsFile.close()
191
       data points = instanceKmeans.retrieveData(data)
192
       instanceKmeans.plotClusters(data_points, centroids, labels)
```

3.2 kmeansAlgorithm.py

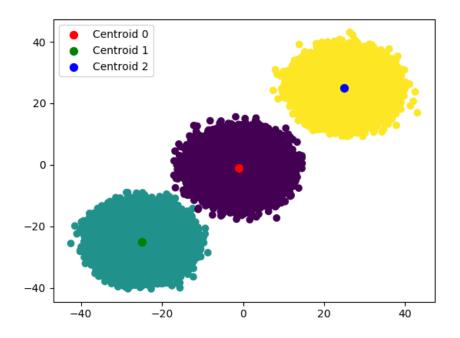
In order to implement k-means algorithm on hadoop mrjob is used. Mrjob is a python package, which allows to write multi-step MapReduce jobs in pure Python and run them on a hadoop cluster. In our case mrjob run on a single-node cluster. (The script can also be run locally by commenting the argument "-r hadoop".)

Algorithm 2 k-means algorithm

- 1. Define the number of clusters, k.
- 2. Select k data points as initial centroids.
- 3. Assign each data object to the closest cluster centroid.
- 4. Recalculate the clusters' centroids.
- 5. If the centroids remain unchanged the algorithm terminates. Otherwise, the steps are repeated from Step 2.

Algorithm 3 k-means algorithm - MapReduce

- 1. The mapper function returns each data point and the cluster, to which it belongs.
- 2. The combiner function returns partial sums of batches of data points belonging to the same cluster.
- 3. The reducer returns the new centroids of each cluster.
- 4. If the centroids remain unchanged the algorithm terminates. Otherwise, the steps are repeated from the beginning.



```
1 """kmeansAlgorithm.py: Implement the k-means clustering
2
      algorithm on the input data."""
4 from mrjob.job import MRJob
5 from mrjob.job import MRStep
6 import numpy as np
7 import re
9 __author__ = "Stratos Gounidellis, Lamprini Koutsokera"
10 __copyright__ = "Copyright 2017, BDSMasters"
11
12
13 class KmeansAlgorithm (MRJob):
14
      def configure_options(self):
15
           """Set the arguments for the class KmeansAlgorithm.
16
17
           :param self: A instance of the class KmeansAlgorithm.
18
           11 11 11
19
           super(KmeansAlgorithm, self).configure_options()
20
           self.add_passthrough_option(
               "--k", type="int", help="Number of clusters.")
21
22
           self.add_file_option("--centroids")
23
24
      def retrieveCentroids(self, file):
25
           """Retrieve the centroids coordinated from the centroids file.
26
27
           :param self: An instance of the class KmeansAlgorithm.
28
           :param file: A file with the centroids.
29
           :return: A list with the centroids.
30
```

```
31
           with open(file, "r") as inputFile:
32
               output_data = inputFile.readlines()
33
34
           centroids = []
35
           for point in output_data:
36
               p = re.search("\setminus [(.*?)\setminus]", point).group()
37
               p = p.replace("[", "").replace("]", "")
38
               p.strip()
39
               axisx, axisy = p.split(",")
40
               axisx = float(axisx)
41
               axisy = float(axisy)
42
               point_list = [axisx, axisy]
43
               centroids.append(point_list)
44
           return centroids
45
46
       def assignPointtoCluster(self, _, line):
47
           """Assign each point to its closest cluster - Mapper Function.
48
49
           :param self: An instance of the class KmeansAlgorithm.
50
           :param line: A line from the input data, with data points in
51
               the form [axisx axisy]
52
           :yield: The identifier of a cluster and a point belonging to it.
           . . . .
53
54
           axisx, axisy = line.split()
55
           data_point = np.array([float(axisx), float(axisy)])
56
           centroids = self.retrieveCentroids(self.options.centroids)
57
           distances = [np.linalg.norm(data_point - centroid)
58
                        for centroid in centroids]
59
           cluster = np.argmin(distances)
60
           yield int(cluster), data_point.tolist()
61
62
       def calculatePartialSum(self, cluster, data_points):
63
           """Calculate the partial sum of the data points belonging to
64
               each cluster - Combiner Function.
65
66
           :param self: An instance of the class KmeansAlgorithm.
67
           :param cluster: An identifier for each cluster.
68
           :param data_points: A list of points belonging to each cluster.
69
           :yield: The identifier of a cluster, the partial sum of its
70
               data points and their number.
71
72
           sum_points = np.array(data_points.next())
73
           counter = 1
74
           for data_point in data_points:
75
               sum_points += data_point
76
               counter += 1
77
           yield cluster, (sum_points.tolist(), counter)
78
79
       def calculateNewCentroids(self, cluster, partial_sums):
80
           """Calculate the new centroids of the clusters - Reduce Function.
81
82
           :param self: An instance of the class KmeansAlgorithm.
83
           :param cluster: An identifier for each cluster.
84
           :param partial_sums: A list with the partial sum of the
```

```
85
               data points of a cluster and their number.
86
            :yield: The identifier of a cluster and its new centroid.
87
88
           total_sum, total_counter = partial_sums.next()
89
           total_sum = np.array(total_sum)
90
           for partial_sum, counter in partial_sums:
91
               total_sum += partial_sum
92
               total counter += counter
93
           yield cluster, (total_sum / total_counter).tolist()
94
95
       def steps(self):
96
            """Set the steps of the MRJob.
97
98
            :param self: An instance of the class KmeansAlgorithm.
99
100
            :return: a list of steps constructed with MRStep().
101
102
           return [MRStep(mapper=self.assignPointtoCluster,
103
                    combiner=self.calculatePartialSum,
104
                    reducer=self.calculateNewCentroids)]
105
106
107 if __name__ == "__main__":
108 KmeansAlgorithm.run()
```

4 Testing Functionality

4.1 test.py

```
1 import unittest
2 from createDataPoints import DataGenerator
3 from kmeans import KmeansRunner
5 __author__ = "Stratos Gounidellis, Lamprini Koutsokera"
6 __copyright__ = "Copyright 2017, BDSMasters"
8
9 class TestStringMethods (unittest.TestCase):
10
11
      def test dataPoints(self):
12
           instanceData = DataGenerator()
13
           fname = "test.txt"
14
           instanceData.generateData(100, fname)
15
           with open(fname) as f:
16
               for i, l in enumerate(f):
17
                   pass
18
           i + 1
19
           self.assertEqual(100, i+1)
20
21
      def test_exceptionClustersNumber(self):
22
           fname = "test.txt"
23
           instanceKmeans = KmeansRunner()
```

```
24
          with self.assertRaises(Exception) as context:
25
               instanceKmeans.initialCentroids(fname, 1)
26
           self.assertIn("Error the number of clusters should be greater" +
27
                         "than or equal to 2!", "".join(context.exception))
28
29
      def test_fileLength(self):
30
           fname = "test.txt"
31
          instanceKmeans = KmeansRunner()
32
          testFile = open(fname, "w+")
33
          testFile.close()
34
          with self.assertRaises(Exception) as context:
35
               instanceKmeans.retrieveData(fname)
36
          self.assertIn("The input file is empty!", "".join(context.exception))
37
38
39 if __name__ == "__main__":
40 unittest.main()
```

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