Energy Conversion and Management 52 (2011) 1721–1727

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Mathematical models of natural gas consumption

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ARTICLE INFO

Article history:
Received 12 December 2009
Received in revised form 19 September 2010
Accepted 15 October 2010
Available online 18 November 2010

Keywords:
Natural gas consumption
Mathematical model
Least Squares
Least Absolute Deviations

ABSTRACT

In this paper we consider the problem of natural gas consumption hourly forecast on the basis of hourly movement of temperature and natural gas consumption in the preceding period. There are various methods and approaches for solving this problem in the literature. Some mathematical models with linear and nonlinear model functions relating to natural gas consumption forecast with the past natural gas consumption data, temperature data and temperature forecast data are mentioned. The methods are tested on concrete examples referring to temperature and natural gas consumption for the area of the city of Osijek (Croatia) from the beginning of the year 2008. The results show that most acceptable forecast is provided by mathematical models in which natural gas consumption and temperature are related explicitly.

1. Introduction

Due to the fact that natural gas emits much less CO_2 than coal and is the cleanest burning of all fossil fuels, it can be considered as an important adjunct to renewable energy sources such as wind or solar, as well as a bridge to the new energy economy [1]. In order to achieve lower emissions of global warming pollution it is important to efficiently use natural gas. EU countries are highly dependent upon import of gas [2]. Besides some other measures, the efficient usage includes building models for accurate prediction of gas consumption, which can directly lower the purchase costs for distributors as well as for final consumers. An accurate prediction of gas consumption is also needed due to the fact that distributors are required (by their suppliers) to nominate the amount of natural gas they will need for the next day within a regulated tolerance interval.

Previous research in the area of energy consumption (gas or electricity) reveals that various deterministic and stochastic models have been applied to describe and forecast natural gas consumption [3]. Past load and weather data were generally used as the model inputs, although some authors show that other data are also relevant. Darbellay and Slama [4] forecasted short-term demand for electricity in the Czech Republic by using neural networks and the ARMA model. They found that forecasting the short-term evolution of the Czech electric load is primarily a linear problem, although there are certain conditions under which neural networks could be superior to linear models. The normalized mean square error (NMSE) and the mean absolute percentage error (MAPE) were used as measures of model successfulness. Beccali et al. [5] predicted daily electric load of a suburban area in Italy. Their input variables included 24-h weather data (hourly dry bulb temperatures, relative humidity, global solar radiation) along with historical load data. Thaler et al. [6] used the radial basis neural network algorithm to build a model for energy consumption in the gas distribution system in Slovenia. Besides calculating the prediction error, the authors estimated the probability distribution of prediction for the one-day time interval, which can be used to estimate the risk of energy demand beyond a certain prescribed value. They also proposed a cost function that includes operation and control costs of a distribution system as well as penalties related to excess energy demand. Monthly gas consumption of residential customers in Croatia was investigated by Gelo [7], and the multivariate regression analysis showed dependance of consumption and the average monthly temperature, while the impact of other input predictors was found less significant. Potočnik et al. [8] use a statistics-based machine forecasting model to predict future consumption of natural gas in Slovenia in 2005 and 2006. They use previous consumption, past weather data, weather forecast, and some additional parameters, such as seasonal effects and nominations as input variables. In addition to that, they emphasize a need to define a control strategy that combines an energy demand forecasting model, an economic incentive model and a risk model. Building such strategy is also highlighted as of capital interest for an optimal management of a gas distribution system, and in

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doi:10.1016/j.enconman.2010.10.037

conjunction with careful planning of a pipeline network, it could also harness energy recovery from pipeline pressure energy [9,10].

It is obvious from the above that models created for different countries and regions vary according to the methodology used, selection of input variables, time horizons, and the accuracy of prediction.

The main focus of this paper is on the methodology regarding prediction of the hourly consumption of natural gas. In order to provide an efficient model, its accuracy is critical. Previous authors mostly used statistical forecasting methods, such as autoregressive moving average (ARMA), cycle analysis, multiple regression, and recently artificial neural networks [4,11] while obtaining different results. Here we use several advanced linear and nonlinear mathematical models such as exponential, Gompertz and logistic model. The methods were tested on a Croatian dataset, and some functional relations among temperature and gas consumption were revealed. The obtained results could be used in explaining the dependencies among variables necessary to build an online system for energy distribution management, not only for gas, but also for electricity or water distribution.

2. Problem statement

Given are the data (t_i,T_i,y_i), $i = 1,\ldots,m$, where t_i is the time (in hours), T_i is the temperature in time t_i, and y_i are quantities of the gas consumed in that particular hour. On the basis of these data natural gas consumption in hourly intervals in the next period should be forecast. This problem is considered in numerous papers (see e.g. [5,12,13,8,14]). Thereby it is also possible to take into account other relevant data (seasonal information, days of the week, holidays, etc.). Similar problems also occur with electricity or water consumption (see [5,4,6]). As an illustration, Fig. 1 shows movement of hourly natural gas consumption and air temperature in the city of Osijek (Croatia) for the first 40 days in 2008. A 24-h periodicity in data can be observed immediately (see also [5,12,8]). In order to confirm this hypothesis, on the basis of data referring to temperature movement (t_i,T_i), $i = 1,\ldots,m$ and natural gas consumption (t_i,y_i), $i = 1,\ldots,m$, respectively, we estimate the best Least Squares (LS) optimal parameters of the model function

$$f(x;a,b,c,d) = a + b\sin\left(\frac{2\pi}{c}x + d\right). \tag{1}$$

LS-optimal parameters of model function (1) are shown in Table 1. These results clearly confirm the hypothesis on a 24-h periodicity with respect to temperature and natural gas consumption movement.

Further, similarly to [14], we will consider natural gas consumption estimation of individual residential and small commercial customers, whose consumption depends mainly on temperature movement. This consideration does not include large industrial consumers that have to nominate their consumption on a daily basis.

Several various approaches for solving this problem can be found in the literature: artificial neural networks, mathematical modelling and regression analysis, as well as various statistical methods (see e.g. [5,12,13,4,6,15,3]).

Table 1
LS-optimal parameters of model function (1).

	a^*	b^*	c^*	d^*
Temperature	−1.85444	2.17864	24.0633	5.31868
Consumption	36.8154	9.08336	24.0126	5.38884

3. Implicit dependence of natural gas consumption on temperature

The simplest way of expressing the dependence of natural gas consumption on temperature is a hypothesis that this dependence is linearly implicitly contained in hourly natural gas consumption data (t_i,y_i), $i = 1,\ldots,m$. Furthermore, assuming that in addition to the basic period $T = 24$ there exist another several shortest periodical influences, on the basis of hourly natural gas consumption data in a few preceding days we can search for optimal parameters of the model function which consists of linear and trigonometric part

$$\begin{aligned} f(t;a,b,c,\gamma) = {} & a_0 + \gamma t + a_1\cos\frac{2\pi}{T}t + b_1\sin\frac{2\pi}{T}t + a_2\cos\frac{2\pi}{c_1}t \\ & + b_2\sin\frac{2\pi}{c_1}t + \cdots + a_r\cos\frac{2\pi}{c_{r-1}}t + b_r \\ & \times \sin\frac{2\pi}{c_{r-1}}t, \qquad T \\ & = 24, \end{aligned} \tag{2}$$

where $a = (a_0,a_1,\ldots,a_r)$, $b = (b_1,\ldots,b_r)$, $c = (c_1,\ldots,c_{r-1})$, $r \geqslant 2$. Thereby optimal parameters a^*,b^*,c^*,γ^* can be searched for by applying the LS-method (see e.g. [16–19]):

$$F_2(a,b,c,\gamma) = \sum_{i=1}^{m} w_i(y_i - f(t_i;a,b,c,\gamma))^2 \to \min_{a,b,c,\gamma}, \tag{3}$$

or by applying the Least Absolute Deviations (LAD) method (see e.g. [16,20–22]):

$$F_1(a,b,c,\gamma) = \sum_{i=1}^{m} w_i|y_i - f(t_i;a,b,c,\gamma)| \to \min_{a,b,c,\gamma}. \tag{4}$$

Thereby the weights of the data $w_i > 0$ are defined such that more recent data are more influential than the older data. This can be achieved by using corresponding weight functions (see [19]):

$$w_i = W\left(\frac{|i-m|}{m}\right), \quad i = 1,\ldots,m, \tag{5}$$

$$W(u) = \begin{cases} (1-u^3)^3, & 0 \leqslant u \leqslant 1, \\ 0, & u > 1. \end{cases} \quad \text{or} \quad W(u) = e^{-\sigma u^2}, \quad \sigma > 0.$$

The LS-method is applied if the data errors are normally distributed, and if we suppose that outliers can appear among the data (e.g. simultaneous switching on of smaller industrial consumers), then

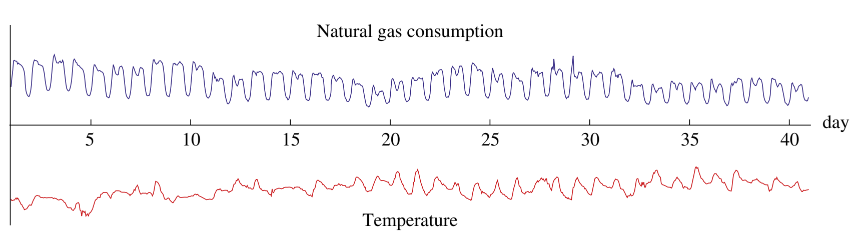

Fig. 1. Air temperature and natural gas consumption in hourly intervals in the city of Osijek in the first 40 days in 2008.

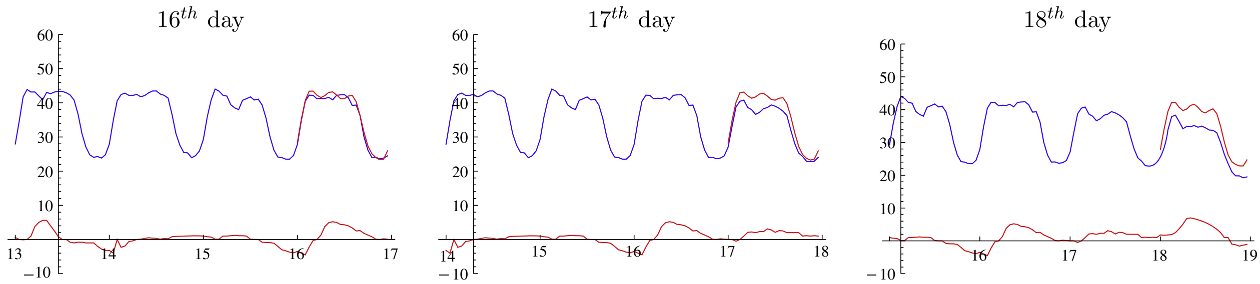

Fig. 2. Forecast on the basis of linear – trigonometric model function.

the application of the LAD-principle is preferred. It should be stressed here that minimizing functionals (3) and (4) in both approaches are not simple and that special methods for their minimization should be used (see e.g. [20–24]). Knowing the optimal model function of the form (2) we can try to forecast the hourly natural gas consumption for the next day.

Example 1. For natural gas consumption data already mentioned for the city of Osijek we will predict natural gas consumption on the basis of linear–trigonometric model functions (2) by applying LAD-principle (4).[1] As an illustration, we will determine optimal parameters of model function (2) on the basis of data from the 13th – 15th,14th – 16th and 15th – 17th day in 2008, for the purpose of consumption forecast for the 16th, 17th and 18th day of the year 2008, respectively. In Fig. 2 actual consumption and forecast consumption are denoted by a blue[2] and a red curve, respectively. It may be noticed that the forecast obtained in this way does not react fast enough to temperature change. Namely, the influence of temperature change on natural gas consumption must be expressed more directly.

4. Explicit dependence of natural gas consumption on temperature

In Example 1 it can be seen that a significant temperature change influences natural gas consumption fast and directly (see also [5,12]). Let us consider this dependence in more detail. Let $\left(T_i^0, y_i^0\right)$, $i = 1, \ldots, m$ be temperature data and natural gas consumption data in preceding m days at a fixed hour t_0. On the basis of these data we will try to identify dependence of natural gas consumption on temperature. Fig. 3 shows the aforementioned data for the city of Osijek at t_0= 6:00 a.m., 9:00 a.m., 12:00. Several jutting dots represent switching on of larger industrial consumers. This dependence can be used in various ways for the purpose of natural gas consumption forecast.

4.1. Applications of fermat – torricelli – weber point

One possibility is the application of the *Fermat – Torricelli – Weber* (FTW) problem (see [23,5,24]). Namely, if for the next day at t_0 temperature T_0 is forecast, then we can try to determine natural gas consumption y_0 at hour t_0 such that

$$F(y_0) = \min_{y \in [0,+\infty\rangle} F(y), \qquad F(y) = \sum_{i=1}^{m} w_i d(P(y), P_i), \tag{6}$$

where $P(y) = (T_0, y), P_i = (T_i^0, y_i^0)$, and $d : \mathbb{R}^2 \to [0, +\infty\rangle$ is some metric function.

Weights of the data $w_i > 0$ can be defined by weight function (5) such that bigger weights are assigned points $P_i = (T_i^0, y_i^0)$ for which temperature T_i^0 is closer to forecast value T_0. If the minimum of functional (6) is attained for y_0, then that value represents natural gas consumption forecast for the next day at t_0 hour. The point $P_0 = (T_0, y_0)$ is the FTW-point.

Let us note that in the literature there are various possibilities referring to selection of metric function d. Also, instead of requiring the sum of distances to be minimal, we can request the maximal distance to be minimal (see [23,24]).

Example 2. On the basis of natural gas consumption data and temperature data for the city of Osijek the application of the FTW-point is illustrated for the purpose of natural gas consumption forecast. Thereby we will use the Euclidean metric function, so that the corresponding functional (6) becomes

$$F(y) = \sum_{i=1}^{m} w_i \sqrt{\left(T_i^0 - T_0\right)^2 + \left(y_i^0 - y\right)^2}.$$

In this case minimization given in (6) can be carried out by the Weiszfeld iterative procedure (see e.g. [25]).

$$y^{(k+1)} = \sum_{i=1}^{m} w_i \frac{y_i^0}{\rho_i(y^{(k)})} \left(\sum_{i=1}^{m} w_i \frac{1}{\rho_i(y^{(k)})} \right)^{-1},$$

$$\rho_i(y^{(k)}) = \sqrt{(T_i^0 - T_0)^2 + (y_i^0 - y^{(k)})^2}, \quad k = 0, 1, \ldots$$

Fig. 4 shows points $\left(T_i^0, y_i^0\right)$ for several days at t_0= 12:00. Thereby the darker black dots represent the data (T_i^0, y_i^0) with bigger weights. This figure also shows forecast temperature and consumption forecast (red dot) and actual temperature and consumption (blue dot). The quality of hourly natural gas consumption forecast (red) and relative day errors in percents for the days mentioned can be seen in Fig. 5 and Table 2, respectively.

4.2. Functional dependence of natural gas consumption and temperature

Another possibility on the basis of data $\left(T_i^0, y_i^0\right)$, $i = 1, \ldots, m$ is to try to functionally link natural gas consumption to temperature at a fixed hour t_0. In Fig. 3 it can be seen that these data can be described e.g. by decreasing exponential model function $T \mapsto be^{-cT}$, $b, c > 0$.

If we take into account that a decrease in temperature causes an increase in consumption to a certain value unknown in advance, and by increasing temperature it is reduced to zero (see e. g. [26], then Gompertz model function $T \mapsto e^{a-be^{cT}}$, $a, b, c > 0$ (see e.g. [3,27]) or logistic model function $T \mapsto \frac{a}{1+be^{cT}}$, $a, b, c > 0$ (see [3,28])

[1] All evaluations and illustrations were done by using *Mathematica* 6 on a PC (CPU: 2.00 GHz Intel Core 2 Duo processor, Memory: 1.99 GB DDR2) on the basis of our own software.

[2] For interpretation of color in all figures, the reader is referred to the web version of this article.

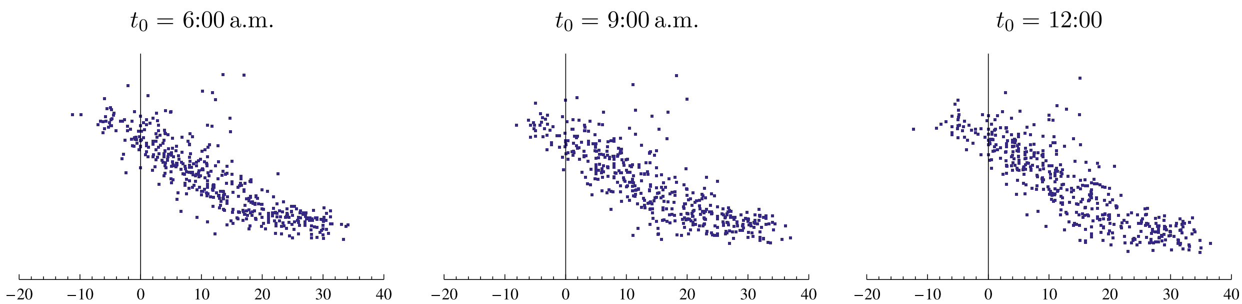

Fig. 3. Dependence of natural gas consumption on temperature in Osijek at t_0 = 6:00 a.m., 9:00 a.m., 12:00.

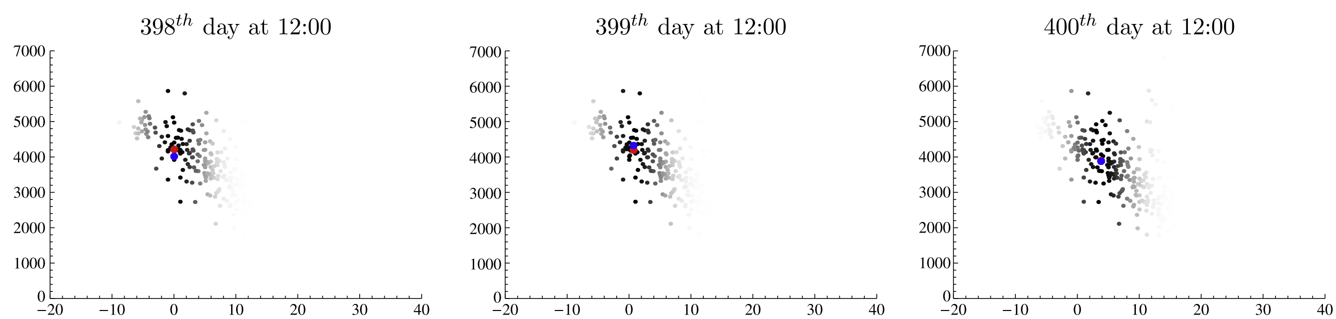

Fig. 4. FTW-point: forecast temperature and consumption forecast (red dot) and actual temperature and consumption (blue dot). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

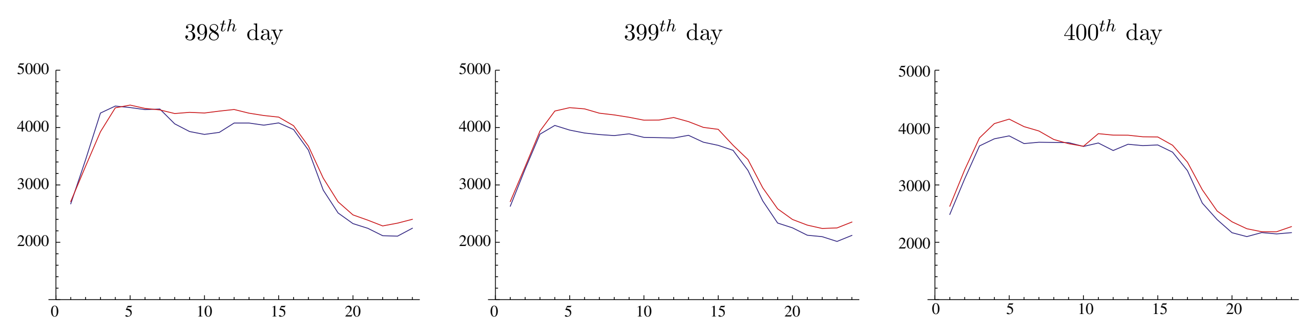

Fig. 5. FTW-point: forecast (red) and actual (blue) natural gas consumption at 12:00. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
Comparison of forecast consumption with actual natural gas consumption.

Day	Actual consumption	FTW estimation		Gompertz model func.		Linear model func.	
		$\mathscr{F}(T_0)$	Relative err. (in %)	$\mathscr{G}(T_0)$	Relative err. (in %)	$\mathscr{L}(T_0)$	Relative err. (in %)
398	83,828	86,767	3.5	86,293	2.9	85,171	1.6
399	78,610	84,300	7.2	82,853	5.4	81,274	3.4
400	76,613	80,160	4.6	79,117	3.3	76158	0.6

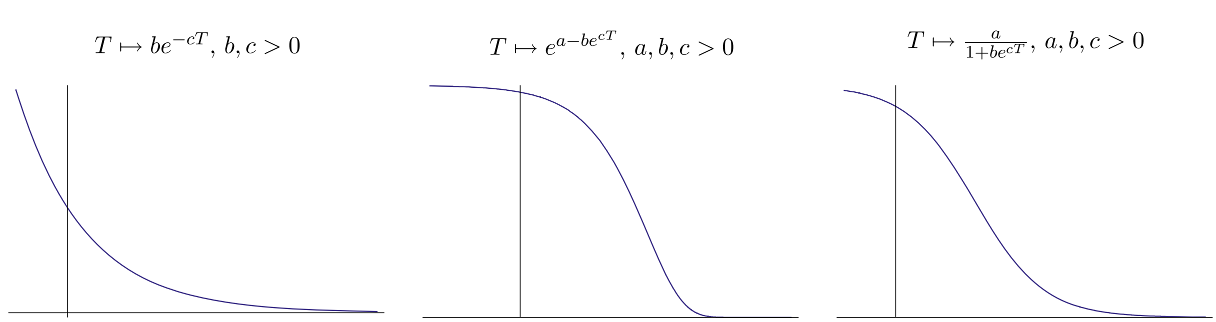

Fig. 6. Exponential, Gompertz and logistic model function.

will be used for the description of this functional dependence as a model. The shapes of these model functions are shown in Fig. 6.

4.2.1. Application of Gompertz model function

On the basis of temperature and natural gas consumption data $\left(T_i^0, y_i^0\right), \quad i = 1, \ldots, m$ in preceding m days at a fixed hour t_0 we should estimate optimal parameters (a^0, b^0, c^0) of the Gompertz model function

$$\mathscr{G}(T; a, b, c) = e^{a - be^{cT}}, \quad a, b, c > 0, \tag{7}$$

in accordance with the assumption that in the next day at t_0 temperature T_0 is forecast. The forecast of natural gas consumption for the next $(m + 1)$th day at t_0 hour will be given by $\mathscr{G}(T_0; a^0, b^0, c^0)$.

Since outliers can appear among the data (see Fig. 3), parameters of Gompertz model function (7) are estimated according to the LAD-principle by minimizing the functional

$$F_1(a, b, c) = \sum_{i=1}^{m} w_i |y_i^0 - e^{a - be^{cT_i^0}}| \to \min_{a,b,c}. \tag{8}$$

Thereby weights of the data $w_i > 0$ can be defined such that more recent data have bigger weights than older data

$$w_i = W\left(\frac{|i - m|}{m}\right), \quad i = 1, \ldots, m, \qquad W(u) = e^{-\sigma u^2}, \quad \sigma > 0, \tag{9}$$

or such that the data $\left(T_i^0, y_i^0\right)$ for which temperature T_i^0 is closer to the forecast value T_0 have bigger weights than the data referring to temperatures that significantly differ from the forecast value T_0

$$w_i = W\left(\frac{\left|T_i^0 - T_0\right|}{T_0}\right), \quad i = 1, \ldots, m, \qquad W(u) = e^{-\sigma u^2}, \quad \sigma > 0. \tag{10}$$

It is of course best to combine these two approaches:

$$w_i = W\left(\frac{|i - m|}{m}, \frac{\left|T_i^0 - T_0\right|}{T_0}\right), \quad i = 1, \ldots, m, \tag{11}$$

$$W(u, v) = e^{-\sigma_1 u^2 - \sigma_1 v^2}, \quad \sigma_1, \sigma_2 \geqslant 0.$$

The problem of minimizing functional (8) is a numerically very demanding nondifferentiable nonlinear minimization problem. For solving this problem there exist general methods (see e.g. [29]) and corresponding ready-made software (*Mathematica, Matlab, SAS, Statistica*). It happens often that by using given software minimization of functional (8) cannot be done or very often it gives wrong solutions. This is the reason why instead of minimizing functional (8) it is proposed (see e.g. [30]) to minimize functional

$$\Phi(a, b, c) = \sum_{i=1}^{m} w_i \left| \ln y_i^0 - a + be^{cT_i^0} \right| \to \min_{a,b,c}. \tag{12}$$

The problem of minimizing functional (12) can be considered as a one-dimensional minimization problem.

$$\min_{c>0} \psi(c), \tag{13}$$

whereby the value of the function ψ in some point $\hat{c}$ is

$$\psi(\hat{c}) = \min_{a,b>0} \sum_{i=1}^{m} w_i \left| \ln y_i - a + be^{\hat{c}T_i^0} \right|. \tag{14}$$

Minimization problem (14) can be solved by applying the Two Points Method[3] (see [21]) and one-dimensional minimization problem (13) can be solved by the Brent method (see [31]) or some of methods mentioned in [29].

As an illustration, consider the data $\left(T_i^0, y_i^0\right), \quad i = 1, \ldots, m$, $m = 399$ at a selected fixed hour t_0 = 10:00 a.m. Thereby, in Fig. 7a data (T_i^0, y_i^0) with bigger weights w_i are shown by darker black points. Point (T_0, y_{m+1}^0), which represents a pair (forecast temperature, actual consumption) on the *400*-th day is given in Fig. 7a by a blue point. The closer the point to the Gompertz curve, the better the forecast.

Example 3. For the data on natural gas consumption for the city of Osijek that were used earlier Fig. 7a shows actual consumption (blue curve) and consumption forecast (red curve) obtained by applying the Gompertz model function for 398th, 399th and 400th day. The quality of hourly natural gas consumption forecast and relative day errors in percents for the mentioned days can be seen in Fig. 8 and Table 2, respectively.

4.2.2. Linear dependence of natural gas consumption and temperature

In Section 4.2.1, on the basis of natural gas consumption data and temperature data (T_i^0, y_i^0), $i = 1, \ldots, m$ in preceding m days at a fixed hour t_0 we estimated optimal parameters (a^0, b^0, c^0) of the Gompertz model function with the assumption that temperature T_0 is forecast for the next day at t_0.

Since we are practically interested only in the behavior of consumption at t_0 hour for the temperature close to T_0, then the Gompertz model function can be approximated by a linear model function $\mathscr{L}(T) = \alpha T + \beta$, whose parameters α, β can be determined by minimizing the functional

$$\Phi(\alpha, \beta) = \sum_{i=1}^{m} w_i |y_i^0 - \alpha T_i^0 - \beta|, \tag{15}$$

where weights of the data $w_i > 0$ can also be determined as in Section 4.2.1. In such way the problem is reduced to the problem of determining a best weighted LAD-line (see [21]).

As an illustration, consider the data $\left(T_i^0, y_i^0\right), \quad i = 1, \ldots, m$, $m = 399$ at a fixed hour t_0 = 10:00 a.m. Thereby, in Fig. 7b data $\left(T_i^0, y_i^0\right)$ with bigger weights w_i are shown by darker black points. Point (T_0, y_{m+1}^0), which represents a pair (forecast temperature, actual consumption) on the 400th day is given in Fig. 7b by a blue point. The closer that point to the straight line, the better the forecast.

Finally, let us compare the results obtained by applying the FTW-point, Gompertz and linear model function. On the basis of data from the preceding period and by the aforementioned methods we will compare results obtained for three selected days (398–400). Table 2 displays actual daily consumption and forecast according to the FTW-point, Gompertz and linear model and relative daily errors in percents, and in Fig. 9 the forecast quality is observed in more detail.

From the given illustrations and conducted experiments it can be noticed that linear approximation gives an acceptable forecast for practical needs.

4.3. Extension of the model

Given are temperature data and natural gas consumption data $\left(T_i^0, y_i^0\right), \quad i = 1, \ldots, m$ in preceding m days at a fixed hour t_0. Suppose that natural gas consumption y_i^0 on the ith day depends on temperatures at a fixed hour t_0 in preceding ν days linearly

$$y_i^0 = \alpha_0 + \alpha_1 T_i^0 + \alpha_2 T_{i-1}^0 + \cdots + \alpha_\nu T_{i-\nu+1}^0 + \varepsilon_i, \quad i = \nu, \ldots, m, \tag{16}$$

or nonlinearly

$$y_i^0 = \alpha_0 \left(T_i^0\right)^{\alpha_1} \left(T_{i-1}^0\right)^{\alpha_2} \cdots \left(T_{i-\nu+1}^0\right)^{\alpha_\nu} + \varepsilon_i, \quad i = \nu, \ldots, m, \tag{17}$$

[3] Our own software available at http://www.mathos.hr/seminar/software/WTP.m.

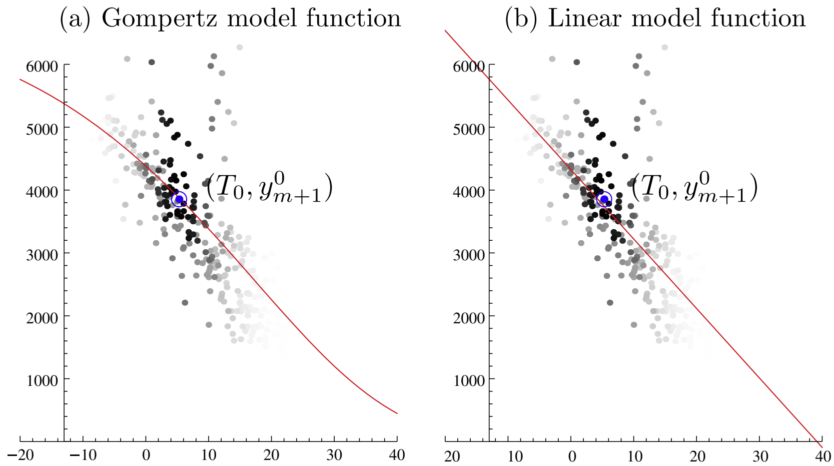

Fig. 7. Data points and Gompertz and linear model function.

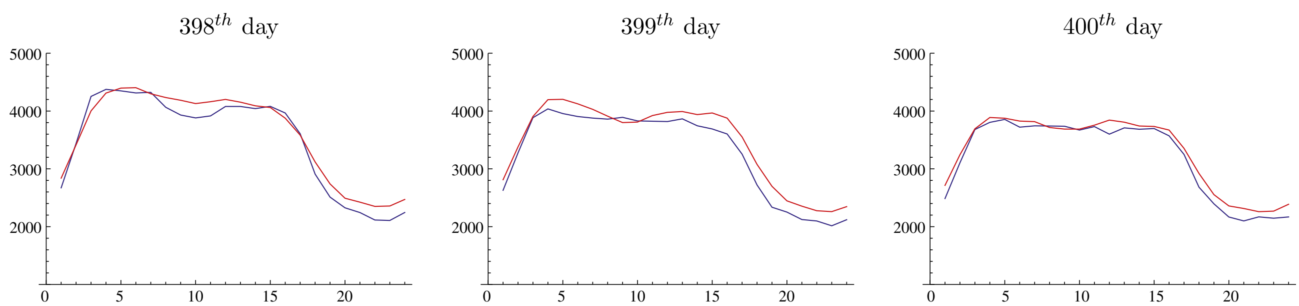

Fig. 8. Actual consumption (blue curve) and consumption forecast (red curve) obtained by applying the Gompertz model function. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

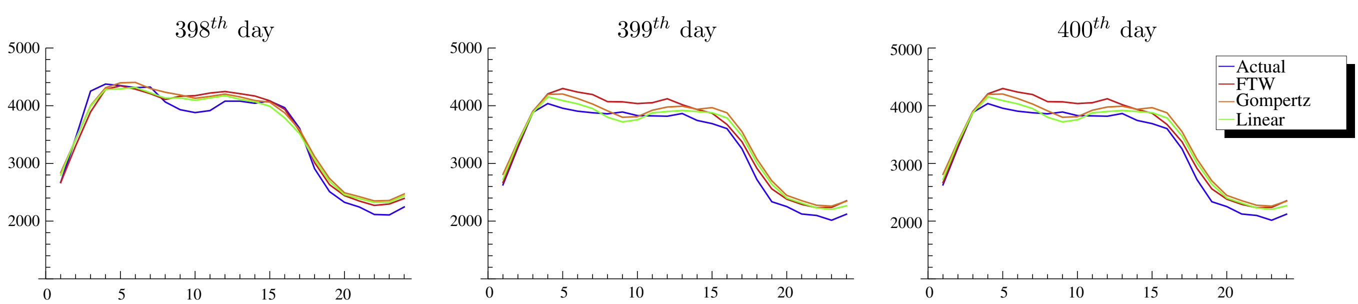

Fig. 9. Comparison of actual daily natural gas consumption for three selected days (blue) and estimation obtained by applying the FTW-point (red), Gompertz model (orange) and linear model (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

or as a linear combination of nonlinear model functions (i.e. Gompertz model functions)

$$y_i^0 = \alpha_1 e^{-b_1 e^{c_1 T_i^0}} + \alpha_2 e^{-b_2 e^{c_2 T_{i-1}^0}} + \cdots + \alpha_\nu e^{-b_\nu e^{c_\nu T_{i-\nu+1}^0}} + \varepsilon_i, \qquad (18)$$

$$b_k, c_k > 0, \qquad i = \nu, \ldots, m.$$

The unknown parameters in (16)–(18) can be searched for as best LAD-solutions of overdetermined systems of equations. Thereby the weights $w_i > 0$ are again defined as in Section 4.2.1. Note that the problem of estimating parameters of model function (16) leads to searching for a best LAD-solution of an overdetermined system of linear equations (see e.g. [16,22]), whereas the problem of estimating parameters of model function (17) and (18) is a difficult nonlinear separable LAD-problem.

5. Conclusions

The paper presents several possible methods for forecasting natural gas hourly consumption on the basis of the past natural gas consumption data, temperature data and temperature forecast data. Based upon mutual comparison of the given methods it can be concluded that practically most acceptable forecast is provided by mathematical models in which natural gas consumption and temperature are related explicitly. Since in the model this dependence is considered at the fixed hour t_0 for the temperature close to T_0, it is shown that linear approximation does not significantly lag behind approximation by means of the Gompertz model function. Application of the FTW-point also yields practically acceptable results.

The observed problem is relevant to gas distribution networks, and the methodological results obtained in this paper could be of interest in planning the management operations of such complex systems which depend not only on hourly behavior of external air temperature but also on other relevant data, such as fluctuations influenced by the control strategy of particular plants. The functional dependencies of other input variables should be investigated in further research. Due to the fact that some plants, mainly addressing residential heating and hot water production, work for a given hour period during a day and their control strategy is of the on/off type, the results could be used for building an online control system that will be able not only to accurately predict the next-hour consumption, but also to react to some internal and external conditions.

Acknowledgements

This work was supported by the Ministry of Science, Education and Sports, Republic of Croatia, through research Grant 235-2352818-1034. The authors wish to thank the Croatian Gas Industry company (namely, Mrs. Marija Somolanji and Mr. Zlatko Tonković) for valuable discussion and for introducing us to the practical problem.

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