CS 512 Homework 0

A) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$.

1. $2A - B =$

$$\begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

3. A. a unit vector in the direction of A

$\|\vec{a}\| = \sqrt{(1)^2 + (2)^2 + (3)^3}$ $\hat{u} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$

$= \sqrt{14}$

4 The direction cosines of a matrix A

$|\bar{A}| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{14}$.

Direction cosines are.

$\cos\alpha = \frac{a_x}{|\bar{A}|} = \frac{1}{\sqrt{14}}$ $\cos\gamma = \frac{3}{\sqrt{14}}$

$\cos\beta = \frac{a_y}{|\bar{A}|} = \frac{2}{\sqrt{14}}$

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5) A·B & B·A

$A \cdot B = 4 + 10 + 18$ $\quad B \cdot A = 4 + 10 + 18$

$= 32 \quad\quad = 32$

6) Angle between A & B

Angle can be given by

$$\cos\alpha = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| \cdot |\bar{b}|}$$

$$\bar{A} \cdot \bar{B} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 4 + 10 + 18 = 32$$

$|\bar{A}| = \sqrt{14}$ $\quad |\bar{B}| = \sqrt{(4)^2 + (5)^2 + (6)^2}$

$= \sqrt{16 + 25 + 36}$

$= \sqrt{77}$

$$\therefore \cos\alpha = \frac{90}{\sqrt{14}\sqrt{77}} = \frac{32}{\sqrt{1078}} = 0.97$$

$\cos\alpha = 0.97$

7) Vector Perpindiculor to A.

Two products are orthogonal if their dot products are zero.

$\vec{A} \cdot \vec{V} = 0$

Let $\vec{V} = [a, b, c]$

$\therefore \vec{A} \cdot \vec{V} = a + 2b + 3c$

Now,

$a + 2b + 3c = 0$

Let $a, b = 1$.

$\therefore 1 + 2 + 3c = 0$

$c = -1$.

$\therefore \vec{V} = (1, 1, -1)$

To check.

$\vec{A} \cdot \vec{V} = (1 + 2 - 3) = 0$

8) A×B & B×A

$A = [1, 2, 3] \qquad B = [4, 5, 6]$

1 4
2 5
3 6

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

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$$= \begin{vmatrix} i & j & K \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$= -3i + 6j - 3K \rightarrow (-3, 6, -3)$

9) vector perpendicular to both A & B.

cross pdt of A & B = $\begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$

Now

$C \cdot A = -3 + 12 - 9 = 0$

|||rly

$C \cdot B = -12 + 30 + 18 = 0$

$\therefore \vec{C} = (-3, 6, -3)$ is perpendicular to both A & B

10) Linear dependency between A, B, C

$$\begin{vmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{vmatrix}$$

$= 1(9) - 4(3) - 1(-3)$

$= 0$

Linearly dependent

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II) A^TB and AB^T

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$A^TB = 32$$

AB^T

$$B^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$\underline{AB^T = 32}$$

B)

1) $$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

$$2A - B = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -3 & 2 \\ -3 & 12 & -3 \end{bmatrix}$$

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2) AB & BA.

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$BA = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 11 \\ -5 & 15 & 2 \end{bmatrix}$$

3) $(AB)^T$ & $B^T A^T$

$$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}.$$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}. \quad A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}.$$

$$B^T . A^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4) $|A|$ & $|C|$

$$|A| = 1(-13) - 2(-4) + 3(20)$$
$$= -13 + 8 + 60.$$
$$= 55$$

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$|C| = 1(9) - 2(18) + 3(9)$

$= 9 + 27 - 36.$

$= 0$

5) the matrix (A, B, C) in which the row vectors form an orthogonal set

→ To check whether A, B, C are an orthogonal if

$A^T B = 0$ etc.

Now,

$$A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$A^T . B = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & \cdots\cdots \\ & \\ & \end{bmatrix} \neq 0.$$

$$A^T C = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & \cdots\cdots \\ \cdots & \\ & \end{bmatrix} \neq 0.$$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}$$

$$B^T C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & \cdots & \\ & \cdots & \\ & & \end{bmatrix} \neq 0.$$

$\therefore$ A, B, C is not an orthogonal set

6) A^{-1} & B^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}.$$

$$|A| = -18 - 2(-4) + 3(20)$$
$$= -6 + 8 + 66$$
$$= \cancel{55}$$

<u>Finding minors</u>

$$A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}.$$

$$M_{11} = \begin{vmatrix} -2 & 5 \\ 3 & -1 \end{vmatrix} = -13 \quad M_{12} = \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} = -17.$$

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$M_{13} = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix} = 12 \quad M_{14} = \begin{vmatrix} 4 & 0 \\ 3 & -1 \end{vmatrix} = -4$

$M_{15} = \begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} = -1 \quad M_{16} = \begin{vmatrix} 1 & 4 \\ 3 & 3 \end{vmatrix} = -9$

$M_{17} = \begin{vmatrix} 4 & 0 \\ -2 & 5 \end{vmatrix} = 20 \quad M_{18} = \begin{vmatrix} 1 & 0 \\ 2 & 5 \end{vmatrix} = 5 \quad M_{19} = \begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix} = -10$

$$Adj(M) = \begin{bmatrix} -13 & -17 & 12 \\ -4 & -1 & -9 \\ 20 & 5 & -10 \end{bmatrix}$$

$$Adj(M) = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

ii) B^{-1}

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$|B| = 1(-7) - 2(14) + 1(-7)$
$= -7 - 28 - 7$
$= -42$

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Finding minors

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}$$

$M_{11} = \begin{vmatrix} 1 & -2 \\ -4 & 1 \end{vmatrix} = -7$ $M_{12} = \begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} = 4$

$M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} = -9$ $M_{14} = \begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix} = 14$

$M_{15} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -2$ $M_{16} = \begin{vmatrix} 1 & 2 \\ 1 & -4 \end{vmatrix} = -6$

$M_{17} = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$ $M_{18} = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = -8$

$M_{19} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -3.$

$$Adj(M) = \begin{bmatrix} -7 & 4 & -9 \\ 14 & -2 & -6 \\ -7 & -8 & -3 \end{bmatrix}$$

$$Adj(M) = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{42} \begin{bmatrix} 7 & 4 & 9 \\ 14 & 2 & -6 \\ 7 & -8 & 3 \end{bmatrix}$$

C let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$.

1. eigen values & corres. eigen vectors of A.

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = 0.$$

$(1-\lambda)(2-\lambda) - 6 = 0$

$(2 - \lambda - 2\lambda + \lambda^2) - 6 = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$(\lambda + 1)(\lambda - 4) = 0$

$\lambda = -1, 4$ Eigen values.

when $\lambda = -1$ (eigen value)

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} u_1 = 0.$$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0.$$

$2u_1 + u_2 = 0$

$u_1 + u_2 = 0.$

$u_1 = -u_2$

$u_1 = -1$ $u_2 = 1.$

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$$u = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\lambda = 4$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} v1 \\ v2 \end{bmatrix} = 0$$

$-3v1 + 2v2 = 0$

$3v1 = 2v2$.

when $v2 = 1$

$v1 = \frac{2}{3}$.

$$V = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

eigen vectors are:

$$u = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

2) The matrix $v^{-1}AV$ where V is composed of eigenvectors of A

$$V = \begin{bmatrix} -1 & 2/3 \\ 1 & 1 \end{bmatrix}$$

$$|v| = \frac{-5}{3}$$

Finding minors

$V_{11} = 1 \qquad V_{12} = 1$

$V_{21} = 2/3 \qquad V_{22} = -1$

$$\text{Adj}(V) = \begin{bmatrix} 1 & 1 \\ 2/3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ -2/3 & -1 \end{bmatrix}$$

$$V^{-1} = \frac{-3}{5}\begin{bmatrix} 1 & -1 \\ -2/3 & -1 \end{bmatrix}$$

$$= \frac{3}{5}\begin{bmatrix} -1 & 1 \\ 2/3 & 1 \end{bmatrix}.$$

$$A \cdot V = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2/3 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 8/3 \\ -1 & 4 \end{bmatrix}$$

$\frac{2}{3} + 4$

$$V^{-1} \cdot AV = \frac{3}{5}\begin{bmatrix} -1 & 1 \\ 2/3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 8/3 \\ -1 & 4 \end{bmatrix}.$$

$\frac{2}{3} - 1$

$\frac{16}{9} + 4$

$$= \frac{3}{5}\begin{bmatrix} -2 & 4/3 \\ -1/3 & 52/9 \end{bmatrix}$$

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3) dot pdt. both eigen vectors of A

$$u = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad v = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}.$$

$$u \cdot v = -2/3 + 1 = \frac{1}{3}$$

4) $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

$$|B - \lambda I| = 0$$

$$\begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(5-\lambda) - 4 = 0$$

$$(10 - 2\lambda - 5\lambda + \lambda^2) - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda = \frac{7 \pm \sqrt{49-24}}{2} \qquad \frac{7 \pm 5}{2} \qquad 6, 1$$

$\lambda = 6, 1$ (Eigen values)

$-2u_1 = u_2.$

when $\lambda = 6$ $\qquad 1 = -2$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0. \qquad u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$-4u_1 - 2u_2 = 0.$ $\qquad u_2 = -2u_1$ $\qquad -2u_1 = u_2$

$2u_1 + u_2 = 0.$ $\qquad u_1 = 2 \quad u_2 = -1$ $\qquad u_1 = 1 \quad u_2 = -2$

1 2

-2 1

When $\lambda = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$$

$V_1 - 2V_2 = 0$ or $-2V_1 + 4V_2 = 0$

$V_1 = 2V_2$ or $V_1 - 2V_2 = 0$

$V_1 = 2, \quad V_2 = 1$

$$V = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad V = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$U \cdot V = 4 - 2 = 2 = 0$

5) $U = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad V = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

The dot product is 0 because the 2 vectors are orthogonal & symmetric.

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D)

1) $f(x) = x^2 + 3$ $\quad g(x,y) = x^2 + y^2$

$f'(x) = 2x$ $\quad f''(x) = 2$

2) partial derivatives : $\frac{\partial g}{\partial x}$ & $\frac{\partial g}{\partial y}$

$\frac{\partial g}{\partial x} = 2x$ $\quad \frac{\partial g}{\partial y} = 2y$

3 $\nabla g(x,y)$

$$\nabla g = \frac{\partial g}{\partial x} i + \frac{\partial g}{\partial y} j$$

$g(x,4) = x^2 + 4^2$.

then $\nabla g(x,4) = (2x, 24) = \underline{\underline{2x i + 24 j}}$.

4) A pdf of a univariate gausian distribution

$$pdf = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}$$

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A.
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$\|A\|$ & angle of A relative to +ve x axis

$$\|A\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{1}{\sqrt{14}}$$

$$\alpha = \cos^{-1}\left(1/\sqrt{14}\right) = 74.5^\circ$$