

Mastermind

LOGICAL ANALYSIS

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Game Rules

2 PLAYERS

- Code maker
- Code breaker

GAME

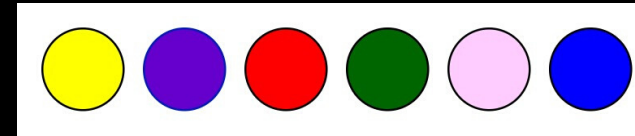
- Code maker makes a secret code.
- Code breaker attempts to guess it in 3 chances.
- Each guess is analysed by code maker, and a feedback is generated and passed on to code breaker.



Code/Guess

Feedback

6 AVAILABLE COLORS IN TOTAL



EACH CODE/GUESS

- Length 4
- No duplicate colours

EACH FEEDBACK

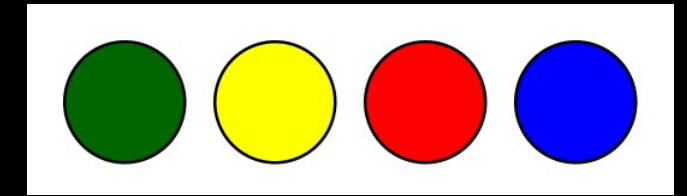
- Length 4
- Each element :
 - 1 : Perfectly guessed colour for that position.
 - -1 : Incorrect colour, it does not belong anywhere in the code.
 - 0 : This colour belongs in the code, but at some other position.



Epistemic Logic Application

Kripke models within S5 Axiom System

Secret Code



REAL WORLD

- Same as secret code.
- Code maker knows only real world.
- Code breaker considers all worlds possible = 360 worlds

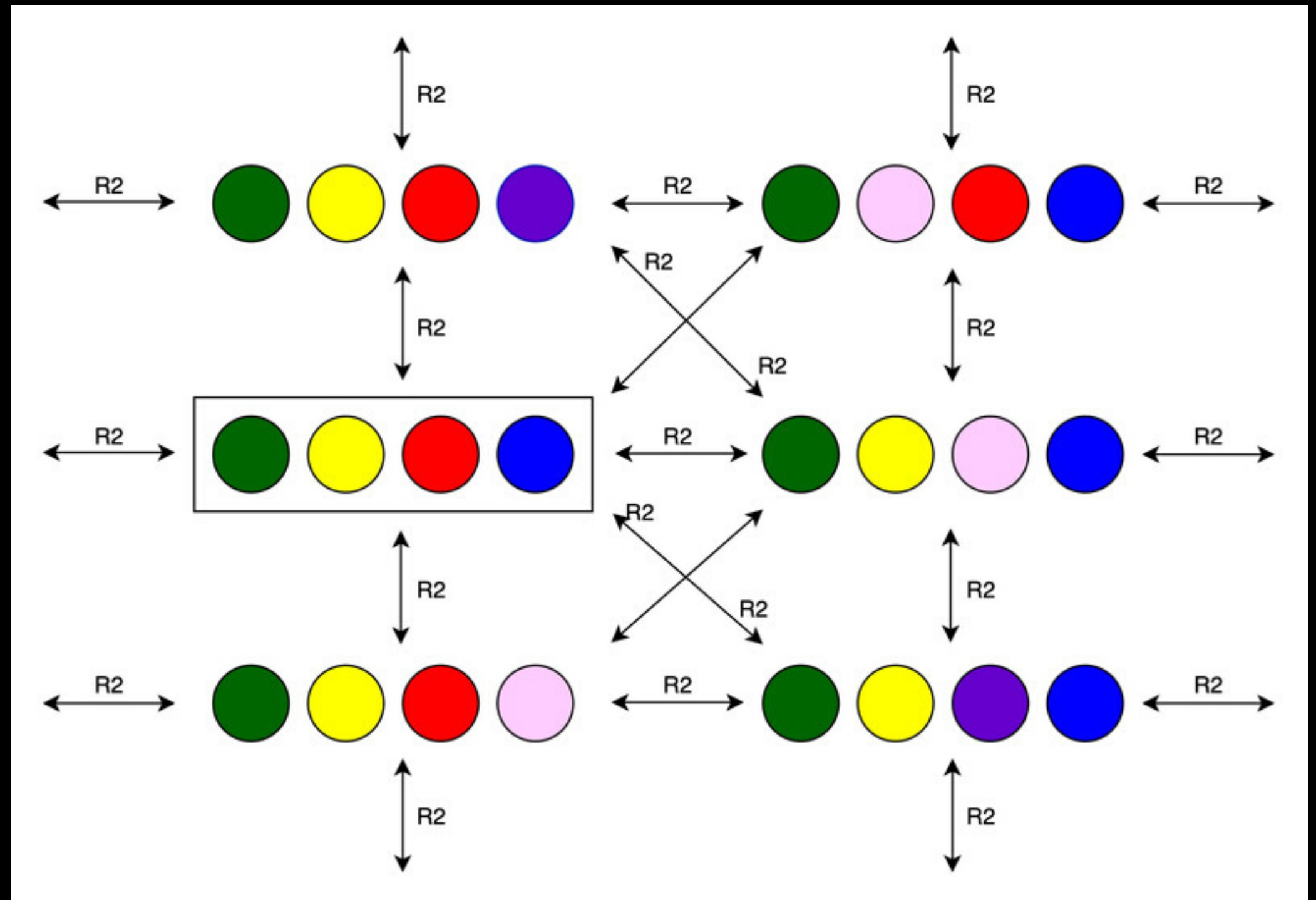
AGENTS

- Code maker = Agent 1
- Code breaker = Agent 2

RELATIONS

- Agent 1 has reflexive relations for every world = 360 relations
- Agent 2 can access all worlds from all worlds = 129600 relations

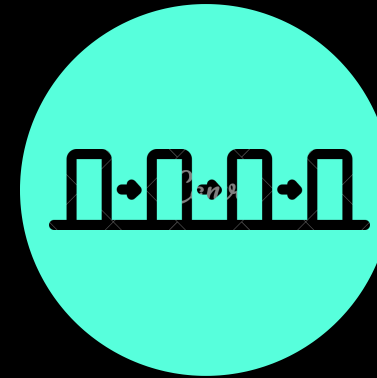
EXAMPLE KRIPKE MODEL



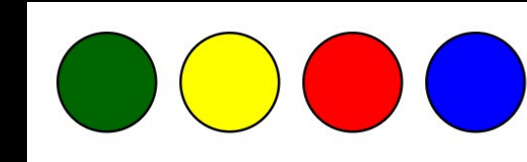
Higher Order Knowledge

For every guess of code breaker, code maker analyses it and generates a feedback.

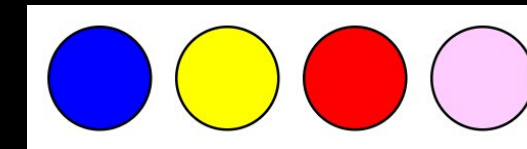
The knowledge derived from this feedback is public announcement, and becomes common knowledge for both agents.



SECRET CODE



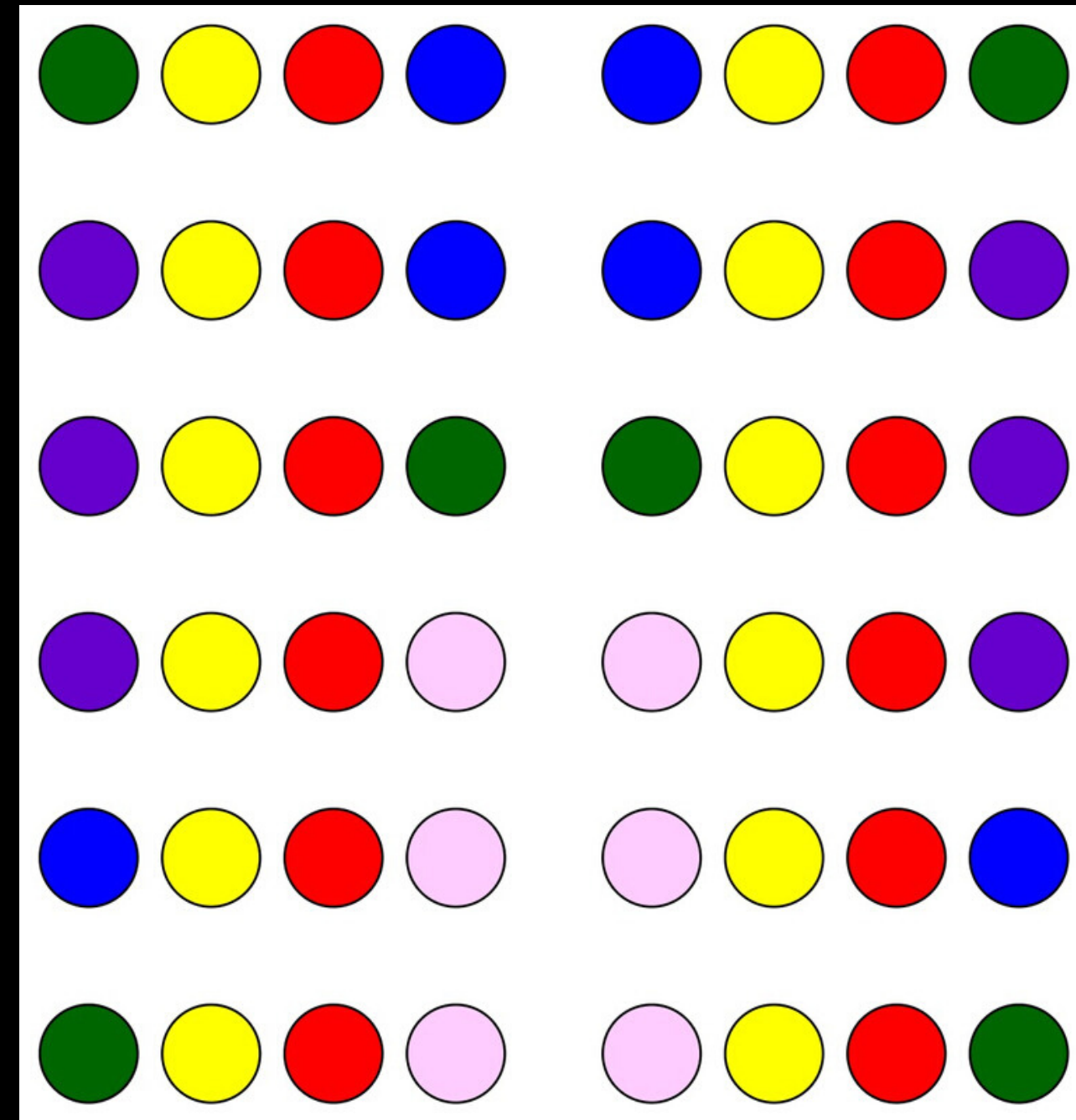
FIRST GUESS



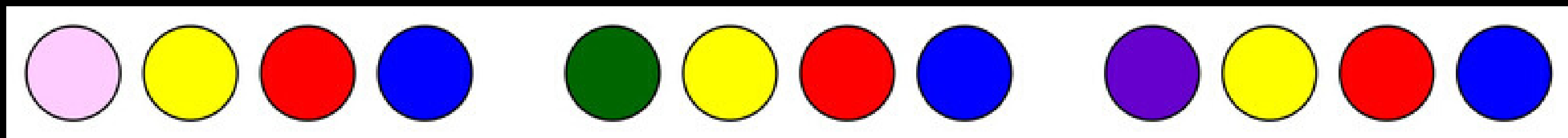
FEEDBACK

[0 1 1 -1]

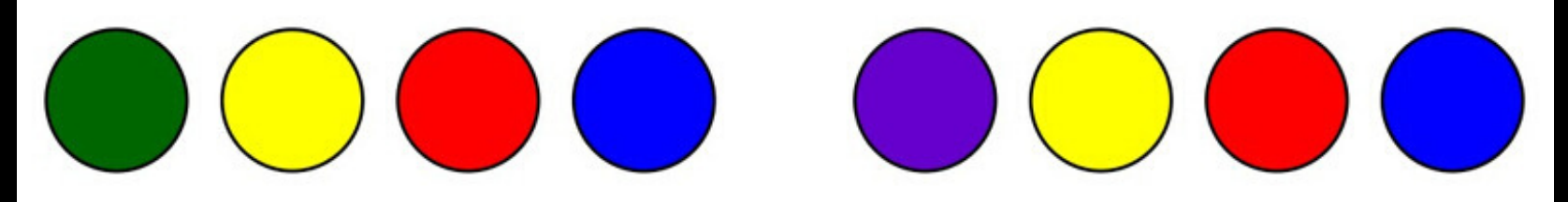
Second and third colours
have to be **yellow** and **red**
respectively. All the worlds
with other possibilities for
second and third colors are
eliminated.



Blue needs to be somewhere other than the 1st position. But since 2nd and 3rd position are already occupied, blue can only occur at 4th position. This eliminates all the worlds with any other colour on 4th position, or blue colour in any other position.

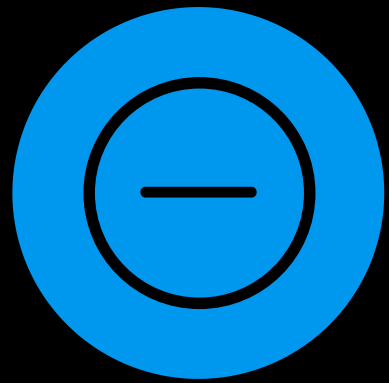


Only place left vacant is the 1st one. This position is occupied by any colour other than pink. Thus, all the worlds with pink in any place on the code are eliminated.





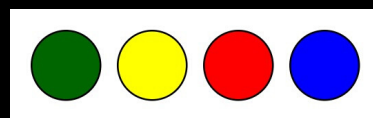
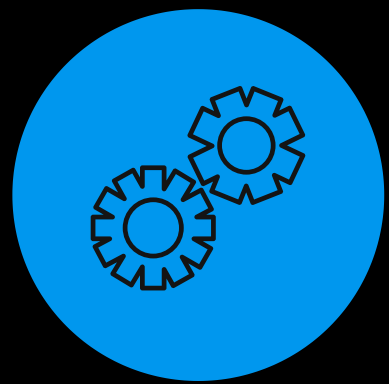
Positively true:
<position>:<color>



Negatively true:
~(<position>:<color>)



May or may not be true:
*(<position>:<color>)

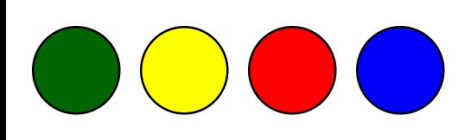


- 1:green
- 2:yellow
- 3:red
- 4:blue

Agent Knowledge

AGENT 1 KNOWLEDGE

- Individual knowledge for agent 1 is only updated once, when he generates the code.
- For the secret code :

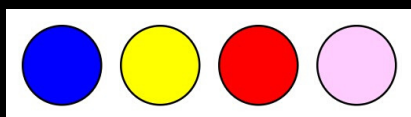


Agent 1 gains following knowledge :

```
[ '1:green', '2:yellow', '3:red', '4:blue', '~(1:yellow)', '~(1:violet)', '~(1:red)', '~(1:pink)', '~(1:blue)', '~(2:violet)', '~(2:red)', '~(2:green)', '~(2:pink)', '~(2:blue)', '~(3:yellow)', '~(3:violet)', '~(3:green)', '~(3:pink)', '~(3:blue)', '~(4:yellow)', '~(4:violet)', '~(4:red)', '~(4:green)', '~(4:pink)' ]
```


Common Knowledge and Agent 2 Knowledge

- Updated with every move and it's feedback.
- For the move :



feedback : [0 1 1 -1]

1ST COLOUR BLUE : FEEDBACK 0

- Common Knowledge : Blue is not present at position 1, i.e., $\sim(1:\text{blue})$.
- Agent 2 Knowledge : Deduce that blue is either at position 2 or 3 or 4, i.e., $*(2:\text{blue}), *(3:\text{blue}), *(4:\text{blue})$.

2ND COLOUR YELLOW : FEEDBACK 1

- Common Knowledge : Yellow is present at position 2, i.e., $(2:\text{yellow})$.
- Agent 2 Knowledge : Deduce that no other colour is present at position 2, i.e., $\sim(2:\text{violet}), \sim(2:\text{red}), \sim(2:\text{green}), \sim(2:\text{pink}), \sim(2:\text{blue})$.

3RD COLOUR RED : FEEDBACK 1

- Common Knowledge : Red is present at position 3, i.e., $(3:\text{red})$.
- Agent 2 Knowledge : Deduce that no other colour is present at position 3, i.e., $\sim(3:\text{violet}), \sim(3:\text{yellow}), \sim(3:\text{green}), \sim(3:\text{pink}), \sim(3:\text{blue})$.

4TH COLOUR PINK : FEEDBACK -1

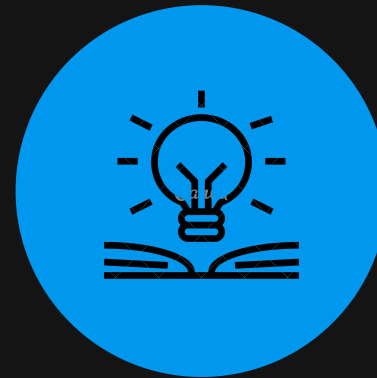
- Common Knowledge : Pink is not present at any position i.e., $\sim(1:\text{pink}), \sim(2:\text{pink}), \sim(3:\text{pink}), \sim(4:\text{pink})$
- Agent 2 Knowledge : Deduce that any other colour may be present at position 4, i.e., $*(4:\text{yellow}), *(4:\text{violet}), *(4:\text{red}), *(4:\text{green}), *(4:\text{blue})$.

Code Breaker Strategies



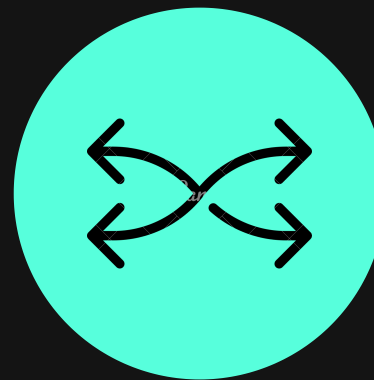
MATHEMATICIAN

- Analyses feedback to generate next move
- Information gained from older moves not maintained



LOGICIAN

- Analyses available knowledge model to generate next move
- Information is never lost



RANDOM GAMBLER

- Does not use any information
- Generates every move randomly

Thank you :D

QUESTIONS?