ASSIGNMENT-5

FINANCIAL MATHEMATICS

Submitted by-

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Consider the equation for geometric Brownian motion, as used to model the path of an underlying asset:

$$dS = \mu S dt + \sigma S dt \tag{1}$$

where dX is the increment of a Wiener process (drawn from a Normal distribution with mean zero and standard deviation $\sqrt{dt}$); we may then write that

$$dX = \phi\sqrt{dt} \tag{2}$$

where ϕ is a random variable drawn from a normalised Normal distribution. Utilising eq(2) and risk neutrally, (1) can be integrated exactly over a timescale δt (NOT necessarily small) to yield

$$S(t+\delta t) = S(t)exp((r - \frac{1}{2}\sigma^2)\delta t + \sigma\phi\sqrt{\delta t}) \tag{3}$$

equation (3) then generates a random path. Since δt need not be small, in the case of European options, it is possible to generate a (random) value of S at expiry (t=T) in just one step (i.e $\delta t = T$). From this value (say S(T)), the payoff can be easily calculated (max(S(T)X,0) in the case of call option). If this payoff is denoted as, $Payoff_i$ (for the i^th simulation), then the value of this payoff at t=0 is

$$C_i(t=0) = Payoff_i e^{-rT} = max(S(T) - X, 0)e^{-rT} \tag{4}$$

If N simulations are performed, then we merely average out the $C_i(t=0)$ to yield an approximation for the value of the call, i.e

$$C = \frac{\sum_{i=1}^{N} C_i(t=0)}{N} \tag{5}$$

We have to calculate the value of a European call and put option, with S(t=0)=5, X=5, r=0.04, σ=0.2, T=0.5 .
We have to plot the value of the two options with increasing N (N=1000,2000,.....,50000 or more!).
We have to compare the obtained values with the exact values.
And we have to determine how accurately the values of our call and put options satisfy the put-call parity relationship with increasing N (N=1000,2000,....,50000 or more).

File Name: **callput.m**

```
function [cpay,ppay,spot] = callput(S0,K,T,r,sigma,N)
%CALLPUT Summary of this function goes here
%   Detailed explanation goes here
phi=randn([1,N]);
dT=T;
for i=1:N
    S(i)=S0*exp((r-.5*sigma.^2)*dT + sigma*sqrt(dT)*phi(i));
    C(i)= exp(-r*T)*max(S(i)-K, 0);
    P(i)=exp(-r*T)*max(K-S(i), 0);
```

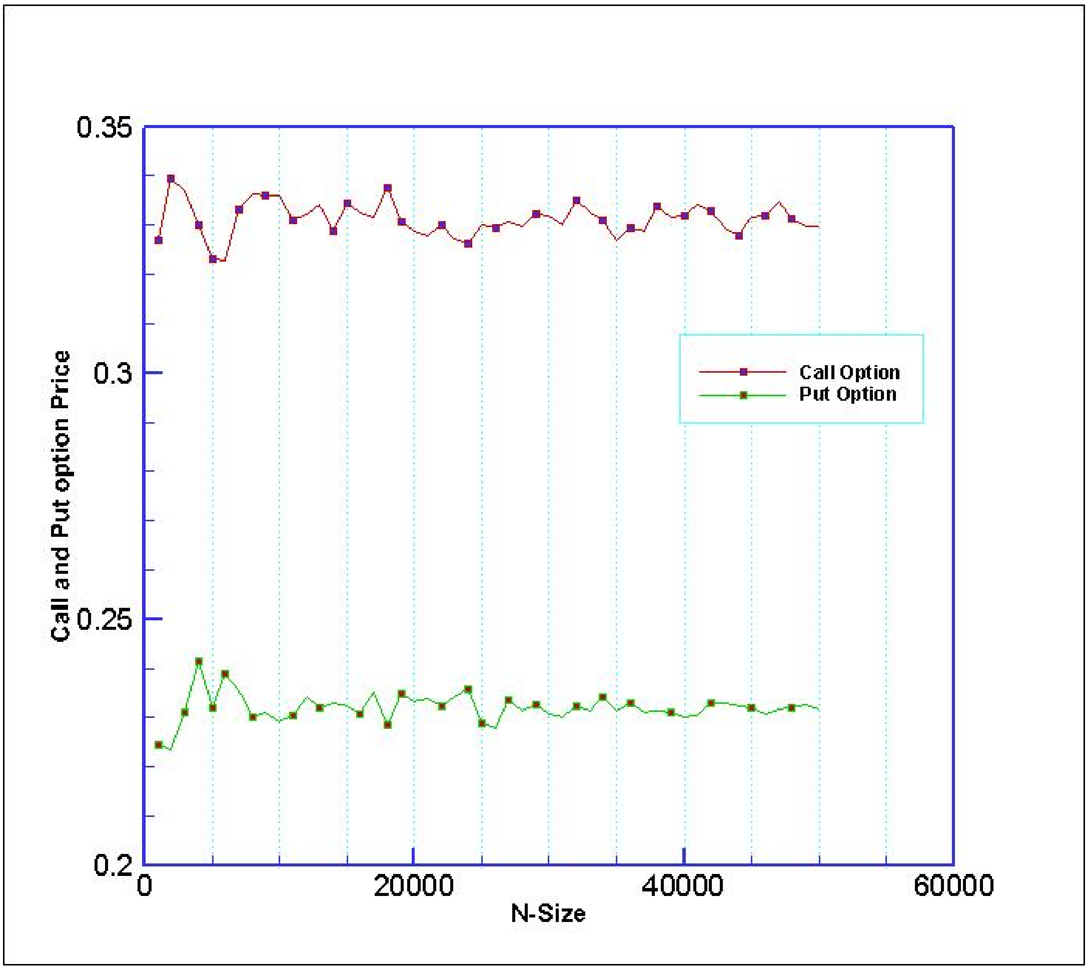
```

end
price1=sum(C)/N;
price2=sum(P)/N;
cpay=price1;
ppay=price2;
spot=sum(S)/N;
end
```

File Name: **cppayoff.m**

```
clc;
clear all;
f=fopen('CPData.txt','w');
f1=fopen('output.txt','w');
[BSCall, BSPut] = blsprice(5,5,.04,.5,.2)
fprintf(f1,'i\t\t    MC-call\t              MC-Put\t
            Error%%\r\n');
for i=1:50
     [cpayoff,ppayoff,spot]=callput(5,5,.5,.04,.2,i
        *1000);
     cp(i)=cpayoff;
     pp(i)=ppayoff;
     sp(i)=spot;
     parity=abs(cp(i)+ 5*exp(-.04*.5)- pp(i)-sp(i))
        /100;
     fprintf(f1,'%d\t                %3.4f\t             %3.4f\
        t             %3.4f%% \r\n',i*1000,cp(i),pp(i),
        parity);
     fprintf(f,'%d\t        %3.4f\t       %3.4f  \r\n',i
        *1000,cp(i),pp(i));
end
N=[1000:1000:50000];
plot(N,cp)
hold on
plot(N,pp)
legend('Call Payoff','Put Payoff')
```

File Name: **output.txt**

i	MC-call	MC-Put	Error%
1000	0.3268	0.2246	0.0010%
2000	0.3396	0.2236	0.0010%
3000	0.3369	0.2312	0.0010%
4000	0.3300	0.2414	0.0010%
5000	0.3233	0.2320	0.0010%
6000	0.3228	0.2388	0.0010%
7000	0.3332	0.2356	0.0010%
8000	0.3363	0.2300	0.0010%
9000	0.3360	0.2311	0.0010%
10000	0.3361	0.2292	0.0010%
11000	0.3311	0.2304	0.0010%
12000	0.3324	0.2341	0.0010%
13000	0.3342	0.2321	0.0010%
14000	0.3288	0.2330	0.0010%
15000	0.3344	0.2322	0.0010%
16000	0.3326	0.2308	0.0010%
17000	0.3315	0.2351	0.0010%
18000	0.3376	0.2286	0.0010%
19000	0.3307	0.2350	0.0010%
20000	0.3289	0.2333	0.0010%
21000	0.3278	0.2338	0.0010%
22000	0.3300	0.2325	0.0010%
23000	0.3271	0.2341	0.0010%
24000	0.3263	0.2358	0.0010%
25000	0.3301	0.2290	0.0010%
26000	0.3295	0.2279	0.0010%
27000	0.3308	0.2335	0.0010%
28000	0.3297	0.2313	0.0010%
29000	0.3323	0.2326	0.0010%
30000	0.3318	0.2308	0.0010%
31000	0.3301	0.2301	0.0010%
32000	0.3350	0.2325	0.0010%
33000	0.3326	0.2313	0.0010%
34000	0.3309	0.2341	0.0010%
35000	0.3270	0.2314	0.0010%
36000	0.3295	0.2329	0.0010%
37000	0.3287	0.2312	0.0010%
38000	0.3338	0.2314	0.0010%
39000	0.3315	0.2310	0.0010%
40000	0.3321	0.2303	0.0010%
41000	0.3343	0.2304	0.0010%
42000	0.3328	0.2331	0.0010%
43000	0.3295	0.2329	0.0010%
44000	0.3280	0.2323	0.0010%
45000	0.3315	0.2319	0.0010%
46000	0.3321	0.2307	0.0010%
47000	0.3347	0.2316	0.0010%
48000	0.3313	0.2319	0.0010%
49000	0.3298	0.2328	0.0010%
50000	0.3298	0.2317	0.0010%


0.35
0.3
0.25
0.2
Call and Put option Price
0
20000
40000
60000
N-Size
Call Option
Put Option