Simple & Multiple Correspondence Analyses

Contingency, categorical, ordinal, continuous and mixed data

Derek Beaton

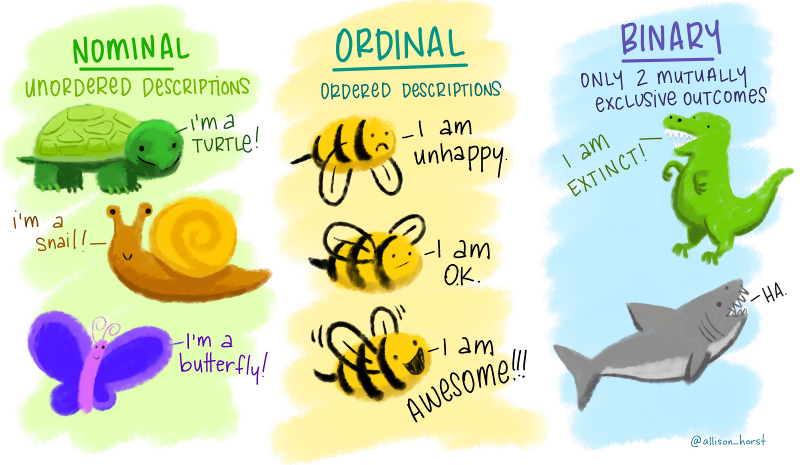
Rotman Research Institute

October 28, 2019

Before we get started

Our new best friends

via @allison_horst



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- Some are *very* difficult and effectively ignored

- What do we do with all of these in a PCA like way?
- Some are *very* difficult and effectively ignored
    - We won't do that!

Motivation for today

- Not everything is a number

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- Sometimes numbers aren't numbers!

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Motivation for today

- Not everything is a number
- Sometimes numbers aren't numbers!
- We need to recognize when this happens
  - And know what to do

Typology

- SS Stevens (not a boat!)

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- Levels of measurement

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- Levels of measurement
- Excellent examples: https://en.wikipedia.org/wiki/Level_of_measurement

Where to find everything

- Generally: https://github.com/derekbeaton/workshops

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- Today:

Overview

- Revisit PCA

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- Looking at some data

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- Simple correspondence analysis

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Revisting PCA

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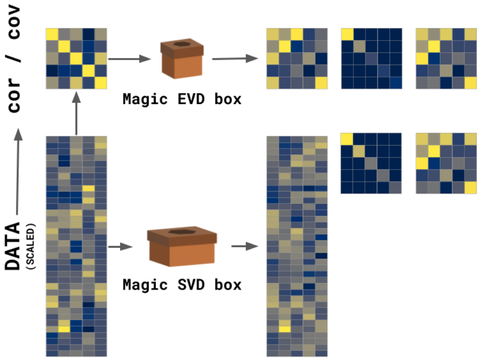
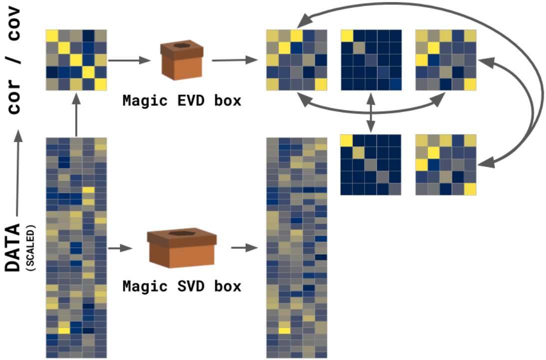
- Break data into components
  - Orthogonal
  - Rank ordered
  - Made of bits & pieces of original measures

Eigen- and singular value decompositions



Eigen- and singular value decompositions



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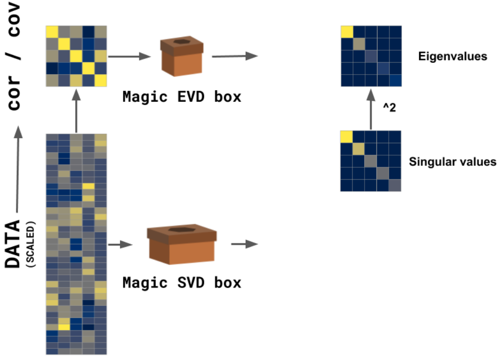

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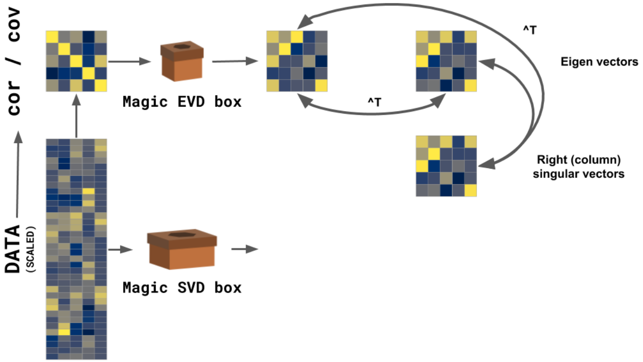

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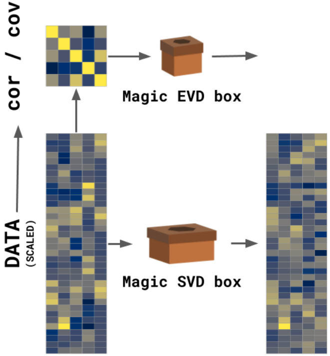

Left (row) singular vectors

Some data

Diagnosis and education

	CN	Dementia	MCI
ADV	39	7	54
B	57	17	75
B+	75	19	113
HS	25	13	46
HS+	39	9	77

- Given a table, and asked for a multivariate analysis

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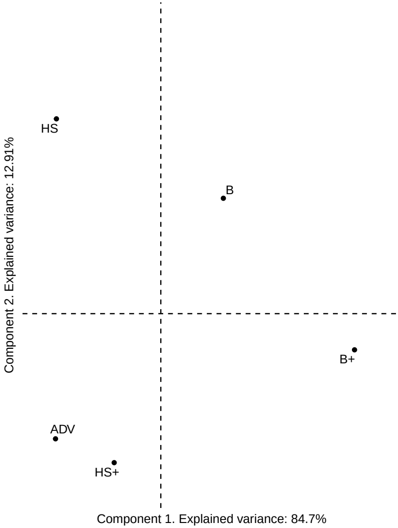
- We do what we know: PCA

PCA:
Row component scores



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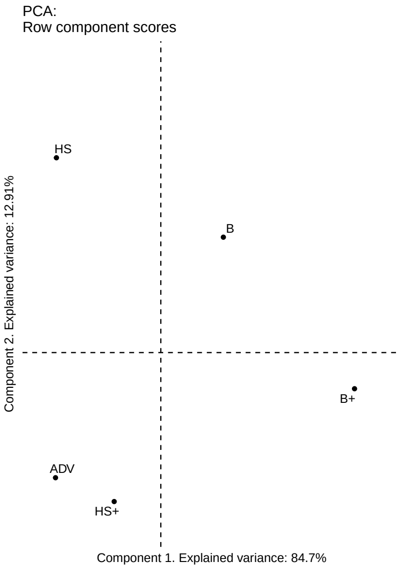
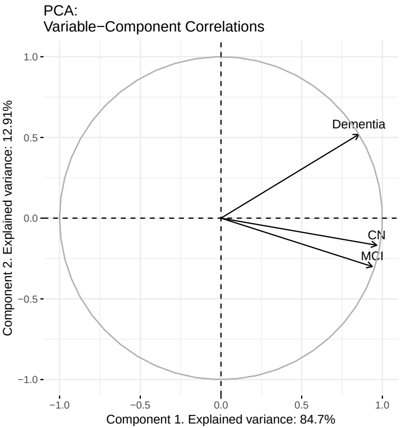
PCA:
Variable Component Correlations


PCA:
Row component scores
HS
B
B+
ADV
HS+
Component 2. Explained variance: 12.91%
Component 1. Explained variance: 84.7%


PCA:
Variable−Component Correlations
Dementia
CN
MCI
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Component 2. Explained variance: 12.91%
Component 1. Explained variance: 84.7%

What did we analyze?

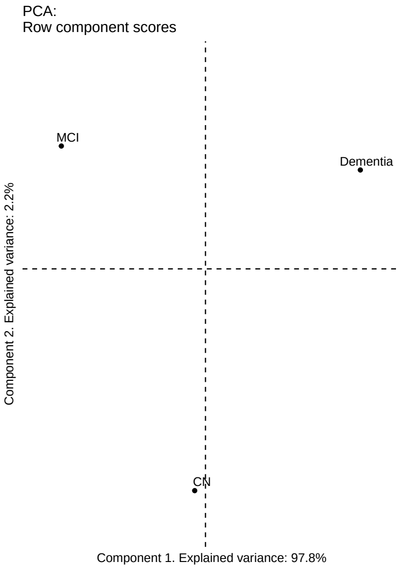
	CN	Dementia	MCI
CN	1.000	0.730	0.921
Dementia	0.730	1.000	0.652
MCI	0.921	0.652	1.000

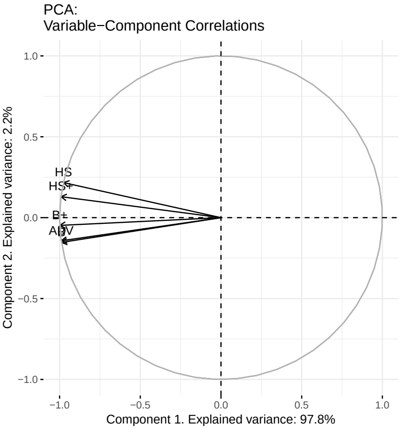
What did PCA detect?

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
B+	75	19	113	***207***
HS	25	13	46	***84***
HS+	39	9	77	***125***

Let's try something different!

	ADV	B	B+	HS	HS+
CN	39	57	75	25	39
Dementia	7	17	19	13	9
MCI	54	75	113	46	77


PCA:
Row component scores
MCI
Dementia
CN
Component 2. Explained variance: 2.2%
Component 1. Explained variance: 97.8%


PCA:
Variable−Component Correlations
HS
HS+
B+
ABV
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Component 2. Explained variance: 2.2%
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What did PCA analyze?

	ADV	B	B+	HS	HS+
ADV	1.000	1.000	0.995	0.935	0.963
B	1.000	1.000	0.994	0.932	0.960
B+	0.995	0.994	1.000	0.965	0.984
HS	0.935	0.932	0.965	1.000	0.996
HS+	0.963	0.960	0.984	0.996	1.000

What did PCA detect?

	ADV	B	B+	HS	HS+	***Row sums***
CN	39	57	75	25	39	***235***
Dementia	7	17	19	13	9	***65***
MCI	54	75	113	46	77	***365***

What is PCA for?

- When we can compute a *meaningful* covariance or correlation matrix

Let's take another look

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
B+	75	19	113	***207***
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Column sums	***235***	***65***	***365***	

- Tell me things about this matrix

Let's take another look

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
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Column sums	***235***	***65***	***365***	

- Tell me things about this matrix
- What kind of problem does this look like?

Simple correspondence analysis

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  - Across virtually every field (except psychology and neuroscience)
- The magic of CA relies on the magic of χ^2
  - And there's some *crazy* magic here

History

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- And then Benzecri (1964) & Escofier (1965)
- Many more very important characters to re-discover CA

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  - A geneaology of CA 2: http://siba-ese.unisalento.it/index.php/ejasa/article/view/19785

We're diving in

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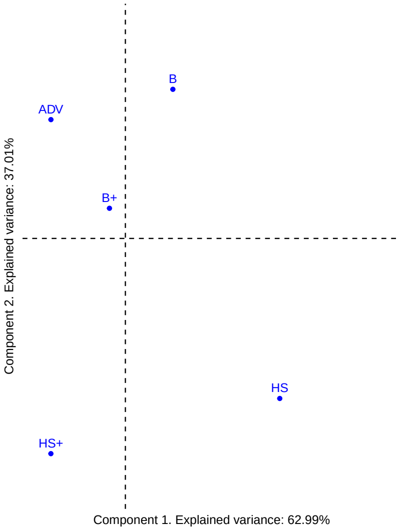
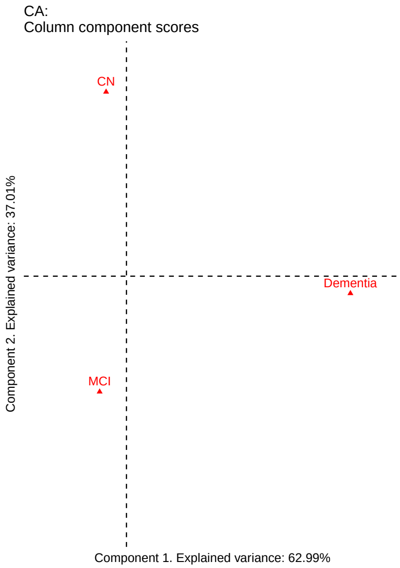
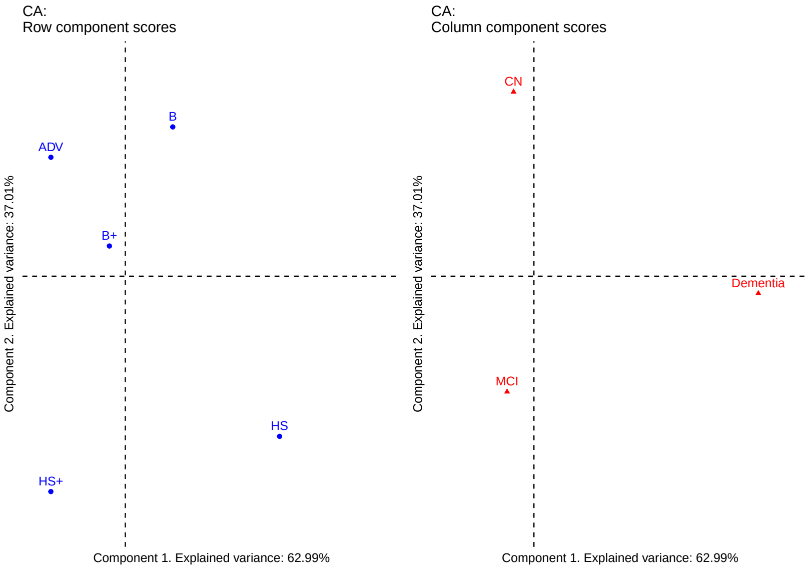



CA:
Column component scores
CN
Dementia
MCI
Component 2. Explained variance: 37.01%
Component 1. Explained variance: 62.99%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Want to see a cool trick?

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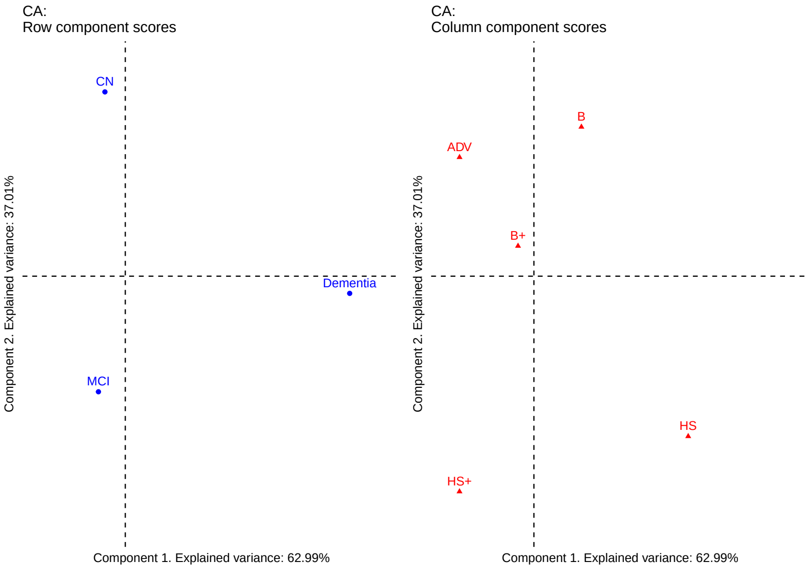
What if we perform CA on this?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How did that happen?

Table 1: Data

	CN	Dementia	MCI
ADV	39	7	54
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Table 2: Observed probabilites

	CN	Dementia	MCI
ADV	0.059	0.011	0.081
B	0.086	0.026	0.113
B+	0.113	0.029	0.170
HS	0.038	0.020	0.069
HS+	0.059	0.014	0.116

Table 3: Observed probabilites and margins

	CN	Dementia	MCI	***Row sums***
ADV	0.059	0.011	0.081	***0.150***
B	0.086	0.026	0.113	***0.224***
B+	0.113	0.029	0.170	***0.311***
HS	0.038	0.020	0.069	***0.126***
HS+	0.059	0.014	0.116	***0.188***
Column sums	***0.353***	***0.098***	***0.549***	

Table 4: Expected probabilites and margins

	CN	Dementia	MCI	***Row sums***
ADV	0.053	0.015	0.083	***0.150***
B	0.079	0.022	0.123	***0.224***
B+	0.110	0.030	0.171	***0.311***
HS	0.045	0.012	0.069	***0.126***
HS+	0.066	0.018	0.103	***0.188***
Column sums	***0.353***	***0.098***	***0.549***	

Table 5: Deviations: Observed - Expected

	CN	Dementia	MCI
ADV	0.006	-0.004	-0.002
B	0.007	0.004	-0.010
B+	0.003	-0.001	-0.001
HS	-0.007	0.008	0.000
HS+	-0.007	-0.004	0.013

Table 6: Row constraints (inverse row margins)

	ADV	B	B+	HS	HS+
ADV	6.65	0.000	0.000	0.000	0.00
B	0.00	4.463	0.000	0.000	0.00
B+	0.00	0.000	3.213	0.000	0.00
HS	0.00	0.000	0.000	7.917	0.00
HS+	0.00	0.000	0.000	0.000	5.32

Table 7: Column constraints (inverse row margins)

	CN	Dementia	MCI
CN	2.83	0.000	0.000
Dementia	0.00	10.231	0.000
MCI	0.00	0.000	1.822

The GSVD

- The generalized SVD

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  - Constraints (weights) for rows & columns of rectangular table

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- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)

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  - Matrix multiplication (by constraints on data)

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- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)
- GSVD is
  - Matrix multiplication (by constraints on data)
  - The SVD
  - More matrix multiplication (by constraints on vectors)

The GSVD

- Notation

The GSVD

- Notation
- Visualize it in SVD form

The GSVD

- Notation
- Visualize it in SVD form
  - That looks familiar...

What we did to the data

- O, wi & wj, E, Z

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- O, wi & wj, E, Z
- Oh look that's Chi2

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- Sum of eigenvalues * sum of table = Chi2.

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What we did to the data

- O, wi & wj, E, Z
- Oh look that's Chi2
- Sum of eigenvalues * sum of table = Chi2.
  - Each component is an additive orthogonal slice of Chi2. WOAH.
  - The eigenvalues are *magic*

CA visualized

- Oh look it's CCA-ish

CA visualized

- Oh look it's CCA-ish
- Oh it really really is CCA-ish!

Rules

- It's like PCA

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  - Variance (singular values, eigen values)

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  - Directions & inter-relationships

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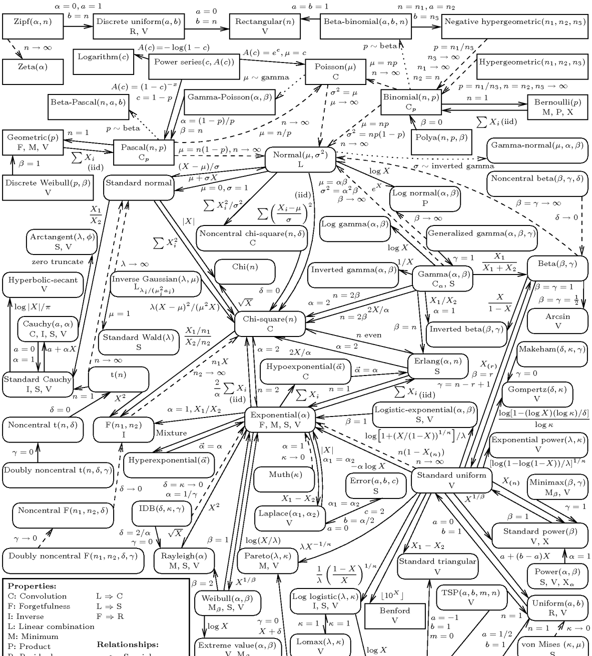
- It's like PCA
  - Variance (singular values, eigen values)
  - Directions & inter-relationships
  - Components scores
- It's unlike PCA
  - Relative interpretation *between* sets
  - Generalized singular vectors
  - Bifactor
    - Rows & columns treated the same
    - Together they help make components, as opposed to PCA

Multiple correspondence analysis

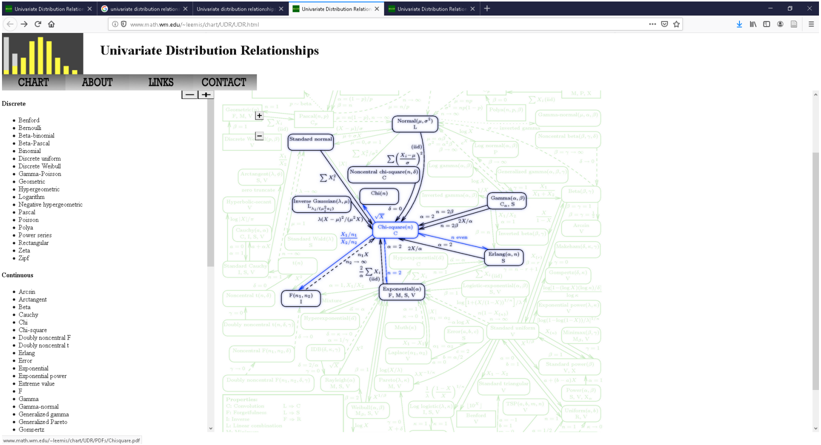
MAGIC!

Chi-squared



Zipf(α, n)
Discrete uniform(a, b) R, V
Rectangular(n) V
Beta-binomial(a, b, n)
Negative hypergeometric(n1, n2, n3)
Zeta(α)
Logarithm(c)
Power series(c, A(c))
Poisson(μ) C
Hypergeometric(n1, n2, n3)
Beta-Pascal(n, a, b)
Gamma-Poisson(α, β)
Binomial(n, p) Cp
Bernoulli(p) M, P, X
Geometric(p) F, M, V
Pascal(n, p) Cp
Polya(n, p, β)
Normal(μ, σ²) L
Gamma-normal(μ, α, β)
Discrete Weibull(p, β) V
Standard normal
Noncentral beta(β, γ, δ)
Log normal(α, β) P
Log gamma(α, β)
Generalized gamma(α, β, γ)
Arctangent(λ, φ) S, V
Noncentral chi-square(n, δ) C
Chi(n)
Inverted gamma(α, β)
Gamma(α, β) Cα, S
Beta(β, γ)
Hyperbolic-secant V
Inverse Gaussian(λ, μ)
Arcsin V
Cauchy(a, α) C, I, S, V
Chi-square(n) C
Standard Wald(λ) S
Inverted beta(β, γ)
Makeham(δ, κ, γ)
Erlang(α, n) S
Standard Cauchy I, S, V
t(n)
Hypoexponential(α⃗) C
Gompertz(δ, κ) V
Logistic-exponential(α, β) S, V
Noncentral t(n, δ)
F(n1, n2) I
Exponential(α) F, M, S, V
Exponential power(λ, κ) V
Mixture
Hyperexponential(α⃗)
Doubly noncentral t(n, δ, γ)
Muth(κ)
Error(a, b, c) S
Standard uniform V
Minimax(β, γ) Mβ, V
Noncentral F(n1, n2, δ)
IDB(δ, κ, γ)
Laplace(α1, α2) V
Standard power(β) V, X
Doubly noncentral F(n1, n2, δ, γ)
Rayleigh(α) M, S, V
Pareto(λ, κ) M, V
Standard triangular V
Power(α, β) S, V, Xα
Weibull(α, β) Mβ, S, V
Log logistic(λ, κ) I, S, V
Benford V
TSP(a, b, m, n) V
Uniform(a, b) R, V
Extreme value(α, β) V, Mα
Lomax(λ, κ) V
von Mises (κ, μ) S
Properties:
C: Convolution
F: Forgetfulness
I: Inverse
L: Linear combination
M: Minimum
P: Product
L ⇒ C
L ⇒ S
F ⇒ R
Relationships:

Chi-squared



See here

Some many bonuses!

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See the reference sections of these

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