Simple & Multiple Correspondence Analyses

Contingency, categorical, ordinal, continuous and mixed data

Derek Beaton

Rotman Research Institute

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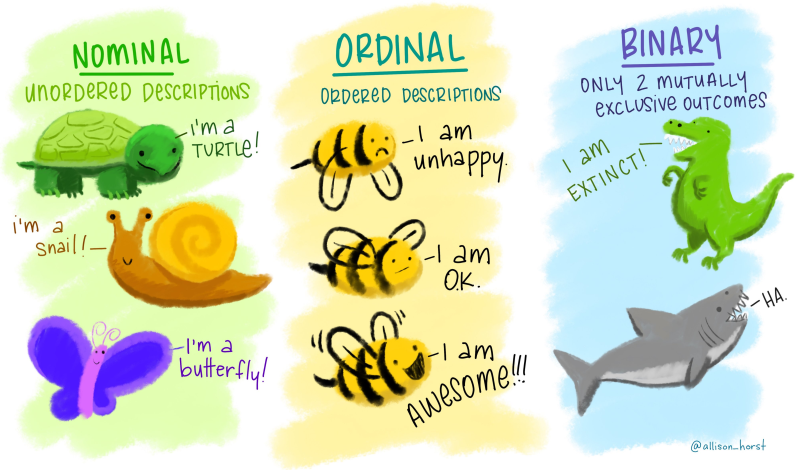
Before we get started

Our new best friends

via @allison_horst



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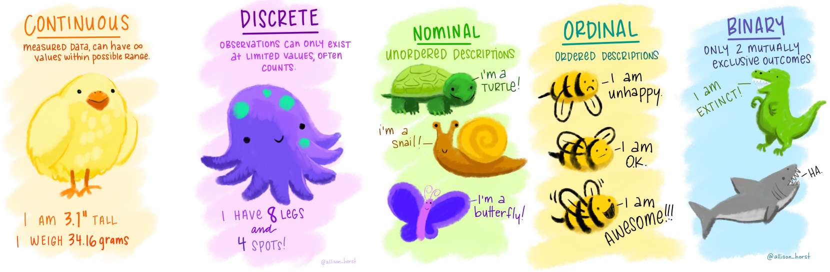

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- See SS Steven's typology: https://en.wikipedia.org/wiki/Level_of_measurement

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- Introduce CA, MCA, and tricks
  - Leave you overwhelmed, but knowing that
  - PCA is sometimes the most wrong approach
  - CA & MCA are suitably less wrong

Where to find everything

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  - Alzheimer's Disease Neuroimaging Initiative (ADNI) data

Overview

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- A whole bunch of bonuses
  - Robustness, PLS, Software

Revisting PCA

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- When we can compute a covariance or correlation matrix
- Break data into components
  - Orthogonal
  - Rank ordered
  - Made of bits & pieces of original measures

Some data

Diagnosis and education

	CN	Dementia	MCI
ADV	39	7	54
B	57	17	75
B+	75	19	113
HS	25	13	46
HS+	39	9	77

- Given a table, and asked for a multivariate analysis

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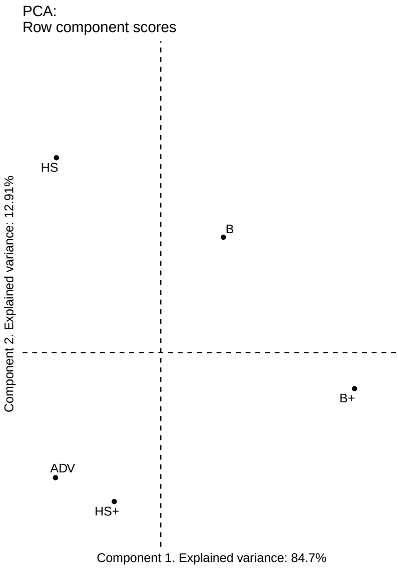
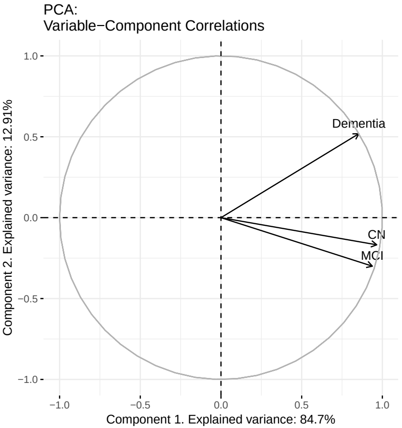
- We do what we know: PCA


PCA:
Row component scores
HS
B
B+
ADV
HS+
Component 1. Explained variance: 84.7%
Component 2. Explained variance: 12.91%


PCA:
Variable−Component Correlations
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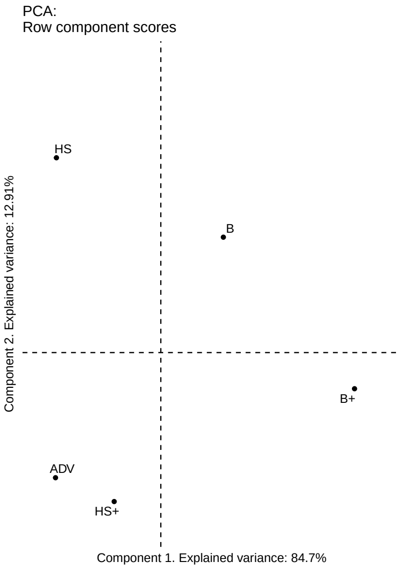
Component 1. Explained variance: 84.7%
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Dementia
CN
MCI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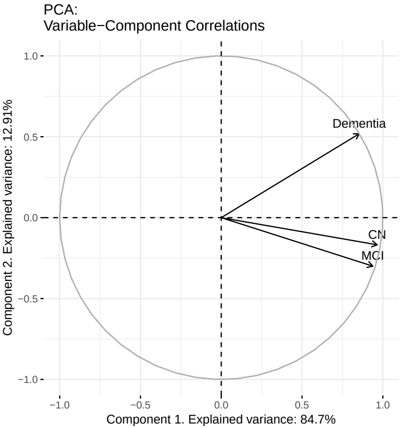
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What did we analyze?

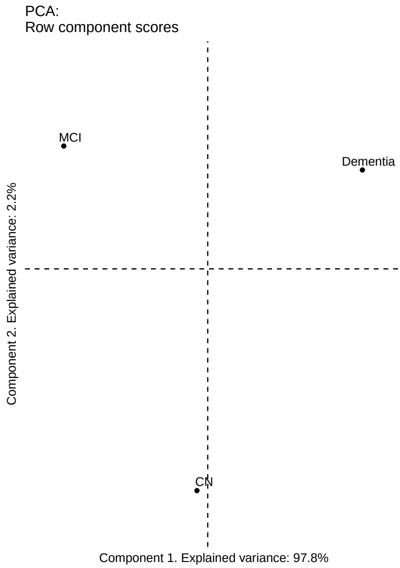
	CN	Dementia	MCI
CN	1.000	0.730	0.921
Dementia	0.730	1.000	0.652
MCI	0.921	0.652	1.000

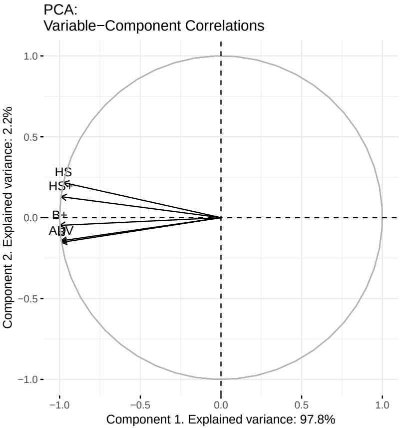
What did PCA detect?

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
B+	75	19	113	***207***
HS	25	13	46	***84***
HS+	39	9	77	***125***

Let's try something different!

	ADV	B	B+	HS	HS+
CN	39	57	75	25	39
Dementia	7	17	19	13	9
MCI	54	75	113	46	77


PCA:
Row component scores
MCI
Dementia
CN
Component 2. Explained variance: 2.2%
Component 1. Explained variance: 97.8%


PCA:
Variable−Component Correlations
HS
HS+
B+
ABV
1.0
0.5
0.0
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0.0
0.5
1.0
Component 2. Explained variance: 2.2%
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What did PCA analyze?

	ADV	B	B+	HS	HS+
ADV	1.000	1.000	0.995	0.935	0.963
B	1.000	1.000	0.994	0.932	0.960
B+	0.995	0.994	1.000	0.965	0.984
HS	0.935	0.932	0.965	1.000	0.996
HS+	0.963	0.960	0.984	0.996	1.000

What did PCA detect?

	ADV	B	B+	HS	HS+	***Row sums***
CN	39	57	75	25	39	***235***
Dementia	7	17	19	13	9	***65***
MCI	54	75	113	46	77	***365***

What is PCA for?

- When we can compute a *meaningful* covariance or correlation matrix

Let's take another look

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
B+	75	19	113	***207***
HS	25	13	46	***84***
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Column sums	***235***	***65***	***365***	

- Tell me things about this matrix

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	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
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Column sums	***235***	***65***	***365***	

- Tell me things about this matrix
- What kind of problem does this look like?

Simple correspondence analysis

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  - From Benzecri (1964) & Escofier (1965)
  - Fully developed by Escofier (1969)
- Explosion of the technique in France
  - Across virtually every field (except psychology and neuroscience)

History

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- And then Benzecri (1964) & Escofier (1965)
- Many more very important characters to re-discover CA

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We're diving in

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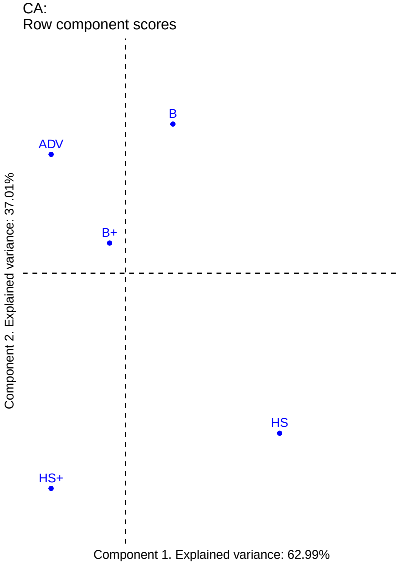
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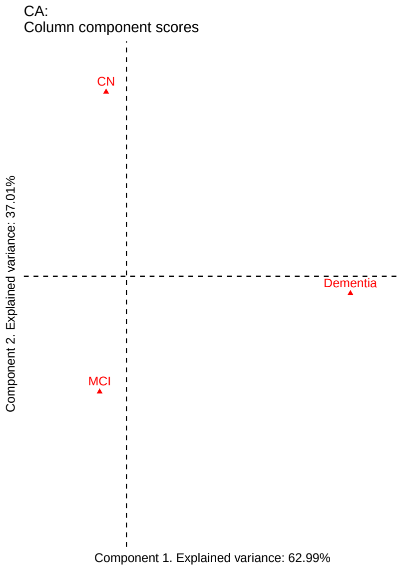
CA:
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Dementia
MCI
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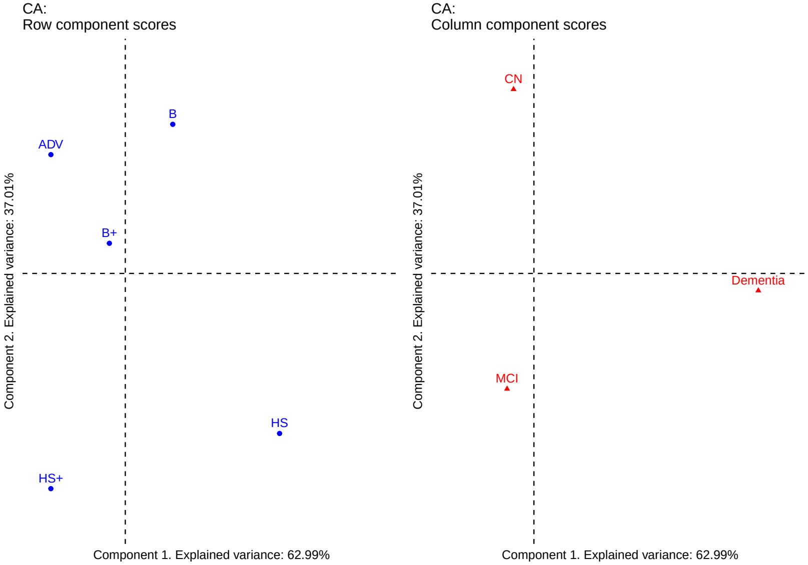
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Want to see a cool trick?

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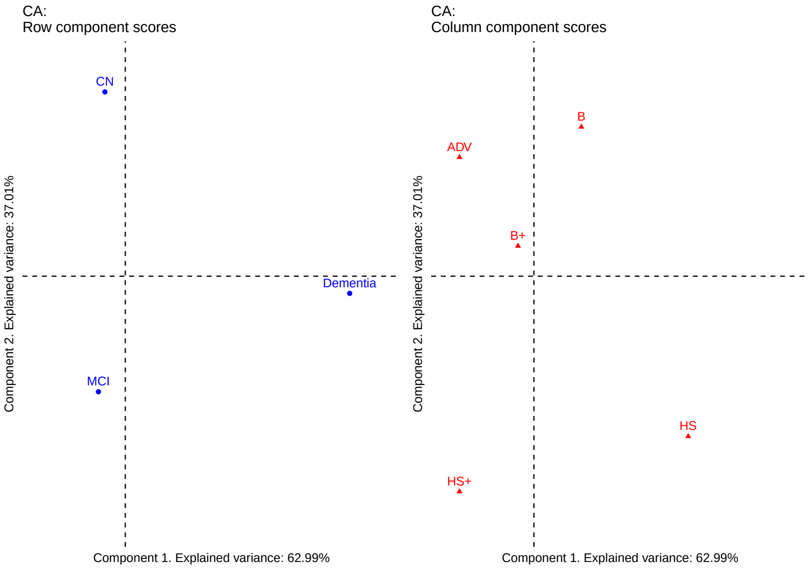
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How did that happen?

Table 1: Data

	CN	Dementia	MCI
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HS+	39	9	77

Table 2: Observed probabilites

	CN	Dementia	MCI
ADV	0.059	0.011	0.081
B	0.086	0.026	0.113
B+	0.113	0.029	0.170
HS	0.038	0.020	0.069
HS+	0.059	0.014	0.116

Table 3: Observed probabilites and margins

	CN	Dementia	MCI	***Row sums***
ADV	0.059	0.011	0.081	***0.150***
B	0.086	0.026	0.113	***0.224***
B+	0.113	0.029	0.170	***0.311***
HS	0.038	0.020	0.069	***0.126***
HS+	0.059	0.014	0.116	***0.188***
Column sums	***0.353***	***0.098***	***0.549***	

Table 4: Expected probabilites and margins

	CN	Dementia	MCI	***Row sums***
ADV	0.053	0.015	0.083	***0.150***
B	0.079	0.022	0.123	***0.224***
B+	0.110	0.030	0.171	***0.311***
HS	0.045	0.012	0.069	***0.126***
HS+	0.066	0.018	0.103	***0.188***
Column sums	***0.353***	***0.098***	***0.549***	

Table 5: Deviations: Observed - Expected

	CN	Dementia	MCI
ADV	0.006	-0.004	-0.001
B	0.007	0.004	-0.010
B+	0.003	-0.002	-0.001
HS	-0.007	0.007	0.000
HS+	-0.008	-0.005	0.013

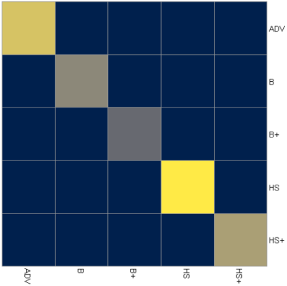
Table 6: Row constraints (inverse row margins)

	ADV	B	B+	HS	HS+
ADV	6.65	0.000	0.000	0.000	0.00
B	0.00	4.463	0.000	0.000	0.00
B+	0.00	0.000	3.213	0.000	0.00
HS	0.00	0.000	0.000	7.917	0.00
HS+	0.00	0.000	0.000	0.000	5.32

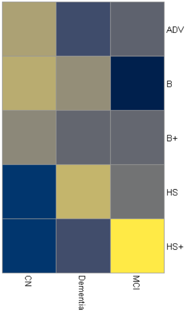
Table 7: Column constraints (inverse column margins)

	CN	Dementia	MCI
CN	2.83	0.000	0.000
Dementia	0.00	10.231	0.000
MCI	0.00	0.000	1.822

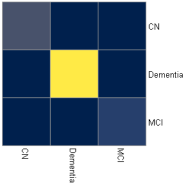
What CA needs



R: Row constraints
(inverse row probabilities)



Z: Deviations



C: Column constraints
(inverse column probabilities)

- GSVD($\mathbf{R}$, $\mathbf{X}$, $\mathbf{C}$)

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- Uses but generalizes the SVD

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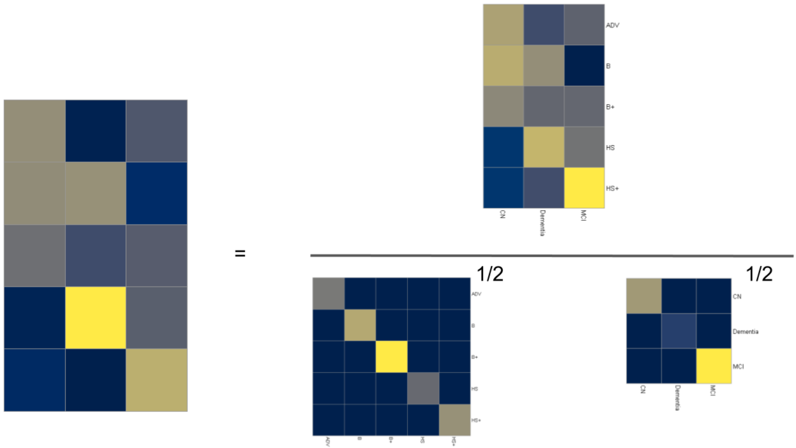
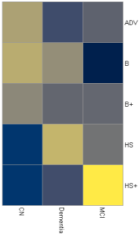
  - *Generalized* singular vectors

What we really decompose



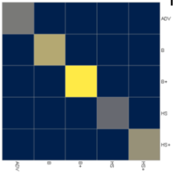
What we really decompose

- A rectangle
- Deviations: Observed - Expected
  - Expected from Observed's margins

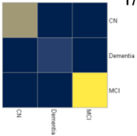


- Two squares
- Row margins and column margins

1/2



1/2



$$\frac{\mathbf{Z}}{\mathbf{R}^{\frac{1}{2}}\mathbf{C}^{\frac{1}{2}}}$$

$$\frac{(\mathbf{O} - \mathbf{E})}{\mathbf{E}^{\frac{1}{2}}}$$

$$\chi^2 = \Sigma \frac{(\mathbf{O} - \mathbf{E})^2}{\mathbf{E}}$$

CA's secrets

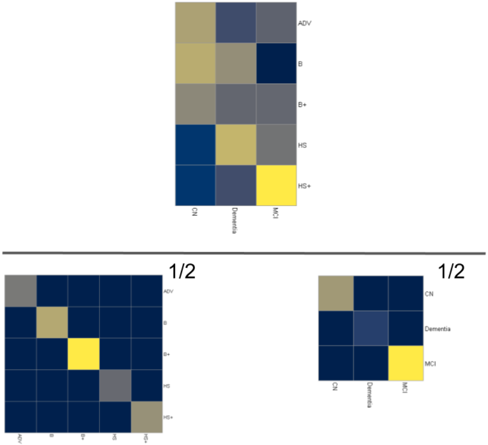
```r
EDU <- amerge_subset$PTEDUCAT
DX <- amerge_subset$DX
edu_dx_table <- table(EDU, DX)

chisq.test(edu_dx_table)
```

```
##
##  Pearson's Chi-squared test
##
## data:  edu_dx_table
## X-squared = 8.648, df = 8, p-value = 0.3729
```

```r
edu_dx_ca <- epCA(edu_dx_table, graphs = F)
sum(edu_dx_ca$ExPosition.Data$eigs) * sum(edu_dx_table)
```

```
## [1] 8.647979
```



Besides χ^2 this looks really familiar. What else are rectangles over squares?

$$r = \frac{cov(\mathbf{x}, \mathbf{y})}{\sigma_{\mathbf{x}} \times \sigma_{\mathbf{y}}}$$

More of CA's secrets

- CA generalizes canonical correlation analysis (CCA)

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- CA is the CCA between two *nominal* tables

More of CA's secrets

- CA generalizes canonical correlation analysis (CCA)
- CA is the CCA between two *nominal* tables
- How do we create a contingency table?

Nominal data

EDU	DX
B	Dementia
B	MCI
B+	Dementia
HS	Dementia
B+	CN

B	B+	ADV	HS+	HS
1	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	0	0	1
0	1	0	0	0

MCI	CN	Dementia
0	0	1
1	0	0
0	0	1
0	0	1
0	1	0

	CN	Dementia	MCI
ADV	39	7	54
B	57	17	75
B+	75	19	113
HS	25	13	46
HS+	39	9	77

B	B+	ADV	HS+	HS
1	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	0	0	1
0	1	0	0	0

MCI	CN	Dementia
0	0	1
1	0	0
0	0	1
0	0	1
0	1	0

B	B+	ADV	HS+	HS
1	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	0	0	1
0	1	0	0	0

MCI	CN	Dementia
0	0	1
1	0	0
0	0	1
0	0	1
0	1	0

How to analyze this *on its own*?

How to analyze nominal data?

- “coding categorical variables with the indicator matrix of dummy variables and considering them as Gaussian, for instance, is almost a crime.” in *Jan de Leeuw and the French School of Data Analysis* (Husson, Josse, Saporta)

How to analyze nominal data?

- "coding categorical variables with the indicator matrix of dummy variables and considering them as Gaussian, for instance, is almost a crime." in *Jan de Leeuw and the French School of Data Analysis* (Husson, Josse, Saporta)
- We *could* perform PCA on nominal data, but what would we get?

	B	B+	ADV	HS+	HS	MCI	CN	Dementia
B	149	0	0	0	0	75	57	17
B+	0	207	0	0	0	113	75	19
ADV	0	0	100	0	0	54	39	7
HS+	0	0	0	125	0	77	39	9
HS	0	0	0	0	84	46	25	13
MCI	75	113	54	77	46	365	0	0
CN	57	75	39	39	25	0	235	0
Dementia	17	19	7	9	13	0	0	65

	B	B+	ADV	HS+	HS	MCI	CN	Dementia
B	1	−0.361	−0.226	−0.259	−0.204	−0.049	0.033	0.03
B+	−0.361	1	−0.283	−0.323	−0.256	−0.004	0.013	−0.013
ADV	−0.226	−0.283	1	−0.202	−0.16	−0.008	0.032	−0.039
HS+	−0.259	−0.323	−0.202	1	−0.183	0.065	−0.042	−0.042
HS	−0.204	−0.256	−0.16	−0.183	1	−0.001	−0.044	0.073
MCI	−0.049	−0.004	−0.008	0.065	−0.001	1	−0.815	−0.363
CN	0.033	0.013	0.032	−0.042	−0.044	−0.815	1	−0.243
Dementia	0.03	−0.013	−0.039	−0.042	0.073	−0.363	−0.243	1

Multiple correspondence analysis

Multiple correspondence analysis

- Two perspectives:

Multiple correspondence analysis

- Two perspectives:
  - *Weighted* PCA for nominal data

Multiple correspondence analysis

- Two perspectives:
  - *Weighted* PCA for nominal data
  - Generalized CA for N-way contingency tables

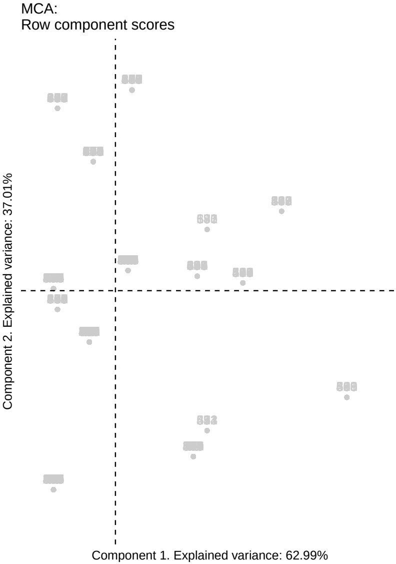
Multiple correspondence analysis

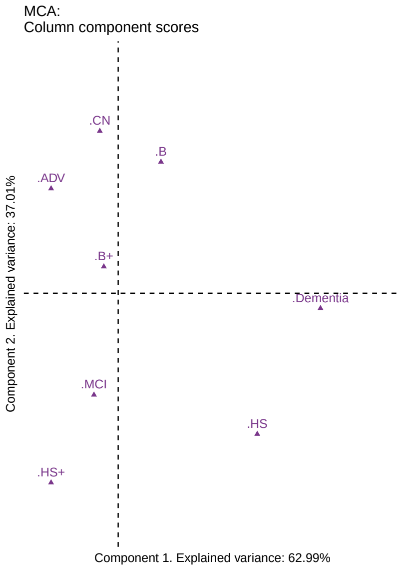
- Two perspectives:
  - *Weighted* PCA for nominal data
  - Generalized CA for N-way contingency tables
- So much more than nominal

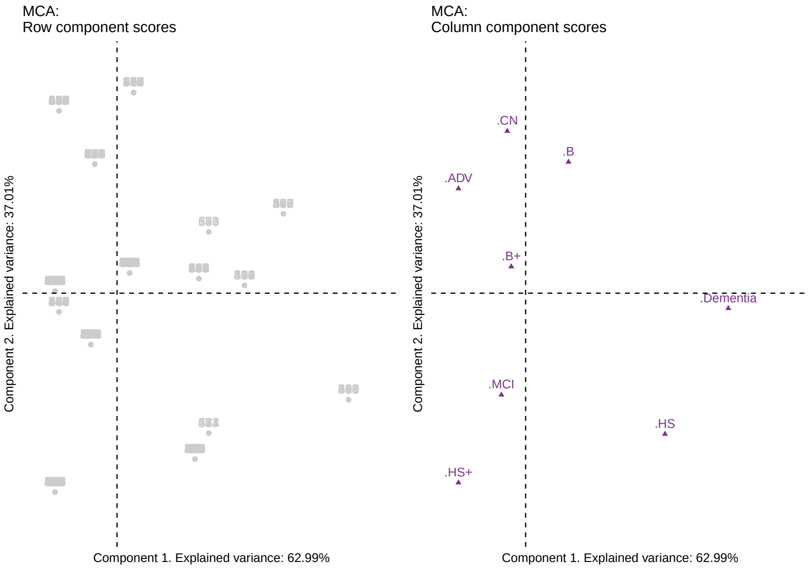
We're diving in

B	B+	ADV	HS+	HS	MCI	CN	Dementia
1	0	0	0	0	0	0	1
1	0	0	0	0	1	0	0
0	1	0	0	0	0	0	1
0	0	0	0	1	0	0	1
0	1	0	0	0	0	1	0

This is the kind of table we're analyzing. It has $N = 665$.


MCA:
Row component scores
Component 2. Explained variance: 37.01%
Component 1. Explained variance: 62.99%


MCA:
Column component scores
.CN
.B
.ADV
.B+
.Dementia
.MCI
.HS
.HS+
Component 1. Explained variance: 62.99%
Component 2. Explained variance: 37.01%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MCA:
Column component scores
.CN
.B
.ADV
.B+
.Dementia
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.HS
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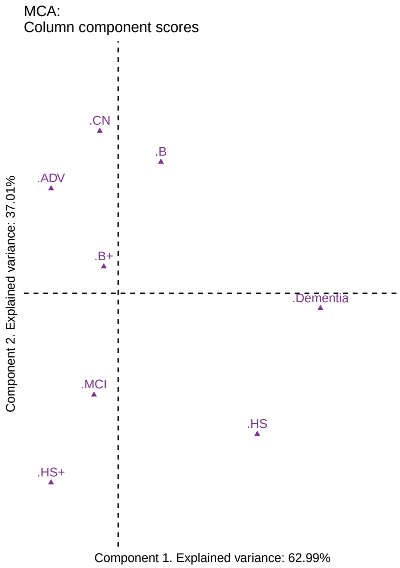
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Column component scores
.CN
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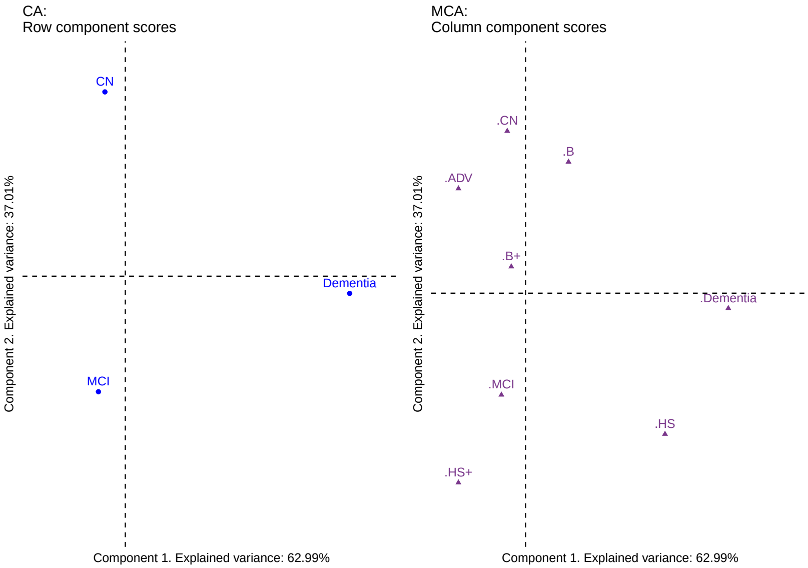

CA:
Row component scores
CN
Dementia
MCI
Component 1. Explained variance: 62.99%
Component 2. Explained variance: 37.01%
MCA:
Column component scores
.CN
.B
.ADV
.B+
.Dementia
.MCI
.HS
.HS+
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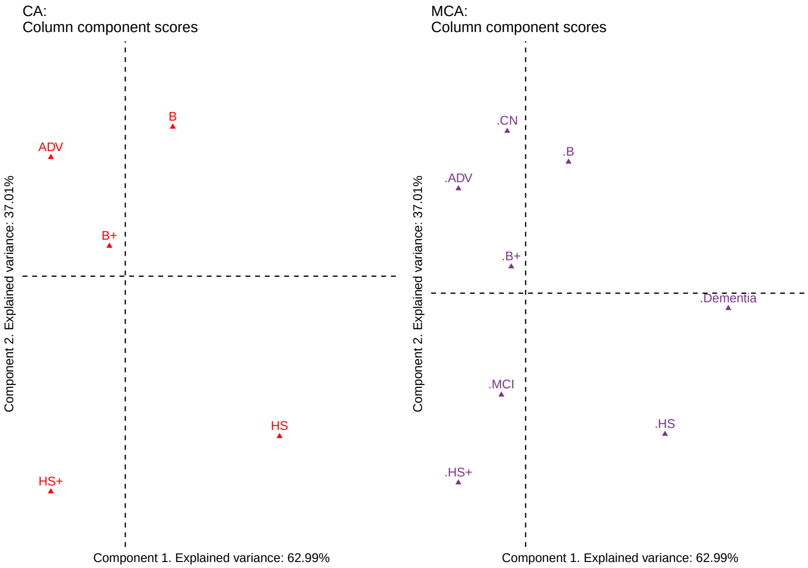

CA:
Column component scores
Component 2. Explained variance: 37.01%
Component 1. Explained variance: 62.99%
B
ADV
B+
HS
HS+
MCA:
Column component scores
Component 2. Explained variance: 37.01%
Component 1. Explained variance: 62.99%
.CN
.B
.ADV
.B+
.Dementia
.MCI
.HS
.HS+

CA & MCA Magic!

	CN	Dementia	MCI
ADV	39	7	54
B	57	17	75
B+	75	19	113
HS	25	13	46
HS+	39	9	77

B	B+	ADV	HS+	HS	MCI	CN	Dementia
1	0	0	0	0	0	0	1
1	0	0	0	0	1	0	0
0	1	0	0	0	0	0	1
0	0	0	0	1	0	0	1
0	1	0	0	0	0	1	0

Same technique on two *different* tables: same result

Scaling up

- Let's bring in ApoE

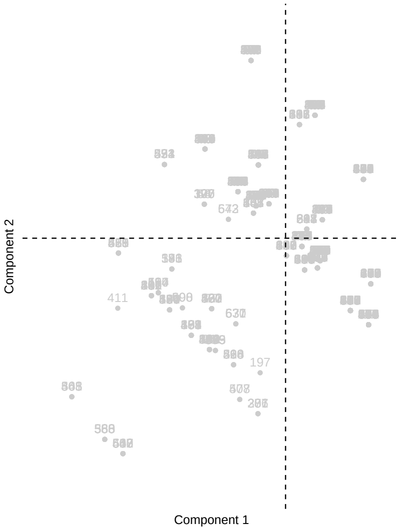
Scaling up

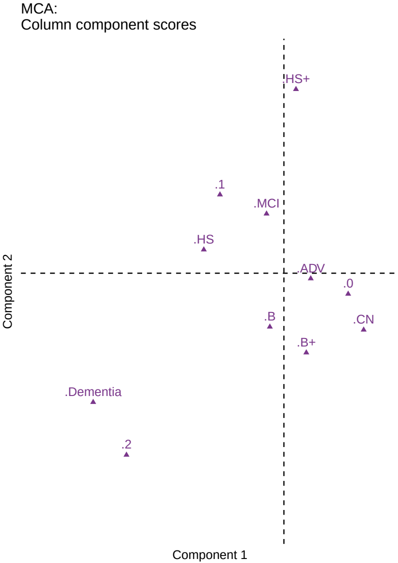
- Let's bring in ApoE
- It has 3 levels: 0 copy, 1 copy, 2 copies

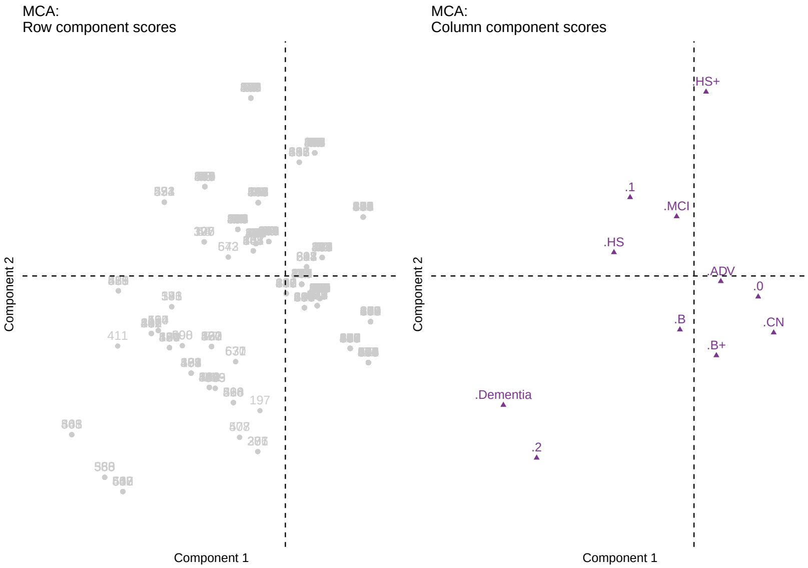
EDU	DX	APOE
B	Dementia	2
B	MCI	0
B+	Dementia	2
HS	Dementia	2
B+	CN	0

B	B+	ADV	HS+	HS	MCI	CN	Dementia	0	1	2
1	0	0	0	0	0	0	1	0	0	1
1	0	0	0	0	1	0	0	1	0	0
0	1	0	0	0	0	0	1	0	0	1
0	0	0	0	1	0	0	1	0	0	1
0	1	0	0	0	0	1	0	1	0	0

MCA:
Row component scores




MCA:
Column component scores
.HS+
.1
.MCI
.HS
.ADV
.0
.B
.CN
.B+
.Dementia
.2
Component 2
Component 1


MCA:
Row component scores
Component 2
Component 1
411
197
MCA:
Column component scores
Component 2
Component 1
.HS+
.1
.MCI
.HS
.ADV
.0
.B
.CN
.B+
.Dementia
.2

Crisp vs. fuzzy coding

EDU	DX	APOE
B	Dementia	2
B	MCI	0
B+	Dementia	2
HS	Dementia	2
B+	CN	0

B	B+	ADV	HS+	HS	MCI	CN	Dementia	0	1	2
1	0	0	0	0	0	0	1	0	0	1
1	0	0	0	0	1	0	0	1	0	0
0	1	0	0	0	0	0	1	0	0	1
0	0	0	0	1	0	0	1	0	0	1
0	1	0	0	0	0	1	0	1	0	0

Crisp vs. fuzzy coding

EDU	DX	APOE
B	Dementia	2
B	MCI	0
B+	Dementia	2
HS	Dementia	2
B+	CN	0

B	B+	ADV	HS+	HS	MCI	CN	Dementia	0	1	2
1	0	0	0	0	0	0	1	0	0	1
1	0	0	0	0	0.5	0	0.5	1	0	0
0	1	0	0	0	0	0	1	0	0	1
0	0	0	0	1	0	0	1	0	0	1
0	1	0	0	0	0	1	0	1	0	0

Our first fuzzy friend

- Modified Hachinksi

- Modified Hachinksi
  - 0, 1, 2, 3 (in these data)

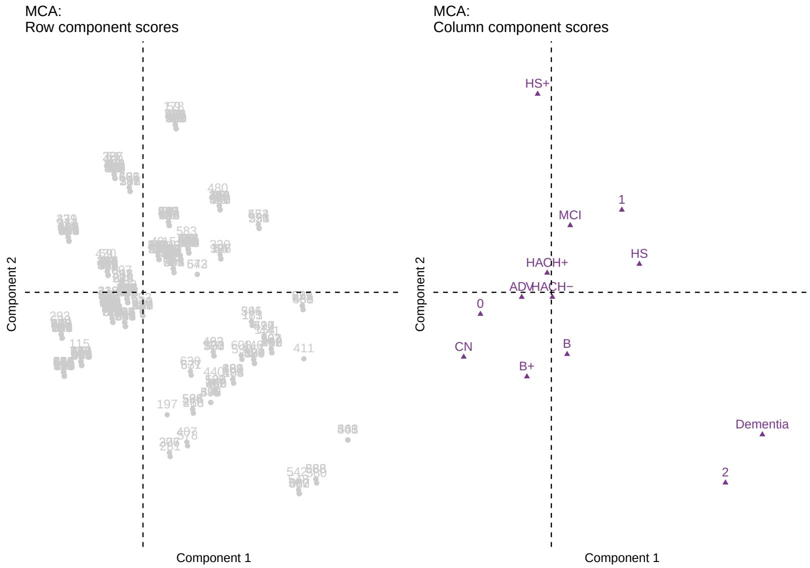
- Modified Hachinksi
    - 0, 1, 2, 3 (in these data)
- Specific form of fuzzy coding: “bipolar”

HACH
0
3
1
2
1

HACH−	HACH+
1	0
0	1
0.667	0.333
0.333	0.667
0.667	0.333

EDU	DX	APOE	HACH
B	Dementia	2	1
B	MCI	0	1
B+	Dementia	2	0
HS	Dementia	2	0
B+	CN	0	0

B	B+	ADV	HS+	HS	MCI	CN	Dementia	0	1	2	HACH−	HACH+
1	0	0	0	0	0	0	1	0	0	1	0.667	0.333
1	0	0	0	0	1	0	0	1	0	0	0.667	0.333
0	1	0	0	0	0	0	1	0	0	1	1	0
0	0	0	0	1	0	0	1	0	0	1	1	0
0	1	0	0	0	0	1	0	1	0	0	1	0


MCA:
Row component scores
Component 1
Component 2
MCA:
Column component scores
Component 1
Component 2
HS+
1
MCI
HS
HACH+
ADV
HACH−
0
CN
B
B+
Dementia
2

Our second fuzzy friend

- Age: 55.00 - 89.60

- Age: 55.00 - 89.60
  - But we need to scale it (Z-score)

- Age: 55.00 - 89.60
    - But we need to scale it (Z-score)
- We use two columns again:

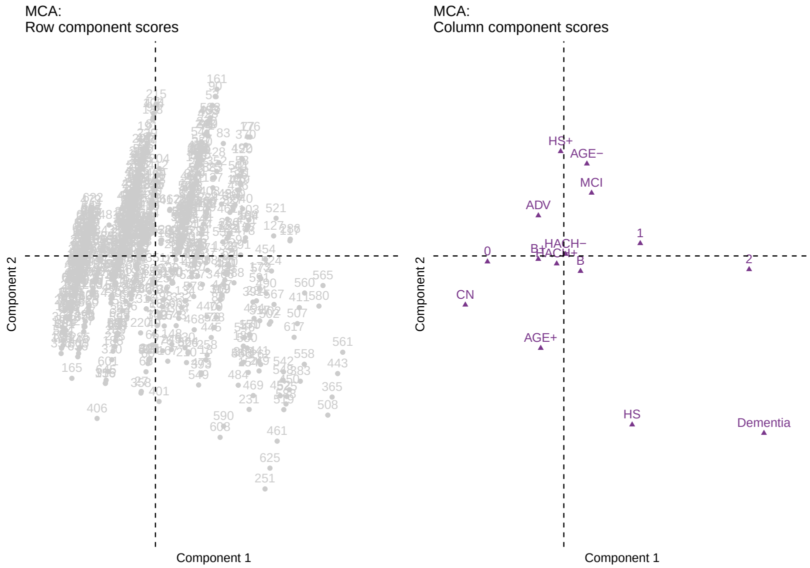
- Age: 55.00 - 89.60
  - But we need to scale it (Z-score)
- We use two columns again:
  - $\frac{(1-x)}{2}$ & $\frac{(1+x)}{2}$

AGE	AGE (Z)
76.3	0.637
76.5	0.666
64.4	−1.095
62.9	−1.314
63.9	−1.168

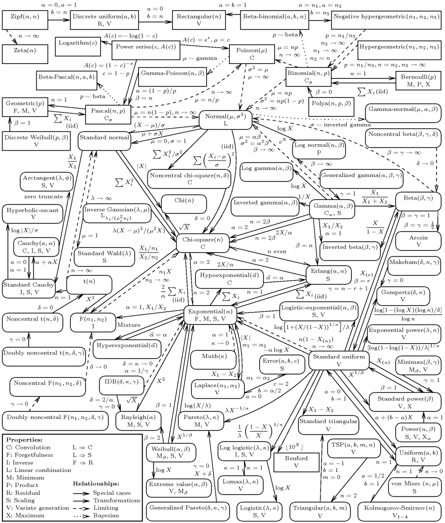
AGE−	AGE+
0.181	0.819
0.167	0.833
1.048	−0.048
1.157	−0.157
1.084	−0.084

EDU	DX	APOE	HACH	AGE
B	Dementia	2	1	76.3
B	MCI	0	1	76.5
B+	Dementia	2	0	64.4
HS	Dementia	2	0	62.9
B+	CN	0	0	63.9

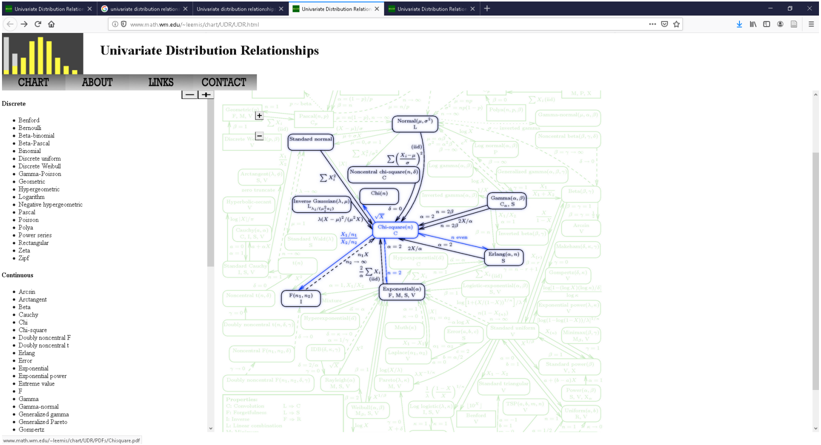
B	B+	ADV	HS+	HS	MCI	CN	Dementia	0	1	2	HACH−	HACH+	AGE−	AGE+
1	0	0	0	0	0	0	1	0	0	1	0.667	0.333	0.181	0.819
1	0	0	0	0	1	0	0	1	0	0	0.667	0.333	0.167	0.833
0	1	0	0	0	0	0	1	0	0	1	1	0	1.048	−0.048
0	0	0	0	1	0	0	1	0	0	1	1	0	1.157	−0.157
0	1	0	0	0	0	1	0	1	0	0	1	0	1.084	−0.084


MCA:
Row component scores
Component 2
Component 1
MCA:
Column component scores
HS+
AGE−
MCI
ADV
1
0
B+
HACH−
HACH+
B
2
CN
AGE+
HS
Dementia
Component 2
Component 1

How is this magic possible?!


Zipf(α, n)
Discrete uniform(a, b) R, V
Rectangular(n) V
Beta-binomial(a, b, n)
Negative hypergeometric(n1, n2, n3)
Zeta(α)
Logarithm(c)
Power series(c, A(c))
Poisson(μ) C
Hypergeometric(n1, n2, n3)
Beta-Pascal(n, a, b)
Gamma-Poisson(α, β)
Binomial(n, p) Cp
Bernoulli(p) M, P, X
Geometric(p) F, M, V
Pascal(n, p) Cp
Normal(μ, σ²) L
Polya(n, p, β)
Gamma-normal(μ, α, β)
Discrete Weibull(p, β) V
Standard normal
Noncentral beta(β, γ, δ)
Log normal(α, β) P
Log gamma(α, β)
Noncentral chi-square(n, δ) C
Generalized gamma(α, β, γ)
Arctangent(λ, φ) S, V
Chi(n)
Inverted gamma(α, β)
Gamma(α, β) Cα, S
Beta(β, γ)
Hyperbolic-secant V
Inverse Gaussian(λ, μ)
Arcsin V
Chi-square(n) C
Cauchy(a, α) C, I, S, V
Standard Wald(λ) S
Inverted beta(β, γ)
Makeham(δ, κ, γ)
Hypoexponential(α⃗) C
Erlang(α, n) S
Standard Cauchy I, S, V
t(n)
Gompertz(δ, κ) V
Noncentral t(n, δ)
F(n1, n2) I
Exponential(α) F, M, S, V
Logistic-exponential(α, β) S, V
Exponential power(λ, κ) V
Doubly noncentral t(n, δ, γ)
Hyperexponential(α⃗)
Muth(κ)
Error(a, b, c) S
Standard uniform V
Minimax(β, γ) Mβ, V
Noncentral F(n1, n2, δ)
IDB(δ, κ, γ)
Laplace(α1, α2) V
Standard power(β) V, X
Doubly noncentral F(n1, n2, δ, γ)
Rayleigh(α) M, S, V
Pareto(λ, κ) M, V
Standard triangular V
Power(α, β) S, V, Xα
Weibull(α, β) Mβ, S, V
Log logistic(λ, κ) I, S, V
Benford V
TSP(a, b, m, n) V
Uniform(a, b) R, V
Extreme value(α, β) V, Mβ
Lomax(λ, κ) V
von Mises (κ, μ) S
Generalized Pareto(δ, κ, γ)
Logistic(λ, κ) S, V
Triangular(a, b, m) V
Kolmogorov-Smirnov(n) V1–4
Properties:
C: Convolution
F: Forgetfulness
I: Inverse
L: Linear combination
M: Minimum
P: Product
R: Residual
S: Scaling
V: Variate generation
X: Maximum
L ⇒ C
L ⇒ S
F ⇒ R
Relationships:
Special cases
Transformations
Limiting
Bayesian

See here


www.math.wm.edu/~leemis/chart/UDR/UDR.html
Univariate Distribution Relationships
CHART
ABOUT
LINKS
CONTACT
Discrete
Benford
Bernoulli
Beta-binomial
Beta-Pascal
Binomial
Discrete uniform
Discrete Weibull
Gamma-Poisson
Geometric
Hypergeometric
Logarithm
Negative hypergeometric
Pascal
Poisson
Polya
Power series
Rectangular
Zeta
Zipf
Continuous
Arcsin
Arctangent
Beta
Cauchy
Chi
Chi-square
Doubly noncentral F
Doubly noncentral t
Erlang
Error
Exponential
Exponential power
Extreme value
F
Gamma
Gamma-normal
Generalized gamma
Generalized Pareto
Gompertz
Normal(μ, σ²)
Standard normal
Noncentral chi-square(n, δ)
Chi(n)
Inverse Gaussian(λ, μ)
Chi-square(n)
Gamma(α, β)
Erlang(α, n)
F(n₁, n₂)
Exponential(α)
www.math.wm.edu/~leemis/chart/UDR/PDFs/Chisquare.pdf

See here

Conclusions

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    - And the *G*SVD needs to be your other new best friend
- Learn to recognize data types
    - Learn what to do with them
    - But

Conclusions

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- What about:
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  - Use a mixture

	ASKS_HELP	NO_TIME_SELF
Ordinal responses		
PARTICIPANT 1	1	2
PARTICIPANT 2	2	3
PARTICIPANT 3	0	1
PARTICIPANT *n*	3	0
PARTICIPANT *N*	1	4

	ASKS_HELP			NO_TIME_SELF		
	NEVER	RARELY	ALWAYS	NEVER	RARELY	ALWAYS
Ordinal responses						
PARTICIPANT 1	0	1	0	0	.667	.333
PARTICIPANT 2	0	.667	.333	0	.333	.667
PARTICIPANT 3	1	0	0	0	1	0
PARTICIPANT *n*	0	.333	.667	1	0	0
PARTICIPANT *N*	0	1	0	0	0	1

Some many bonuses!

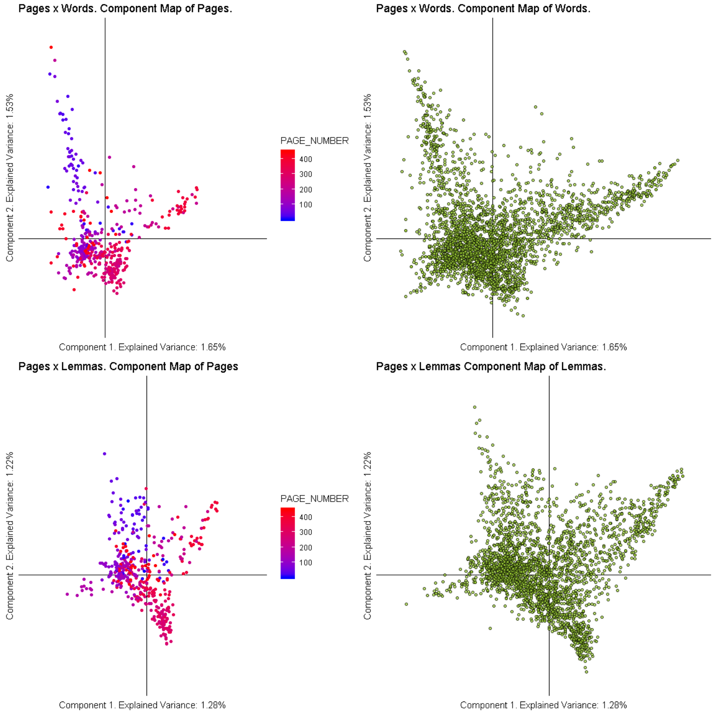
A workshop

- https://github.com/derekbeaton/Workshops/tree/master/RTC/PCA_MCA_Resampling

A workshop

- https://github.com/derekbeaton/Workshops/tree/master/RTC/PCA_MCA_Resampling
  - Extraordinary detail on all of this

The Mueller report


Pages x Words. Component Map of Pages.
Component 2. Explained Variance: 1.53%
Component 1. Explained Variance: 1.65%
PAGE_NUMBER
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Component 2. Explained Variance: 1.22%
Component 1. Explained Variance: 1.28%
PAGE_NUMBER
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Pages x Lemmas Component Map of Lemmas.
Component 2. Explained Variance: 1.22%
Component 1. Explained Variance: 1.28%

The Marvel Cinematic Universe

- Actually super cool

The Marvel Cinematic Universe

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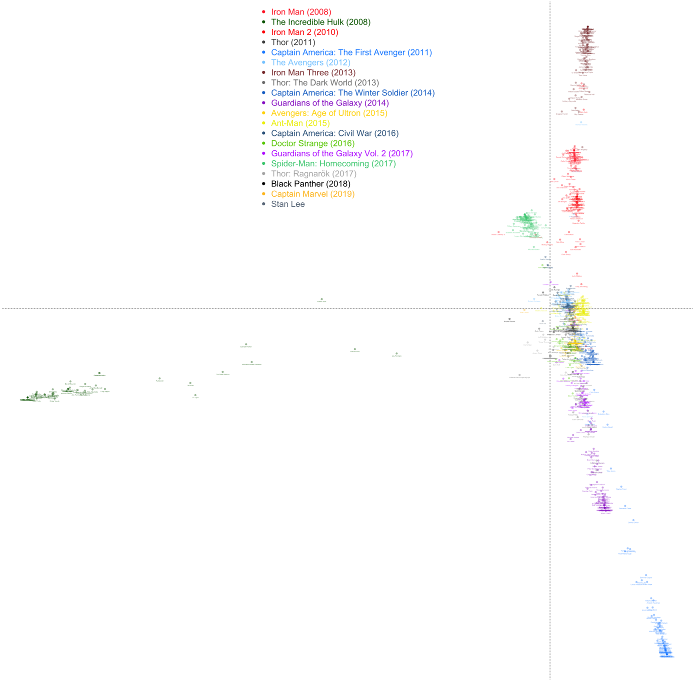
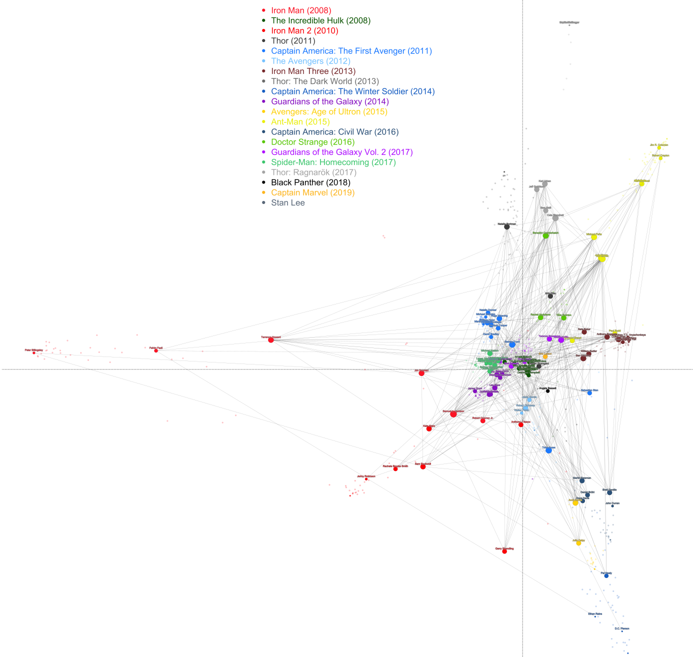
- CA naturally handles networks

The Marvel Cinematic Universe

- Actually super cool
- CA naturally handles networks
- https://github.com/derekbeaton/Marvel-Cinematic-Universe_Network/


Iron Man (2008)
The Incredible Hulk (2008)
Iron Man 2 (2010)
Thor (2011)
Captain America: The First Avenger (2011)
The Avengers (2012)
Iron Man Three (2013)
Thor: The Dark World (2013)
Captain America: The Winter Soldier (2014)
Guardians of the Galaxy (2014)
Avengers: Age of Ultron (2015)
Ant-Man (2015)
Captain America: Civil War (2016)
Doctor Strange (2016)
Guardians of the Galaxy Vol. 2 (2017)
Spider-Man: Homecoming (2017)
Thor: Ragnarök (2017)
Black Panther (2018)
Captain Marvel (2019)
Stan Lee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Stan Lee

GMCD & ours

- Generalized minimum covariance determinant

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- ours (another R package)

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  - Has some important bells-and-whistles

GMCD & ours

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  - https://github.com/derekbeaton/ours

GPLS

- First: a PLS for mixed data types

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  - Including those not discussed here

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- Second: unify the “two-table” techniques

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  - PLS, CCA, RRR/RDA

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  - PLS, CCA, RRR/RDA
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  - https://github.com/derekbeaton/gpls

GPLS

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  - Including those not discussed here
- Second: unify the “two-table” techniques
  - PLS, CCA, RRR/RDA
- Package & preprint
  - https://github.com/derekbeaton/gpls
  - Github issues where I routinely call myself a “dummy”

ExPosition

- ExPosition

ExPosition

- ExPosition
  - Family of packages

ExPosition

- ExPosition
  - Family of packages
  - Includes resampling

ExPosition

- ExPosition
  - Family of packages
  - Includes resampling
  - Lots of PCA & CA techniques

Some alternatives

- FactoMineR

Some alternatives

- FactoMineR
- ade4

Some alternatives

- FactoMineR
- ade4
- ca

Some alternatives

- FactoMineR
- ade4
- ca
- MASS

Some alternatives

- FactoMineR
- ade4
- ca
- MASS
- psych

Some alternatives

- FactoMineR
- ade4
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- MASS
- psych
- factoextra (visualization)

Some alternatives

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- ade4
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- So many others

(Some) References

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