Simple & Multiple Correspondence Analyses

Contingency, categorical, ordinal, continuous and mixed data

Derek Beaton

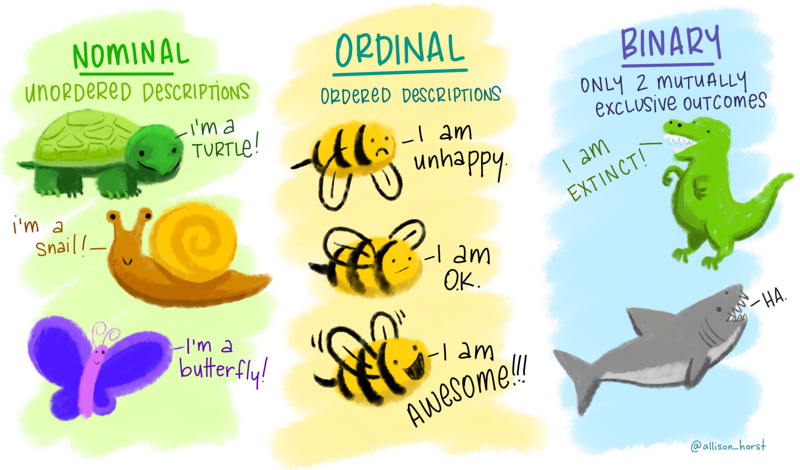
Rotman Research Institute

October 28, 2019

Before we get started

Our new best friends

via @allison_horst



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- Some are *very* difficult and effectively ignored

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- Some are *very* difficult and effectively ignored
  - We won't do that!

Motivation for today

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- Sometimes numbers aren't numbers!

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Motivation for today

- Not everything is a number
- Sometimes numbers aren't numbers!
- We need to recognize when this happens
  - And know what to do

Typology

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- Levels of measurement

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- Levels of measurement
- Excellent examples: https://en.wikipedia.org/wiki/Level_of_measurement

Where to find everything

- Generally: https://github.com/derekbeaton/workshops

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- Today:

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- Looking at some data

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Revisting PCA

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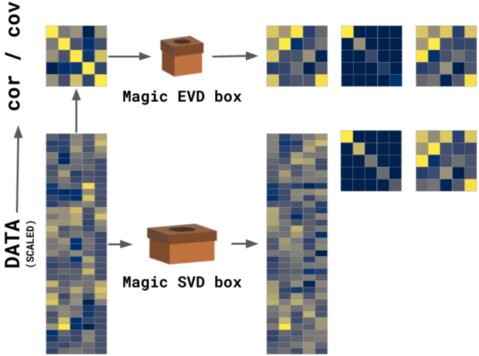
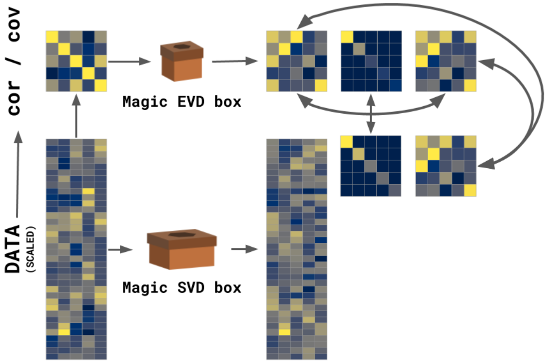
- When we can compute a covariance or correlation matrix
- Break data into components
  - Orthogonal
  - Rank ordered
  - Made of bits & pieces of original measures

Eigen- and singular value decompositions



Eigen- and singular value decompositions



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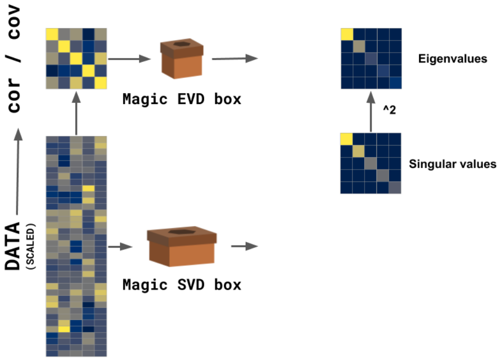

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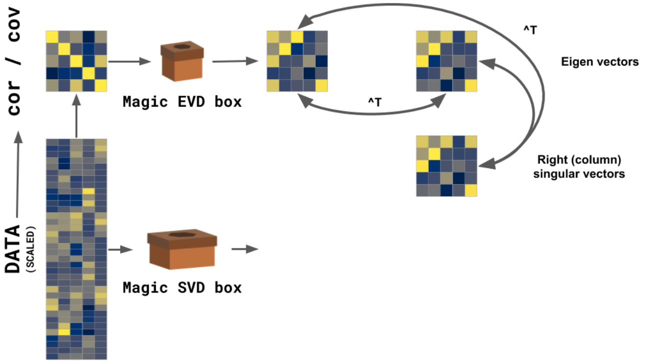

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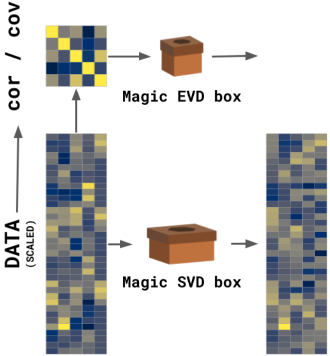

Left (row) singular vectors

Some data

Diagnosis and education

	CN	Dementia	MCI
ADV	39	7	54
B	57	17	75
B+	75	19	113
HS	25	13	46
HS+	39	9	77

- Given a table, and asked for a multivariate analysis

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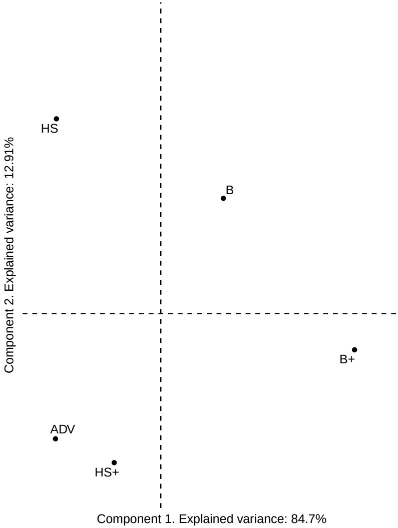
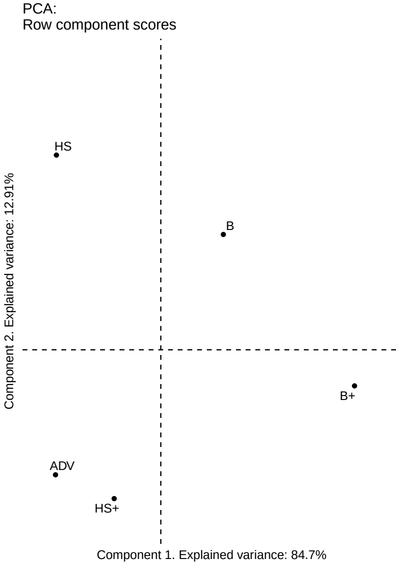
- We do what we know: PCA

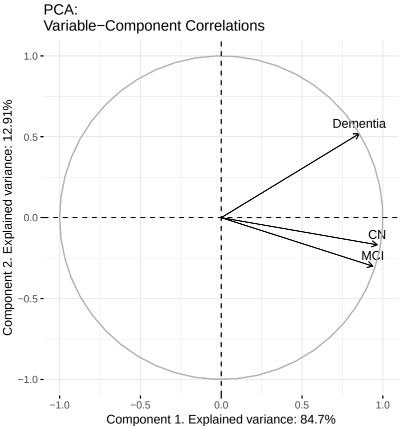
PCA:
Row component scores



***

PCA:
Variable- Component Correlations


PCA:
Row component scores
HS
B
B+
ADV
HS+
Component 2. Explained variance: 12.91%
Component 1. Explained variance: 84.7%


PCA:
Variable−Component Correlations
Dementia
CN
MCI
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Component 2. Explained variance: 12.91%
Component 1. Explained variance: 84.7%

What did we analyze?

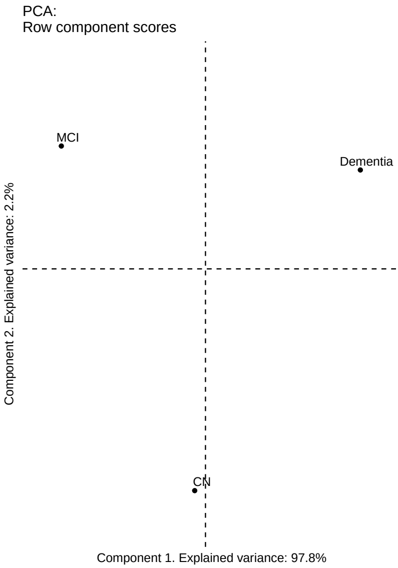
	CN	Dementia	MCI
CN	1.000	0.730	0.921
Dementia	0.730	1.000	0.652
MCI	0.921	0.652	1.000

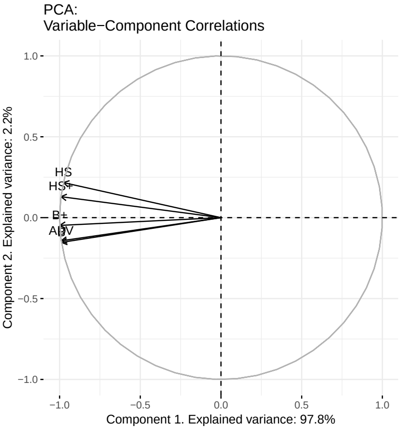
What did PCA detect?

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
B+	75	19	113	***207***
HS	25	13	46	***84***
HS+	39	9	77	***125***

Let's try something different!

	ADV	B	B+	HS	HS+
CN	39	57	75	25	39
Dementia	7	17	19	13	9
MCI	54	75	113	46	77


PCA:
Row component scores
MCI
Dementia
CN
Component 1. Explained variance: 97.8%
Component 2. Explained variance: 2.2%


PCA:
Variable−Component Correlations
HS
HS+
B+
ABV
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Component 1. Explained variance: 97.8%
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What did PCA analyze?

	ADV	B	B+	HS	HS+
ADV	1.000	1.000	0.995	0.935	0.963
B	1.000	1.000	0.994	0.932	0.960
B+	0.995	0.994	1.000	0.965	0.984
HS	0.935	0.932	0.965	1.000	0.996
HS+	0.963	0.960	0.984	0.996	1.000

What did PCA detect?

	ADV	B	B+	HS	HS+	***Row sums***
CN	39	57	75	25	39	***235***
Dementia	7	17	19	13	9	***65***
MCI	54	75	113	46	77	***365***

What is PCA for?

- When we can compute a *meaningful* covariance or correlation matrix

Let's take another look

	CN	Dementia	MCI	***Row sums***
ADV	39	7	54	***100***
B	57	17	75	***149***
B+	75	19	113	***207***
HS	25	13	46	***84***
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Column sums	***235***	***65***	***365***	

- Tell me things about this matrix

Let's take another look

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Column sums	***235***	***65***	***365***	

- Tell me things about this matrix
- What kind of problem does this look like?

Simple correspondence analysis

What is CA?

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  - Across virtually every field (except psychology and neuroscience)
- The magic of CA relies on the magic of χ^2
  - And there's some *crazy* magic here

History

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- And then Benzecri (1964) & Escofier (1965)
- Many more very important characters to re-discover CA

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We're diving in

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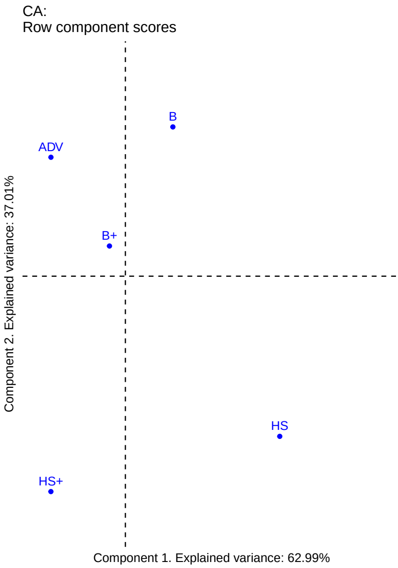
CA:
Row component scores
B
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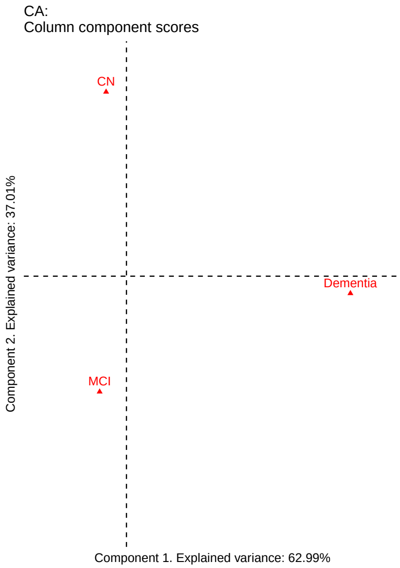
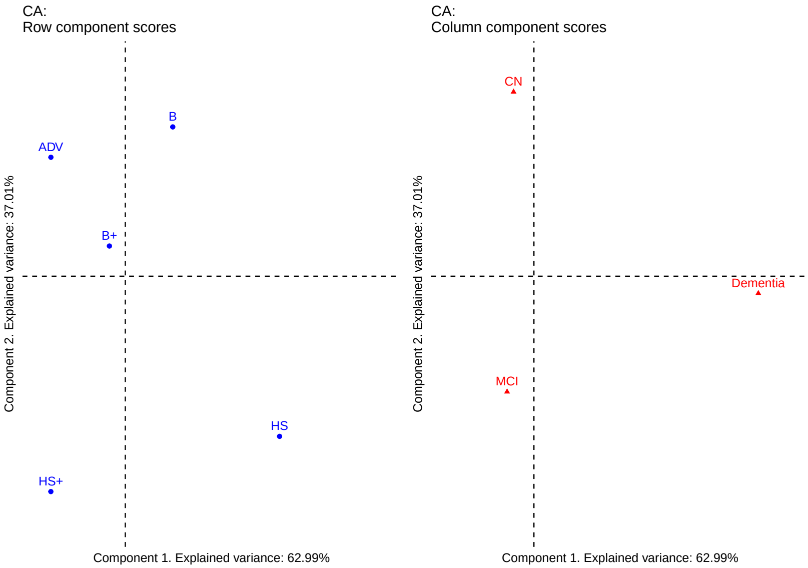
Component 2. Explained variance: 37.01%
Component 1. Explained variance: 62.99%


CA:
Column component scores
CN
Dementia
MCI
Component 2. Explained variance: 37.01%
Component 1. Explained variance: 62.99%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Want to see a cool trick?

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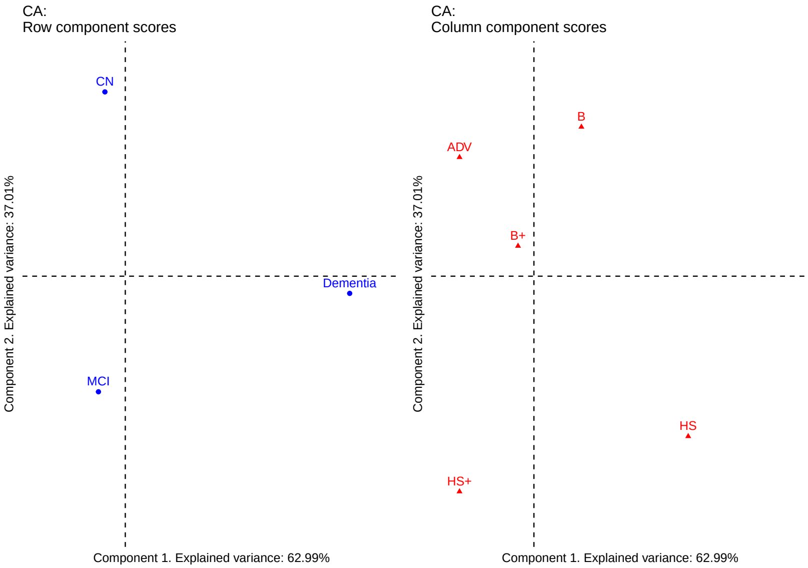
What if we perform CA on this?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How did that happen?

Table 1: Data

	CN	Dementia	MCI
ADV	39	7	54
B	57	17	75
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HS	25	13	46
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Table 2: Observed probabilites

	CN	Dementia	MCI
ADV	0.059	0.011	0.081
B	0.086	0.026	0.113
B+	0.113	0.029	0.170
HS	0.038	0.020	0.069
HS+	0.059	0.014	0.116

Table 3: Observed probabilites and margins

	CN	Dementia	MCI	***Row sums***
ADV	0.059	0.011	0.081	***0.150***
B	0.086	0.026	0.113	***0.224***
B+	0.113	0.029	0.170	***0.311***
HS	0.038	0.020	0.069	***0.126***
HS+	0.059	0.014	0.116	***0.188***
Column sums	***0.353***	***0.098***	***0.549***	

Table 4: Expected probabilites and margins

	CN	Dementia	MCI	***Row sums***
ADV	0.053	0.015	0.083	***0.150***
B	0.079	0.022	0.123	***0.224***
B+	0.110	0.030	0.171	***0.311***
HS	0.045	0.012	0.069	***0.126***
HS+	0.066	0.018	0.103	***0.188***
Column sums	***0.353***	***0.098***	***0.549***	

Table 5: Deviations: Observed - Expected

	CN	Dementia	MCI
ADV	0.006	-0.004	-0.001
B	0.007	0.004	-0.010
$B+$	0.003	-0.002	-0.001
HS	-0.007	0.007	0.000
$HS+$	-0.008	-0.005	0.013

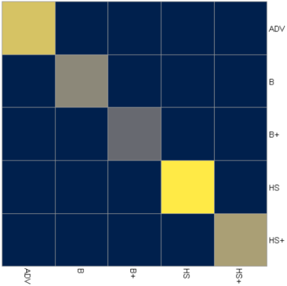
Table 6: Row constraints (inverse row margins)

	ADV	B	B+	HS	HS+
ADV	6.65	0.000	0.000	0.000	0.00
B	0.00	4.463	0.000	0.000	0.00
B+	0.00	0.000	3.213	0.000	0.00
HS	0.00	0.000	0.000	7.917	0.00
HS+	0.00	0.000	0.000	0.000	5.32

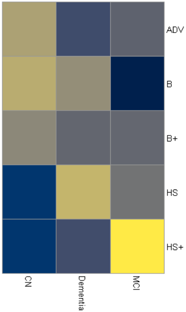
Table 7: Column constraints (inverse column margins)

	CN	Dementia	MCI
CN	2.83	0.000	0.000
Dementia	0.00	10.231	0.000
MCI	0.00	0.000	1.822

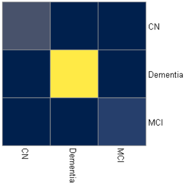
What CA needs



R: Row constraints
(inverse row probabilities)



Z: Deviations



C: Column constraints
(inverse column probabilities)

- GSVD($\mathbf{R}$, $\mathbf{X}$, $\mathbf{C}$)

- GSVD(**R**, **X**, **C**)
- Uses but generalizes the SVD

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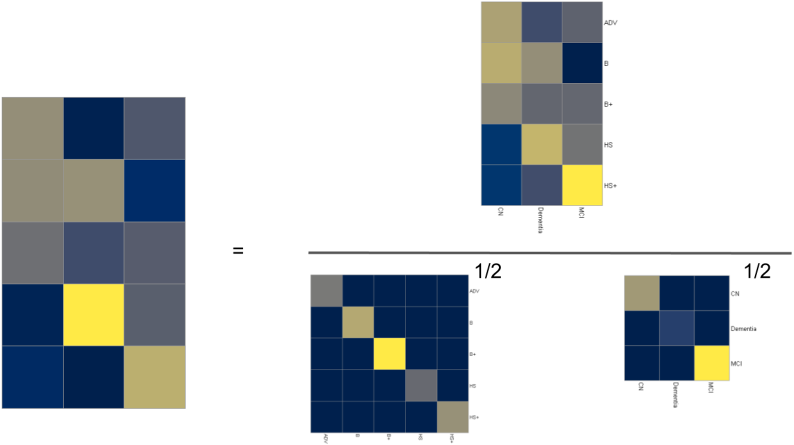
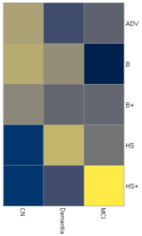
- Uses but generalizes the SVD
  - Uses row & column weights (constraints)
- Gives back
  - Component (factor) scores
  - Eigenvalues, singular values, & singular vectors
  - *Generalized* singular vectors

What we really decompose

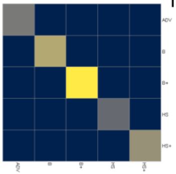


- A rectangle
- Deviations: Observed - Expected
    - Expected from Observed's margins

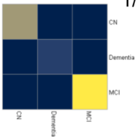


- Two squares
- Row margins and column margins

1/2



1/2



$$\frac{\mathbf{Z}}{\mathbf{R}^{\frac{1}{2}}\mathbf{C}^{\frac{1}{2}}}$$

$$\frac{(\mathbf{O} - \mathbf{E})}{\mathbf{E}^{\frac{1}{2}}}$$

$$\chi^2 = \sum \frac{(\mathbf{O} - \mathbf{E})^2}{\mathbf{E}}$$

CA's first secrets

```r
EDU <- amerge_subset$PTEDUCAT
DX <- amerge_subset$DX
edu_dx_table <- table(EDU, DX)

chisq.test(edu_dx_table)
```

```
##
##  Pearson's Chi-squared test
##
## data:  edu_dx_table
## X-squared = 8.648, df = 8, p-value = 0.3729
```

```r
edu_dx_ca <- epCA(edu_dx_table, graphs = F)
sum(edu_dx_ca$ExPosition.Data$eigs) * sum(edu_dx_table)
```

```
## [1] 8.647979
```

The GSVD

Simple quick magic Then visualize it (as 3 matrices, then 1 over 2 which is just the probs not inverse) Then swing back to Chi2 Then swing to CCA Then expand it & transition to MCA

[[[pick up here and drop most of the stuff below]]]

The GSVD

- The generalized SVD

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  - Constraints (weights) for rows & columns of rectangular table

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  - Constraints (weights) for rows & columns of rectangular table
  - Required for CA and fancier PCA-like techniques & extensions

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  - Constraints (weights) for rows & columns of rectangular table
  - Required for CA and fancier PCA-like techniques & extensions
- For some matrix $\mathbf{X}$ with I rows and J columns

The GSVD

- The generalized SVD
  - Constraints (weights) for rows & columns of rectangular table
  - Required for CA and fancier PCA-like techniques & extensions
- For some matrix $\mathbf{X}$ with I rows and J columns
- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)

The GSVD

- The generalized SVD
  - Constraints (weights) for rows & columns of rectangular table
  - Required for CA and fancier PCA-like techniques & extensions
- For some matrix $\mathbf{X}$ with I rows and J columns
- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)
- GSVD is

The GSVD

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  - Required for CA and fancier PCA-like techniques & extensions
- For some matrix $\mathbf{X}$ with I rows and J columns
- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)
- GSVD is
  - Matrix multiplication (by constraints on data)

The GSVD

- The generalized SVD
  - Constraints (weights) for rows & columns of rectangular table
  - Required for CA and fancier PCA-like techniques & extensions
- For some matrix $\mathbf{X}$ with I rows and J columns
- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)
- GSVD is
  - Matrix multiplication (by constraints on data)
  - The SVD

The GSVD

- The generalized SVD
  - Constraints (weights) for rows & columns of rectangular table
  - Required for CA and fancier PCA-like techniques & extensions
- For some matrix $\mathbf{X}$ with I rows and J columns
- SVD($\mathbf{X}$) vs. GSVD($\mathbf{W}_I$,$\mathbf{X}$,$\mathbf{W}_J$)
- GSVD is
  - Matrix multiplication (by constraints on data)
  - The SVD
  - More matrix multiplication (by constraints on vectors)

What we did to the data

- O, wi & wj, E, Z

What we did to the data

- O, wi & wj, E, Z
- Oh look that's Chi2

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- Sum of eigenvalues * sum of table = Chi2.

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What we did to the data

- O, wi & wj, E, Z
- Oh look that's Chi2
- Sum of eigenvalues * sum of table = Chi2.
  - Each component is an additive orthogonal slice of Chi2. WOAH.
  - The eigenvalues are *magic*

CA visualized

- Oh look it's CCA-ish

CA visualized

- Oh look it's CCA-ish
- Oh it really really is CCA-ish!

Rules

- It's like PCA

Rules

- It's like PCA
  - Variance (singular values, eigen values)

Rules

- It's like PCA
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  - Directions & inter-relationships

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Rules

- It's like PCA
  - Variance (singular values, eigen values)
  - Directions & inter-relationships
  - Components scores
- It's unlike PCA
  - Relative interpretation *between* sets
  - Generalized singular vectors
  - Bifactor

Rules

- It's like PCA
  - Variance (singular values, eigen values)
  - Directions & inter-relationships
  - Components scores
- It's unlike PCA
  - Relative interpretation *between* sets
  - Generalized singular vectors
  - Bifactor
    - Rows & columns treated the same

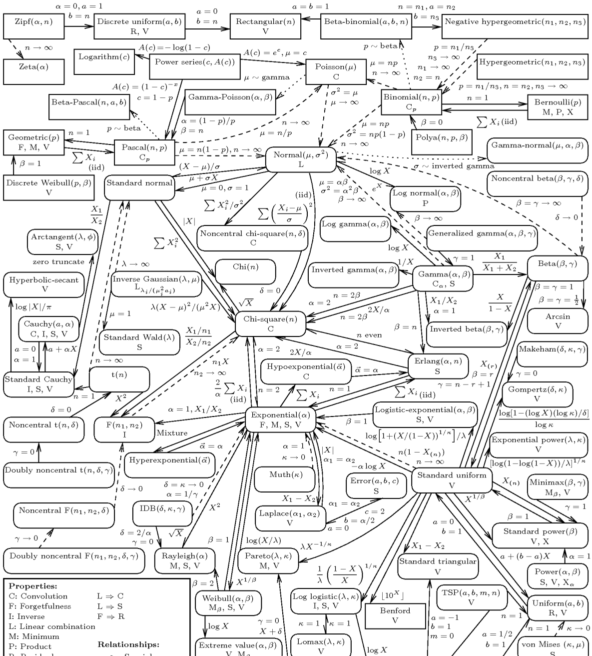
Rules

- It's like PCA
  - Variance (singular values, eigen values)
  - Directions & inter-relationships
  - Components scores
- It's unlike PCA
  - Relative interpretation *between* sets
  - Generalized singular vectors
  - Bifactor
    - Rows & columns treated the same
    - Together they help make components, as opposed to PCA

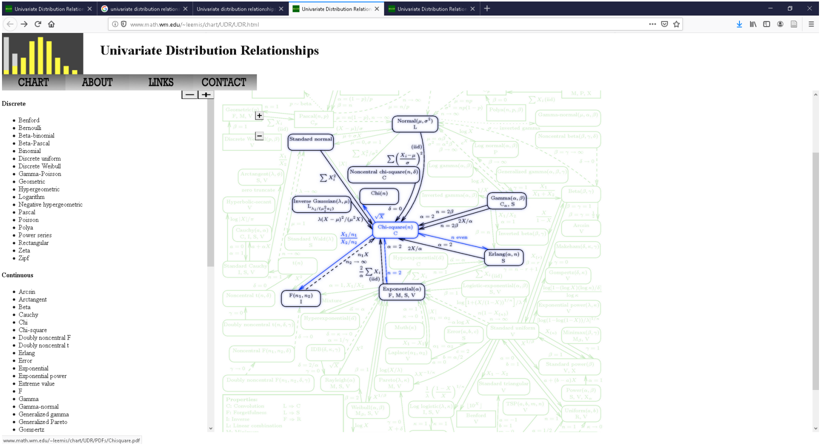
Multiple correspondence analysis

MAGIC!

Chi-squared


Zipf(α, n)
Discrete uniform(a, b)
R, V
Rectangular(n)
V
Beta-binomial(a, b, n)
Negative hypergeometric(n_1, n_2, n_3)
Zeta(α)
Logarithm(c)
Power series(c, $A(c)$)
Poisson(μ)
C
Hypergeometric(n_1, n_2, n_3)
Beta-Pascal(n, a, b)
Gamma-Poisson(α, β)
Binomial(n, p)
C_p
Bernoulli(p)
M, P, X
Geometric(p)
F, M, V
Pascal(n, p)
C_p
Polya(n, p, β)
Normal(μ, σ^2)
L
Gamma-normal(μ, α, β)
Discrete Weibull(p, β)
V
Standard normal
Noncentral beta(β, γ, δ)
Log normal(α, β)
P
Arctangent(λ, ϕ)
S, V
Noncentral chi-square(n, δ)
C
Log gamma(α, β)
Generalized gamma(α, β, γ)
Chi(n)
Hyperbolic-secant
V
Inverse Gaussian(λ, μ)
Inverted gamma(α, β)
Gamma(α, β)
C_α, S
Beta(β, γ)
Chi-square(n)
C
Arcsin
V
Cauchy(a, α)
C, I, S, V
Standard Wald(λ)
S
Inverted beta(β, γ)
Makeham(δ, κ, γ)
Standard Cauchy
I, S, V
t(n)
Hypoexponential($\vec{\alpha}$)
C
Erlang(α, n)
S
Gompertz(δ, κ)
V
Noncentral t(n, δ)
F(n_1, n_2)
I
Exponential(α)
F, M, S, V
Logistic-exponential(α, β)
S, V
Exponential power(λ, κ)
V
Doubly noncentral t(n, δ, γ)
Hyperexponential($\vec{\alpha}$)
Muth(κ)
Error(a, b, c)
S
Standard uniform
V
Minimax(β, γ)
M_β, V
Noncentral F(n_1, n_2, δ)
IDB(δ, κ, γ)
Laplace(α_1, α_2)
V
Standard power(β)
V, X
Doubly noncentral F(n_1, n_2, δ, γ)
Rayleigh(α)
M, S, V
Pareto(λ, κ)
M, V
Standard triangular
V
Power(α, β)
S, V, X_α
Weibull(α, β)
M_β, S, V
Log logistic(λ, κ)
I, S, V
Benford
V
TSP(a, b, m, n)
V
Uniform(a, b)
R, V
Extreme value(α, β)
Lomax(λ, κ)
V
von Mises (κ, μ)
Properties:
C: Convolution
F: Forgetfulness
I: Inverse
L: Linear combination
M: Minimum
P: Product
L ⇒ C
L ⇒ S
F ⇒ R
Relationships:

Chi-squared



See here

Some many bonuses!

(Some) References

See the reference sections of these

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