Appendix

Appendix A: Alternative generative process

Algorithm 4 Generative process: one receiver and one or more senders

Input: number of events and nodes (E, A), covariates $(\boldsymbol{x}, \boldsymbol{y})$, and coefficients $(\boldsymbol{b}, \boldsymbol{\eta})$

```
for e = 1 to E do
  for j = 1 to A do
    for i = 1 to A (i ≠ j) do
      set λ_iej = b^⊤ x_iej
    end for
    draw u_je ~ MB_G(λ_je)
    set μ_je = g^{-1}(η^⊤ y_je)
    draw τ_je ~ f_τ(μ_je, V(μ))
  end for
  if n ≥ 2 tied events then
    set r_e, ..., r_{e+n-1} = argmin_j(τ_je)
    set s_e = u_{r_e e}, ..., s_{e+n-1} = u_{r_{e+n-1} d}
    set t_e, ..., t_{e+n-1} = t_{e-1} + min_j τ_je
    jump to e = e + n
  else
    set r_e = argmin_j(τ_je)
    set s_e = u_{r_e e}
    set t_e = t_{e-1} + min_j τ_je
  end if
end for
```

Appendix B: Normalizing constant of MB$_G$

Our probability measure "MB$_G$"—the multivariate Bernoulli distribution with non-empty Gibbs measure—defines the probability of sender i selecting the binary receiver vector $\boldsymbol{u}_{ie}$ as

$$\Pr(\boldsymbol{u}_{ie}|\,\boldsymbol{b}, \boldsymbol{x}_{ie}) = \frac{1}{Z(\boldsymbol{\lambda}_{ie})} \exp\Big(\log\big(\mathrm{I}(\|\boldsymbol{u}_{ie}\|_1 > 0)\big) + \sum_{j \neq i} \lambda_{iej} u_{iej}\Big),$$

where the receiver intensity is a linear combination of receiver selection features—i.e., $\lambda_{iej} = \boldsymbol{b}^\top \boldsymbol{x}_{iej}$—as defined in Secton 2.1.

To use this distribution efficiently, we derive a closed-form expression for $Z(\boldsymbol{\lambda}_{ie})$ that does not require brute-force summation over the support of $\boldsymbol{u}_{ie}$ (i.e., $\forall \boldsymbol{u}_{ie} \in [0,1]^A$). We recognize that if $\boldsymbol{u}_{ie}$ were drawn via independent Bernoulli distributions in which $\Pr(u_{iej} = 1|\,\boldsymbol{b}, \boldsymbol{x}_{ie})$ was given by $\mathrm{logit}(\lambda_{iej})$, then

$$\Pr(\boldsymbol{u}_{ie}|\,\boldsymbol{b}, \boldsymbol{x}_{ie}) \propto \exp\Big(\sum_{j \neq i} \lambda_{iej} u_{iej}\Big).$$

This is straightforward to verify by looking at

$$\Pr(u_{iej} = 1 | \, \boldsymbol{u}_{ie\setminus j}, \boldsymbol{b}, \boldsymbol{x}_{ie}) = \frac{\exp{(\lambda_{iej})}}{\exp{(\lambda_{iej})} + 1},$$

where the subscript "$\setminus j$" denotes a quantity excluding data from position j. Now we denote the logistic-Bernoulli normalizing constant as $Z^l(\boldsymbol{\lambda}_{ie})$, which is defined as

$$Z^l(\boldsymbol{\lambda}_{ie}) = \sum_{\boldsymbol{u}_{ie} \in [0,1]^A} \exp\Big(\sum_{j \neq i} \lambda_{iej} u_{iej}\Big).$$

Now, since

$$\exp\Big(\log\Big(\mathrm{I}(\|\boldsymbol{u}_{ie}\|_1 > 0)\Big) + \sum_{j \neq i} \lambda_{iej} u_{iej}\Big) = \exp\Big(\sum_{j \neq i} \lambda_{iej} u_{iej}\Big),$$

except when $\|\boldsymbol{u}_{ie}\|_1 = 0$, we note that

$$\begin{aligned} Z(\boldsymbol{\lambda}_{ie}) &= Z^l(\boldsymbol{\lambda}_{ie}) - \exp\Big(\sum_{\forall u_{iej} = 0} \lambda_{iej} u_{iej}\Big) \\ &= Z^l(\boldsymbol{\lambda}_{ie}) - 1. \end{aligned}$$

We can therefore derive a closed form expression for $Z(\boldsymbol{\lambda}_{ie})$ via a closed form expression for $Z^l(\boldsymbol{\lambda}_{ie})$. This can be done by looking at the probability of the zero vector under the logistic-Bernoulli model:

$$\frac{1}{Z^l(\boldsymbol{\lambda}_{ie})} \exp\Big(\sum_{\forall u_{iej} = 0} \lambda_{iej} u_{iej}\Big) = \prod_{j \neq i} \Big(1 - \frac{\exp{(\lambda_{iej})}}{\exp{(\lambda_{iej})} + 1}\Big).$$

Then, we have

$$\frac{1}{Z^l(\boldsymbol{\lambda}_{ie})} \quad = \prod_{j \neq i} \frac{1}{\exp(\lambda_{iej}) + 1}.$$

Finally, the closed form expression for the normalizing constant is

$$Z(\boldsymbol{\lambda}_{ie}) = \prod_{j \neq i} \big(\exp(\lambda_{iej}) + 1\big) - 1.$$

Appendix C: Comparison of PPC results: log-normal vs. exponential

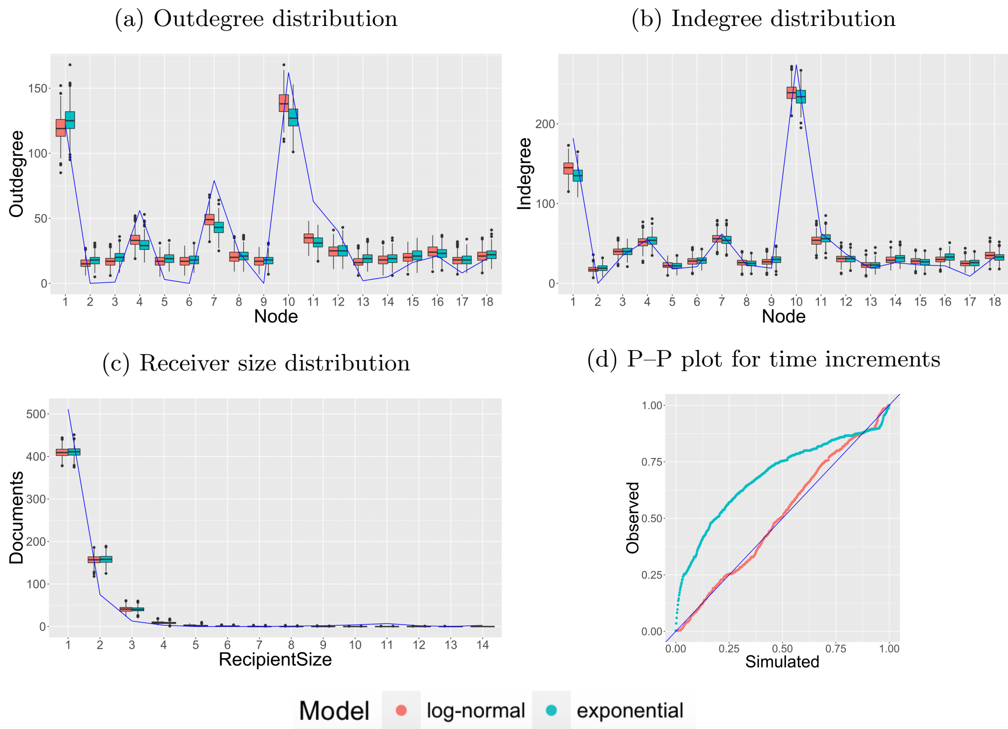

Figure 8: Comparison of PPC results between log-normal (*red*) and exponential (*green*) distributions. Blue lines denote the observed statistics in (a)–(c) and denotes the diagonal line in (d).

Appendix D: Convergence diagnostics

(a) Traceplots of $\boldsymbol{b}$ (b) Traceplot of $\boldsymbol{\eta}$

(c) Geweke diagnostics for $\boldsymbol{b}$ (d) Geweke diagnostics for $\boldsymbol{\eta}$ and σ_τ^2

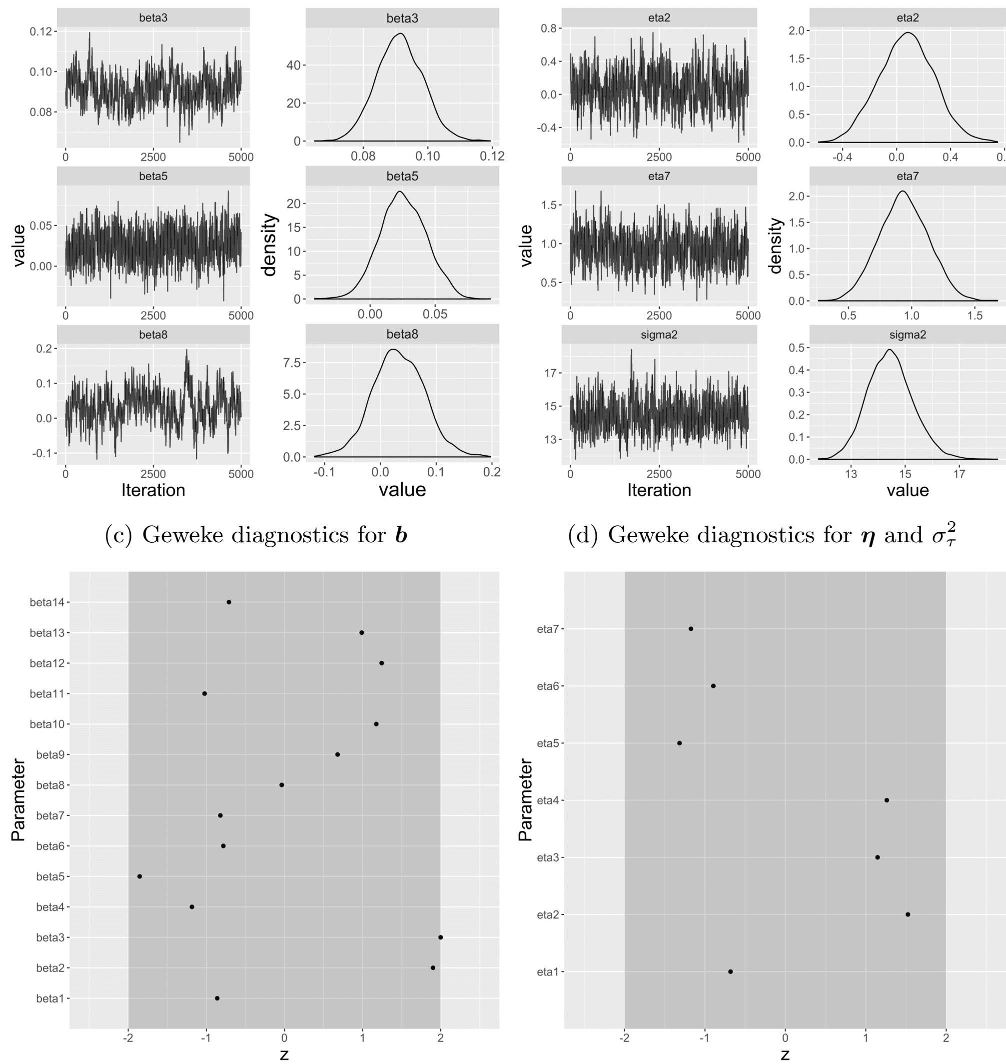

Figure 9: Convergence diagnostics from log-normal distribution.