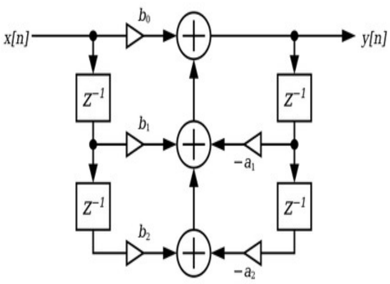
Goal: To determine the time domain response of the transfer function

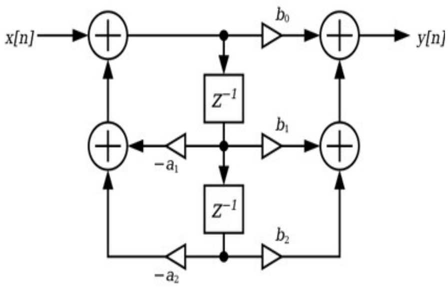
The transfer function for a linear, time-invariant, digital filter can be expressed as a transfer function in the Z-domain; if it is causal, then it has the form:[1]

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$

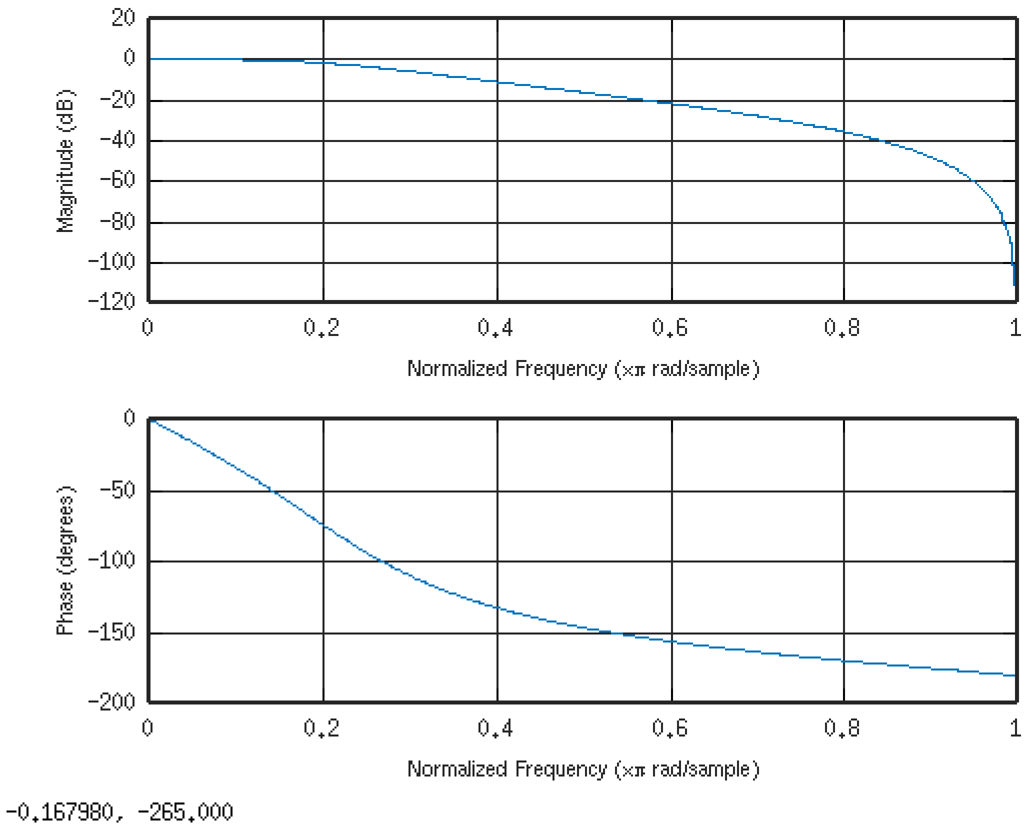
Direct Form 1




Direct Form II

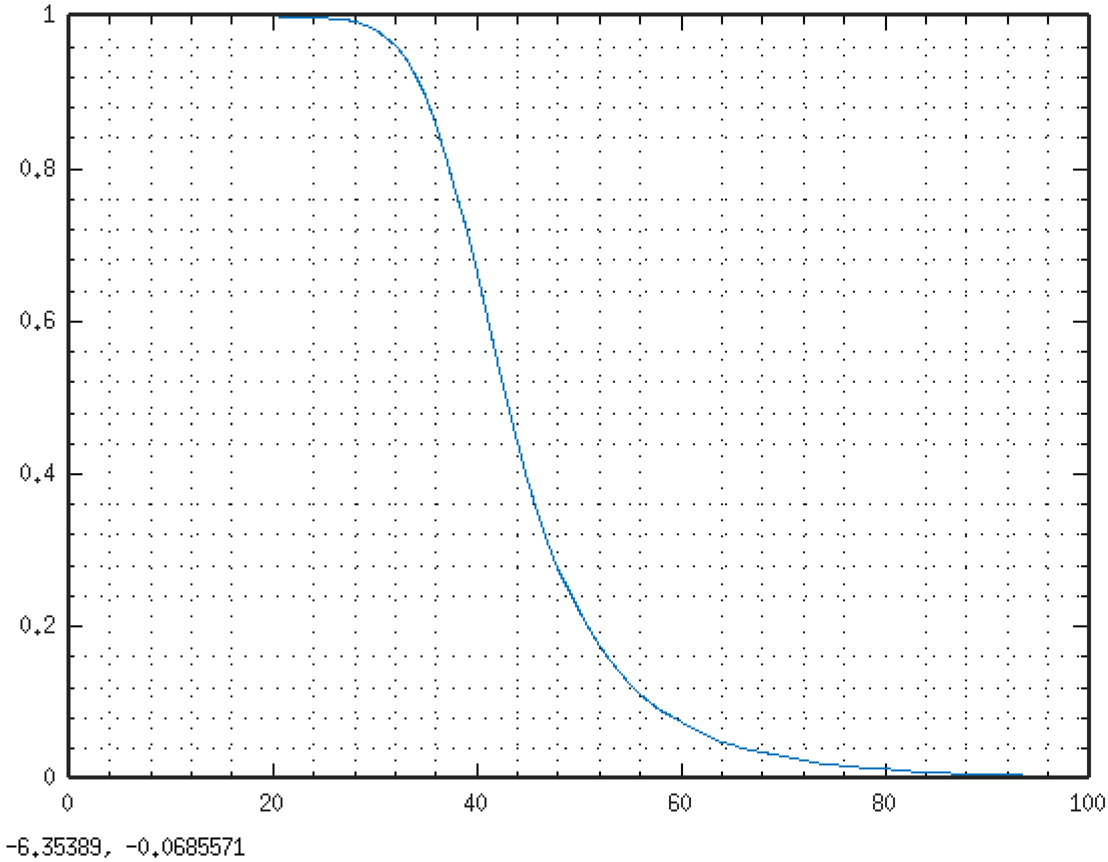



Octave filter




https://www.youtube.com/watch?v=vikFFw6Hn0o
Adding to butttr from video above

```
%|H(w)| = Ao/sqrt(1+(w/wo)2n
ff =  (0:1000-1)*T;
ww = 2*pi*ff;
pow = 2*order;
ww = (ww/nf);
ww = ww(1:1000).^pow;
ww = sqrt(ww + 1);
ww = (ww).^-1;

figure
%subplot(2,1,1)
plot((1:100),ww(1:100))
grid minor on
```



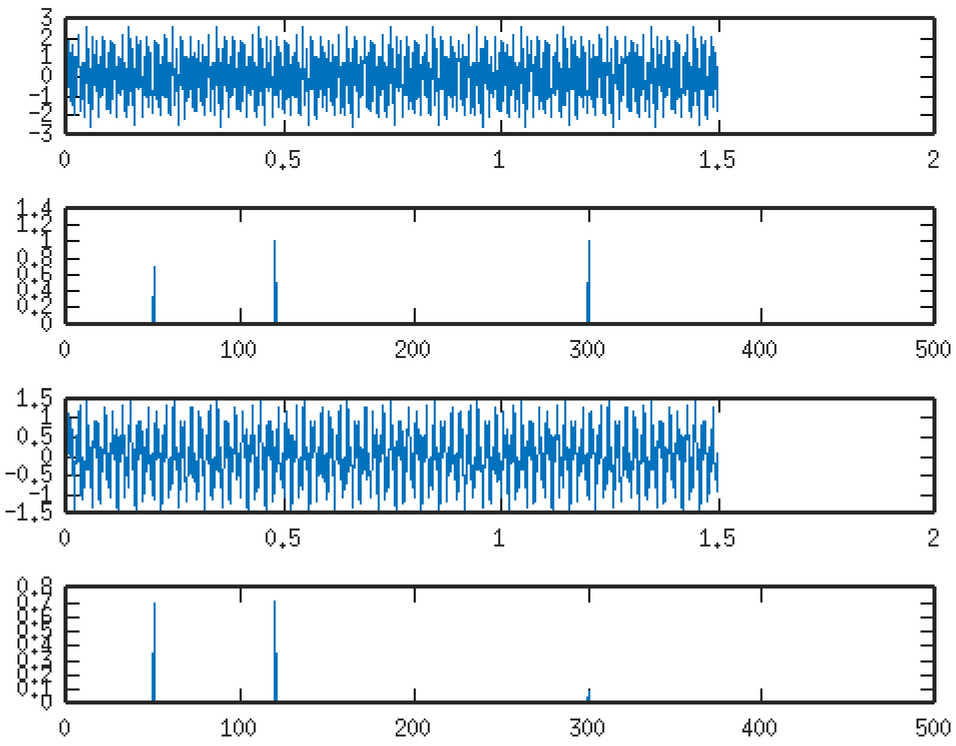

The above is order equal 6

Testcase
The first  signal x is 3 sine waves 50, 120, and 300 Hz

The 2nd is the FFT of the signal x.

The 3rd is the filterd with the Butterworth filter.

The 4th show that only the 50 & 120 Hz are present.




421.525, 4.77541

Starting first with order 2
a = 1.00000 -0.98241 0.34767

b = 0.091315 0.182630 0.091315

[A,B,C,D] = tf2ss(b,a);

A =

```
   5.5511e-17   3.4767e-01
  -1.0000e+00   9.8241e-01
```

B =

```
  -0.059568
   0.272338
```

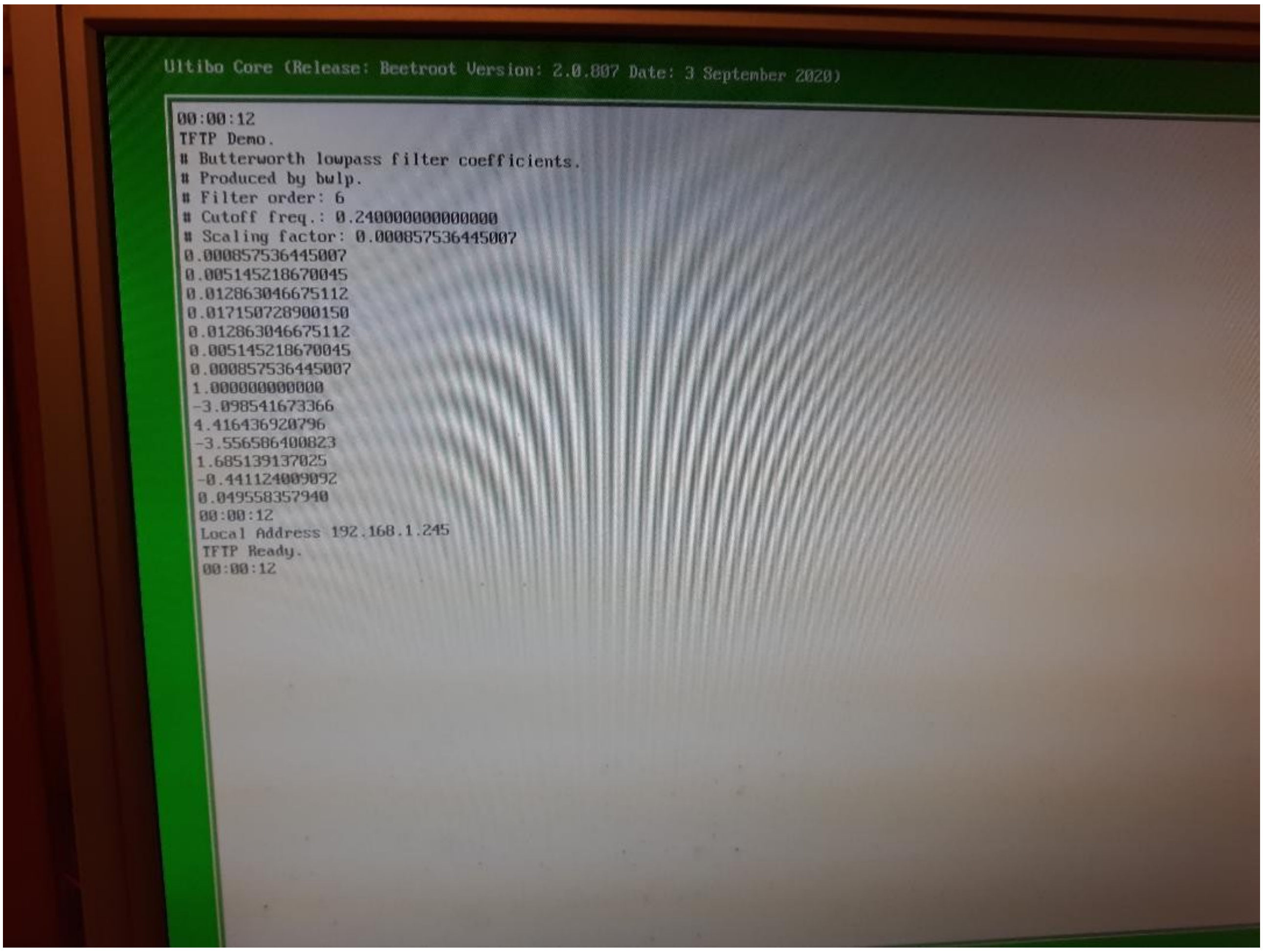
C =

```
   0   1
```

D = 0.091315

butt6120lp
normalize freq
nf = 0.24000
zeros
ans =

```
  -1
  -1
  -1
  -1
  -1
  -1

poles
ans =

   0.61925 + 0.56170i
   0.49120 + 0.32617i
   0.43881 + 0.10665i
   0.43881 - 0.10665i
   0.49120 - 0.32617i
   0.61925 - 0.56170i

theta =

   0.73670
   0.58617
   0.23842
  -0.23842
  -0.58617
  -0.73670

b
b =

 Columns 1 through 6:

   0.00085754   0.00514522   0.01286305   0.01715073   0.01286305   0.00514522

 Column 7:

   0.00085754

a
a =

   1.000000  -3.098542   4.416437  -3.556586   1.685139  -0.441124   0.049558
```

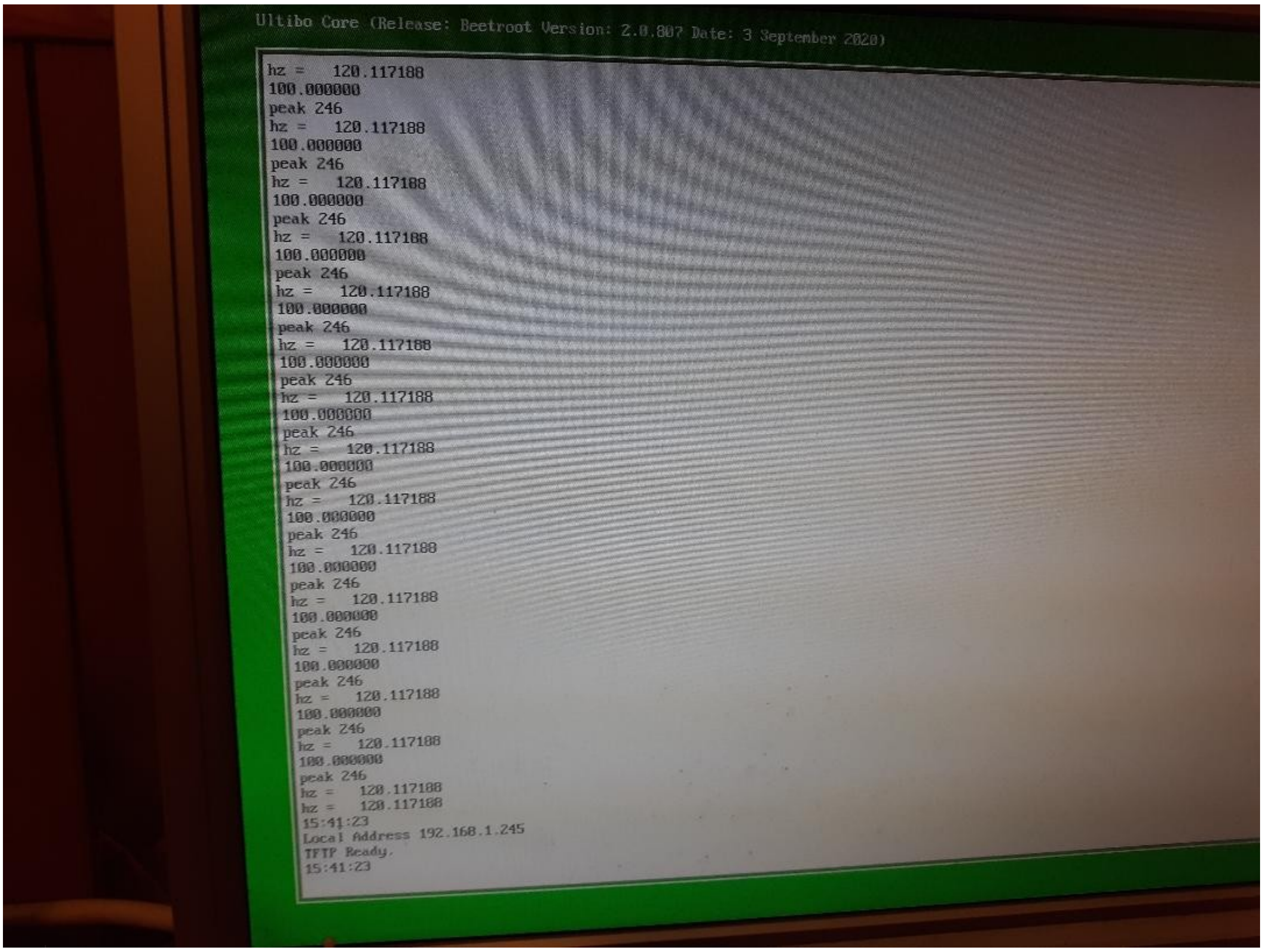



FFT 50 120 300

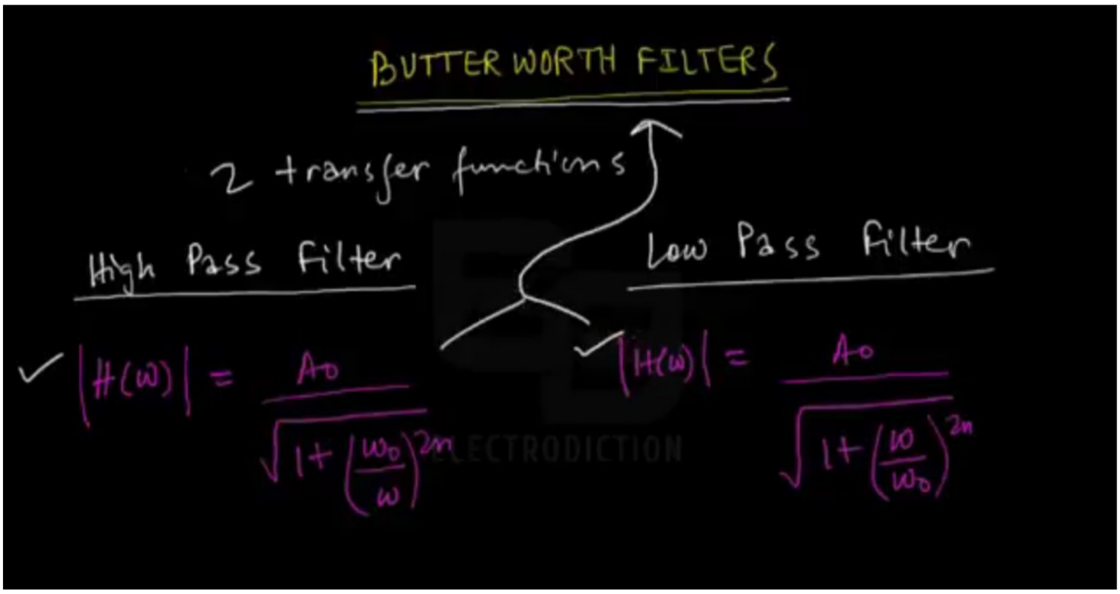
delta f 0.48828125 samples 2048 fs 1000

sample = 10*(sin(2*pi*50*t[i]) + sin(2*pi*120*t[i]) + sin(2*pi*300*t[i])); //no DC

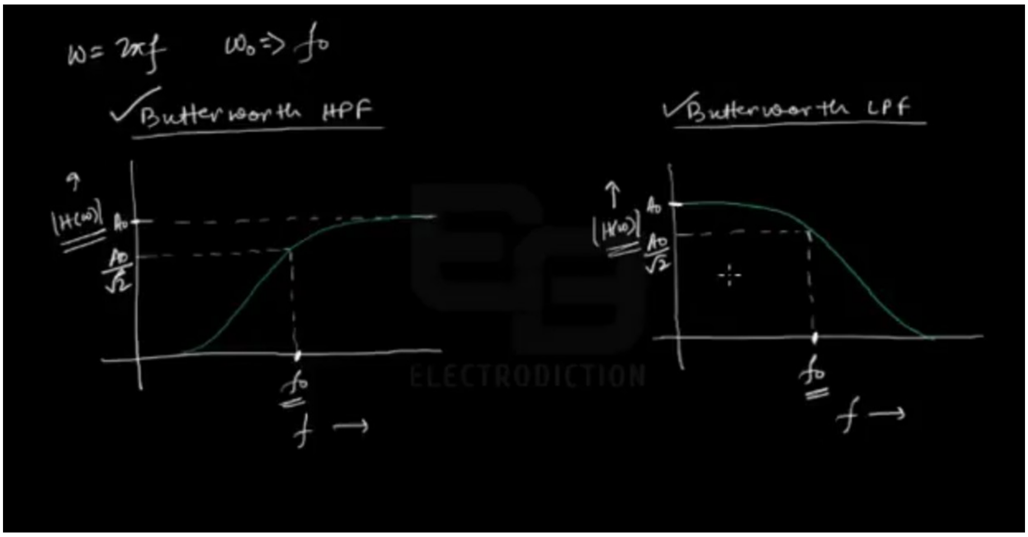
| | | | |
|---|---|---|---|
| ( 2.104689, | 0.000000) | 102 | 49.8046875 |
| ( 7.311564, | 0.000000) | 103 | |
| ( 4.836622, | 0.000000) | 104 | |
| | | | |
| ( 2.750056, | 0.000000) | 246 | 120.1171875 |
| ( 8.715654, | 0.000000) | 247 | |
| ( 1.689716, | 0.000000) | 248 | |
| | | | |
| ( 2.071553, | 0.000000) | 614 | 299.8046875 |
| ( 7.257226, | 0.000000) | 615 | |
| ( 4.845451, | 0.000000) | 616 | |



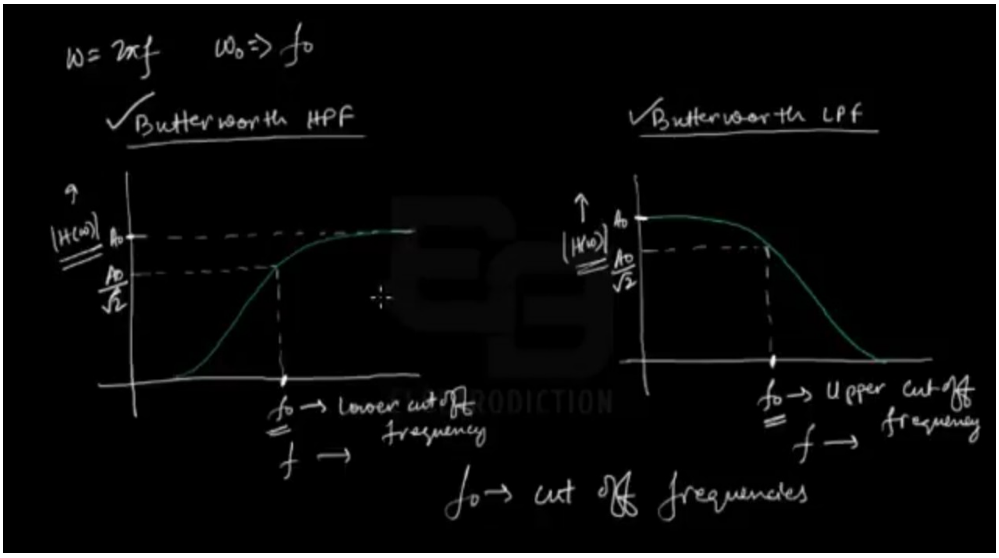

https://www.youtube.com/watch?v=vikFFw6Hn0o



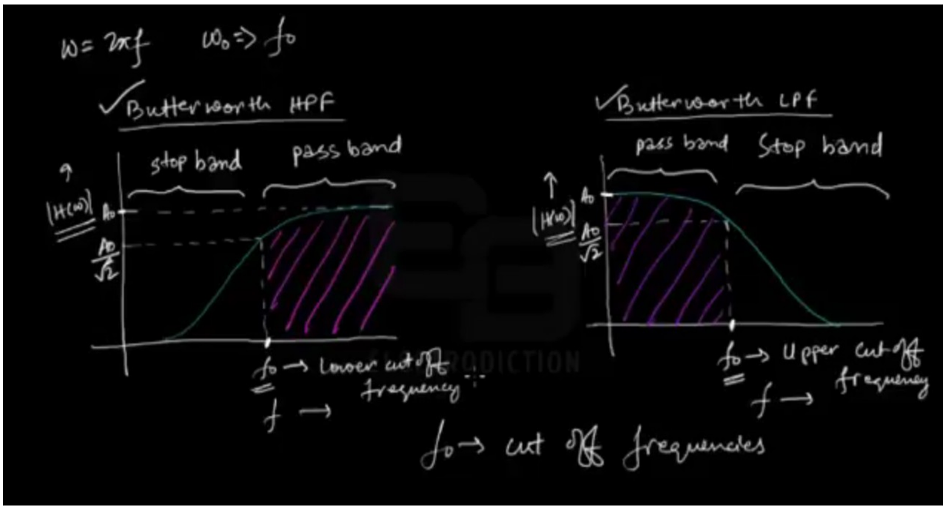

Ao Maximum gain in passbands
|H(w)| normalized gain w = 2pif wo = fo
wo Lower cutoff angular freqency (HPF)
upper cutoff angular freqency (LPF)
angular freqency of input signal
n order(interger 1,2,3…)



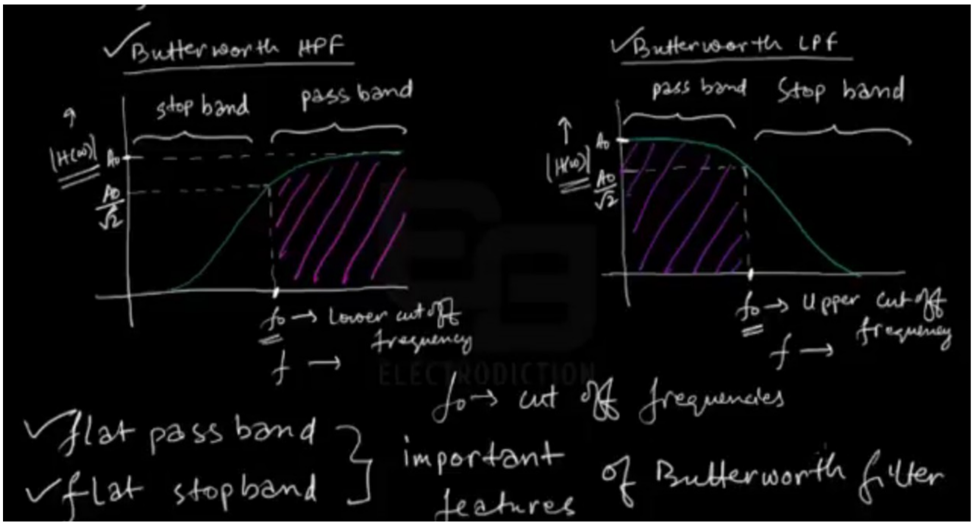

bw2.png




bw3.png

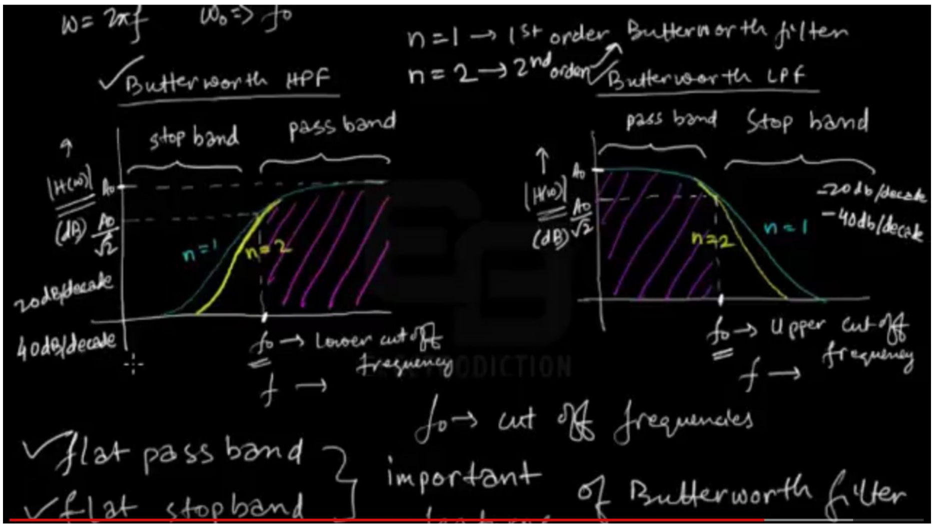



bw4.png



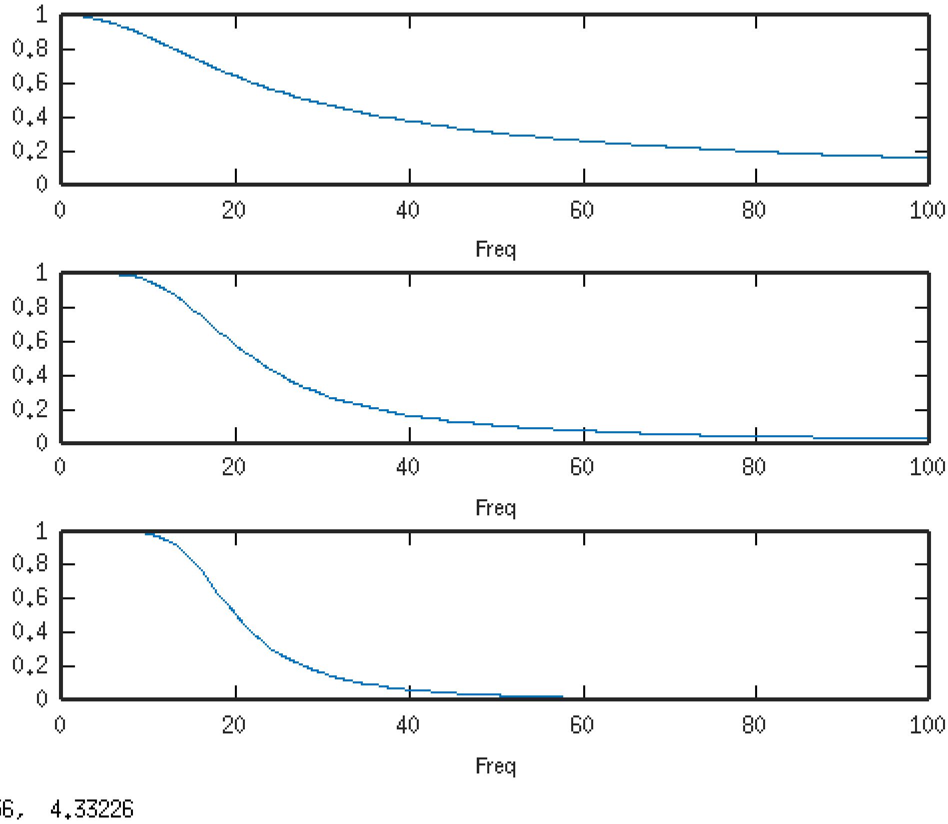
Butterworth HPF
stop band
pass band
|H(ω)|
Ao
Ao/√2
fo → Lower cut off frequency
f →
Butterworth LPF
pass band
Stop band
fo → Upper cut off frequency
f →
fo → cut off frequencies
flat pass band
flat stop band
important features of Butterworth filter


bw5.png

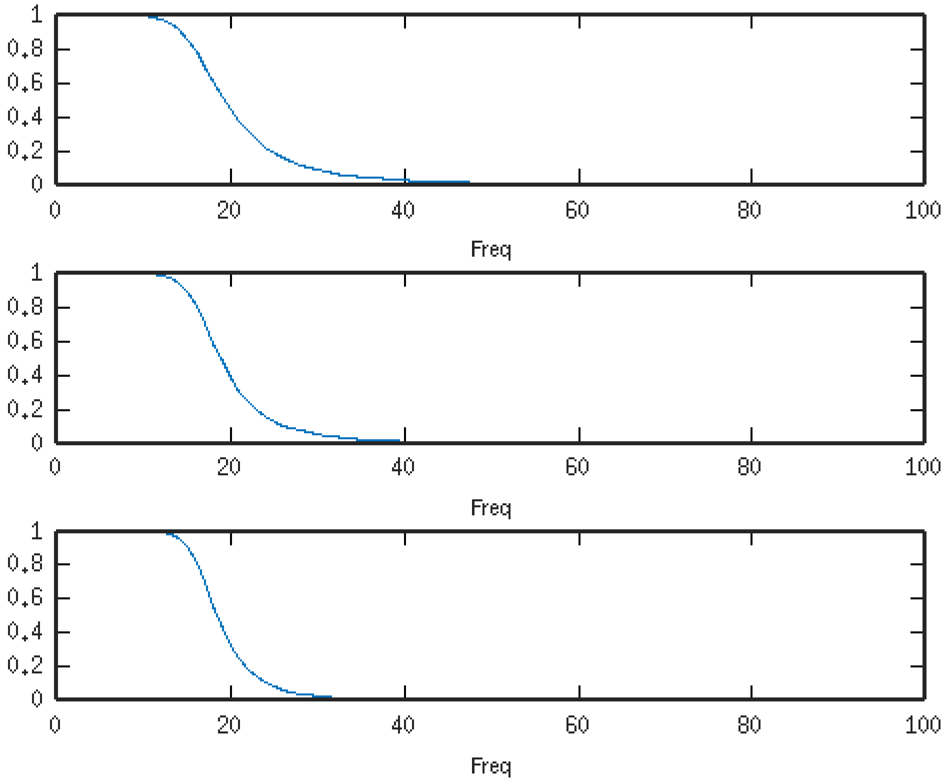


ω = 2πf
ω0 => fo
n = 1 → 1st order Butterworth filter
n = 2 → 2nd order
Butterworth HPF
stop band
pass band
|H(ω)|
(dB)
Ao
Ao/√2
n=1
n=2
20dB/decade
40dB/decade
fo → Lower cut off frequency
f →
Butterworth LPF
pass band
Stop band
-20db/decade
-40db/decade
n=2
n=1
fo → Upper cut off frequency
f →
fo → cut off frequencies
flat pass band
flat stop band
important
of Butterworth filter

For Order 1, 2, & 3 50 Hz

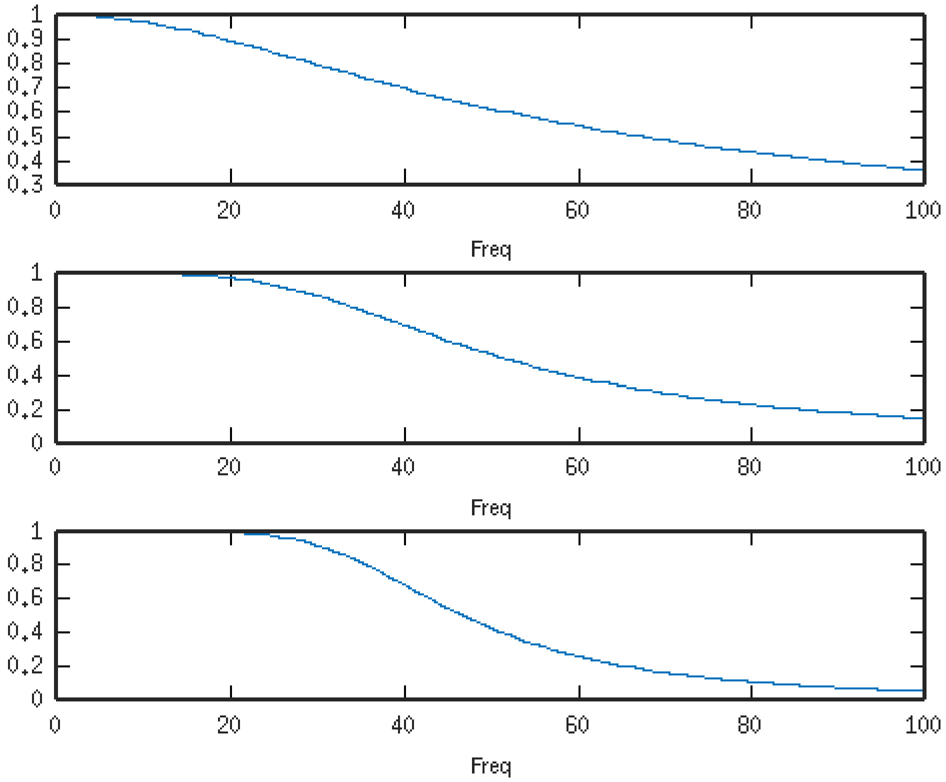



For Order 4, 5, & 6 50 Hz



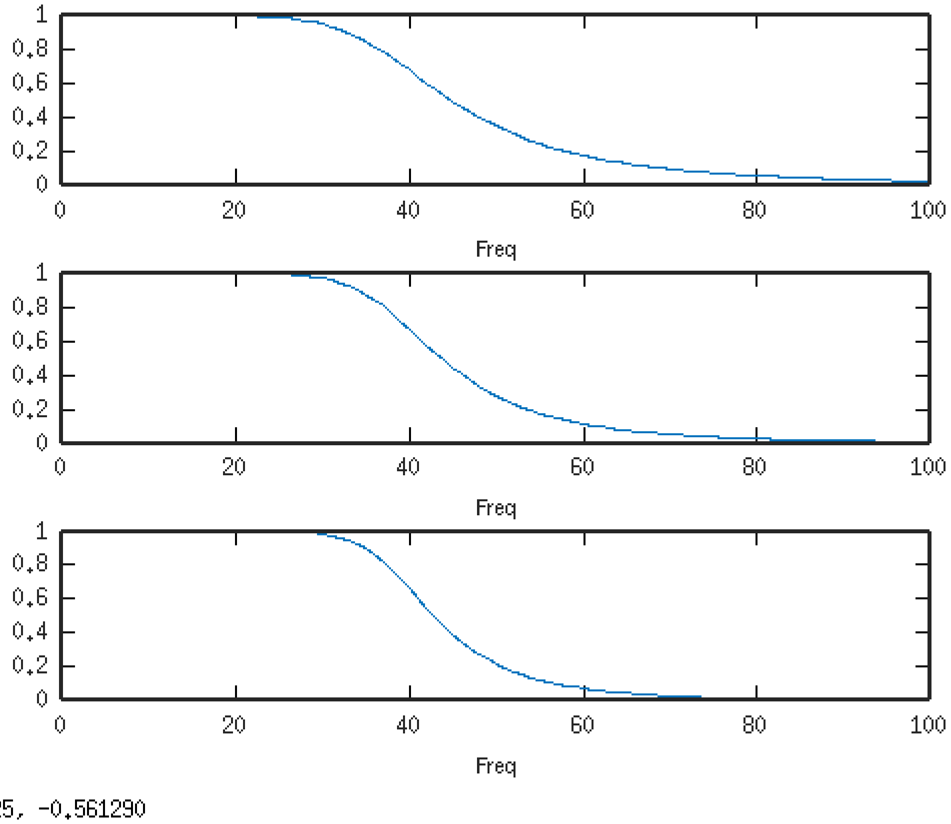

102.049, 0.227419

For Order 1, 2, & 3 120 Hz

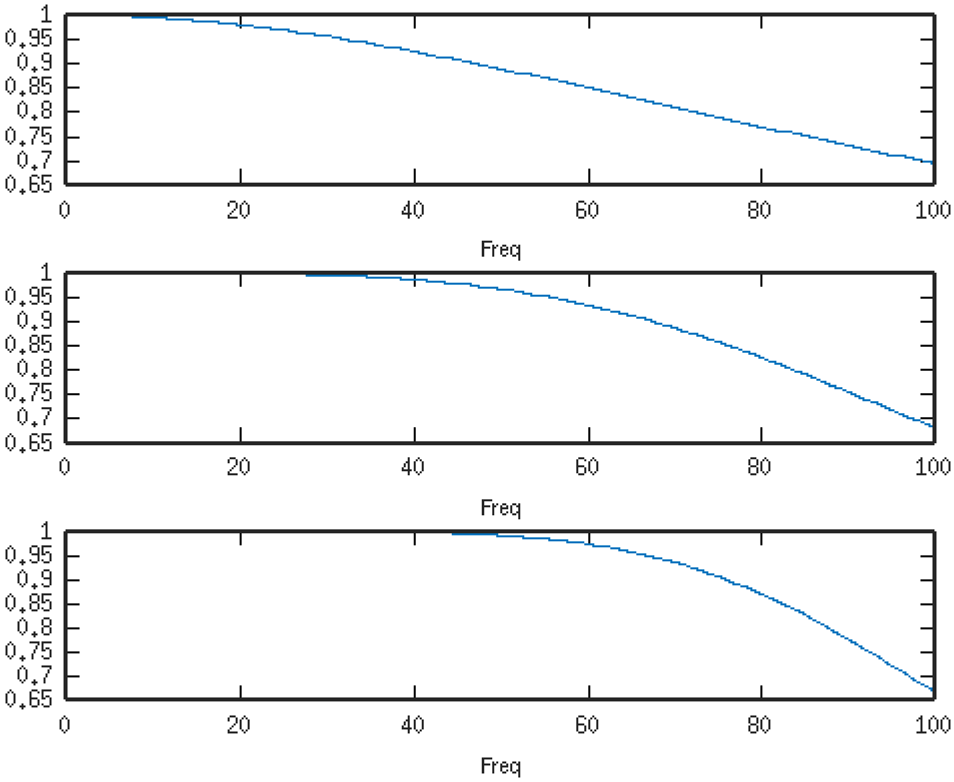



103.908, 1.10000

For Order 4, 5, & 6 120 Hz

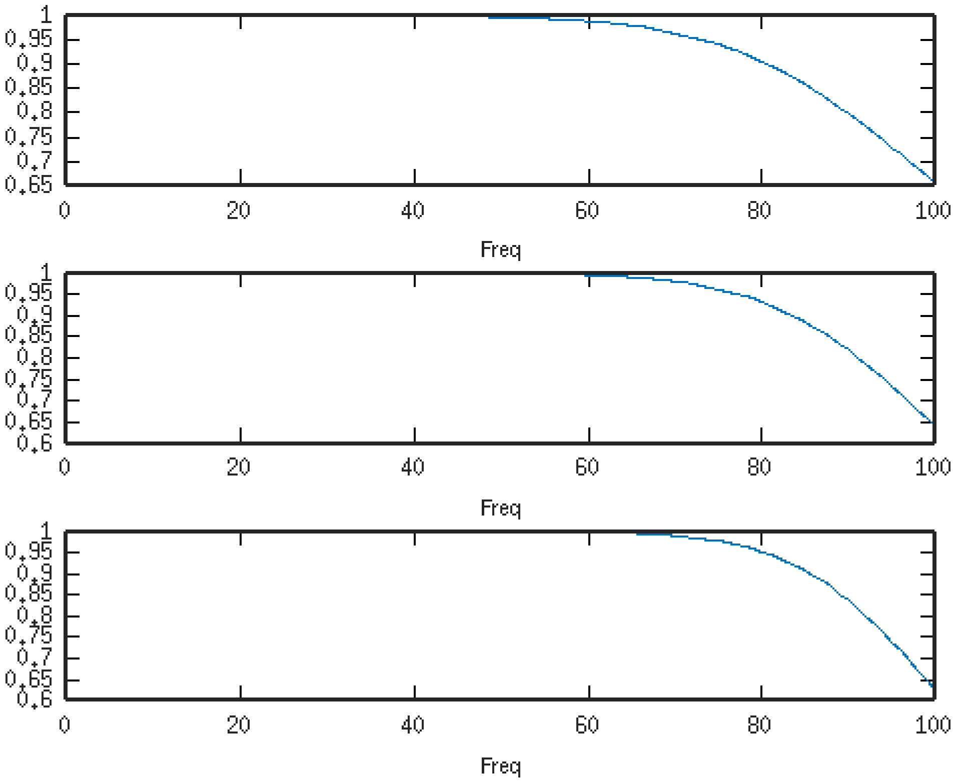



For Order 1, 2, & 3 300 Hz




94.8944, 1.89363

For Order 4, 5, & 6 300 Hz




106.209, 0.430968