TAX EVASION ON A SOCIAL NETWORK

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CONTENT

INTRODUCTION

COMPLIANCE AND REFERENCE DEPENDENCE

→ We relate non compliant behaviour to a body of evidence on the **importance of positional concerns** (keeping up with the Jones)

→ Tax evasion may be used to improve agents' relative standing

→ As a consequence, the choice of **how much to evade is affected by social interaction**

→ New project studying **tax evasion** that builds on a previous TARC project on **tax avoidance**

TAX EVASION - RELEVANCE AND RESEARCH

→ Tax evasion causes **significant losses of public revenues** (4.4 bn. £ in UK)

→ Growing interest by tax agencies on understanding evasion so to **design efficient deterrence measures**

→ **Rich literature** using different approaches to study evasion decision and optimal policies

RELATED LITERATURE

- Kahneman and Tversky 1979
  Reference dependence of utility
- Gali 1994
  "Keeping up with the Jones"
- Myles and Naylor 1996
  Tax evasion and group conformity
- Ballester, Calvo, Zenou 2006
  Network game with local payoff complementarities
- Quah 2007
  Monotone comparative statics on network games

MODELLING FEATURES

Provide a model where:

→ Agents differ in **income**, **reference group** and **probability of detection**

→ Taxpayers may engage in **risky** tax evasion

→ **Self** and **social** comparison shape the **reference income**

→ **Social** comparison depends on agents' **social network**

RESEARCH QUESTIONS

→ Our analysis has focused on **four** questions:

1. Is it possible to characterize **optimal evasion** and how do **changes in the exogenous parameters** (income, risk aversion, etc.) affect it?

2. Is **self comparison** able to replicate the **dynamic profile of the response** of evasion to an effective anti-evasion intervention as observed empirically?

3. Is it possible to characterize the direct and indirect **revenue effects** of interventions?

4. How much does the **availability of more information** (especially related to social network) improves the capacity of a tax authority to **infer revenue effects**?

MODEL

MODELLING OF EVASION

- We define evasion E_{it} as the **liabilities under-declarerd** by taxpayer i at time t
- Evasion is a **risky** activity:
  - The tax agency may detect evasion
  - If evasion is detected, a **fine** f proportional to the evaded tax debt is also imposed

TAXPAYERS CHARACTERISTICS

- Taxpayers are **distinguished** by:
  - Exogenous Income W_i
  - Probability of being audited p_i
    - Probability that taxpayer i is discovered under declaring
    - Probability that the tax agency is successful in prosecuting
  - Who they compare to in the social network: their **reference group**

REFERENCE INCOME

→ Taxpayers determine their reference R_{it} income based on **Social**-related and **Self**-related considerations

→ **Social**:

The (weighted) **average consumption** of taxpayer's **reference group**

→ **Self**:

Their habit consumption $h_{it} = f(C_{it-\mathbf{1}} \ldots C_{it-\mathbf{T}})$

OPTIMAL EVASION

THE TAXPAYER'S PROBLEM

$$\max_{E_i} \mathbb{E}\left(U_{it}\right) \equiv [1-p_i]\, U\left(C_{it}^n - R_{it}\right) + p_i\left[U\left(C_{it}^a - R_{it}\right)\right]$$

After-tax income **if not audited**

$C_{it}^n \equiv X_i + E_{it}$

After-tax income **if audited**

$C_{it}^a \equiv C_{it}^n - (1+f)E_{it}$

Utility is linear-quadratic

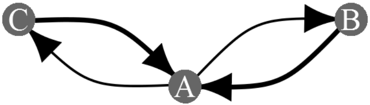
$U(z) = z[b - \frac{az}{2}]$

Optimal Evasion at an interior solution is:

$$E_{it}^* = \frac{1-p_i f}{a\zeta_i}\{a[R_{it} - X_i] + b\}, \zeta_i > 0$$

A SIMPLE EXAMPLE

Taxpayer interaction through the reference income leads to the rise of a game



$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0 & .5 & .5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \equiv \mathbf{G}$$

$$\begin{cases} E_A^* & = \frac{1-p_i f}{a\zeta_A}\{a[R_A(h_A; E_B^*, E_C^*) - X_A] + b\} \\ E_B^* & = \frac{1-p_i f}{a\zeta_B}\{a[R_B(h_B; E_A^*) - X_B] + b\} \\ E_C^* & = \frac{1-p_i f}{a\zeta_C}\{a[R_C(h_C; E_A^*) - X_C] + b\} \end{cases}$$

REFERENCE DEPENDENCE

Taxpayer i expected after-tax income when evading E_{it} is:

$$q_{it} = X_i + [1 - p_i f] E_{it}$$

We can then define:

$$Z_{it} = \iota_h h_{it} + \iota_s \mathbf{g}_i \mathbf{q}_t$$

And reference income:

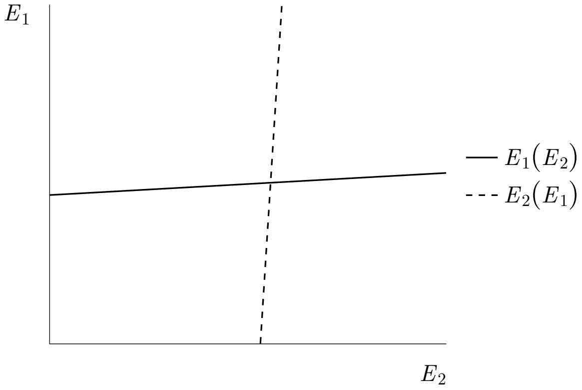
$$R_{it} = R_{it}(h_{it}; \mathbf{q}_t(\mathbf{E}_t)) = R_{i,t-1} + \varsigma_R \left[Z_{it} - R_{i,t-1} \right]$$

where:

$X_i = (1-t) W_i$	Honest after-tax income
ι_h, ι_s	Self and social comparison parameters
$\mathbf{g}_i$	Weights of i's reference group
$\varsigma_R \in (0,1)$	Reference consumption reactiveness

BEST RESPONSE

Quadratic utility leads to linear best response



Positive slope of best response functions follows from strategic complementarity in E_{it}, E_{jt}

WEIGHTED BONACICH CENTRALITY AND EVASION

Expanding E^*_{it} using the definitions of R_{it}, Z_{it} and q_{it} we can rewrite:

$$\begin{cases} E^*_A & = & \eta_i\{a[R_A(h_A; E^*_B, E^*_C) - X_A] + b\} \\ E^*_B & = & \eta_i\{a[R_B(h_B; E^*_A) - X_B] + b\} \\ E^*_C & = & \eta_i\{a[R_C(h_C; E^*_B) - X_C] + b\} \end{cases}$$

$$\mathbf{E}_t = \boldsymbol{\alpha}_t + \mathbf{M}\beta\mathbf{E}$$

And solve à la **Cournot-Nash**:

$$\mathbf{E}_t = [\mathbf{I} - \mathbf{M}\boldsymbol{\beta}]^{-1}\boldsymbol{\alpha}_t = b(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}_t)$$

Where $b(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}_t)$ is the weighted Bonacich centrality measure:

OPTIMAL EVASION

- Key theoretical result is that **evasion is closely related to the concept of “Bonacich” Network Centrality**
  - More “central” taxpayers evade more
- Network centrality is a concept developed in sociology
  - Measures the amount of influence/power players have within a network

COROLLARIES

Corollary 1

If the probability of audit is equal among taxpayers, i.e. $p_i = p$, then:

$$\mathbf{E_t} = b(\mathbf{M}, \boldsymbol{\omega}, \boldsymbol{\alpha}_t)$$

Where:

$$\omega_{ii} = \frac{\iota_s \varsigma_R [1-pf]^2}{\zeta}$$

Corollary 2

In a steady state of the model consumption satisfies $\mathbf{C}^{SS} = \mathbf{C}^{n,SS} = \mathbf{X} + \mathbf{E}^{SS}$.

Steady state evasion $\mathbf{E}^{SS}$, is then given by the vector of Bonacich centralities, $\mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}^{SS})$, with

$$\alpha_i^{SS} = \frac{1 - p_i f}{a\zeta_i} \left\{ b - a \left[X_i - R\left(h_i^{SS}, \mathbf{X} \right) \right] \right\}$$

MONOTONE COMPARATIVE STATICS IN TIME

A marginal parameter change entails contemporaneous and delayed effects on the steady state of the model:

1. The contemporaneous effect $\frac{\partial E_i^{SS}}{\partial z}$ is not accounting for delayed effects
2. The full effect $\frac{dE_i^{SS}}{dz}$ includes also the delayed effect caused by adjustments of habit consumption

Lemma 1

$$\text{if} \quad \frac{\partial X_i}{\partial z}\frac{\partial E_i^{SS}}{\partial z} \geq 0 \quad \text{then} \quad sign\left(\frac{dE_i^{SS}}{dz}\right) = sign\left(\frac{\partial E_i^{SS}}{\partial z}\right)$$

It is sufficient to have same sign for $\partial E_i^{SS}/\partial z$, and steady state consumption, $\partial C_i^{SS}/\partial z$

MONOTONE COMPARATIVE STATICS RESULTS

Habit consumption	$+$	Other's Income	$+/0$
Own comparison	$+$	Social comparison	$+/0$
Own audit prob.	$-$	Others audit prob.	$-/0$
Risk Aversion	$-$	Tax rate	$+$
Fine	$-$		

Monotone comparative statics for interior E_i^*

These results apply both to contemporaneous and full effects

EVASION AND INCOME

→ In the case of **income, contemporary and delayed effects have opposite signs**

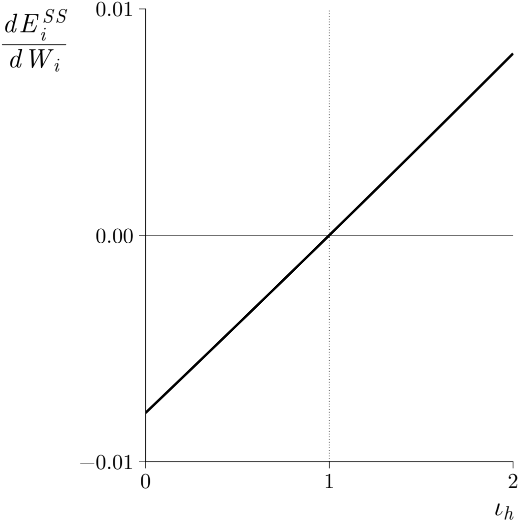
→ The **contemporaneous effect causes evasion to fall** due to the increased income, i.e. $\frac{\partial E_i^{SS}}{\partial X_i} < 0$

→ However, **the delayed effect causes an increase in habit consumption** $\frac{dC_i^{SS}}{dX_i} < 0$ that as a positive effect on evasion.

This allows our model to replicate the observed behaviour of evasion increasing in income $\frac{dE_i^{SS}}{dX_i} > 0$

EVASION VS. CONCERN FOR HABIT

The higher a taxpayer's concern for habit i_h the more evasion increases in income



RESPONSES TO INTERVENTION

DYNAMIC RESPONSE TO LEGAL INTERVENTION

→ Empirical evidence shows a **persistent dynamic behavioural response** to interventions

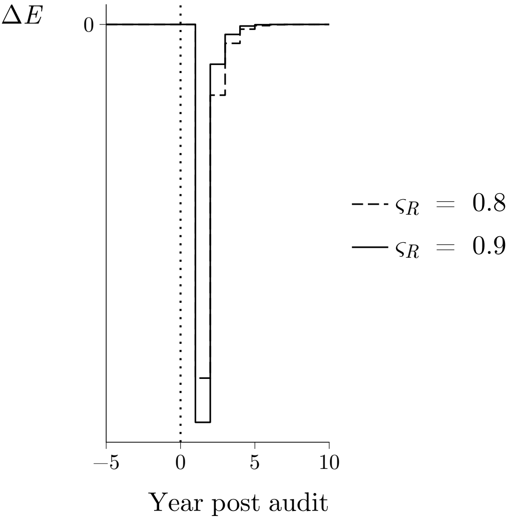
→ The literature argued that **belief updating may be driving** this evidence

→ We show that **self-comparison is able to replicate the same dynamic**

→ Calibrating the persistence ς_R it is possible to **closely match the behaviour observed in reality**

RESPONSE TO LEGAL INTERVENTION VS. PERSISTENCE

- Here periods interpreted as years
- **Deterrence is maximal after the intervention and slowly fades**
- With high levels of persistence the dynamic behavioural response lasts ≈ 4 years



INTERVENTION REVENUE EFFECTS

How does an audit to a taxpayer affect the steady-state evasion of the model?

1. **Direct effect $\mathbf{E}_i^{SS}$**

   On targeted taxpayer, by **averting attempted evasion**
2. **Indirect effects I_{ij}**

   Expected additional revenue that arises **from future changes in evasion behaviour** (**negative externality**)

   - → I_{ii} from the audited tapayer
   - → I_{ij} from non-audited taxpayers
     - → $\boldsymbol{\Sigma_i} = \sum_{j \in \mathcal{N} \setminus i} I_{ij}$ **aggregate cross indirect effect**

→ Indirect effects **2X-6X** direct ones

TAX AGENCY'S INFERENCE PROBLEM

- Tax authorities engage in inferring both **direct effects** $\mathbf{E}^{SS}$ and **aggregate gross indirect effects** $\mathbf{\Sigma}$
  - Taxpayers usually ranked by discriminant function and audited sequentially until budget is exhausted
- Crucial information for tax authorities is correct rank of $\mathbf{E}^{SS}$ and $\mathbf{\Sigma}$
  - Optimal audit targeting if tax authorities were able to exactly infer **rankings** of direct and indirect effects.

Tax authorities require measures that are ordinally equivalent to direct and indirect effects

$$\mathbf{A} \sim \mathbf{B} \iff A_{i1} \gtrless A_{j1} \Leftrightarrow B_{i1} \gtrless B_{j1} \forall i, j$$

MEASURES ORDINALLY EQUIVALENT TO REVENUE EFFECTS

The indirect revenue effects of conducting a single audit of i satisfy:

$$\mathbf{I}_i \sim Diag[\mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}^{SS})]\mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})$$

where $\mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}^{SS}) \equiv \mathbf{E}_i^{SS}$ and $\boldsymbol{\rho}_i^{SS} = \frac{\partial \boldsymbol{\alpha}^{SS}}{\partial C_i^{SS}}$

Sizes of the **own** and **cross indirect** effects are **ordinally equivalent** to the product of the steady state level of evasion and a new measure of **Bonacich centrality**

INFERENCE OF REVENUE EFFECTS

- When there is full observability $\mathcal{F}$ it is possible to exactly determine direct ($\mathbf{E}^{SS}$) and cross indirect ($\mathbf{\Sigma}$) effects
- Tax agencies infer revenue effects under **limited observability**

How valuable is **network information**?

- Two cases considered:
  1. **Partial observability** ($\mathcal{P}$): The tax agency observes the reference groups of taxpayers but has no information on the comparison intensity
  2. **No observability** ($\emptyset$): Everybody attaches equal importance to all the other taxpayers
- We assess the role of network information in prediction using a the **Spearman rank correlation coefficient**, i.e. $\rho^S_{\mathbf{E}^{\mathcal{P}},\mathbf{E}^{\emptyset}}$

NETWORK STRUCTURE

- We generate a static network using the Bianconi-Barabási **fitness** model
  - Taxpayers with **higher wealth** have a higher probability of making new connections
  - Taxpayers already **heavily connected** have a higher probability of making new connections (sublinear preferential attachment, $\phi < 1$)

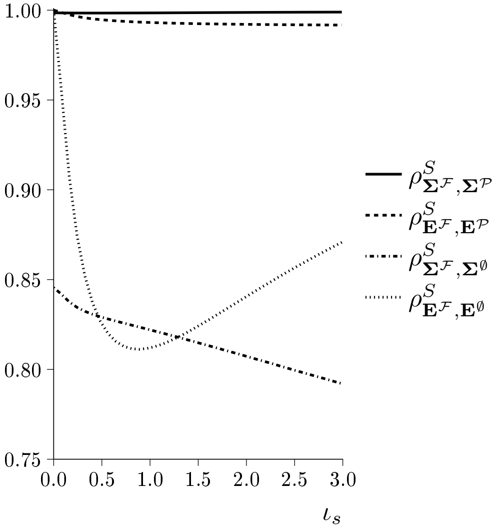
Formally:

$$\Pi_i = \frac{W_i[d^{in}(i)]^{\phi}}{\sum_{j \in \mathcal{N}} W_j[d^{in}(j)]^{\phi}}$$

The resulting **static** social networks used in our simulations resembles the ones observed empirically

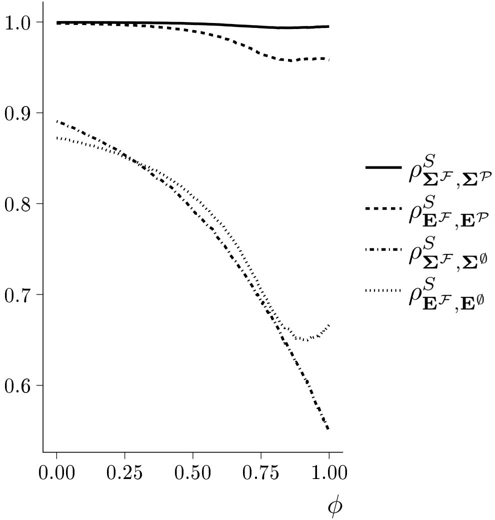
INFERENCE ACCURACY AND SOCIAL COMPARISON

- **Accuracy improves** significantly from the **no network observability $\emptyset$ to partial observability $\mathcal{P}$**
- Higher concern for social comparison ι_s decrease accuracy



INFERENCE ACCURACY AND PREFERENTIAL ATTACHMENT

- **Accuracy improves** significantly from the **no network observability** $\emptyset$ **to partial observability** $\mathcal{P}$
- Stronger preferential attachment ϕ decreases accuracy



CONCLUSIONS

CONCLUDING REMARKS

→ Social interaction may heavily affect evasion behaviour

→ **Self comparison** is able to **replicate** the persistent dynamic **response of evasion** to intervention observed empirically

→ Different **Bonacich** measures of centrality characterize optimal **evasion** and **revenues effects** from auditing

→ **Social network information** improve significantly the **prediction** of revenues effects from interventions

FURTHER RESEARCH

→ Quantify the additional **revenue recovered using network information**

→ Extend the analysis to **crime** as a whole

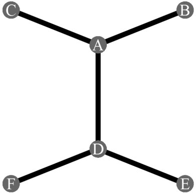
→ Analyse how adding or **removing taxpayers (detention)** may affect compliance

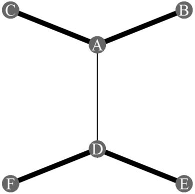
Thank You!

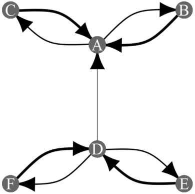
Questions?

SOCIAL NETWORK AND MATRIX REPRESENTATION

Undirected Network	Weighted Network	Directed Network







$$\begin{array}{c c} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

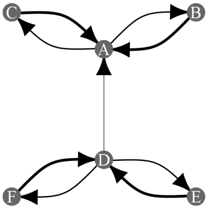
$$\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & .2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & .5 & .5 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & .4 & .4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

SOCIAL NETWORK AS AN ADJACENCY MATRIX

Matrix form of a **weighted directed** network

Directed Network



$$\begin{array}{c c} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & .5 & .5 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & .4 & .4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

ACCOUNTING FOR SOCIAL NETWORK

Expanding E^*_{it} using the definitions of R_{it}, Z_{it} and q_{it} we solve à la **Cournot-Nash**:

$$E_{it} = \alpha_{it} + \varsigma_R \iota_s \sum_{j \neq i} m_{ij} E_{jt} =$$

$$\mathbf{E} = \boldsymbol{\alpha}_t + \mathbf{M} \boldsymbol{\beta} \mathbf{E}$$

Where:

$$m_{ij} = \frac{[1 - p_i f][1 - p_j f]}{\zeta_i} g_{ij}$$

$$\beta_{ii} = \varsigma_R \iota_s$$

$$\alpha_{i1,t} = \frac{1 - p_i f}{a \zeta_i} \{b - a[X_i - R(h_{it}, \mathbf{X})]\}$$

WEIGHTED BONACICH CENTRALITY AND EVASION

Expanding E^*_{it} using the definitions of R_{it}, Z_{it} and q_{it} we can rewrite we solve à la **Cournot-Nash**:

$$\mathbf{E_t} = [\mathbf{I} - \mathbf{M}\boldsymbol{\beta}]^{-1}\boldsymbol{\alpha}_t = b(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}_t)$$

$b(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}_t)$ is the weighted Bonacich centrality defined on:

$\mathbf{M}$	Edge weights scaled by relative ER of E_i
$\boldsymbol{\beta}$	Scales weight of longer paths
$\boldsymbol{\alpha}_t$	Weights centrality by agent characteristics

GENERALIZATION OF OPTIMAL EVASION RESULT

For an **arbitrary** twice differentiable **utility function** considering the FO linear approximation around a Nash equilibrium to the set of FOC, it is:

$$\mathbf{E}_t = \mathbf{J}\mathbf{E}_t + \widehat{\boldsymbol{\alpha}}_t = [\mathbf{I} - \mathbf{J}]^{-1}\,\widehat{\boldsymbol{\alpha}}_t = \left[\sum_{k=0}^{\infty} \mathbf{J}^k\right]\widehat{\boldsymbol{\alpha}}_t$$

Where $\mathbf{J}$ is a matrix of coefficients measuring actions' interactions

A solution is a again in the form of a

weighted Bonacich centrality measure

TAXPAYERS' INTERACTION AS A GAME

The game arising from taxpayers interaction is:

Smooth Supermodular Game (Milgrom and Roberts 1990)

Bounds on strategies	$E_{it} \in (0, tW_i)$
Differentiability	$\mathbb{E}[U_i]$ is of class C^2
Strategic Complements	$\frac{\partial^2 \mathbb{E}[U]_i}{\partial E_{it} \partial E_{jt}} \geq 0$

MONOTONE COMPARATIVE STATICS

Smooth Supermodular Games can be analyzed using **Monotone comparative statics**

Following Quah (2007) we exploit the **weaker** condition of **local supermodularity** around the Nash equilibrium point:

Then, for a given parameter z, it holds:

$$\left.\frac{\partial^2 \mathbb{E}[U]_i}{\partial E_i \partial z}\right|_{E_i=E_i^*} \geq 0 \Leftrightarrow \frac{\partial E_i^*}{\partial z} \begin{cases} > 0 \text{ if } \left.\frac{\partial^2 \mathbb{E}[U]_i}{\partial E_i \partial z}\right|_{E_i=E_i^*} > 0 \\ \geq 0 \text{ if } \left.\frac{\partial^2 \mathbb{E}[U]_i}{\partial E_i \partial z}\right|_{E_i=E_i^*} = 0 \end{cases}$$

MONOTONE COMPARATIVE STATICS

	E_i^*		E_i^*
h_{it}	$+$	X_j	$+/0$
ι_h	$+$	ι_s	$+/0$
p_i	$-$	p_j	$-/0$
f	$-$	t	$+$
a	$-$	b	$+$

Monotone comparative statics for interior E_i^*

These results apply in the short and long run

MEASURES ORDINALLY EQUIVALENT TO REVENUE EFFECTS

Understanding why:

$$\mathbf{I}_i \sim \mathbf{E}_i^{SS}\mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})$$

- The size of the indirect effect I_{ij} is ordinally equivalent to the size of the initial deviation
  - convergence of evasion back to its steady state value is at a uniform rate for all affected taxpayers
- Initial effect can be decomposed linearly as the product of:
  - marginal effect of a change in i's consumption on j's evasion $\partial E_j^{SS}/\partial C_i^{SS} = b_{j1}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})$
  - change in i's consumption $b_{j1}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})E_i^{SS}$

Corollary 3

$$\boldsymbol{\Sigma} \sim \boldsymbol{\chi} \text{ where } \chi_{i1} = \textstyle\sum_{k\in\mathcal{N}} b_{k1}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})E_i^{SS}$$