Tax Evasion on a Social Network

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Introduction

- Tax evasion causes **significant losses of public revenues** (£4.4 bn. in UK)

- Growing interest among tax authorities in **how social attitudes to tax evasion are formed**

- "Big data" information systems potentially allow tax authorities to **perceive social networks to an unprecedented degree**

- Predictive tools find **patterns in data arising due to the determinants of subjects' decisions**

- We investigate the **impact of social network on tax evasion decisions** and develop a framework to **asses the value of social network data**
  - Is it worthwhile for a tax authority to invest in this technology?

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Literature

- **Standard model** of tax evasion treats it as a **private decision**
- **More recent** work allows for **social interactions** to affect compliance (Myles and Naylor, 1996 ; Hashimzade ***et al.***, 2014 ; Goerke, 2013)

Limitations of Existing Literature

- Taxpayers typically assumed to know aggregate-level statistics
- Implicitly **presupposes** that the **network is the complete** network
  - but taxpayers may rely on **heterogeneous "local" information**
  - Also ruling out **heterogeneity in social connectedness**
- Other papers relax the complete network, but maintain other rigidities, i.e., **fixed pattern of connectivity**, **undirected network**

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Contribution

- The **social networks so far used** in the literature seem to **deviate importantly from real-world networks**
- We study a model allowing for an **arbitrary network**
- **Local relative consumption externalities**, heterogeneous across taxpayers
- Theoretical underpinnings to **network equilibria**

Research Questions

Our analysis has focused on **two** questions:

1. Is it possible to characterize **optimal evasion** in presence of relative utility and how do **social interactions** affect it?
2. How much does the **availability of more information** (especially related to social network) improves the capacity of a tax authority to **infer audit revenue effects**?

Preliminaries

- A taxpayer $\imath$ has true income $W_\imath$ on which they **should pay tax** $\theta\left(W_\imath\right)$.
- Taxpayer **may choose to evade** an amount of tax $E_\imath \in \left(0, \theta\left(W_\imath\right)\right)$
- Evasion is a **risky** activity:
  - The **tax agency** is actively seeking to detect and **shut-down** evasion
  - There is a compound probability $p_\imath$ that:
    - **The taxpayer is discovered** under declaring
    - **The tax agency is successful** in shutting down evasion
- The tax authoritiy levies a **fine** $f > 1$ proportional to the evaded tax debt upon successful action
- Taxpayers care about **relative utility**
  - they benchmark consumption against a reference level R

The taxpayer's problem

$$\max_{E_\imath} \mathbb{E}(U_\imath) \equiv [1 - p_\imath] U(C_\imath^n - R_\imath) + p_\imath [U(C_\imath^a - R_\imath)]$$

After-tax income ***if not audited***

$C_\imath^n \equiv X_\imath + E_\imath$

After-tax income ***if audited***

$C_\imath^a \equiv C_\imath^n - fE_\imath$

Utility is linear-quadratic

$U(z) = z[b_\imath - \frac{a_\imath z}{2}]$

The **Privately Optimal Evasion** at an interior solution is:

$$E_\imath = \frac{1 - p_\imath f}{a_\imath \zeta_\imath}\{b_\imath - a_\imath[X_\imath - R_\imath]\}$$

$\zeta_\imath = [1 - p_\imath f]^2 + p_\imath [1 - p_\imath] f^2 > 0$

Endogenising Reference Consumption

- **Observability of consumption** summarised by a **directed network** (graph), where a link (edge) from taxpayer (node) $\imath$ to taxpayer $\jmath$ indicates that $\imath$ observes $\jmath$'s consumption

- Links are **subjectively weighted**
  - some members of the reference group may be more focal comparators

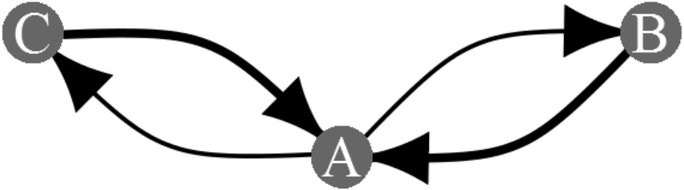
- **Network** of links is represented as an $N \times N$ (adjacency) matrix, $\boldsymbol{G}$, **of subjective comparison intensity weights** $g_{\imath\jmath} \in [0, 1]$,

- The weights satisfy

$$g_{\imath\imath} = 0; \qquad \sum_{\jmath \in \mathcal{R}_\imath} g_{\imath\jmath} = 1$$

- The **set of taxpayers** whose consumption is **observed** by taxpayer $\imath$ is termed $\imath$'s **reference group**, $\mathcal{R}_\imath$

An Example



$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0 & .5 & .5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \equiv \boldsymbol{G}$$

Endogenising Reference Consumption

- Reference consumption taken as the **weighted average of expected consumption** over the members **of the taxpayer reference group** $\mathcal{R}$

$$R_\imath = \sum_{j \in \mathcal{R}_\imath} g_{\imath\jmath} \mathbb{E}\left(\tilde{C}_\jmath\right)$$

Where:

$$\begin{aligned} \mathbb{E}\left(\tilde{C}_\jmath\right) &= [1 - p_\jmath]\, C_\jmath^n + p_\jmath C_\jmath^a \\ &= X_\jmath + [1 - p_\jmath f]\, E_\jmath \end{aligned}$$

Nash Equilibrium – Bonacich Centrality

- **Network centrality** is a concept developed in sociology to quantify the **influence or power** of actors in a network
- **Multiple definitions**: Bonacich centrality (Bonacich, 1987) relevant in our setting

Definition

Consider a network with (weighted) adjacency matrix $\mathbf{A}$. For a scalar β and weight vector α, the weighted Bonacich centrality vector is given by $\mathbf{b}(\mathbf{A}, \beta, \alpha) = [\mathbf{I} - \beta\mathbf{A}]^{-1}\,\alpha$ provided that $[\mathbf{I} - \beta\mathbf{A}]^{-1}$ is well-defined and non-negative.

- The weighted Bonacich centrality computes the (α-weighted) sum of paths originating from a taxpayer in the network

- Longer paths are discounted by the (geometric) factor β

Nash Equilibrium

Proposition

If

(i) utility is linear-quadratic, $U_\imath(z) = \left[b_\imath - \frac{a_\imath z}{2}\right] z$, with $a_\imath \in \left(0, \frac{b_\imath}{W_\imath}\right)$ and $b_\imath > 0$ for all $\imath \in \mathcal{N}$;

(ii) $1 > \rho(\boldsymbol{M})$; $\quad [\mathbf{I} - \boldsymbol{M}]\,\theta(\mathbf{W}) - \alpha > \mathbf{0}$;

then there is a unique interior Nash equilibrium, at which the optimal amount of tax evaded is given by

$$\mathbf{E} = \mathbf{b}(\boldsymbol{M}, 1, \alpha),$$

where

$$\begin{aligned} m_{\imath\jmath} &= \frac{[1 - p_\imath f][1 - p_\jmath f]}{\zeta_\imath} g_{\imath\jmath}; \\ \alpha_{\imath 1} &= \{[1 - p_\imath f] / [a_\imath \zeta_\imath]\} \{b_\imath - a_\imath [X_\imath - R_\imath(\mathbf{X})]\}. \end{aligned}$$

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Generalization of optimal evasion result

What **if utility is not linear-quadratic**?

For an **arbitrary** twice differentiable **utility function**
considering the FO linear approximation around a Nash equilibrium to the set of FOC, it is:

$$\mathbf{E} = \mathbf{J}\mathbf{E} + \widehat{\boldsymbol{\alpha}} = [\mathbf{I} - \mathbf{J}]^{-1}\, \widehat{\boldsymbol{\alpha}}$$

Where $\mathbf{J}$ is a matrix of coefficients measuring actions' interactions

A solution is a again in the form of a
weighted Bonacich centrality measure

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Comparative Statics: Local Strategic Complementarity

- The model exhibits **strategic complementaries in evasion choices**
  - an increase in evasion by one taxpayer induces others to do likewise.
- Formally, **expected utility is supermodular in cross evasion choices**:

$$\frac{\partial^2 \mathbb{E}\left(U_\imath\right)}{\partial E_\imath \partial E_\jmath} = a_\imath g_{\imath\jmath}[1 - p_\imath f][1 - p_\jmath f] > 0 \qquad \jmath \in \mathcal{R}_\imath$$

Comparative Statics: Optimal Evasion

- How is optimal evasion impacted by information carried through the social network?

$$\begin{aligned}\frac{\partial E_\imath}{\partial W_\jmath} &= b_{1\imath}\left(\boldsymbol{M}, 1, \frac{\partial \alpha}{\partial X_\jmath}\right) \geq 0;\\ \frac{\partial E_\imath}{\partial p_\jmath} &= b_{1\imath}\left(\boldsymbol{M}, 1, \frac{\partial \boldsymbol{M}}{\partial p_\jmath}\mathbf{E} + \frac{\partial \alpha}{\partial p_\jmath}\right) \leq 0.\end{aligned}$$

- Results can be strengthened to strict inequalities if $\boldsymbol{G}$ is *connected*

The Value of Network Information

- **Observing links in social networks** ought to help tax authorities to **target better** their limited **audit** resources

- Can tax authorities observe links in social networks?
  - Some individuals – celebrities – for whom it is common knowledge that many people observe them
  - “big data”

- The UK tax authority (HMRC) uses a system known as “Connect”
  - cross-checks public sector and third-party information
  - system produces “spider diagrams” linking individuals to other individuals and to legal entities such as “property addresses, companies, partnerships

- IRS also known to have also invested in big data heavily
  - but much more reticent in revealing its capabilities

Audit targeting

- Tax authority chooses **audit targets conditional** on observing each taxpayers' self-reported **income declaration** ($d_\imath$)

- By definition

$$E_\imath = \theta\left(W_\imath\right) - \theta\left(d_\imath\right)$$

- So

$$d_\imath \equiv \hat{d}_\imath\left(\boldsymbol{G}\right) = \theta^{-1}\left(\theta\left(W_\imath\right) - E_\imath\left(W_\imath;\boldsymbol{G}\right)\right).$$

- We invert this function to obtain

$$W_\imath \equiv \hat{W}\left(d_\imath;\boldsymbol{G}\right) = \hat{d}_\imath^{-1}\left(d_\imath\right)$$

- This gives the true income $W_\imath$ of a taxpayer who optimally declares an income $d_\imath$.

Limited network information

- If tax authority observes $\boldsymbol{G}$ (and the remaining model parameters) it can use $\hat{d}_\imath^{-1}(d_\imath)$ to recover the true incomes

$$\hat{W}(d_\imath; \boldsymbol{G}) = W_\imath$$

- If the tax authority **does not perfectly observe $\boldsymbol{G}$,** but instead some (related) network $\boldsymbol{G}'$, **estimates** of the $W_\imath$ **will be incorrect**

$$\hat{W}(d_\imath; \boldsymbol{G}') \neq W_\imath$$

- Noise in the $\hat{W}$ feeds through into noise in the $\hat{E} = \theta(\hat{W}_\imath) - \theta(d_\imath)$

- Suppose the tax **authority observes only a subset of the links** in the network
  - $\kappa \in [0, 1]$ is the **probability** that the tax authority **observes a given link** in the social network
  - **Network observed** by the tax authority denoted $\boldsymbol{G}(\kappa)$ generated by randomly deleting links (with probability $1 - \kappa$)

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Audit targeting

- Audits targeted to the $100\bar{p}\%$ of taxpayers with the **highest** $\hat{E}$
  - Reminiscent of US "DIF score", and similar to UK audit selection rules
- Full-information auditing gives revenue (in tax and fines) $\mathfrak{R}_{\max} = \mathfrak{R}(G(1))$
- No-information (random) auditing gives $\mathfrak{R}_{RA} = f\boldsymbol{p}\boldsymbol{E}$
- Metric used to assess value of **social network information**:

$$\Psi(\kappa) \equiv \frac{\mathfrak{R}(\boldsymbol{G}(\kappa)) - \mathfrak{R}_{RA}}{\mathfrak{R}_{\max} - \mathfrak{R}_{RA}} \times 100.$$

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The Social Network

- We generate a static network using the Bianconi-Barabási **fitness** model
  - Taxpayers with **higher wealth** have a higher probability of making new connections
  - Taxpayers already **heavily connected** have a higher probability of making new connections
    (sublinear preferential attachment, $\phi < 1$)

Formally:

$$\Pi_i = \frac{W_i [d^{in}(i)]^{\phi}}{\sum_{j \in \mathcal{N}} W_j [d^{in}(j)]^{\phi}}$$

The resulting **static** social networks used in our simulations resembles the ones observed empirically

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Model functions and parameters

- Tax system is linear: $\theta(W) = \theta W$
- Power law distribution of income
- Baseline parameter values
  - $\phi = 0.43$ (Pham ***et al.***, 2016)
  - $N = 200$
  - $a = 2$
  - $b = 80$
  - pf calibrated to achieve evasion of 10%

Predicted wealth

Lemma

Under a linear income tax, the income of a taxpayer who declares income optimally is given by

$$\hat{\mathbf{W}}\left(\mathbf{d};\boldsymbol{G}\right)=\mathbf{b}(\mathbf{V},\theta,\gamma),$$

where

$$\begin{aligned}
v_{\imath\jmath} &= \frac{\zeta_\imath}{\xi_\imath}m_{\imath\jmath}; \qquad \xi_\imath=\left[1-\theta\right]\left[1-p_\imath f\right]+\theta\left\{1+\left[f-2\right]p_\imath f\right\}>0;\\
\gamma_{\imath\jmath} &= \frac{\left\{1+\left[f-2\right]p_\imath f\right\}\theta a_\imath d_\imath+b_\imath\left[1-p_\imath f\right]}{a_\imath\xi_\imath}\\
&\quad+\frac{\left[1-p_\imath f\right]R(\mathbf{X}-\theta\left[1-p_\imath f\right]\mathbf{d})}{\xi_\imath}.
\end{aligned}$$

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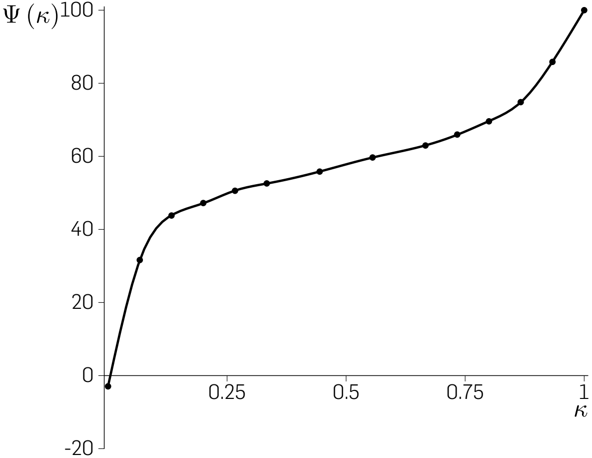
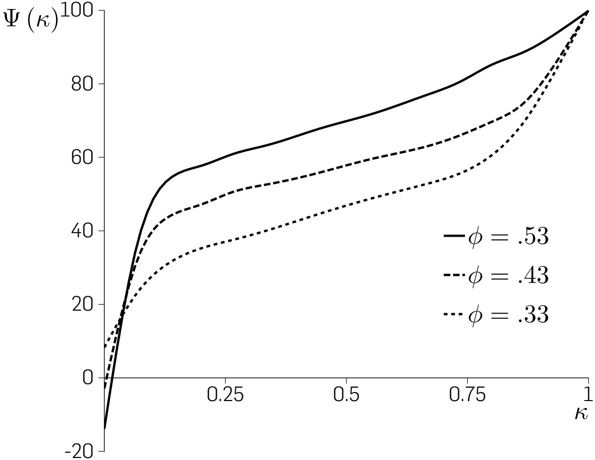
Findings – Baseline effects

- **Initial efforts** in collecting network information are characterized by **high returns**



Findings – Effects of network structure

- The value of network information is **higher if preferential attachment ϕ is stronger**
- Using **predictive tools when little is know may be counterproduc- tive** in highly concentrated networks



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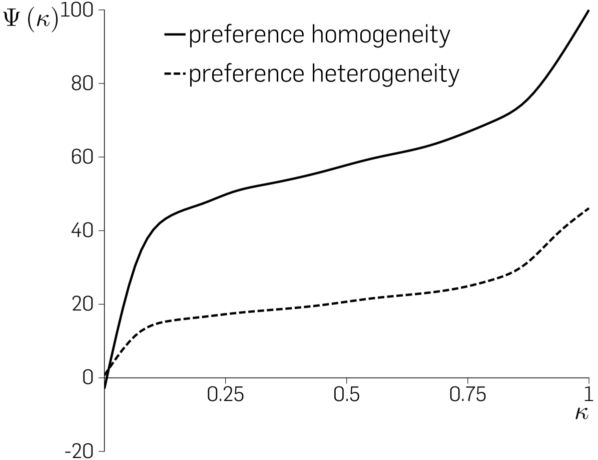
Findings – Effects of unobserved preference heterogeneity

- **Limited interaction** between uncertainty over **preference** and uncertainty over the **network**



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Conclusions

- Our model provides a rich framework for understanding how information conveyed through a (arbitrary) social network influences optimal evasion behavior
- We show that **network information can be of value** to a tax authority
  - **strong gains to knowing a little** about the social network
  - **may actually be counterproductive** to utilize highly incomplete network information
- Some network information is **especially important in highly concentrated networks**

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Further Research

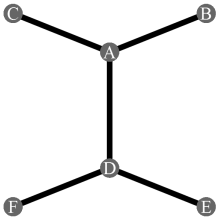
- Introduce **habit** (memory) dependence in reference income
  - Investigate **dynamic response** to audit interventions
  - Study **direct and indirect effects** of audit interventions
- Extend the analysis to **avoidance** and **crime** as a whole
- Analyse how adding or **removing taxpayers** from the network (detention) may affect compliance

Thank You!

Questions?

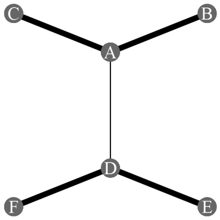
Social Network and matrix representation

Undirected Network



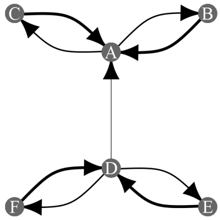
$$\begin{array}{c c} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Weighted Network



$$\begin{array}{cccccc} A & B & C & D & E & F \end{array}$$
$$\begin{pmatrix} 0 & 1 & 1 & .2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Directed Network

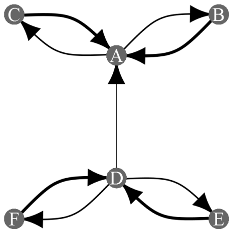


$$\begin{array}{cccccc} A & B & C & D & E & F \end{array}$$
$$\begin{pmatrix} 0 & .5 & .5 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & .4 & .4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Social Network as an Adjacency matrix

Matrix form of a **weighted directed** network

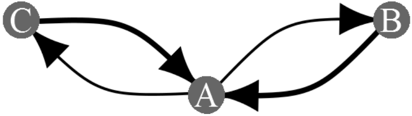
Directed Network



$$
\begin{array}{c c}
 & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} &
\begin{pmatrix}
0 & .5 & .5 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
.2 & 0 & 0 & 0 & .4 & .4 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\end{array}
$$

A simple example

Taxpayer interaction through the reference income leads to the rise of a network game



$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0 & .5 & .5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \equiv \boldsymbol{G}$$

$$\begin{cases} E_A = \frac{1-p_\imath f}{a\zeta_A}\{a[R_A(E_B, E_C) - X_A] + b\} \\ E_B = \frac{1-p_\imath f}{a\zeta_B}\{a[R_B(E_A) - X_B] + b\} \\ E_C = \frac{1-p_\imath f}{a\zeta_C}\{a[R_C(E_A) - X_C] + b\} \end{cases}$$

Weighted Bonacich Centrality and Evasion

Given the linearity of $R_\imath$ the system of equation of optimal evasion is linear:

$$\mathbf{E} = \boldsymbol{\alpha} + \mathbf{ME} \;\equiv\; \begin{cases} E_A = \eta_\imath\{a[R_A(h_A; E_B, E_C) - X_A] + b\} \\ E_B = \eta_\imath\{a[R_B(h_B; E_A) - X_B] + b\} \\ E_C = \eta_\imath\{a[R_C(h_C; E_B) - X_C] + b\} \end{cases}$$

And we can solve à la **Cournot-Nash**:

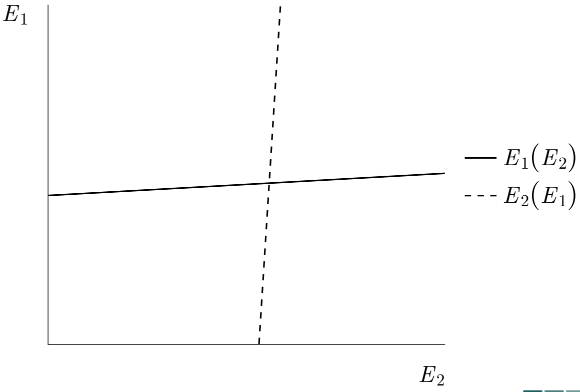
$$\mathbf{E} = [\mathbf{I} - \boldsymbol{M}]^{-1}\boldsymbol{\alpha} = b(\boldsymbol{M}, 1, \boldsymbol{\alpha})$$

Where $b(\boldsymbol{M}, 1, \boldsymbol{\alpha})$ is a weighted Bonacich centrality measure

Best response

- **Linear best response** follows from **quadratic utility** and **linearity of reference income**

Strategic complementarity of E_{it}, E_{jt} leads to **positive slope** of best response functions



Taxpayers' interaction as a game

The game arising from taxpayers interaction is:

Smooth Supermodular Game (Milgrom and Roberts 1990)

Bounds on strategies	$E_{it} \in (0, tW_\imath)$
Differentiability	$\mathbb{E}[U_\imath]$ is of class C^2
Strategic Complementarity	$\frac{\partial^2 \mathbb{E}[U]_\imath}{\partial E_{it} \partial E_{jt}} \geq 0$

Monotone comparative statics

Smooth Supermodular Games can be analyzed using **monotone comparative statics**

Following Quah (2007) we exploit the **supermodularity** of the game to sign **globally** the CS of every parameter except a

To sign the CS of a we exploit the **weaker** condition of **local supermodularity** around the Nash equilibrium point :

$$\left.\frac{\partial^2 \mathbb{E}[U]_\imath}{\partial E_\imath \partial a}\right|_{E_\imath = E_\imath^*} \geq 0 \Leftrightarrow \frac{\partial E_\imath^*}{\partial a} \begin{cases} > 0 \text{ if } \left.\frac{\partial^2 \mathbb{E}[U]_\imath}{\partial E_\imath \partial a}\right|_{E_\imath = E_\imath^*} > 0 \\ \geq 0 \text{ if } \left.\frac{\partial^2 \mathbb{E}[U]_\imath}{\partial E_\imath \partial a}\right|_{E_\imath = E_\imath^*} = 0 \end{cases}$$

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The Social Network

- Utilise a class of **generative network models** developed in the natural sciences
- Networks generated by **incremental addition of nodes and edges to a seed** network

Two key processes:

1. ***node-degree*** (or ***preferential attachment***) process – makes the probability that a new taxpayer added to the network observes an existing taxpayer, $\imath$, a **positive function of** $\imath$'s **in-degree** (the number of taxpayers who already observe $\imath$)
2. ***node-fitness*** process – makes the probability that a new taxpayer added to the network observes an existing taxpayer, $\imath$, a **positive function of** $\imath$'s ***fitness*** (an exogenous and time-invariant characteristic of node $\imath$)

The Social Network

- At step s of the generative process consider a taxpayer $\imath$ with degree $\mathfrak{d}_{\imath s}$, and fitness $\eta_\imath > 0$. Entwine the node-degree and node-fitness processes by allowing the probability that taxpayer $\imath$ is observed by the taxpayer added at step s to be proportional to the product

$$\eta_\imath A\left(\mathfrak{d}_{\imath s}\right) \qquad A'\left(.\right) > 0$$

- Special cases of this approach include
  - Barabási-Albert: $\eta_\imath$ equal across taxpayers
  - Bianconi-Barabási: $A\left(\mathfrak{d}\right) = \mathfrak{d}$
- We generate a static network using the Bianconi-Barabási **fitness** model using $\eta_\imath = W_\imath$ and $A\left(\mathfrak{d}\right) = \mathfrak{d}^\phi \qquad \phi < 1$

$$\Pi_\imath = \frac{\mathfrak{d}_{\imath s}^{\phi} W_\imath}{\sum_{\jmath \in \mathcal{N}} \mathfrak{d}_{\jmath s}^{\phi} W_\jmath}$$

<u>The resulting **static** social networks used in our simulations resembles the ones observed empirically</u>

Network structure

- $\Delta\mathfrak{R}\left(\boldsymbol{G}\right)$ is defined for a **single draw** of the network $\boldsymbol{G}$
- It **may be misleading** when the realized $\boldsymbol{G}$ is unrepresentative of the generative process
- Evaluation of the average $\Delta\mathfrak{R}\left(\boldsymbol{G}\right)$ is **computationally infeasible** (at least at the moment)
- **We instead consider** the $\boldsymbol{G}$ obtained by **averaging until convergence** independent draws from the **network generative process**
    - Each element $g_{\imath\jmath}$ of $\boldsymbol{G}$ is the **mean of comparison intensity weights** across different networks