

TAX EVASION ON A SOCIAL NETWORK

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INTRODUCTION

COMPLIANCE AND REFERENCE DEPENDENCE

- We relate non compliant behaviour to a body of evidence on the **importance of positional concerns** (keeping up with the Jones)
- Tax evasion may be used to improve agents' relative standing
- The choice of **how much to evade is affected by social interaction** and depends on **reference income**
- New project studying **tax evasion** that builds on a previous TARC project on **tax avoidance**

TAX EVASION - RELEVANCE AND RESEARCH

- Tax evasion causes **significant losses of public revenues** (4.4 bn. £ in UK)
- Growing interest by tax agencies on understanding evasion so to **design efficient deterrence measures**
- **Rich literature** using different approaches to study evasion decision and optimal policies

RELATED LITERATURE

- Kahneman and Tversky 1979
Reference dependence of utility
- Gali 1994
"Keeping up with the Jones"
- Myles and Naylor 1996
Tax evasion and group conformity
- Ballester, Calvo, Zenou 2006
Network game with local payoff complementarities
- Quah 2007
Monotone comparative statics on network games

MODELLING FEATURES

Provide a model where:

- Agents differ in **income**, **reference group** and **probability of detection**
- Taxpayers may engage in **risky** tax evasion
- **Self** and **social** comparison shape the **reference income**
- **Social** comparison depends on agents' **social network**

RESEARCH QUESTIONS

→ Our analysis has focused on **three** questions:

1. Is it possible to characterize **optimal evasion** and how do **changes in the exogenous parameters** (income, risk aversion, etc.) affect it?
2. Is it possible to characterize the direct and indirect **revenue effects** of interventions?
3. How much does the **availability of more information** (especially related to social network) improves the capacity of a tax authority to **infer revenue effects**?

MODEL

MODELLING OF EVASION

- We define evasion E_{it} as the **liabilities under-declared** by taxpayer i at time t
- Evasion is a **risky** activity:
 - The tax agency may detect evasion
 - If evasion is detected, a **fine** f proportional to the evaded tax debt is also imposed

REFERENCE INCOME

→ Taxpayers determine their reference R_{it} income based on **Social**-related and **Self**-related considerations

→ **Social:**

The (weighted) **average consumption** of taxpayer's **reference group**

→ **Self:**

Their habit consumption $h_{it} = f(C_{it-1} \dots C_{it-T})$

OPTIMAL EVASION

THE TAXPAYER'S PROBLEM

$$\max_{E_i} \mathbb{E}(U_{it}) \equiv [1 - p_i] U(C_{it}^m - R_{it}) + p_i [U(C_{it}^a - R_{it})]$$

After-tax income **if not audited**

$$C_{it}^m \equiv X_i + E_{it}$$

After-tax income **if audited**

$$C_{it}^a \equiv C_{it}^m - (1 + f)E_{it}$$

Utility is linear-quadratic

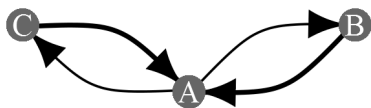
$$U(z) = z[b - \frac{az}{2}]$$

Optimal Evasion at an interior solution is:

$$E_{it}^* = \frac{1 - p_i f}{a\zeta_i} \{a[R_{it} - X_i] + b\}, \zeta_i > 0$$

A SIMPLE EXAMPLE

Taxpayer interaction through the reference income leads to the rise of a game



$$\begin{matrix} & A & B & C \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{pmatrix} 0 & .5 & .5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \equiv \mathbf{G} \end{matrix}$$

$$\begin{cases} E_A^* &= \frac{1-p_{if}}{a\zeta_A} \{a[R_A(h_A; E_B^*, E_C^*) - X_A] + b\} \\ E_B^* &= \frac{1-p_{if}}{a\zeta_B} \{a[R_B(h_B; E_A^*) - X_B] + b\} \\ E_C^* &= \frac{1-p_{if}}{a\zeta_C} \{a[R_C(h_C; E_A^*) - X_C] + b\} \end{cases}$$

REFERENCE DEPENDENCE

Taxpayer i expected after-tax income when evading E_{it} is:

$$q_{it} = X_i + [1 - p_i]E_{it}$$

We can then define:

$$Z_{it} = \iota_h h_{it} + \iota_s \mathbf{g}_i \mathbf{q}_t$$

And reference income:

$$R_{it} = R_{it}(h_{it}; \mathbf{q}_t(\mathbf{E}_t)) = R_{i,t-1} + \varsigma_R [Z_{it} - R_{i,t-1}]$$

where:

$$X_i = (1 - t) W_i$$

Honest after-tax income

$$\iota_h, \iota_s$$

Self and social comparison parameters

$$\mathbf{g}_i$$

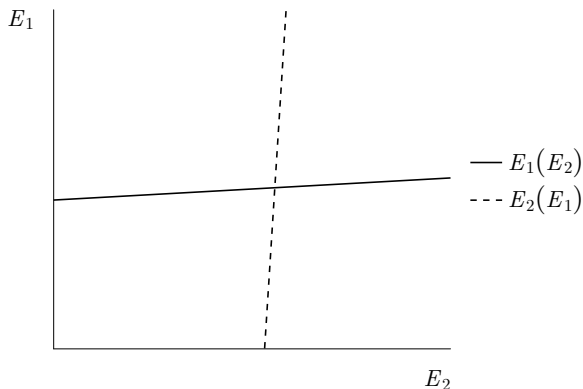
Weights of i 's reference group

$$\varsigma_R \in (0, 1)$$

Reference consumption reactivity

BEST RESPONSE

Quadratic utility leads to linear best response



Positive slope of best response functions follows from strategic complementarity in E_{it}, E_{jt}

WEIGHTED BONACICH CENTRALITY AND EVASION

Expanding E_{it}^* using the definitions of R_{it} , Z_{it} and q_{it} we can rewrite:

$$\begin{cases} E_A^* &= \eta_i \{ a[R_A(h_A; E_B^*, E_C^*) - X_A] + b \} \\ E_B^* &= \eta_i \{ a[R_B(h_B; E_A^*) - X_B] + b \} \\ E_C^* &= \eta_i \{ a[R_C(h_C; E_B^*) - X_C] + b \} \end{cases}$$

$$\mathbf{E}_t = \boldsymbol{\alpha}_t + \mathbf{M}\boldsymbol{\beta}\mathbf{E}$$

And solve à la **Cournot-Nash**:

$$\mathbf{E}_t = [\mathbf{I} - \mathbf{M}\boldsymbol{\beta}]^{-1} \boldsymbol{\alpha}_t = b(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}_t)$$

Where $b(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\alpha}_t)$ is the weighted Bonacich centrality measure:

OPTIMAL EVASION

- Key theoretical result is that **evasion is closely related to the concept of “Bonacich” Network Centrality**
 - More “central” taxpayers evade more
- Network centrality is a concept developed in sociology
 - Measures the amount of influence/power players have within a network

STEADY STATE EVASION

Corollary 2

In a steady state of the model consumption satisfies

$$\mathbf{C}^{SS} = \mathbf{C}^{n,SS} = \mathbf{X} + \mathbf{E}^{SS}.$$

Steady state evasion \mathbf{E}^{SS} , is then given by the vector of Bonacich centralities, $\mathbf{b}(\mathbf{M}, \beta, \alpha^{SS})$, with

$$\alpha_i^{SS} = \frac{1 - p_i^f}{a\zeta_i} \left\{ b - a \left[X_i - R \left(h_i^{SS}, \mathbf{X} \right) \right] \right\}$$

MONOTONE COMPARATIVE STATICS IN TIME

A **permanent** marginal parameter change entails **contemporaneous** and **delayed effects** on steady state evasion:

1. The contemporaneous effect $\frac{\partial E_i^{SS}}{\partial z}$ is not accounting for delayed effects
2. The **full effect** $\frac{dE_i^{SS}}{dz}$ **includes** also the **delayed effect** caused by adjustments of **habit consumption**

MONOTONE COMPARATIVE STATICS RESULTS

Habit consumption	+	Other's Income	+ / 0
Own comparison	+	Social comparison	+ / 0
Own audit prob.	—	Others audit prob.	— / 0
Risk Aversion	—	Tax rate	+
Fine	—		

Monotone comparative statics for interior E_i^*

These results apply both to contemporaneous and full effects

EVASION AND INCOME

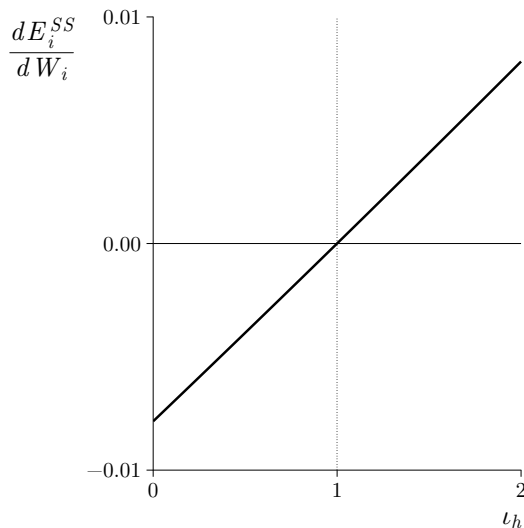
- In the case of **income, contemporary and delayed effects have opposite signs**
- The **contemporaneous effect causes evasion to fall** due to the increased income, i.e. $\frac{\partial E_i^{SS}}{\partial X_i} < 0$
- However, **the delayed effect causes an increase in habit consumption** $\frac{dC_i^{SS}}{dX_i} < 0$ that as a positive effect on evasion.

This allows our model to replicate the observed behaviour

of evasion increasing in income $\frac{dE_i^{SS}}{dX_i} > 0$

EVASION VS. CONCERN FOR HABIT

The higher a taxpayer's concern for habit i_h the more evasion increases in income



RESPONSES TO INTERVENTION

INTERVENTION REVENUE EFFECTS

How does an audit to a taxpayer affect the steady-state evasion of the model?

1. **Direct effect** E_i^{SS}

On targeted taxpayer, by **averting attempted evasion**

2. **Indirect effects** I_{ij}

Expected additional revenue that arises **from future changes in evasion behaviour (negative externality)**

→ I_{ii} from the audited taxpayer

→ I_{ij} from non-audited taxpayers

→ $\Sigma_i = \sum_{j \in \mathcal{N} \setminus i} I_{ij}$ **aggregate cross indirect effect**

→ Indirect effects **2X-6X** direct ones

TAX AGENCY'S INFERENCE PROBLEM

- Tax authorities engage in inferring both **direct effects** \mathbf{E}^{SS} and **aggregate gross indirect effects** Σ
 - Taxpayers usually ranked by discriminant function and audited sequentially until budget is exhausted
- Crucial information for tax authorities is correct rank of \mathbf{E}^{SS} and Σ
 - Optimal audit targeting if tax authorities were able to exactly infer **rankings** of direct and indirect effects.

Tax authorities require measures that are ordinally equivalent to direct and indirect effects

$$\mathbf{A} \sim \mathbf{B} \iff A_{i1} \geq A_{j1} \iff B_{i1} \geq B_{j1} \forall i, j$$

MEASURES ORDINALLY EQUIVALENT TO REVENUE EFFECTS

The indirect revenue effects of conducting a single audit of i satisfy:

$$\mathbf{I}_i \sim \mathbf{E}_i^{SS} \mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})$$

where \mathbf{E}_i^{SS} is an $n \times n$ diagonal matrix and $\boldsymbol{\rho}_i^{SS} = \frac{\partial \boldsymbol{\alpha}^{SS}}{\partial C_i^{SS}}$

Sizes of the **own** and **cross indirect** effects are **ordinally equivalent** to the product of the steady state level of evasion and a new measure of **Bonacich centrality**

INFERENCE OF REVENUE EFFECTS

- When there is full observability \mathcal{F} it is possible to exactly determine direct (\mathbf{E}^{SS}) and cross indirect ($\mathbf{\Sigma}$) effects
- Tax agencies infer revenue effects under **limited observability**

How valuable is **network information**?

- Two cases considered:
 1. **Partial observability** (\mathcal{P}): The tax agency observes the reference groups of taxpayers but has no information on the comparison intensity
 2. **No observability** (\emptyset): Everybody attaches equal importance to all the other taxpayers
- We assess the role of network information in prediction using a the **Spearman rank correlation coefficient**, i.e. $\rho_{\mathbf{E}^{\mathcal{P}}, \mathbf{E}^{\emptyset}}^S$

NETWORK STRUCTURE

- We generate a static network using the Bianconi-Barabási **fitness** model
 - Taxpayers with **higher wealth** have a higher probability of making new connections
 - Taxpayers already **heavily connected** have a higher probability of making new connections (sublinear preferential attachment, $\phi < 1$)

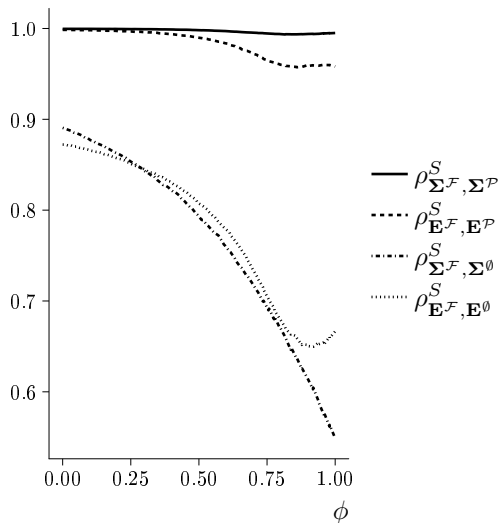
Formally:

$$\Pi_i = \frac{W_i [d^{in}(i)]^\phi}{\sum_{j \in \mathcal{N}} W_j [d^{in}(j)]^\phi}$$

The resulting **static** social networks used in our simulations
resembles the ones observed empirically

INFERENCE ACCURACY AND PREFERENTIAL ATTACHMENT

- **Accuracy improves** significantly from the **no network observability \emptyset to partial observability \mathcal{P}**
- Stronger preferential attachment ϕ decreases accuracy



CONCLUSIONS

CONCLUDING REMARKS

- Social interaction may heavily affect evasion behaviour
- Different **Bonacich** measures of centrality characterize optimal **evasion** and **revenues effects** from auditing
- **Social network information** improve significantly the **prediction** of revenues effects from interventions

FURTHER RESEARCH

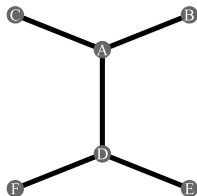
- Quantify the importance of network information in terms of **revenue recovered**
- Extend the analysis to **crime** as a whole
- Analyse how adding or **removing taxpayers (detention)** may affect compliance

Thank You!

Questions?

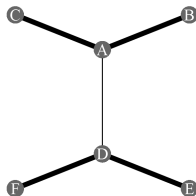
SOCIAL NETWORK AND MATRIX REPRESENTATION

Undirected Network



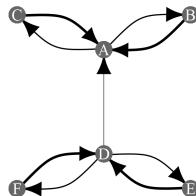
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	1	1	1	0	0
<i>B</i>	1	0	0	0	0	0
<i>C</i>	1	0	0	0	0	0
<i>D</i>	1	0	0	0	1	1
<i>E</i>	0	0	0	1	0	0
<i>F</i>	0	0	0	1	0	0

Weighted Network



	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	1	1	.2	0	0
<i>B</i>	1	0	0	0	0	0
<i>C</i>	1	0	0	0	0	0
<i>D</i>	.2	0	0	0	1	1
<i>E</i>	0	0	0	1	0	0
<i>F</i>	0	0	0	1	0	0

Directed Network

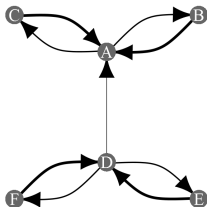


	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	.5	.5	0	0	0
<i>B</i>	1	0	0	0	0	0
<i>C</i>	1	0	0	0	0	0
<i>D</i>	.2	0	0	0	.4	.4
<i>E</i>	0	0	0	1	0	0
<i>F</i>	0	0	0	1	0	0

SOCIAL NETWORK AS AN ADJACENCY MATRIX

Matrix form of a **weighted directed** network

Directed Network



$$\begin{array}{c}
 A \quad B \quad C \quad D \quad E \quad F \\
 \begin{array}{l}
 A \\
 B \\
 C \\
 D \\
 E \\
 F
 \end{array}
 \begin{pmatrix}
 0 & .5 & .5 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 \\
 .2 & 0 & 0 & 0 & .4 & .4 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0
 \end{pmatrix}
 \end{array}$$

ACCOUNTING FOR SOCIAL NETWORK

Expanding E_{it}^* using the definitions of R_{it} , Z_{it} and q_{it}
we solve à la **Cournot-Nash**:

$$E_{it} = \alpha_{it} + \varsigma_R \iota_s \sum_{j \neq i} m_{ij} E_{jt} =$$

$$\mathbf{E} = \boldsymbol{\alpha}_t + \mathbf{M} \beta \mathbf{E}$$

Where:

$$m_{ij} = \frac{[1 - p_{if}][1 - p_{jf}]}{\zeta_i} g_{ij}$$

$$\beta_{ii} = \varsigma_R \iota_s$$

$$\alpha_{i1,t} = \frac{1 - p_{if}}{a \zeta_i} \{b - a[X_i - R(h_{it}, \mathbf{X})]\}$$

WEIGHTED BONACICH CENTRALITY AND EVASION

Expanding E_{it}^* using the definitions of R_{it} , Z_{it} and q_{it} we can rewrite we solve à la **Cournot-Nash**:

$$\mathbf{E}_t = [\mathbf{I} - \mathbf{M}\beta]^{-1} \alpha_t = b(\mathbf{M}, \beta, \alpha_t)$$

$b(\mathbf{M}, \beta, \alpha_t)$ is the weighted Bonacich centrality defined on:

\mathbf{M}	Edge weights scaled by relative ER of E_i
β	Scales weight of longer paths
α_t	Weights centrality by agent characteristics

GENERALIZATION OF OPTIMAL EVASION RESULT

For an **arbitrary** twice differentiable **utility function** considering the FO linear approximation around a Nash equilibrium to the set of FOC, it is:

$$\mathbf{E}_t = \mathbf{J}\mathbf{E}_t + \hat{\alpha}_t = [\mathbf{I} - \mathbf{J}]^{-1} \hat{\alpha}_t = \left[\sum_{k=0}^{\infty} \mathbf{J}^k \right] \hat{\alpha}_t$$

Where \mathbf{J} is a matrix of coefficients measuring actions' interactions

A solution is again in the form of a
weighted Bonacich centrality measure

TAXPAYERS' INTERACTION AS A GAME

The game arising from taxpayers interaction is:

Smooth Supermodular Game (Milgrom and Roberts 1990)

Bounds on strategies

$$E_{it} \in (0, tW_i)$$

Differentiability

$$\mathbb{E}[U_i] \text{ is of class } C^2$$

Strategic Complements

$$\frac{\partial^2 \mathbb{E}[U]_i}{\partial E_{it} \partial E_{jt}} \geq 0$$

MONOTONE COMPARATIVE STATICS

Smooth Supermodular Games can be analyzed using **Monotone comparative statics**

Following Quah (2007) we exploit the **weaker** condition of **local supermodularity** around the Nash equilibrium point:

Then, for a given parameter z , it holds:

$$\frac{\partial^2 \mathbb{E}[U]_i}{\partial E_i \partial z} \bigg|_{E_i = E_i^*} \geq 0 \Leftrightarrow \frac{\partial E_i^*}{\partial z} \begin{cases} > 0 \text{ if } \frac{\partial^2 \mathbb{E}[U]_i}{\partial E_i \partial z} \bigg|_{E_i = E_i^*} > 0 \\ \geq 0 \text{ if } \frac{\partial^2 \mathbb{E}[U]_i}{\partial E_i \partial z} \bigg|_{E_i = E_i^*} = 0 \end{cases}$$

MONOTONE COMPARATIVE STATICS IN TIME

A marginal parameter change entails contemporaneous and delayed effects on the steady state of the model:

1. The contemporaneous effect $\frac{\partial E_i^{SS}}{\partial z}$ is not accounting for delayed effects
2. The full effect $\frac{dE_i^{SS}}{dz}$ includes also the delayed effect caused by adjustments of habit consumption

Lemma 1

$$\text{if } \frac{\partial X_i}{\partial z} \frac{\partial E_i^{SS}}{\partial z} \geq 0 \quad \text{then} \quad \text{sign} \left(\frac{dE_i^{SS}}{dz} \right) = \text{sign} \left(\frac{\partial E_i^{SS}}{\partial z} \right)$$

It is sufficient to have same sign for $\partial E_i^{SS} / \partial z$, and steady state consumption, $\partial C_i^{SS} / \partial z$

MONOTONE COMPARATIVE STATICS

E_i^*		E_i^*	
h_{it}	+	X_j	+ / 0
ι_h	+	ι_s	+ / 0
p_i	−	p_j	− / 0
f	−	t	+
a	−	b	+

Monotone comparative statics for interior E_i^*

These results apply in the short and long run

MEASURES ORDINALLY EQUIVALENT TO REVENUE EFFECTS

Understanding why:

$$\mathbf{I}_i \sim \mathbf{E}_i^{SS} \mathbf{b}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})$$

- The size of the indirect effect I_{ij} is ordinally equivalent to the size of the initial deviation
 - convergence of evasion back to its steady state value is at a uniform rate for all affected taxpayers
- Initial effect can be decomposed linearly as the product of:
 - marginal effect of a change in i 's consumption on j 's evasion
 $\partial E_j^{SS} / \partial C_i^{SS} = b_{j1}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS})$
 - change in i 's consumption $C_i^{n,SS} - C_i^{a,SS} = [1 + f] E_i^{SS}$

Corollary 3

$$\boldsymbol{\Sigma} \sim \boldsymbol{\chi} \text{ where } \chi_{i1} = \sum_{k \in \mathcal{N}} b_{k1}(\mathbf{M}, \boldsymbol{\beta}, \boldsymbol{\rho}_i^{SS}) E_i^{SS}$$

DYNAMIC RESPONSE TO LEGAL INTERVENTION

- Empirical evidence shows a **persistent dynamic behavioural response** to interventions
- The literature argued that **belief updating may be driving** this evidence
- We show that **self-comparison is able to replicate the same dynamic**
- Calibrating the persistence ς_R it is possible to **closely match the behaviour observed in reality**

RESPONSE TO LEGAL INTERVENTION VS. PERSISTENCE

- Here periods interpreted as years
- **Deterrence is maximal after the intervention and slowly fades**
- With high levels of persistence the dynamic behavioural response lasts ≈ 4 years

