Marble Roller Coaster

Purdue University Polytechnic Institute
MET 213 Dynamics

April 2019

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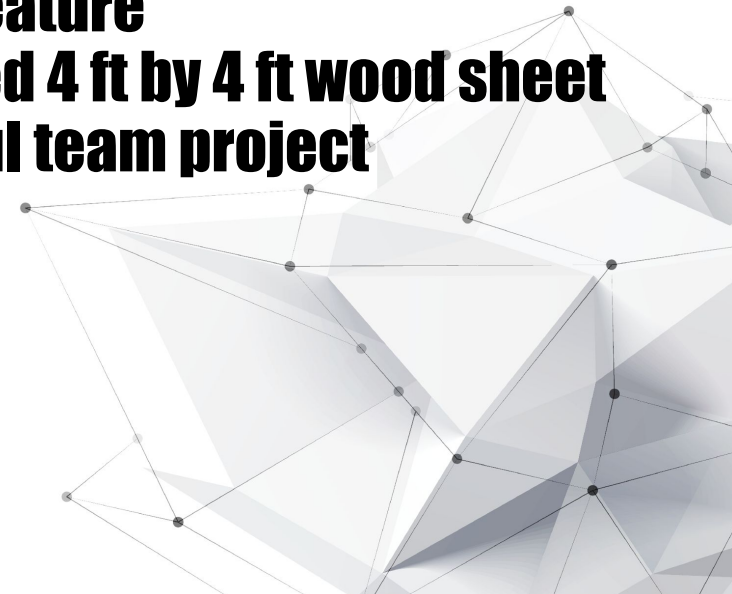
Assignment Instructions

Design a marble roller coaster:

1. **The marble must complete a 2 ft loop**
2. **The marble must complete a second feature**
3. **The marble may only touch the supplied 4 ft by 4 ft wood sheet**
4. **Record a video to document successful team project**



Our Successful Design and Build

Start: Release of ball
Stop: Swish past horizon of hoop

The Dynamics Calculations

Lots and lots of math...

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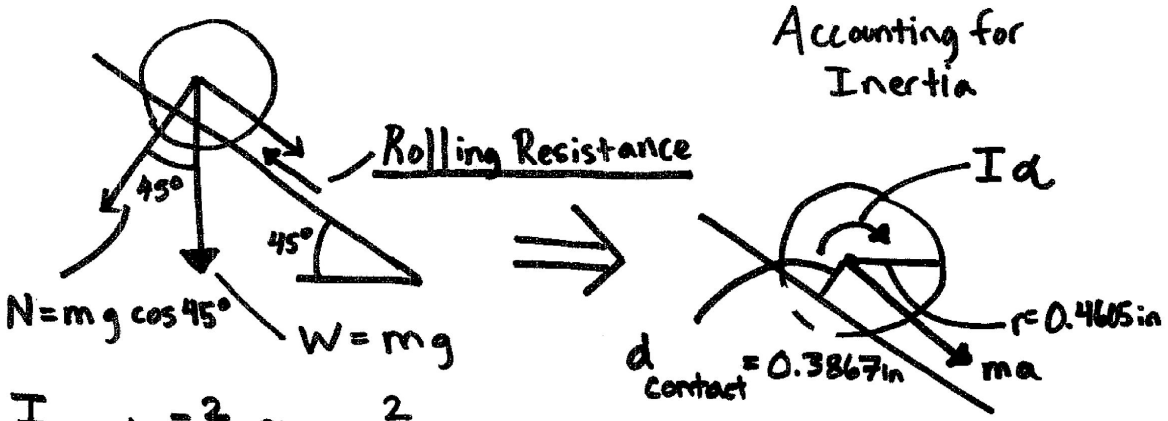


$$I_{marble} = \frac{2}{5} m r_{size}^2$$

Assume no slip

$$I_{roll} = I_{marble} + m d_{contact}^2$$

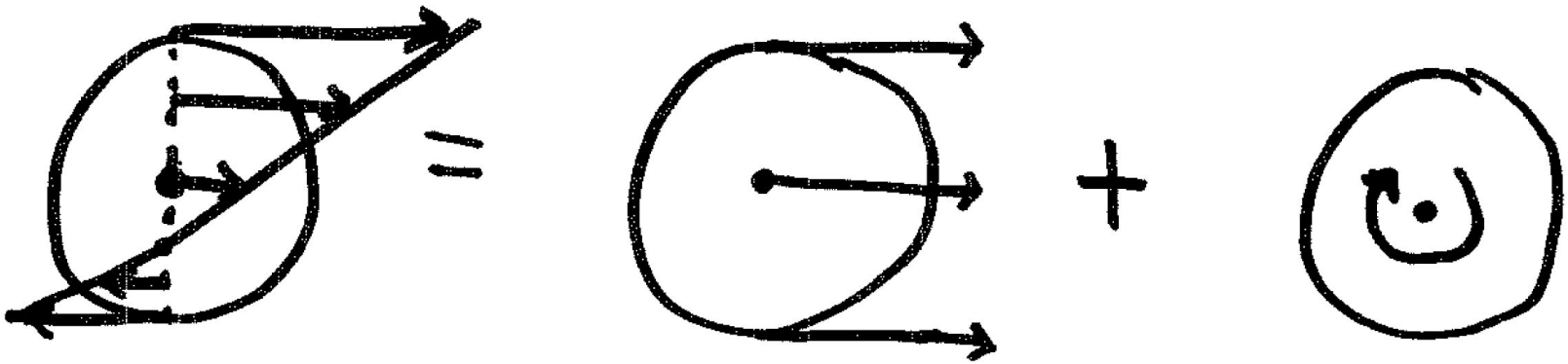
$$= m\left(\frac{2}{5} r^2 + d_{contact}^2\right)$$

$$m = \frac{W}{g} = \frac{0.6\,oz}{16\,oz/lb_m}\left(\frac{1\,lb_f\,s^2}{32.2\,lb_m\,ft}\right)$$

$W = 0.6\,oz \therefore m = 0.00116\ \frac{lb_f \cdot s^2}{ft}$

$$I_{roll} = \left(1.16E\text{-}3\ \frac{lb_f\,s^2}{ft}\right)\left[\frac{2}{5}\left(\frac{0.4605}{12}\,ft\right)^2 + \left(\frac{0.3867}{12}\,ft\right)^2\right]$$

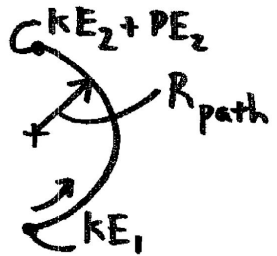
$$I_{roll} = 1.896E\text{-}6\ lb_f \cdot ft \cdot s^2$$



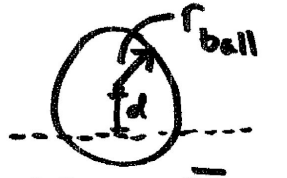
$$KE_1 = KE_2 + PE_2$$

$$KE_1 = mgh = \frac{1}{2}mv^2$$

$$KE_1 = gh = \frac{1}{2}v^2$$



$$M = Fd = I_0\alpha = I_0$$



$$\omega_{bot} = \frac{v_{bot}}{d}$$

$$a_{top} = \frac{v_{top}^2}{R_{path}} \Rightarrow v_{top} = \sqrt{gR_{path}}$$

$$\omega_{top} = \frac{v_{top}}{d} \qquad I_0 = \frac{2}{5}mr^2 + md^2$$

$$KE_2 = \frac{1}{2}mv_{top}^2 + \frac{1}{2}I_0\omega_{top}^2$$

$$= \frac{1}{2}mv_{top}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2 + md^2\right)\left(\frac{v_{top}}{d}\right)$$

$$= \frac{1}{2}mgR\left(2 + \frac{0.4r_{ball}^2}{d^2}\right)$$

$$PE_2 = mg2R_{path} + M_{spin}$$

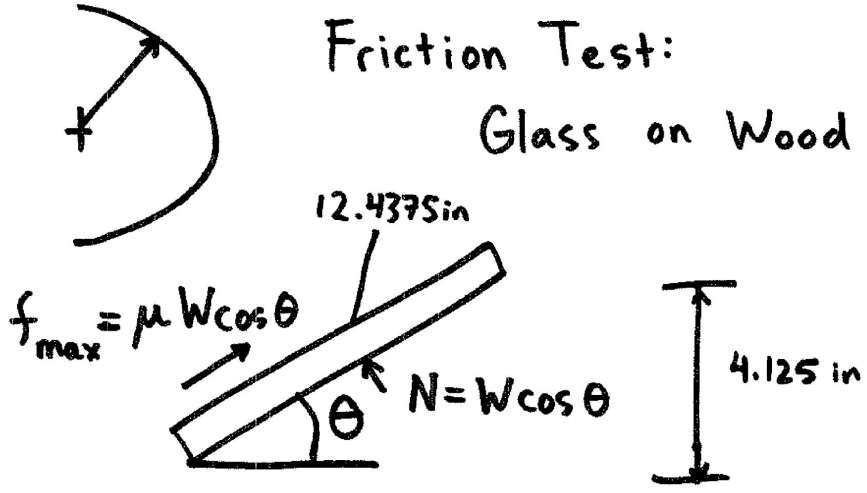
$$KE_1 = \frac{1}{2}mv_{bottom}^2 + \frac{1}{2}I_0\omega_{bot}^2$$

$$= \frac{1}{2}mv_{bottom}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2 + md^2\right)\left(\frac{v_{bottom}}{d}\right)$$

$$= \frac{1}{2}mv_{bot}^2\left(1 + \frac{0.4r^2}{d^2} + 1\right)$$

Friction Test:

Glass on Wood



$$\mu_s \text{ glass wood} = \tan\left[\sin^{-1}\left(\frac{4.125\text{ in}}{12.4375\text{ in}}\right)\right]$$

$$\mu_s = 0.352$$

$\mu_s = 0.352$ $\quad U_{bot} = m\theta = f_f\, d_{contact}\, \Phi_{path}\, R$ $\quad \Phi_{path} R = d\theta_{marble}$

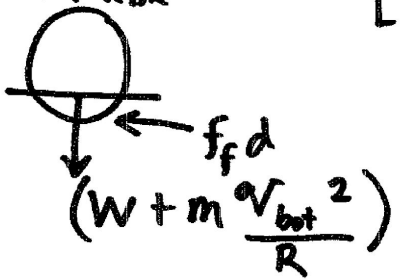
$PE_{bot} = f_f\, \Phi_{path}\, R = \mu_s m\left(g + \frac{v_{bot}^2}{R}\right)\Phi R$ $\quad \therefore \Phi = 180° = \pi\ rad$

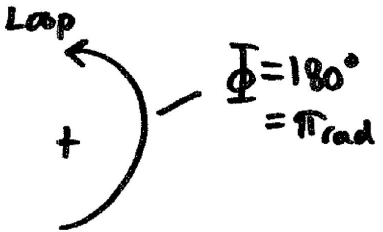
Bottom

$KE + PE = \frac{1}{2} m v_{bot}^2\left(2 + \frac{2r^2}{5d^2}\right) + \mu_s m\left(g + \frac{v_{bot}^2}{R}\right)\Phi R$

$= m\left[\frac{1}{2} v_{bot}^2\left(2 + \frac{2r^2}{5d^2}\right) + \mu_s \Phi\left(gR + v_{bot}^2\right)\right]$

FBD Marble





$m\left[\frac{1}{2} v_{bot}^2\left(2 + \frac{2r^2}{5d^2}\right) + \pi\mu_s\left(gR + v_{bot}^2\right)\right] = 2mgR + \frac{1}{2} mgR\left(2 + \frac{2r^2}{5d^2}\right)$

$$m\left[\frac{1}{2}v_{bot}^2\left(2+\frac{2r^2}{5d^2}\right)+\pi\mu_s(gR+v_{bot}^2)\right]=$$

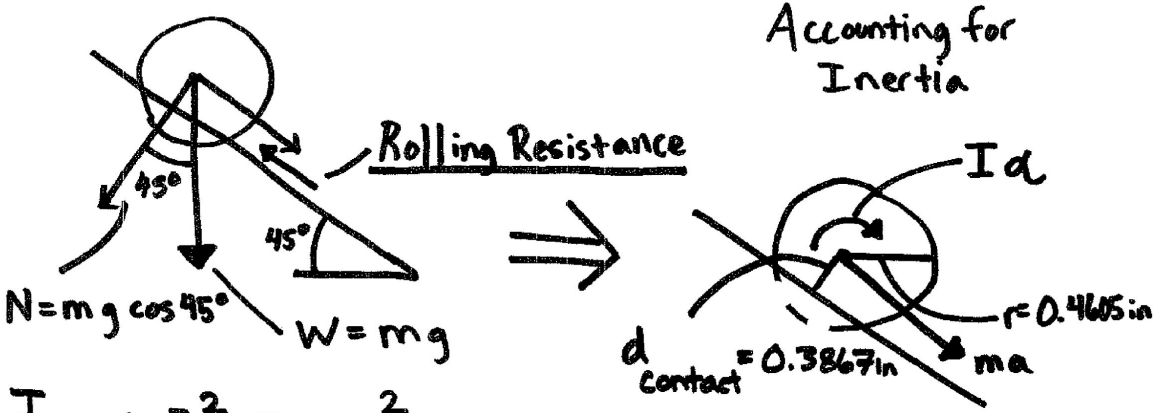
$$2mgR+\frac{1}{2}mgR\left(2+\frac{2r^2}{5d^2}\right)$$

$$v_{bot}^2\left(1+\pi\mu_s+\frac{r^2}{5d^2}\right)+\pi\mu_s gR=$$

$$2gR+\frac{1}{2}gR\left(2+\frac{2r^2}{5d^2}\right)$$

$$v_{bot}^2=\frac{2gR+gR\left(1+\frac{r^2}{5d^2}\right)-\pi\mu_s gR}{1+\pi\mu_s+\frac{r^2}{5d^2}}$$

$$v_{bot}=\sqrt{\frac{gR\left(3-\pi\mu_s+\frac{r^2}{5d^2}\right)}{1+\pi\mu_s+\frac{r^2}{5d^2}}}$$



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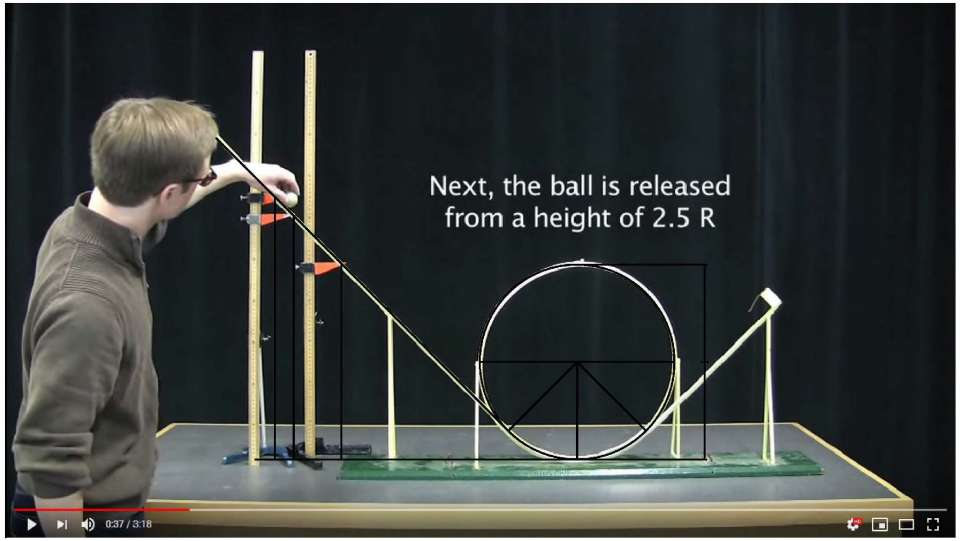
$$m a_{real} h = 2.7 m v^2$$

$$h = \frac{2.7 v^2}{a_{real}}$$

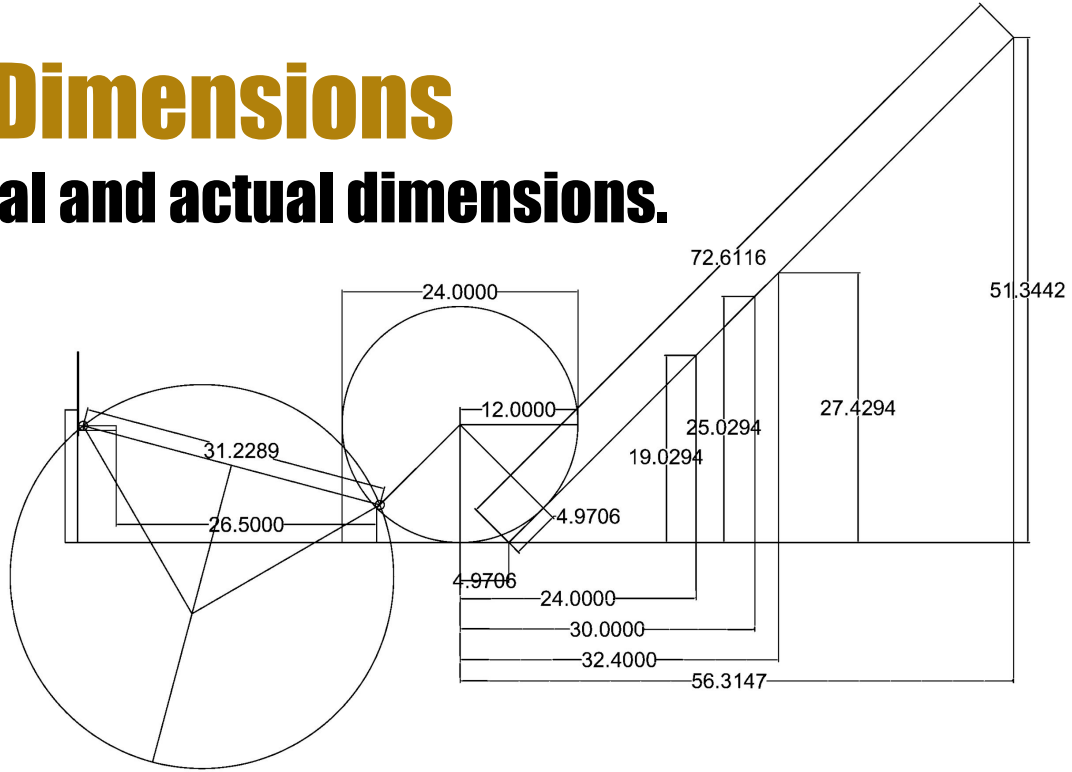
Design Based on MIT Youtube Video



Loop the Loop - YouTube
https://youtu.be/dA_UO86MjLY

General Dimensions

Shows general and actual dimensions.



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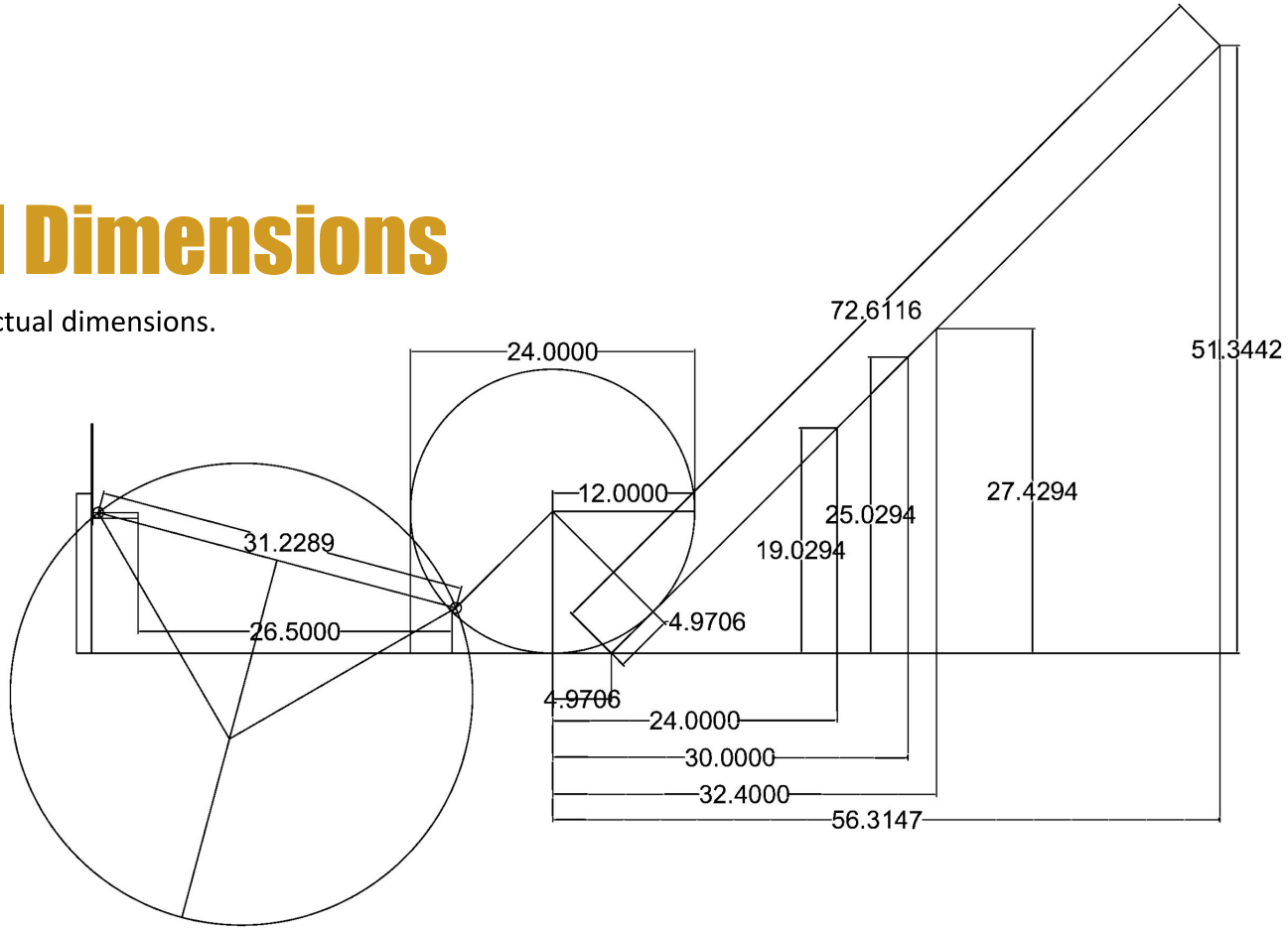
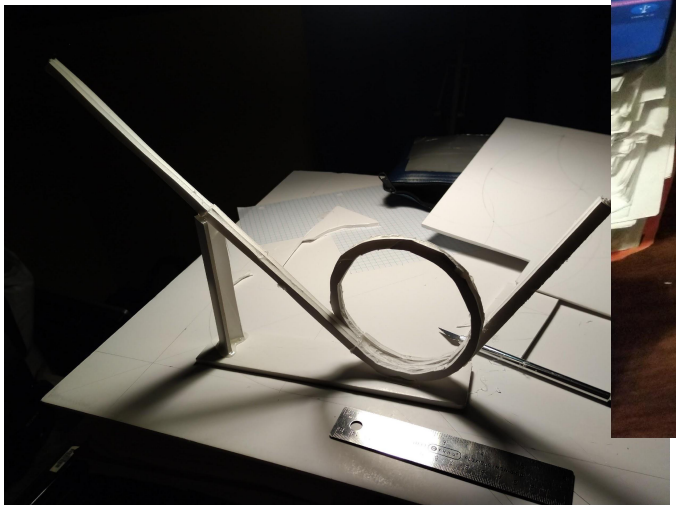
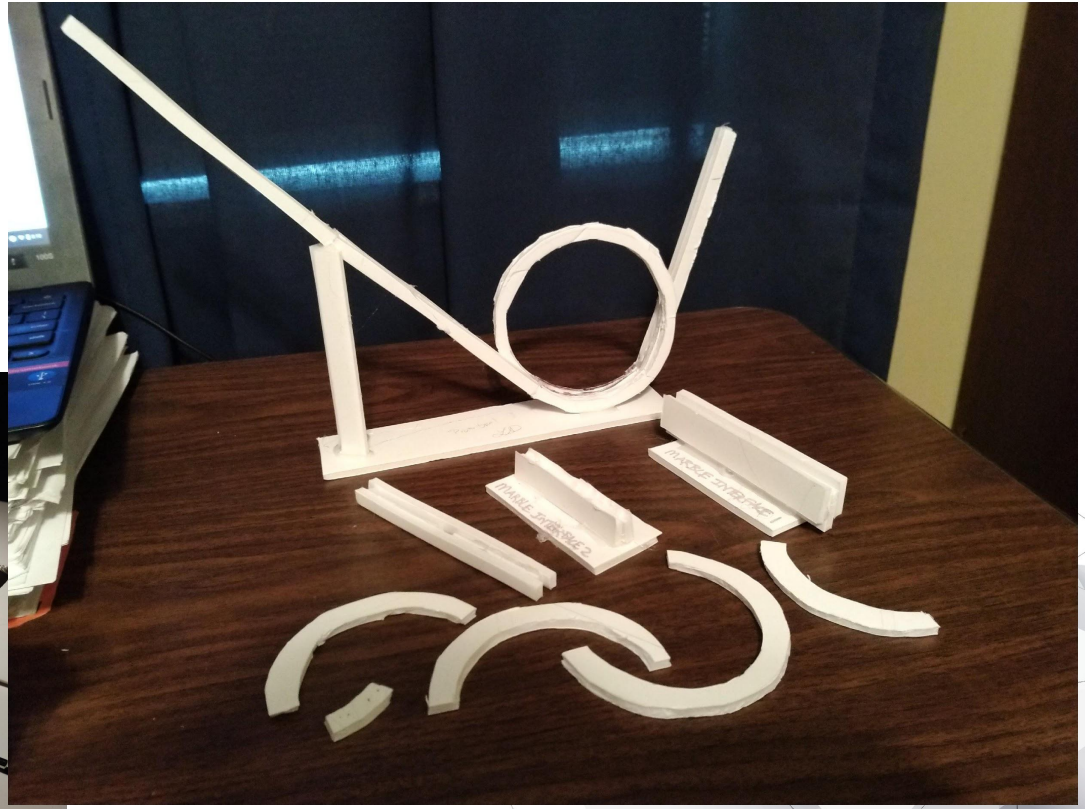


Scale Model

For team discussion





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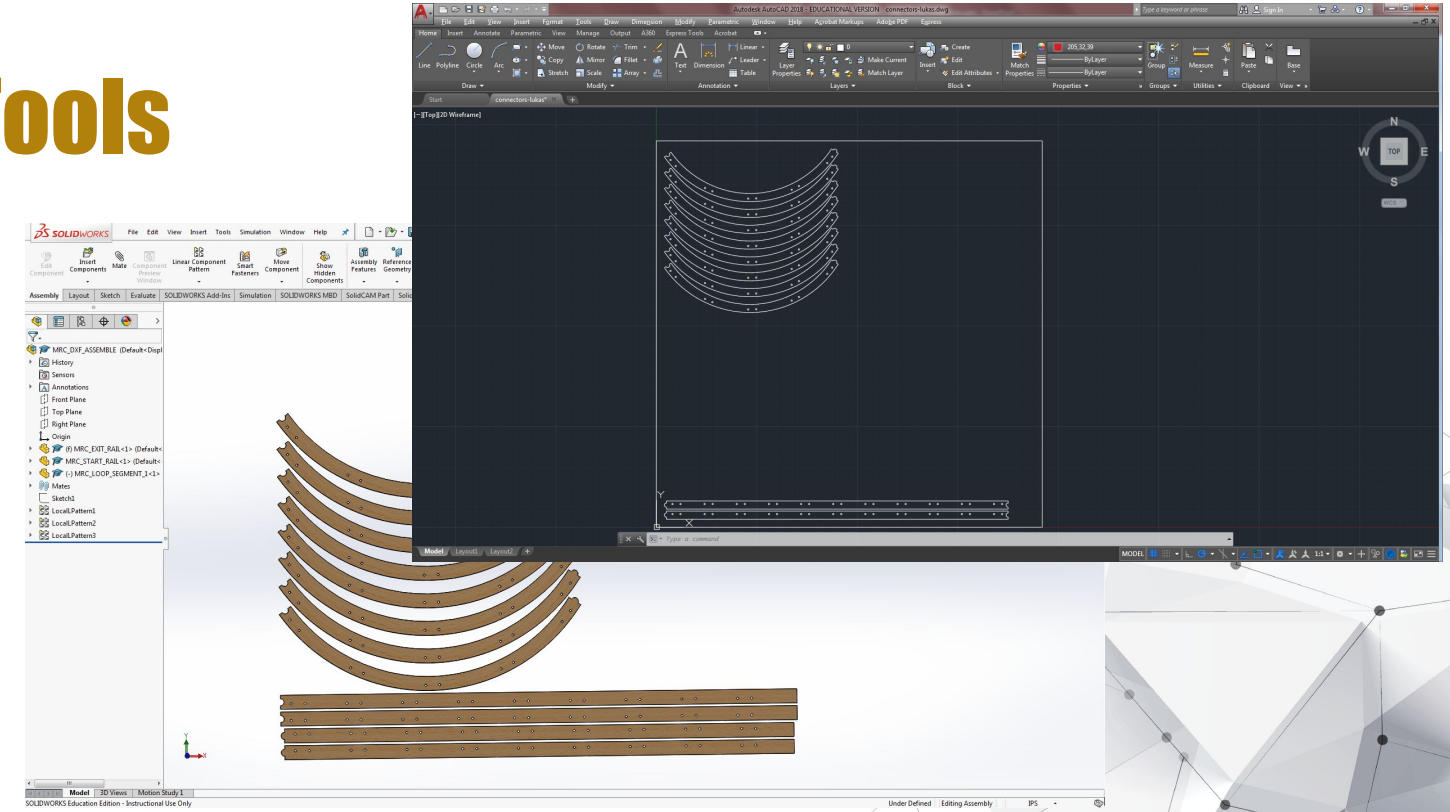
Design Tools

Autocad
Solidworks



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Build Photos

Cut out on a Laser
Assembled
Guide “Wires”
Tested

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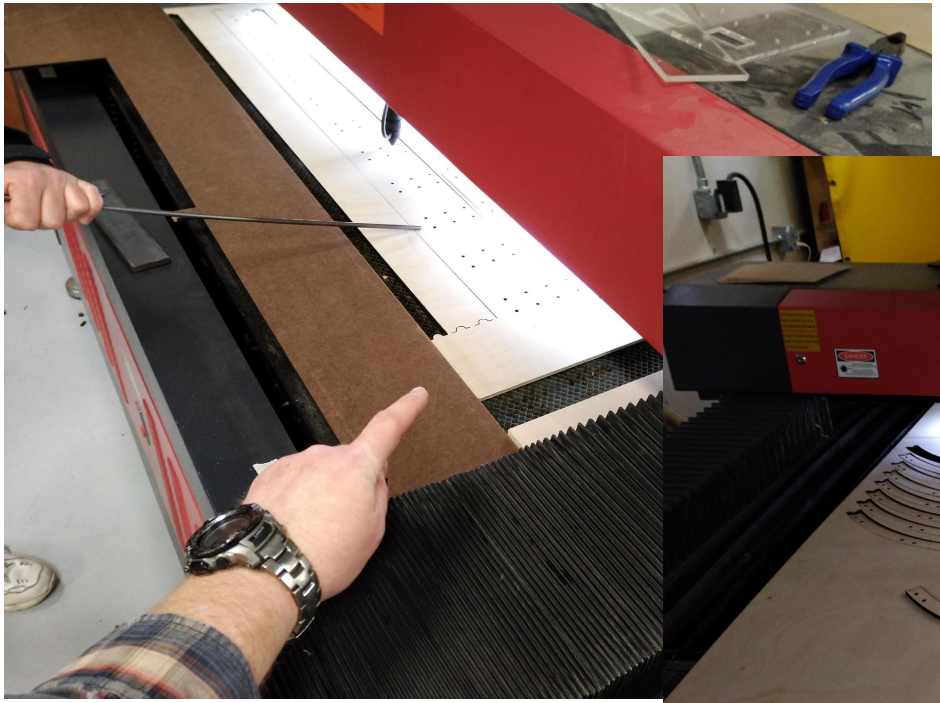

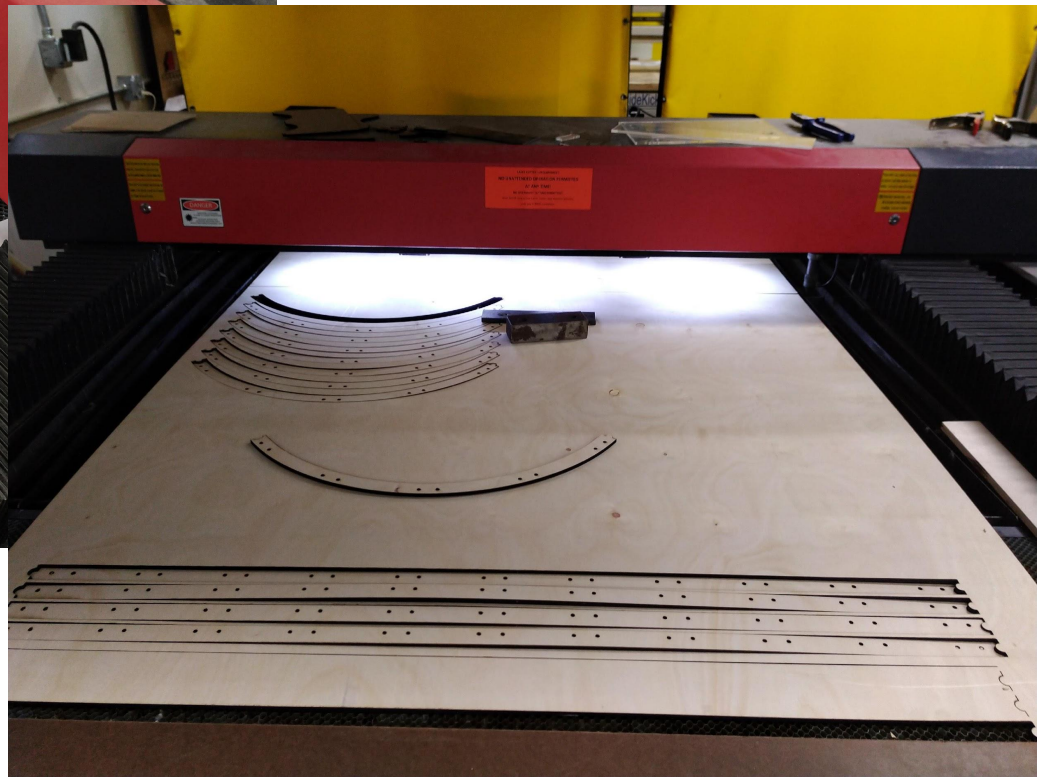

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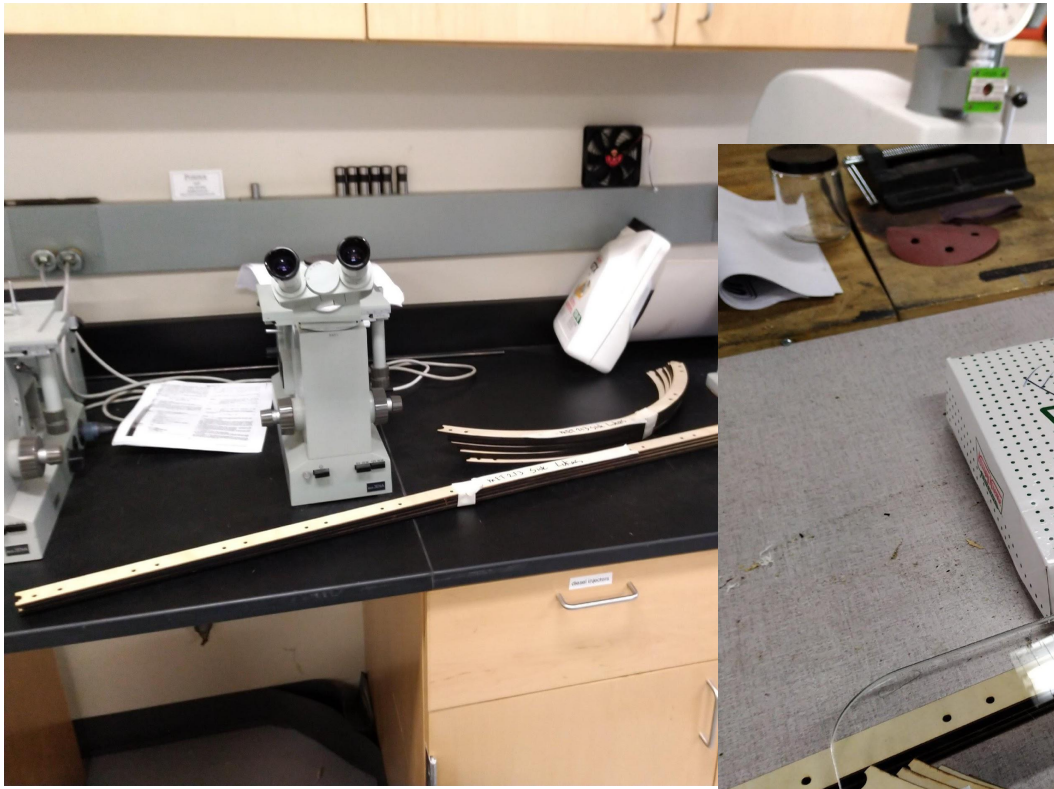

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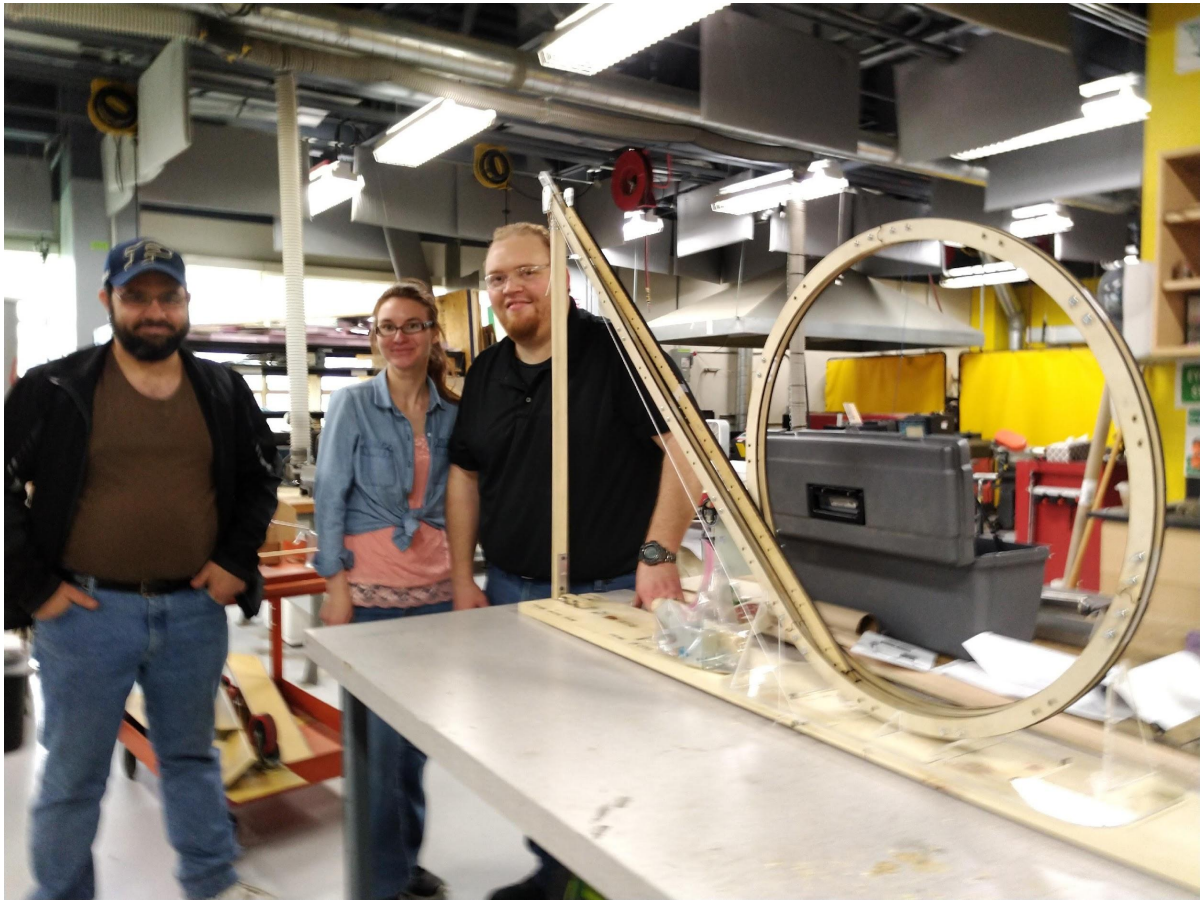

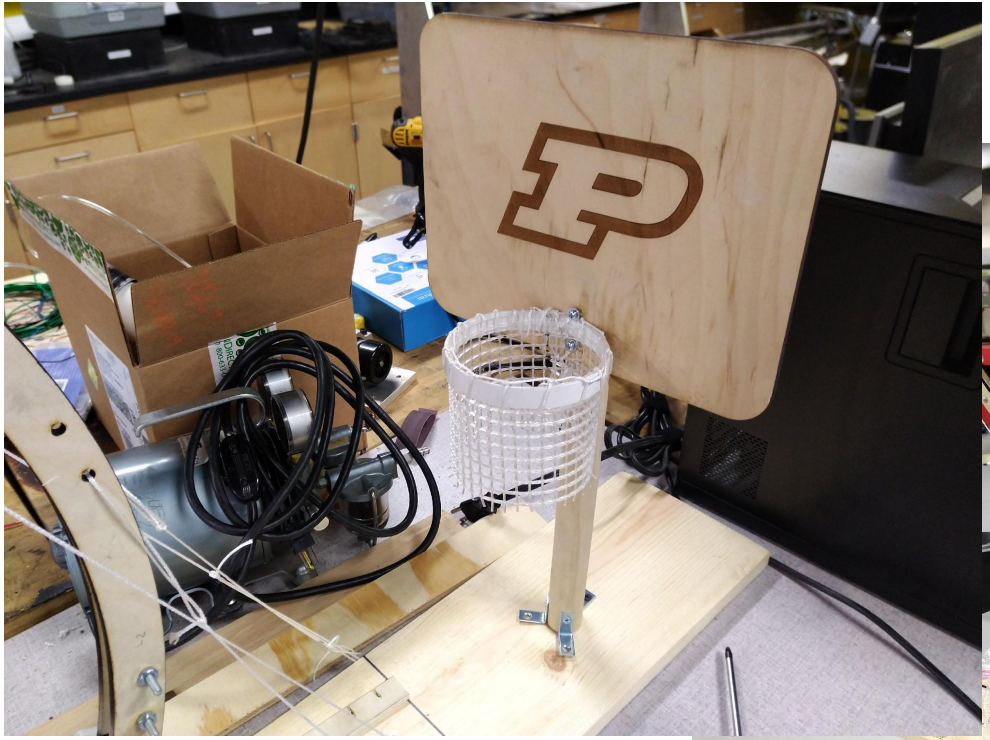

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Purdue MET 213 Dynamics Project Marble Roller Coaster - YouTube
https://www.youtube.com/watch?v=TO-KujscaWc

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