Control in Nonlinear Systems

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Abstract—This document contains the process to obtain a nonlinear system control design, where different types of controllers have to be designed and analyzed for the automation of the process to be controlled, such as regulators, trackers, smith predictor, and slider method.

Keywords; linear; system ; Mat Lab ; control; Control in Nonlinear Systems.

II. .INTRODUCTION

Control systems theory deals with the analysis and design of interacting components of a system in a configuration that provides the desired behavior. The essential configuration used in control systems theory is based on the fundamental concept of feedback, which consists of the process of measuring the variables of interest in the system and using that information to control its behavior. Control theory and practice have a wide range of applications in the fields of aeronautical, chemical, mechanical, environmental, civil, and electrical engineering, as well as in many other non-engineering disciplines. The benefits of efficient control in the industry are immense, including improvements in product quality, reduction in energy consumption, minimization of waste material, higher levels of safety, and pollution reduction. [1]

III. OBJECTIVES

GENERAL OBJECTIVES:

*Control a plant or non-linear system.

SPECIFIC OBJECTIVES.

* Design and analyze different types of controllers for the nonlinear system.

*Design continuous and discrete regulator controller for nonlinear plant

*Design continuous and discrete tracker controller for the nonlinear plant

* Design continuous and discrete predictor Smith controller for nonlinear plant

* Design of control with slider method

IV. THEORETICAL FRAMEWORK

In the 1950s, new tools capable of solving truly interesting problems from the control point of view began to emerge, nonlinear tools that attempted to extend results from linear systems theory to nonlinear systems, results such as those referring to controllability and observability of the system, thus becoming an important alternative to classical linear control. The methods for the design and analysis of control systems for nonlinear processes arise from the need to broaden the knowledge of the behavior of real dynamic systems, since, when obtaining a model that describes the dynamics of the system, some aspects are often omitted. in order to reduce inconveniences presented when applying some classic control technique, either to analyze a process or to design a system that controls it. On the path of designing control systems, we are faced in the first instance with how we should mathematically represent the dynamics of the process to be controlled, this is where the first obstacle could arise, as the vast majority of systems in the industry already know. real-life have non-linear characteristics, opting for linearization of the system around an equilibrium point would be the most obvious solution, but we would obtain limited knowledge of the real system since we would only be analyzing the system around an operating point, but it turns out that the nonlinear systems present multiple isolated equilibrium points.[2]

A solution was proposed by Alexander Mikhailovich Lyapunov in Lyapunov's Stability Theory, based on this theory nonlinear control techniques arise for linear and nonlinear systems, Sliding Mode Control and Adaptive Control. are two of the most relevant control techniques, and with them, it is possible to start from a nonlinear model of the process to design a nonlinear controller. [Khalil, H. K.], [Slotine, J. J. E., 1991] In addition to these techniques, in the 1980s Isidori demonstrated that it was not only possible to extend the results of linear systems theory to nonlinear systems, but also managed to demonstrate the possibility of extending the geometric control theory of linear systems under the use of a set of useful tools in the treatment of linear systems, one of these most important tools is Feedback Linearization. [Isidori, A.][2]

The Jacobian matrix collects all first-order partial derivatives of a multivariate function. Specifically, consider first a function that maps u real inputs, to a single real output:

$$f : \mathbb{R}^u \rightarrow \mathbb{R}$$

Then, for an input vector, x, of length, u, the Jacobian vector of size, u × 1, can be defined as follows:

$$\mathbf{J} = \frac{df(x)}{dx} = \left[\frac{\partial f(x)}{\partial x_1} \dots \frac{\partial f(x)}{\partial x_u} \right]$$

Now, consider another function that maps u real inputs, to v real outputs:

$$\mathbf{f} : \mathbb{R}^u \rightarrow \mathbb{R}^v$$

Then, for the same input vector, x, of length, u, the Jacobian is now a v × u matrix, J ∈ ℝv×u, that is defined as follows:

$$\mathbf{J} = \frac{d\mathbf{f}(\mathbf{x})}{d\mathbf{x}} = \left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_1} \dots \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_u} \right] = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_u} \\ \vdots & & \vdots \\ \frac{\partial f_v(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_v(\mathbf{x})}{\partial x_u} \end{bmatrix}$$

Reframing the Jacobian matrix into the machine learning problem considered earlier, while retaining the same number of u real inputs and v real outputs, we find that this matrix would contain the following partial derivatives [4]:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial z_k^{(1)}}{\partial w_k^{(1)}} & \cdots & \frac{\partial z_k^{(1)}}{\partial w_k^{(u)}} \\ \vdots & & \vdots \\ \frac{\partial z_k^{(v)}}{\partial w_k^{(1)}} & \cdots & \frac{\partial z_k^{(v)}}{\partial w_k^{(u)}} \end{bmatrix}$$

V. DESCRIPTION OF THE PROBLEM

The Plant or Nonlinear Liquid Level System in a set of tanks arranged in a cascade as shown in the following figure 1.

Modelo 15: Sistema de nivel de líquido en un conjunto de tanques dispuestos en cascada
Considere el problema general de controlar la altura del líquido en el último tanque T_n, de una serie de n tanques idénticos y no interactuantes, cuya entrada $u(t)$ está representada por el flujo (no negativo), $u \geq 0$, entregado al primer tanque y la salida está constituida por la altura del líquido en el n-ésimo tanque. Si designamos por x_i la altura en el i-ésimo tanque, el modelo dinámico que describe el sistema es el siguiente:

$$\begin{aligned} \dot{x}_1 &= -\frac{c}{A}\sqrt{x_1} + \frac{1}{A}u \\ \dot{x}_i &= -\frac{c}{A}\sqrt{x_i} + \frac{c}{A}\sqrt{x_{i-1}} \;;\; i = 2, 3, \dots, n \\ y &= x_n \end{aligned} \tag{1.34}$$

donde c es una constante que representa la resistencia a la salida de líquido y A es el área de la base de cualquiera de los tanques.

Ⓜ

Figure 1. Model of liquid level system in a set of tanks arranged in cascade [4]

For this case to be analyzed, it was analyzed with three tanks arranged in cascade, with selected or assumed input and selected or assumed constants and the output to be found, the third tank was selected, so that the equation of the liquid level system in a set of three tanks arranged in a shell is as follows:

$$\begin{aligned} \dot{x}_1 &= -\frac{c}{A}\sqrt{x_1} + \frac{1}{A}u \\ \dot{x}_2 &= -\frac{c}{A}\sqrt{x_2} + \frac{c}{A}\sqrt{x_1} \\ \dot{x}_3 &= -\frac{c}{A}\sqrt{x_3} + \frac{c}{A}\sqrt{x_2} \\ y &= x_3 \end{aligned}$$

Figure 2. Model of liquid level system in a set of three tanks arranged in cascade

VI. RESULT AND ANALYSIS

To observe the nonlinearity of the plant, we proceed to make the plant in the Simulink program, using the corresponding block that allows us to do it with the fcn block and other blocks such as constat, mux, integrator, etc. Taking as constants A=10 and C=5.

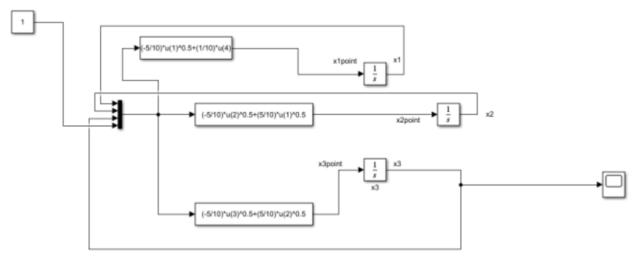

Figure 3. Model of liquid level system in a set of three tanks arranged in cascade made in Simulink

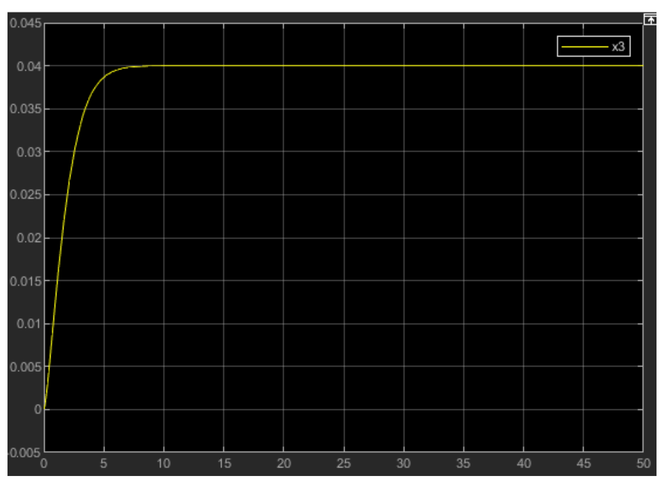

Figure 4. Graphic of the liquid level system model in a set of three tanks arranged in cascade made in Simulink

Observing the previous figure 4 at the moment of giving it an input value u=1, this non-linear plant at its output does not result in the input value, but it returns a value of 0.04, so this non-linear plant needs to perform control.

But to confirm its non-linearity, its input is a sine signal

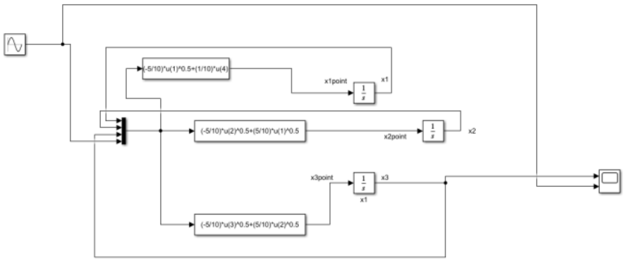

Figure 5. Model of liquid level system in a set of three tanks arranged in cascade made in Simulink with sine input

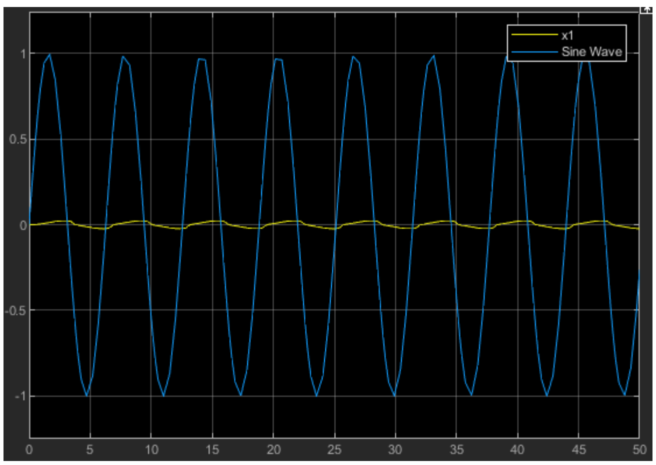

Figure 6. Graphic of the liquid level system model in a set of three tanks arranged in cascade made in Simulink with sine input

VII. CODE IN MATLAB

```
%% Definición de constantes del sistema , Definition of system constants
A=10;
C=5;
```

The constants to be used in the liquid level system are defined in a set of three tanks arranged in cascade, which within these are the variable A and the variable C.

```
%% Puntos de equilibrio ,Balance points
syms x1 x2 x3 u
%Con u=1 , with u=1
x1punto = (-C/A)*x1^0.5 + (1/A);
x2punto = (-C/A)*x2^0.5 + (C/A)*x1^0.5;
x3punto = (-C/A)*x3^0.5 + (C/A)*x2^0.5;
%S = solve(eqn,var) solves the equation eqn for the variable var.

x1e = solve(x1punto==0,x1);%S = solve(eqn,var)
x2e = solve(x2punto==0,x2);%S = solve(eqn,var)
x3e = solve(x3punto==0,x3);%S = solve(eqn,var)

x1e,x2e,x3e
```

Define the equilibrium points to be treated using the Matlab 'solve' function makes the function equal to 0 and thus clears the unknown to be found of x1, x2, and x3.

```
%% Linealizacion del sistema no lineal,Linearization of the nonlinear system
syms x1 x2 x3 u
aj=jacobian([(-C/A)*x1^0.5 + (1/A)*u;(-C/A)*x2^0.5 + (C/A)*x1^0.5;(-C/A)*x3^0.5 + (C/A)*x2^0.5],[x1,x2,x3]);
bj=jacobian([(-C/A)*x1^0.5 + (1/A)*u;(-C/A)*x2^0.5 + (C/A)*x1^0.5;(-C/A)*x3^0.5 + (C/A)*x2^0.5],[u]);

x1 = 1/25; %Valor de x1e ,Value of  x1e
x2 = 1/25; %Valor x2e,Value  of  x1e
x3 = 1/25; %Valor de  x3e,Value of x1e

a=eval(aj);
b=bj;
b = [1/10;0;0];
c=[0,0,1];
d=0;

a,b,c,d
```

Linearization of the plant is performed through the Jacobian arrays in which the previously found equilibrium parts were evaluated.

```
%% Contrabilidad y observabilidad del sistema ,Controllability and observability of the system
 co=ctrb(a,b)
 rangoco=rank(co);
 if rangoco == length(a)
     disp ('el sistema si es controlable')
 else
     disp('el sistema no es controlable')
 end

  ob=obsv(a,c)
 rangoob=rank(ob);
 if rangoco == length(a)
     disp ('el sistema si es observable')
 else
     disp('el sistema no es observable')
 end
```

In this part, it is necessary to know if the plant to be treated is controllable or not and if it is observable or not, these two factors are very important in the controllers to know what type of controller is needed or not.

```
%% Regulador  en  tiempo continuo ,Continuous time regulator
pd = [-10,-11,-12];
k = place(a,b,pd);
```

Specify the appropriate poles for the regulator to improve the system reaction time; these poles must be negative and far away from the center. Finally, the algorithm generates the proportional gain variable k.

```
%% Seguidor en tiempo continuo ,Continuous time tracker
aa=[a,zeros(3,1);-c,0];
ba=[b;0];

pda=[pd,-9];

kt=place(aa,ba,pda);
kp=kt(1:1,1:3);
ki=kt(1:1,4:4);
```

The augmented matrix is produced, and the necessary poles, as well as another pole that must be increased because the system moved up in order, are given. Finally, the program provides us with the integral and proportional gains in the variable kt.

```
%% Observador,Observer
po=[-20,-21,-22]
h=place(a',c',po);
h=h'
```

```
%% Discretizacion,Discretization
tm=0.1; % sampling time ,tiempo de muestreo
[adis,bdis,cdis,ddis]=c2dm(a,b,c,d,tm,'zoh');

pololdisdes=exp(pd(1)*tm);
polo2disdes=exp(pd(2)*tm);
polo3disdes=exp(pd(3)*tm);
PoloAdisdes = exp(pda(4)*tm);

polosdiscredes=[pololdisdes,polo2disdes,polo3disdes];
polosdiscredesaum=[polosdiscredes,PoloAdisdes];
```

The observer employs the same mechanism as the regulator, except that the poles are moved further away from 0 to increase the observer's response time and guarantee it is faster than the plant's. The sample time and discretization approach must be carefully chosen in the discretization portion of the procedure; for the time of sampling is 0.1 and the discretization method Zero-order hold because is one method very appropriate for this case of discretization.

```
%% Regulador en tiempo discreto ,Discrete-time regulator tracker
kdis = place(adis,bdis,polosdiscredes);
```

```
%% Seguidor en tiempo discreto ,Discrete-time tracker
%aadis=[adis,bdis;zeros(1,3),0];
%badis=[zeros(3,1);1]
aadis=[adis,zeros(3,1);-cdis,1]
badis=[bdis;0]
ktdis=place(aadis,badis,polosdiscredesaum);
kpdis=ktdis(:,1:3);
kidis=ktdis(:,4:4);
```

It is the same process that was carried out for a continuous time, unlike the fact that it is carried out for a discrete-time, so it takes the plant in discrete in the cases of the regulator and tracker, the commented part is a second way to carry out the tracker.

```
%% Observador en tiempo discreto,Discrete time observer
podis=[exp(-20*tm),exp(-21*tm),exp(-22*tm)];
hdis=place(adis',cdis',podis);
hdis=hdis';

%time of retarded =1;
```

Because the plant is taken independently, the discrete observer is computed, which is identical to the continuous observer in its computation. And the time of retarded is of 1.

VIII. CONTROLLER DESIGN

For said non-linear plant, the respective controllers were carried out

DESING OF REGULATING CONTROLLERS

The regulator controller is responsible for sending the signal to the system's equilibrium point, which is typically zero. This controller must set the starting circumstances because it lacks an input. To do this, we will design many types of regulatory controllers for this non-linear system.

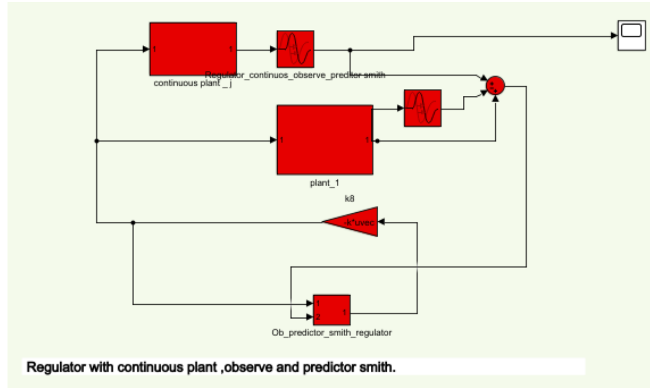

Figure 7. Block diagram of Regulator with continuous plant, observe, and predictor smith.

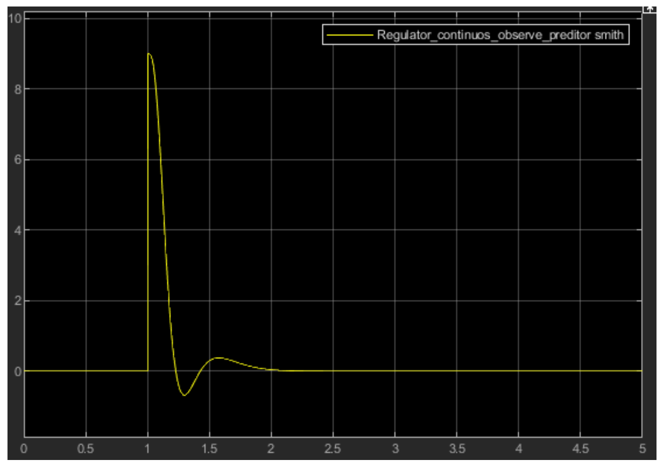

Figure 8. Graph of Regulator with continuous plant, observe, and predictor smith.

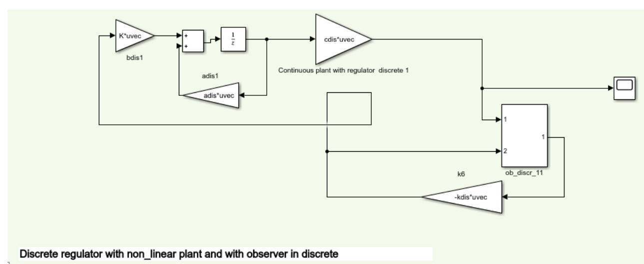

Figure 9. Block diagram of Discrete regulator with non_linear plant and with the observer in discrete

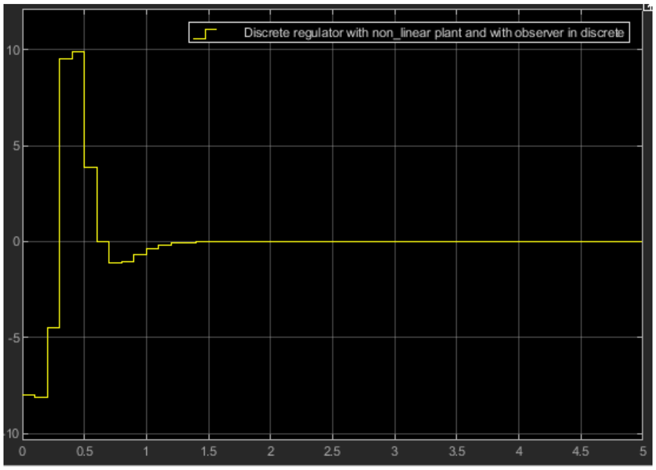

Figure 10. Graph of Discrete regulator with non_linear plant and with the observer in discrete

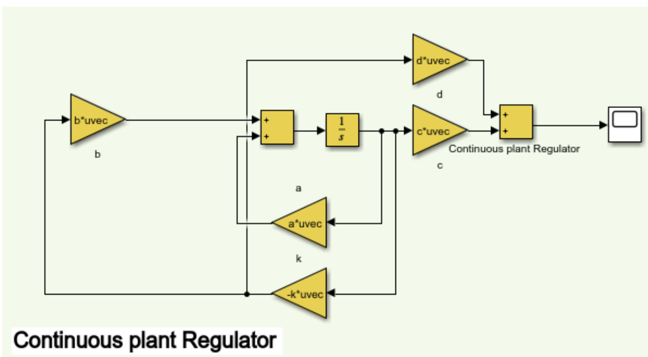

Figure 11. Block diagram of Continuous Plant Regulator

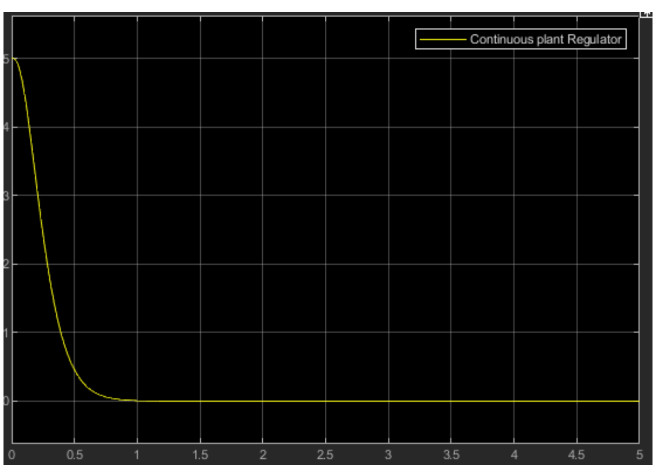

Figure 12. Graph of Continuous Plant Regulator

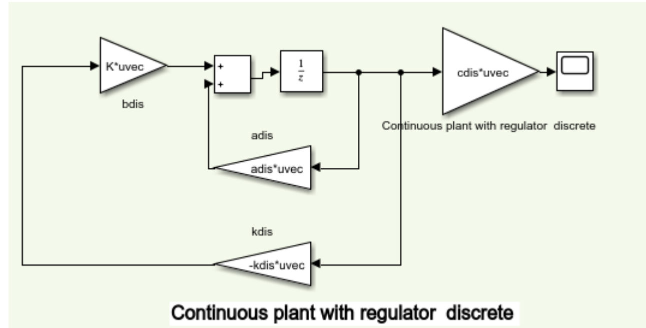

Figure 13. Block diagram of Continuous plant with regulator discrete

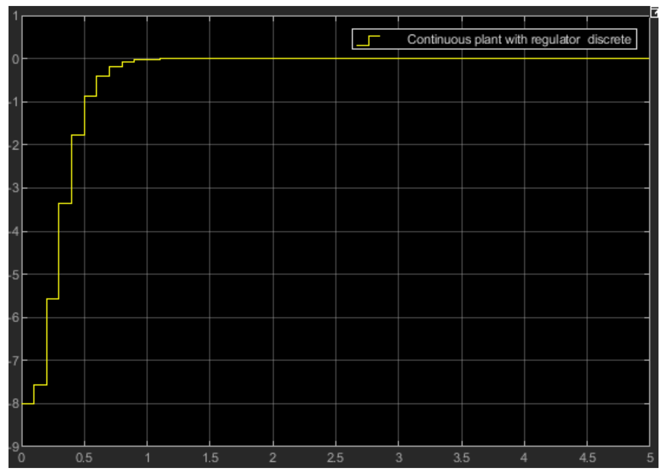

Figure 14. Graph of Continuous plant with regulator discrete

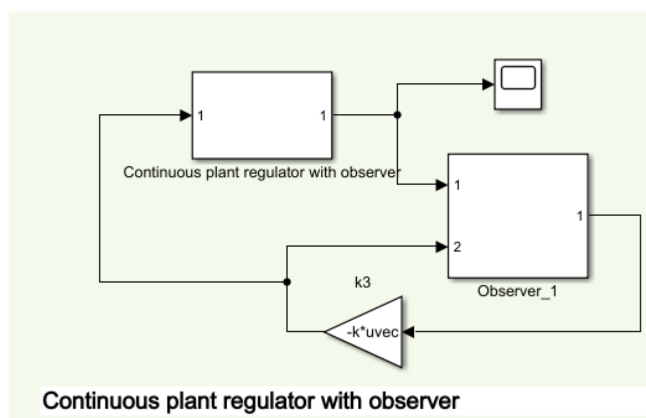

Figure 15. Block diagram of Continuous plant regulator with observer

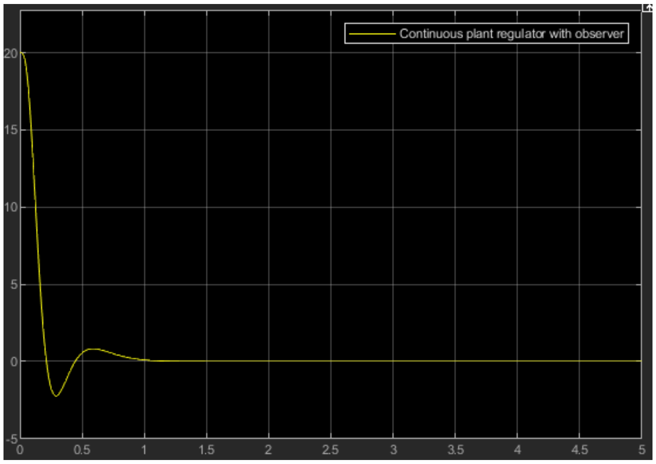

Figure 16. Graph of Continuous plant regulator with observer

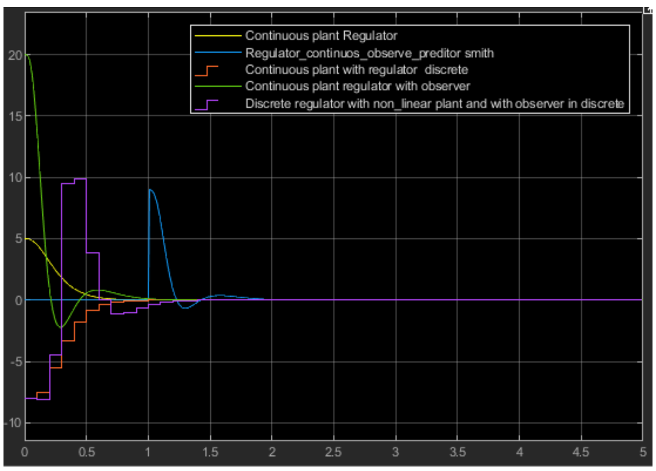

Figure 17. Graph of all regulating controllers

As mentioned above, the regulator controllers are responsible for sending the signal to the balance point of the system, observing graph 17 shows the operation of the regulators for plants or non-linear systems, and for this case in the liquid level system in a set of three tanks arranged in a cascade it is possible to use regulators in different ways depending on the desired stabilization time.

DESING TRACKERS CONTROLLERS

Tracking controllers are well-known and widely used around the world, as well as in a range of industrial applications, to set a system on a value and, as the name implies, to make the system follow an input. But first, it should be mentioned that at the time of taking data from the plant there may be a delay and this must be defined, so if that delay is not taken the following could happen.

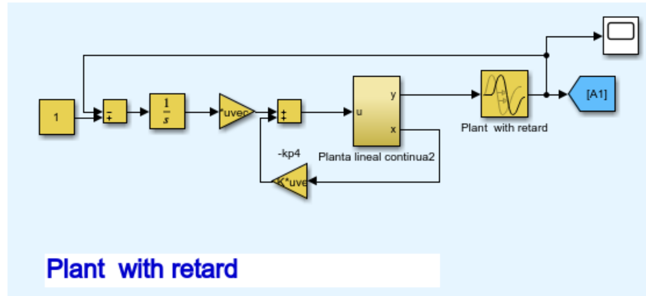

Figure 18. Block diagram of Plant with retard

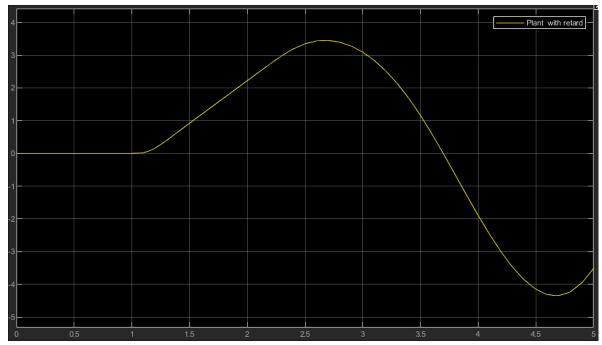

Figure 19. Graph of Plant with retard

Now having this clear, the design of trackers controllers begins.

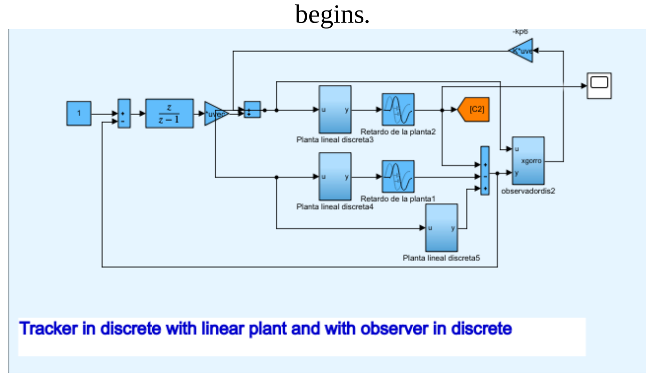

Figure 20. Block diagram of Tracker in discrete with linear plant and with the observer in discrete

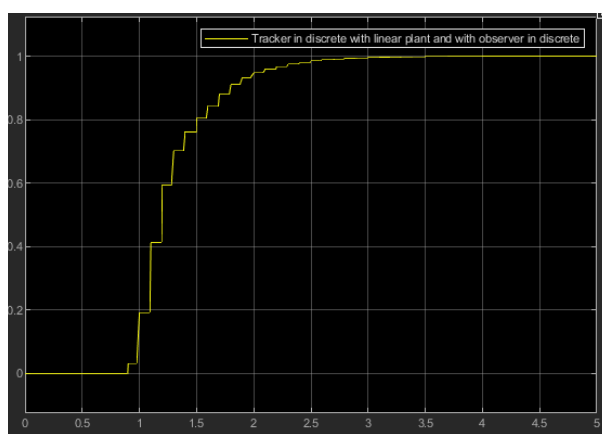

Figure 21. Graph of Tracker in discrete with linear plant and with the observer in discrete

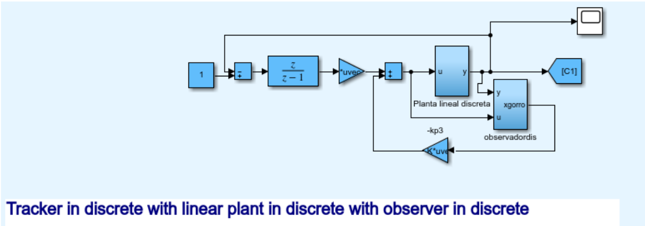

Figure 22. Block diagram of Tracker in discrete with linear plant in discrete with observer in discrete

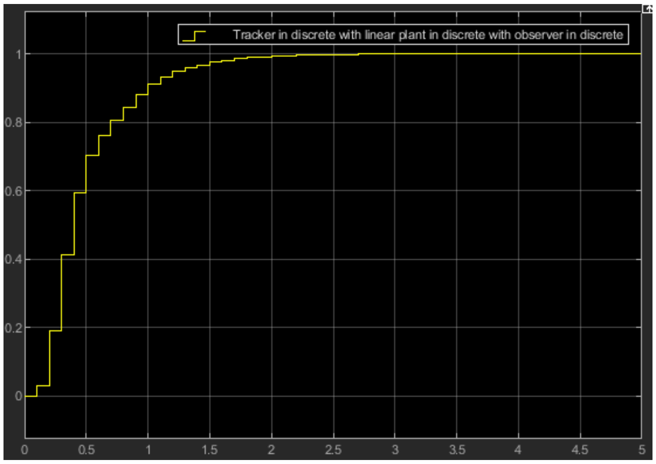

Figure 23. Graph of Tracker in discrete with linear plant in discrete with observer in discrete

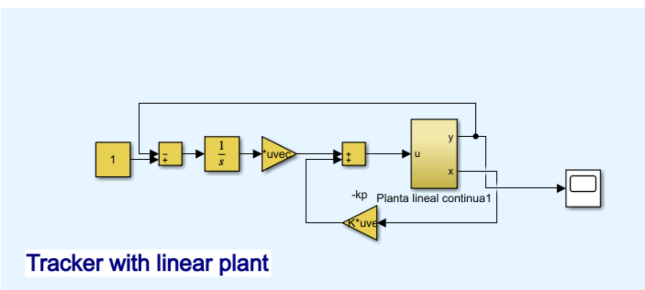

Figure 24. Block diagram of Tracker with linear plant

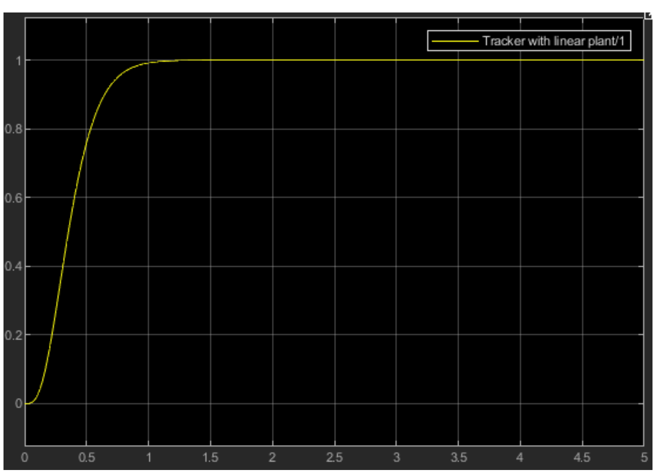

Figure 25. Graph of Tracker with linear plant

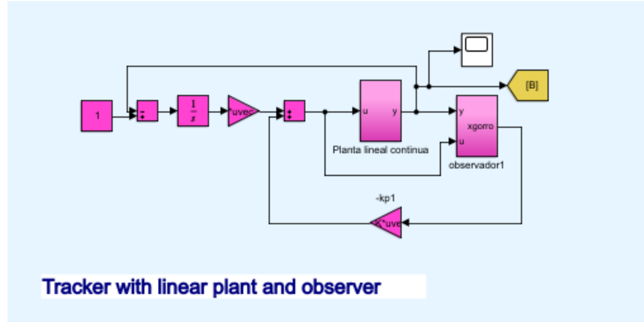

Figure 26. Block diagram of Tracker with linear plant and observer

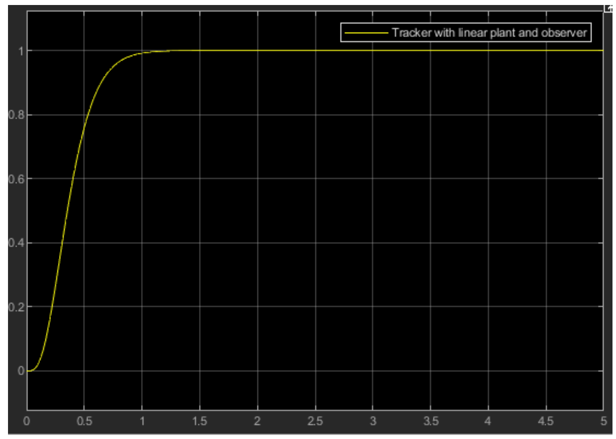

Figure 27. Graph of Tracker with linear plant and observer

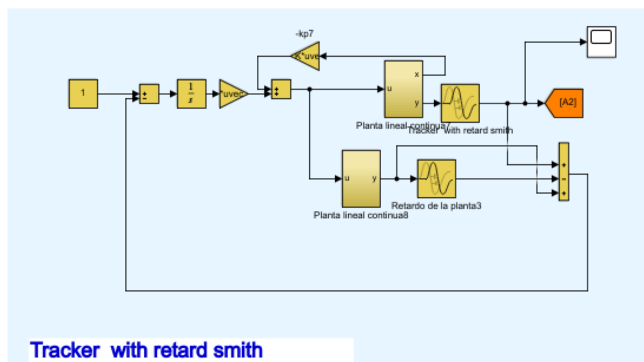

Figure 28. Block diagram of Tracker with retard smith

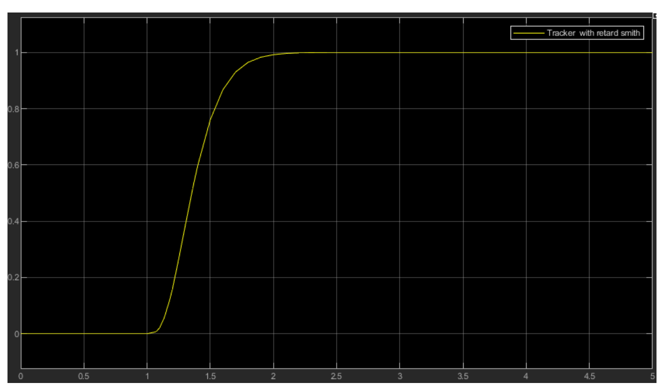

Figure 29. Graph of Tracker with retard smith

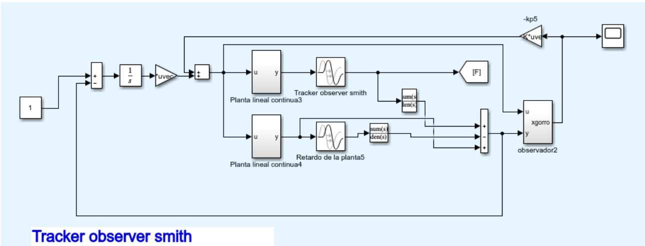

Figure 30. Block diagram of Tracker observer smith

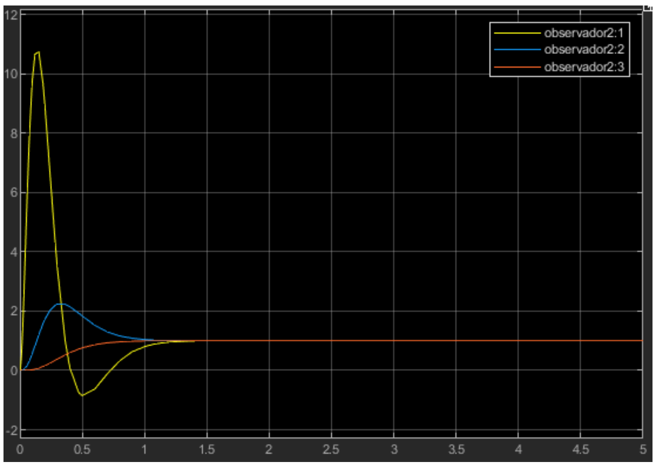

Figure 31. Graph of Tracker observer smith

In this figure 31 It is very peculiar because you can see the different responses that are being given to the states that are in turn controlled

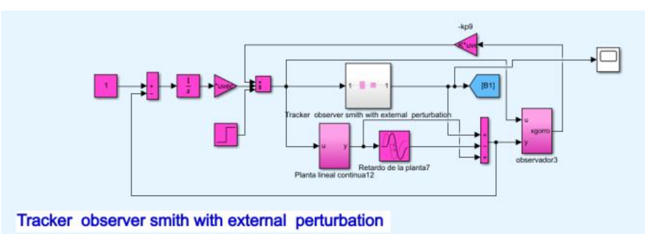

Figure 32. Block diagram of Tracker observer smith with external perturbation

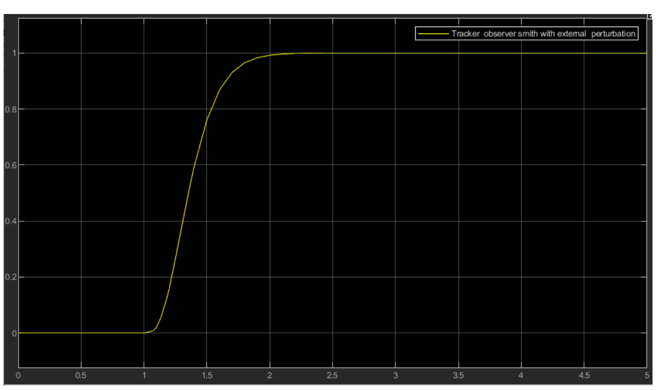

Figure 33. Graph of Tracker observer smith with external perturbation

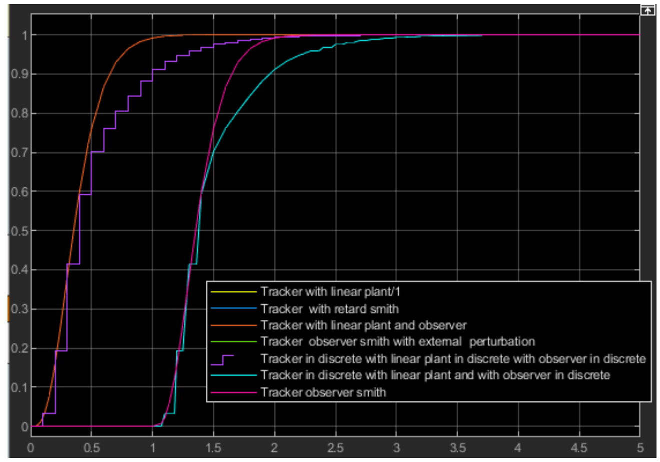

Figure 34. Graph of all trackers controllers

As mentioned above, the trackers' controllers are responsible for the value of the input is equal to the value of the output of the system, observing figure 34 of the graph, it is evident that the trackers work for plants or non-linear systems and this case in the system. of liquid level in a set of three tanks arranged in cascade, it is possible to use trackers in different ways depending on the desired stabilization time.

SLIDER METHOD

For greater ease in the calculations to demonstrate the slider method, it is chosen to put y=x1 at the output. The first thing is to find the relative degree to which a derivative is performed at the output

$$y = x_1$$

$$\dot{y} = \dot{x}_1$$

$$\dot{y} = \dot{x}_1 = -\frac{C}{A}\sqrt{x_1} + \frac{1}{A} * u$$

Since only one derivative is used to find where u was in the equation, then r=1, α1=1. After this process, S(e) must be found

$$S(e) = \sum_{i=-1}^{i-1} \alpha C i + 1) * e^{i}$$

$$S(e) = \sum_{i=-1}^{0} \alpha C i + 1) * e^{i}$$

$$S(e) = \alpha_0 e^{-1} + \alpha_1 e^{0}$$

Taking into account the rules

$$e^{-1} = \int_0^{\infty} e(t) * dt$$

$$e^{0} = e(t)$$

$$e^{1} = \frac{de(t)}{dt}$$

$$S(e) = \alpha_0 \int e(t) + e(t)$$

With which it is observed that S(e) is a sum of an integral and a proportional, that is, a PI

For the following, calculate the derivative of S(e)

$$\dot{S}(e) = \alpha_0 e + \dot{e}$$

With $e = y - reference$

$$\dot{e} = \dot{y}$$

$$\dot{e} = \dot{y} = \dot{x}_1 = -\frac{C}{A}\sqrt{x_1} + \frac{1}{A} * u$$

It is replaced in the $\dot{S}(e)$

$$\dot{S}(e) = \alpha_0 e + -\frac{C}{A}\sqrt{x_1} + \frac{1}{A} * u$$

We know that $\dot{S}(e) = -\eta * sign(s)$

$$\dot{S}(e) = \alpha_0 e + -\frac{C}{A}\sqrt{x_1} + \frac{1}{A} * u = -\eta * sign(s)$$

$$\alpha_0 e + -\frac{C}{A}\sqrt{x_1} + \frac{1}{A} * u = -\eta * sign(s)$$

We clear the value of u

$$u = \left(-\alpha_0 e + \frac{C}{A}\sqrt{x_1} - \eta * sign(s)\right) * A$$

So the block diagram is implemented in Simulink with

$\alpha_0 = 30$ and $\eta = 10$, α_0 *and* η can be any positive value

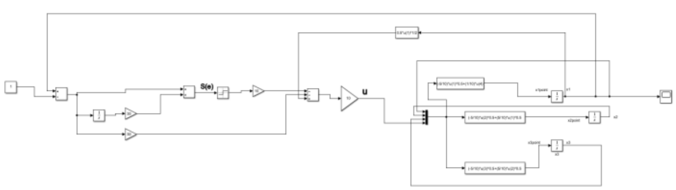

Figure 35. Block diagram of slider method with y=x1

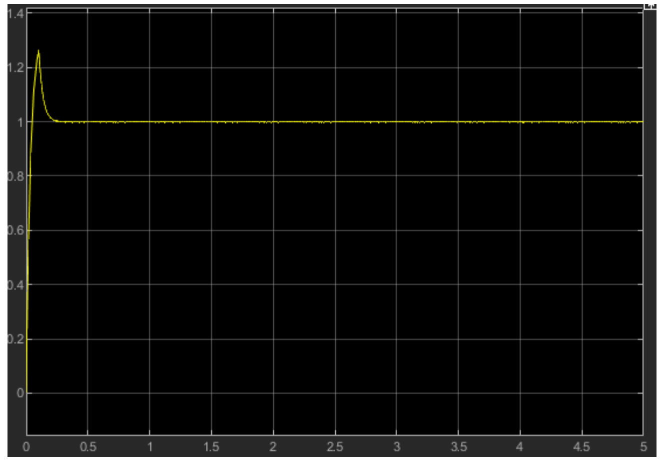

Figure 36. Graph of slider method with y=x1

Previously it was said that α0 n η can be any positive value for this, we will change α0=1 to see its result

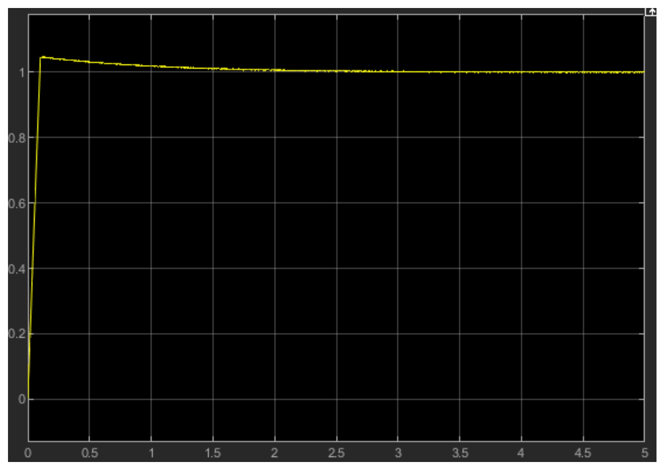

Figure 37. Graph of slider method α0=1

With α0=100

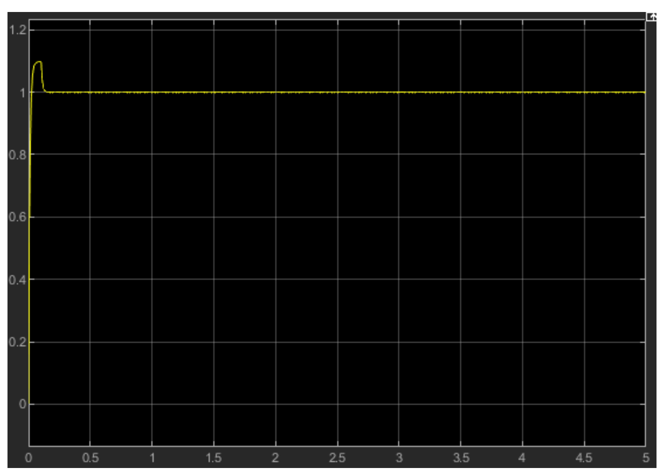

Figure 38. Graph of slider method α0=100

When analyzing the different values of α0 , it can be seen that the value that is put on α0 modifies the shadow impulse that the output will have.

Now changing the value of η=1, leaving α0=30

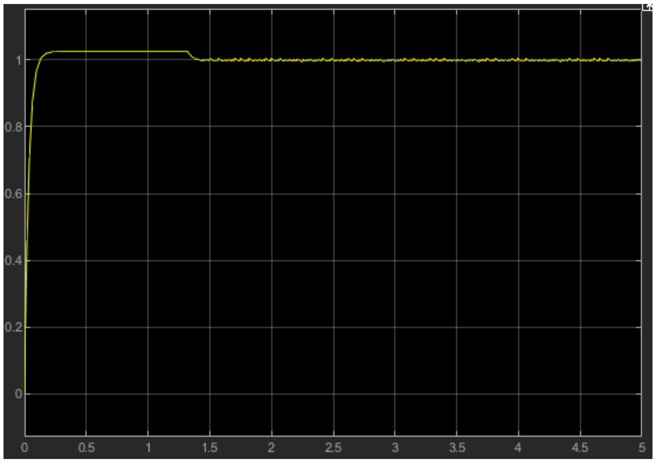

Figure 39. Graph of slider method η*= 1*

With η*= 100*

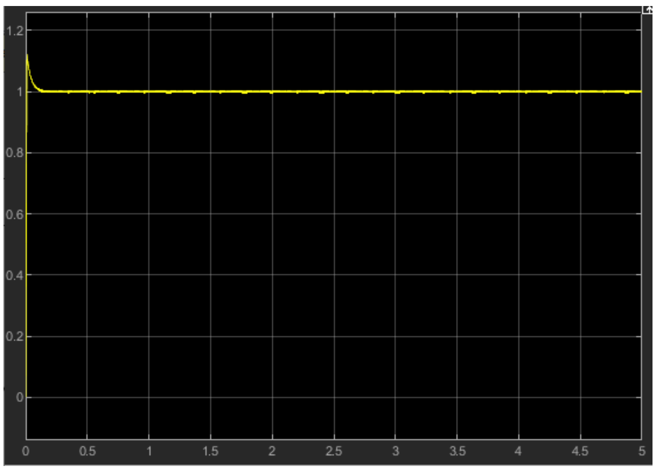

Figure 40. Graph of slider method η*= 100*

When analyzing the different values of η, it can be seen that the value set to η modifies the stabilization value that the output will have.

CONCLUTIONS

* For a non-linear system plant, it is possible to make regulators and trackers depending on it.

* There are different ways of making trackers for non-linear plants, the choice of the tracker that we choose will depend on the needs we have, such as the stabilization time and external disturbances that may occur.

* Realizing trackers controllers in continuous and discrete-time gives the possibility of implementation being a form of choice towards the costs that may occur in the manufacture of these controllers.

* When control is carried out towards the non-linear plant, the energy that is consumed must be taken into account, since this energy factor represents very important costs towards the implemented place.

* The regulator controllers as a control option towards the stability state for the situation that needs to be used, as is the case of satellites, for example.

* The delay time in non-linear plants is important data for its control, so if it is not taken into account, it can be difficult to carry out control.

*Sampling time is vital for systems where controllers are performed in the discrete time since when the time is varied, the control result will be different and may not be controlled.

* Trackers and regulators as effective and efficient when performed in non-linear plants and linear plants.

*In slider method the value set to η modifies the stabilization value that the output will have.

* In slider method the value that is put on α0 modifies the shadow impulse that the output will have.

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