

Introduction
ooooo

Entropy
o

GPs
oo

VI
oo

Theory
oooooooo

Experiments
ooooooo

Conclusion
o

Variational Inference for Inverse Reinforcement Learning with Gaussian Processes

Paulius Dilkas

8th April 2019



H. Kretzschmar, M. Spies, C. Sprunk and W. Burgard, "Socially compliant mobile robot navigation via inverse reinforcement learning", *I. J. Robotics Res.*, 2016



H. Kretzschmar, M. Spies, C. Sprunk and W. Burgard, "Socially compliant mobile robot navigation via inverse reinforcement learning", *I. J. Robotics Res.*, 2016



H. Kretzschmar, M. Spies, C. Sprunk and W. Burgard, "Socially compliant mobile robot navigation via inverse reinforcement learning", *I. J. Robotics Res.*, 2016



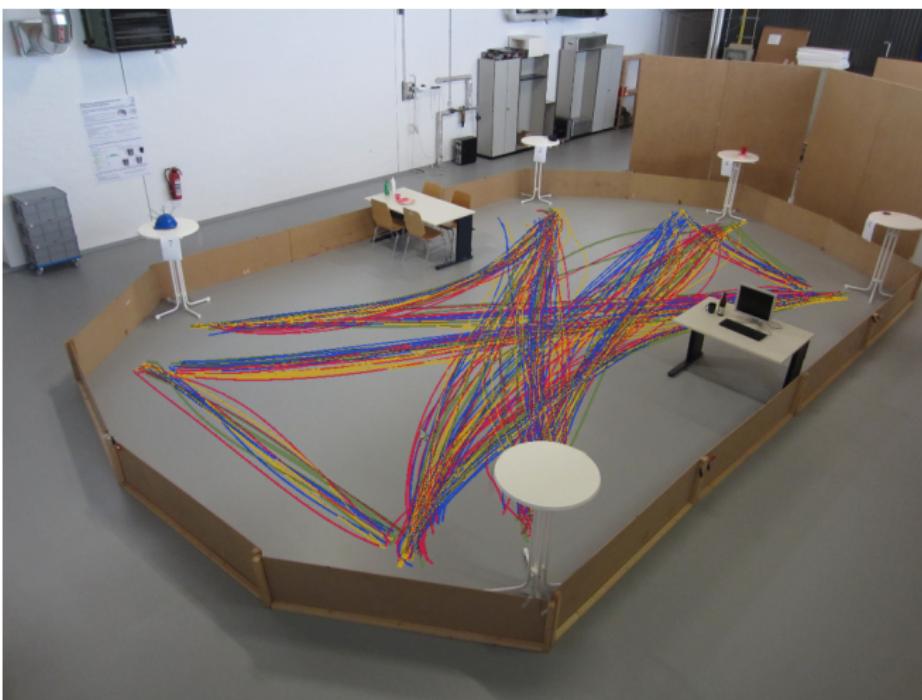
H. Kretzschmar, M. Spies, C. Sprunk and W. Burgard, "Socially compliant mobile robot navigation via inverse reinforcement learning", *I. J. Robotics Res.*, 2016

Inverse Reinforcement Learning (IRL)

Model (MDP)



Demonstrations



H. Kretzschmar, M. Spies, C. Sprunk and W. Burgard, "Socially compliant mobile robot navigation via inverse reinforcement learning", *I. J. Robotics Res.*, 2016

Definition (Markov Decision Process)

An MDP is a set $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, \gamma, \mathbf{r}\}$ that consists of:

- states \mathcal{S}
- actions \mathcal{A}
- transition function $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$
- discount factor $\gamma \in [0, 1)$
- reward function/vector $\mathbf{r} \in \mathbb{R}^{|\mathcal{S}|}$ (or $r : \mathcal{S} \rightarrow \mathbb{R}$)

Definition (Markov Decision Process)

An MDP is a set $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, \gamma, \mathbf{r}\}$ that consists of:

- states \mathcal{S}
- actions \mathcal{A}
- transition function $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$
- discount factor $\gamma \in [0, 1)$
- reward function/vector $\mathbf{r} \in \mathbb{R}^{|\mathcal{S}|}$ (or $r : \mathcal{S} \rightarrow \mathbb{R}$)

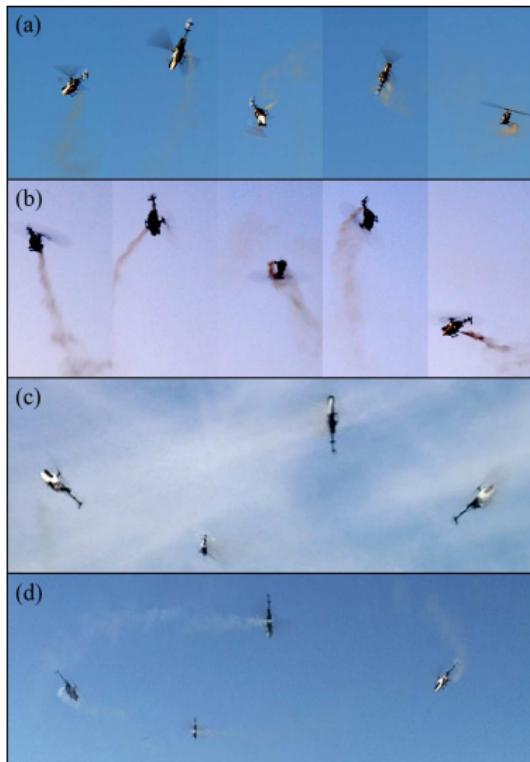
Definition (Inverse Reinforcement Learning (Russell 1998))

Given:

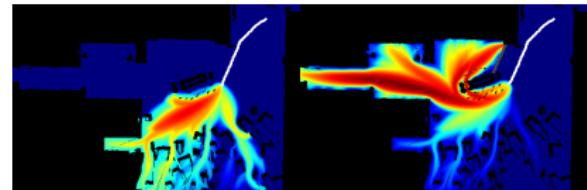
- $\mathcal{M} \setminus \{\mathbf{r}\}$,
- demonstrations $\mathcal{D} = \{\zeta_i\}_{i=1}^N$, where $\zeta_i = \{(s_{i,t}, a_{i,t})\}_{t=1}^T$,
- features $\mathbf{X} \in \mathbb{R}^{|\mathcal{S}| \times d}$,

find \mathbf{r} .

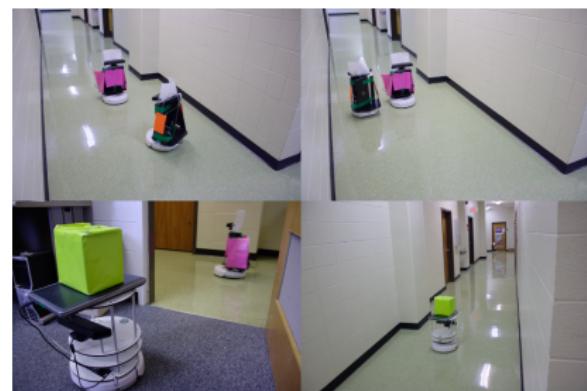
Other Applications



P. Abbeel, A. Coates, M. Quigley and A. Y. Ng, "An application of reinforcement learning to aerobatic helicopter flight", in *NIPS*, 2006

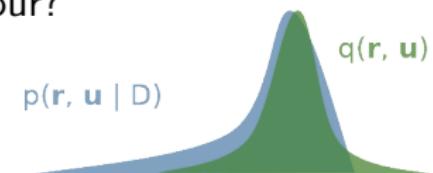


B. D. Ziebart, N. D. Ratliff, G. Gallagher, C. Mertz, K. M. Peterson, J. A. Bagnell, M. Hebert, A. K. Dey and S. S. Srinivasa, "Planning-based prediction for pedestrians", in *IROS*, 2009

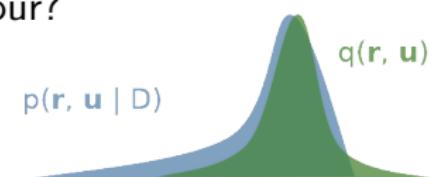


K. D. Bogert and P. Doshi, "Multi-robot inverse reinforcement learning under occlusion with interactions", in *AAMAS*, 2014

- Has the model learned optimal behaviour?
- Can it recognise its own weak spots?
- Solution: variational inference (VI)



- Has the model learned optimal behaviour?
- Can it recognise its own weak spots?
- Solution: variational inference (VI)



Outline for the rest of the talk

- Maximum causal entropy and stochastic policies
- Reward function as a Gaussian process (GP)
- Variational approximation of the posterior distribution
- Theoretical results: how can we compute the gradient?
- Empirical results: does it work?
- Further work: what comes next?

Maximum Causal Entropy

Standard MDP

$$V_r(s) := r(s) + \gamma \max_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} \mathcal{T}(s, a, s') V_r(s')$$

Maximum Causal Entropy MDP¹

$$V_r(s) := \log \sum_{a \in \mathcal{A}} \exp \left(r(s) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s, a, s') V_r(s') \right)$$

¹B. D. Ziebart, J. A. Bagnell and A. K. Dey, “Modeling interaction via the principle of maximum causal entropy”, in *ICML*, 2010.

Reward Function as a Gaussian Process

Automatic Relevance Determination Kernel

For any two states $\mathbf{x}_i, \mathbf{x}_j \in \mathbb{R}^d$,

$$k_{\lambda}(\mathbf{x}_i, \mathbf{x}_j) = \lambda_0 \exp \left(-\frac{1}{2} (\mathbf{x}_i - \mathbf{x}_j)^T \boldsymbol{\Lambda} (\mathbf{x}_i - \mathbf{x}_j) - \mathbb{1}[i \neq j] \sigma^2 \text{tr}(\boldsymbol{\Lambda}) \right)$$

where $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_d)$, $\sigma^2 = 10^{-2}/2$,

$$\mathbb{1}[b] = \begin{cases} 1 & \text{if } b \text{ is true} \\ 0 & \text{otherwise.} \end{cases}$$

Reward Function as a Gaussian Process

Inducing Points

- $m \ll |\mathcal{S}|$ states,
- their features \mathbf{X}_u
- and rewards \mathbf{u} .

The GP Then Gives...

- Kernel/covariance matrices: $\mathbf{K}_{u,u}$, $\mathbf{K}_{r,u}$, $\mathbf{K}_{r,r}$
- Prior probabilities:
 - $p(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{K}_{u,u})$
 - $p(\mathbf{r} | \mathbf{u}) = \mathcal{N}(\mathbf{r}; \mathbf{K}_{r,u}^T \mathbf{K}_{u,u}^{-1} \mathbf{u}, \mathbf{K}_{r,r} - \mathbf{K}_{r,u}^T \mathbf{K}_{u,u}^{-1} \mathbf{K}_{r,u})$

Variational Approximation

Previous Work

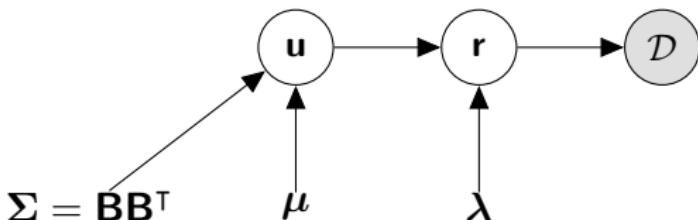
- Levine et al. (2011) assume that $\mathbf{r} = \mathbf{K}_{\mathbf{r}, \mathbf{u}}^T \mathbf{K}_{\mathbf{u}, \mathbf{u}}^{-1} \mathbf{u}$ and maximise the likelihood
- Jin et al. (2017) add more assumptions and use a deep GP model
- Wulfmeier et al. (2015) use a neural network

$$p(\mathbf{r}, \mathbf{u} \mid \mathcal{D}) = \frac{p(\mathcal{D} \mid \mathbf{r}) p(\mathbf{r} \mid \mathbf{u}) p(\mathbf{u})}{p(\mathcal{D})}$$

can be approximated with $q(\mathbf{r}, \mathbf{u}) = q(\mathbf{r} \mid \mathbf{u})q(\mathbf{u})$, where

- $q(\mathbf{r} \mid \mathbf{u}) = p(\mathbf{r} \mid \mathbf{u})$
- $q(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$

Variational Approximation



Goal: **minimise** the *Kullback-Leibler divergence*:

$$D_{\text{KL}}(q(\mathbf{r}, \mathbf{u}) \parallel p(\mathbf{r}, \mathbf{u} \mid \mathcal{D})) = \mathbb{E}_{q(\mathbf{r}, \mathbf{u})} [\log q(\mathbf{r}, \mathbf{u}) - \log p(\mathbf{r}, \mathbf{u} \mid \mathcal{D})]$$

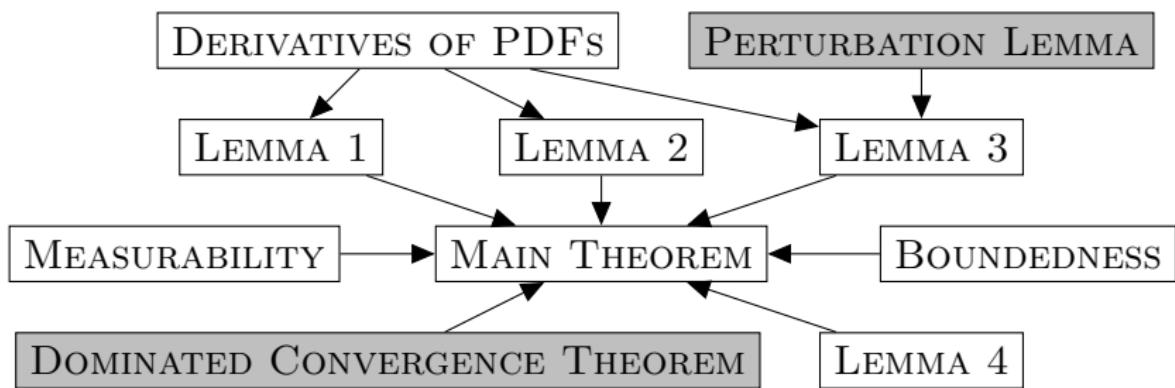
Equivalently, **maximise** the *evidence lower bound*:

$$\begin{aligned} \mathcal{L} &= \mathbb{E}_{q(\mathbf{r}, \mathbf{u})} [\log p(\mathcal{D}, \mathbf{r}, \mathbf{u}) - \log q(\mathbf{r}, \mathbf{u})] \\ &= \mathbf{t}^T \mathbf{K}_{\mathbf{r}, \mathbf{u}}^T \mathbf{K}_{\mathbf{u}, \mathbf{u}}^{-1} \boldsymbol{\mu} - \textcolor{red}{\mathbb{E}[\nu]} - D_{\text{KL}}(q(\mathbf{u}) \parallel p(\mathbf{u})) \end{aligned}$$

where

$$\nu = \sum_{i=1}^N \sum_{t=1}^T V_{\mathbf{r}}(s_{i,t}) - \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s_{i,t}, a_{i,t}, s') V_{\mathbf{r}}(s').$$

Theoretical Results



Mathematical Preliminaries

Vector norms

$$\|\mathbf{x}\|_1 = \sum_i |x_i|$$

$$\|\mathbf{x}\|_\infty = \max_i |x_i|$$

Matrix norms

$$\|\mathbf{A}\|_p = \sup_{\mathbf{x} \neq \mathbf{0}} \frac{\|\mathbf{Ax}\|_p}{\|\mathbf{x}\|_p}$$

$$\|\mathbf{A}\|_\infty = \max_i \sum_j |A_{i,j}|$$

Lemma (Perturbation Lemma)

Let $\|\cdot\|$ be any matrix norm, and let \mathbf{A} and \mathbf{E} be matrices such that \mathbf{A} is invertible and $\|\mathbf{A}^{-1}\| \|\mathbf{E}\| < 1$, then $\mathbf{A} + \mathbf{E}$ is invertible, and

$$\|(\mathbf{A} + \mathbf{E})^{-1}\| \leq \frac{\|\mathbf{A}^{-1}\|}{1 - \|\mathbf{A}^{-1}\| \|\mathbf{E}\|}.$$

Theoretical Results

Seeing V as $V : \mathcal{S} \rightarrow \mathbb{R}^{|\mathcal{S}|} \rightarrow \mathbb{R}$...

Proposition

MDP value functions $V(s) : \mathbb{R}^{|\mathcal{S}|} \rightarrow \mathbb{R}$ (for $s \in \mathcal{S}$) are Lebesgue measurable.

Proposition

If the initial values of the MDP value function satisfy the following bound, then the bound remains satisfied throughout value iteration:

$$|V_{\mathbf{r}}(s)| \leq \frac{\|\mathbf{r}\|_{\infty} + \log |\mathcal{A}|}{1 - \gamma}.$$

Theoretical Results

Theorem

Whenever the derivative exists,

$$\frac{\partial}{\partial t} \iint V_r(s) q(\mathbf{r} \mid \mathbf{u}) q(\mathbf{u}) d\mathbf{r} d\mathbf{u} = \iint \frac{\partial}{\partial t} [V_r(s) q(\mathbf{r} \mid \mathbf{u}) q(\mathbf{u})] d\mathbf{r} d\mathbf{u},$$

where t is any scalar part of μ , Σ , or λ .

A Note on Polynomials

Definition

Let $\mathbb{R}_d[\mathbf{x}]$ denote the vector space of polynomials with degree at most d , where variables are elements of \mathbf{x} , and coefficients are in \mathbb{R} .

Example

$$2x_1^2 + \pi x_2 \in \mathbb{R}_2[\mathbf{x}]$$

$$x_1 x_2 \in \mathbb{R}_2[\mathbf{x}]$$

$$-3x_1 + 1 \in \mathbb{R}_2[\mathbf{x}]$$

$$0 \in \mathbb{R}_2[\mathbf{x}]$$

Helpful Lemmas

Lemma

$$\int \| \mathbf{r} \|_{\infty} q(\mathbf{r} | \mathbf{u}) d\mathbf{r} \leq a + \| \mathbf{K}_{\mathbf{r}, \mathbf{u}}^T \mathbf{K}_{\mathbf{u}, \mathbf{u}}^{-1} \mathbf{u} \|_1,$$

where a is a constant independent of \mathbf{u} .

Lemma

Let $c : \mathbb{R}^{|S|} \times \mathbb{R}^m \rightarrow (a, b) \subset \mathbb{R}$ be an arbitrary bounded function.
Then, for $i = 0, \dots, d$,

$$\frac{\partial q(\mathbf{r} | \mathbf{u})}{\partial \lambda_i} \Big|_{\lambda_i = c(\mathbf{r}, \mathbf{u})}$$

has upper and lower bounds of the form $q(\mathbf{r} | \mathbf{u})d(\mathbf{u})$, where
 $d(\mathbf{u}) \in \mathbb{R}_2[\mathbf{u}]$.

Helpful Lemmas

Lemma

Let $c : \mathbb{R}^{|S|} \times \mathbb{R}^m \rightarrow (a, b) \subset \mathbb{R}$ be an arbitrary bounded function.
Then, for $i = 1, \dots, m$, every element of

$$\frac{\partial q(\mathbf{u})}{\partial \mu} \Big|_{\mu_i=c(\mathbf{r}, \mathbf{u})}$$

has upper and lower bounds of the form $q(\mathbf{u})d(\mathbf{u})$, where
 $d(\mathbf{u}) \in \mathbb{R}_1[\mathbf{u}]$.

Helpful Lemmas

Lemma

Let $i, j = 1, \dots, m$, and let $\epsilon > 0$ be arbitrary. Furthermore, let

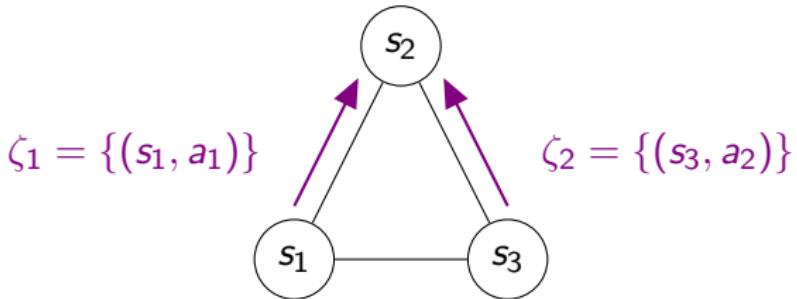
$$c : \mathbb{R}^{|\mathcal{S}|} \times \mathbb{R}^m \rightarrow (\Sigma_{i,j} - \epsilon, \Sigma_{i,j} + \epsilon) \subset \mathbb{R}$$

be a function with a codomain arbitrarily close to $\Sigma_{i,j}$. Then every element of

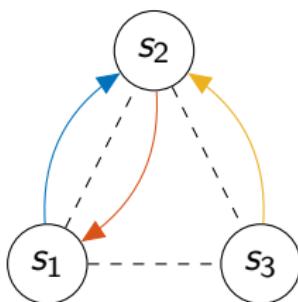
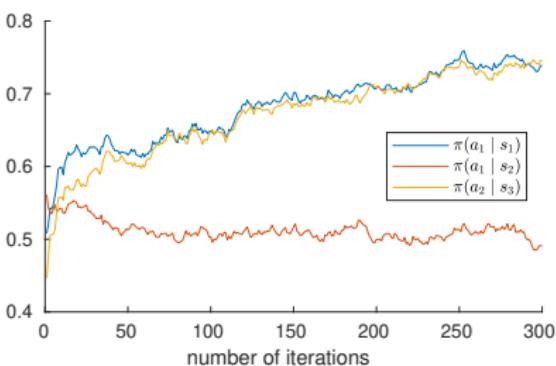
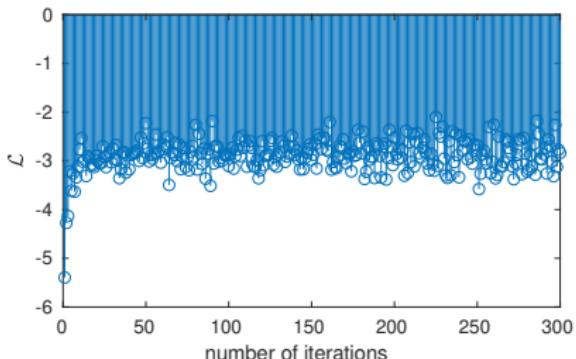
$$\left. \frac{\partial q(\mathbf{u})}{\partial \Sigma} \right|_{\Sigma_{i,j}=c(\mathbf{r}, \mathbf{u})}$$

has upper and lower bounds of the form $q(\mathbf{u})d(\mathbf{u})$, where $d(\mathbf{u}) \in \mathbb{R}_2[\mathbf{u}]$.

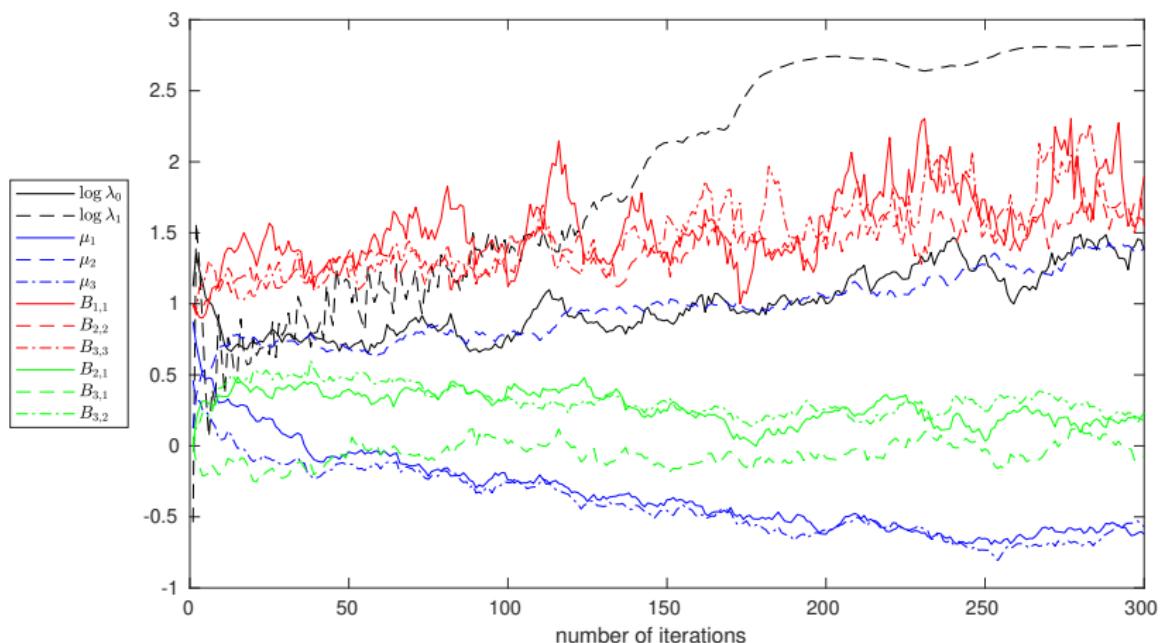
Experimental Scenario



Convergence

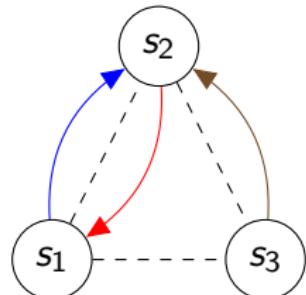
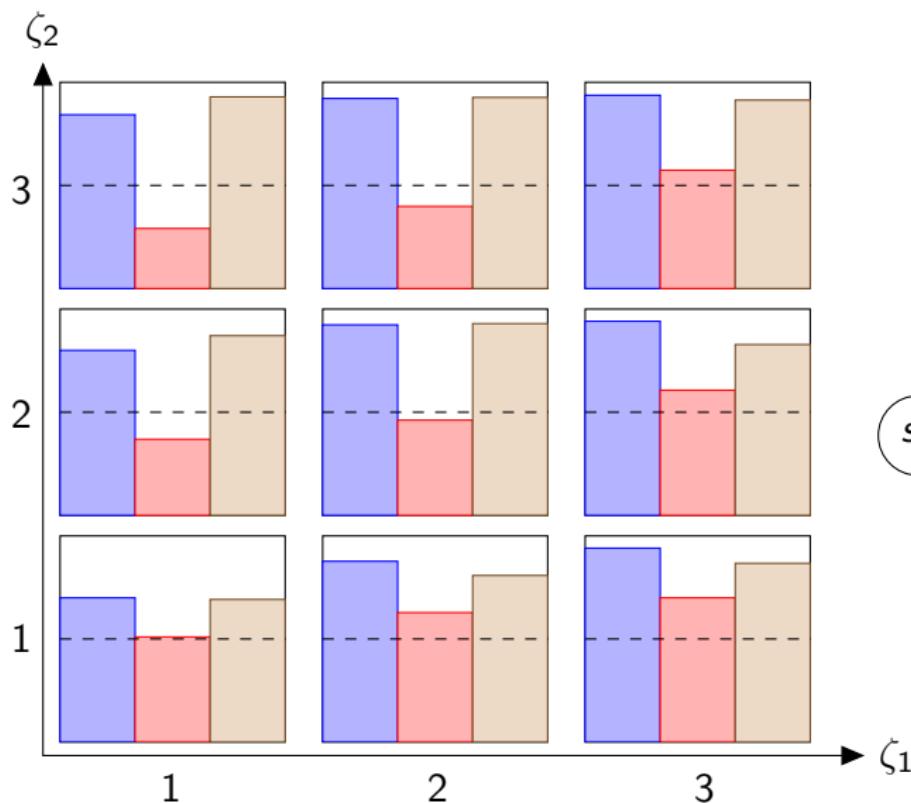


Parameter Convergence

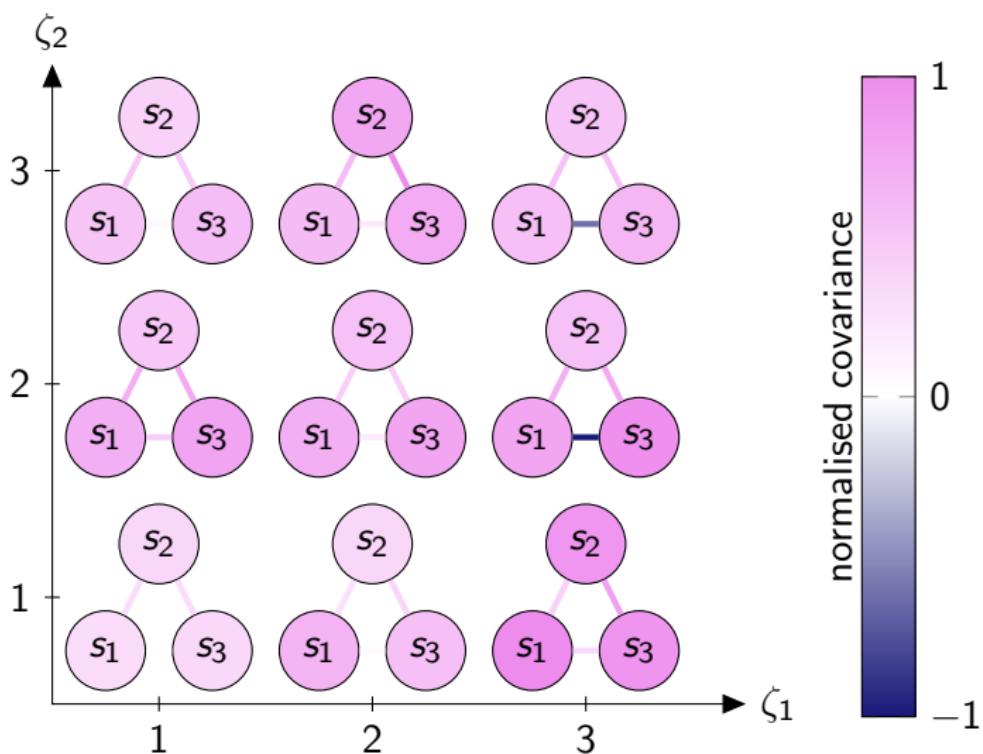


Introduction
oooooEntropy
oGPs
ooVI
ooTheory
ooooooooExperiments
ooo●oooConclusion
o

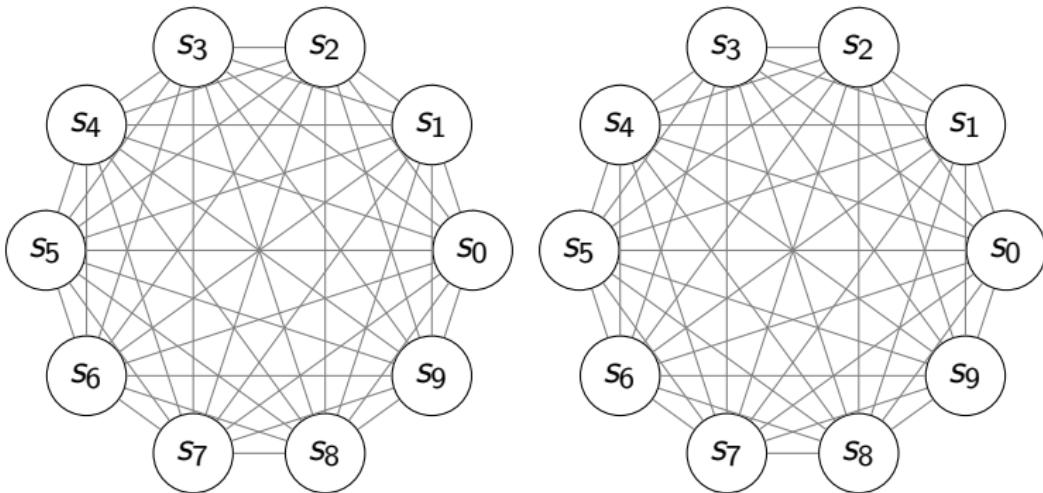
Policy



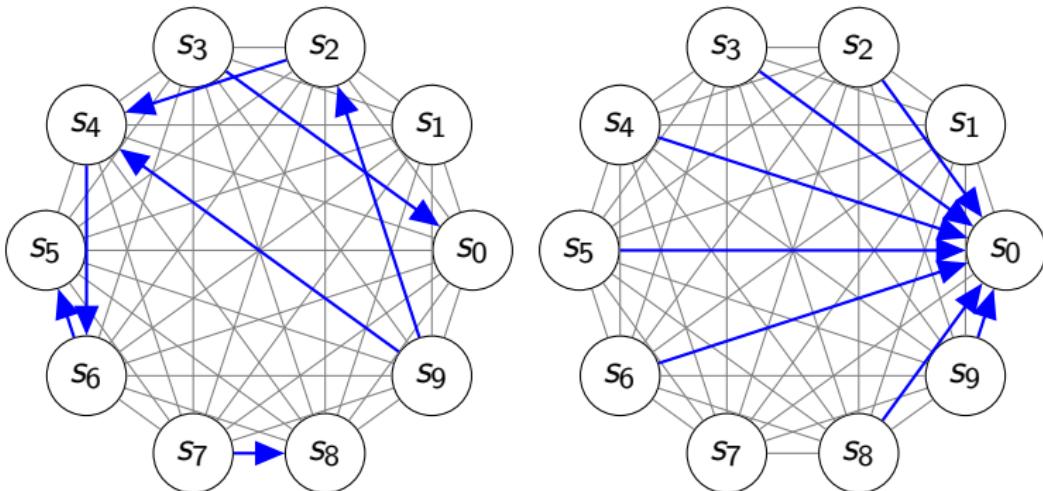
Covariance: Attempt 1



Covariance: Attempt 2 (with Cliques!)



Covariance: Attempt 2 (with Cliques!)



Introduction
ooooo

Entropy
o

GPs
oo

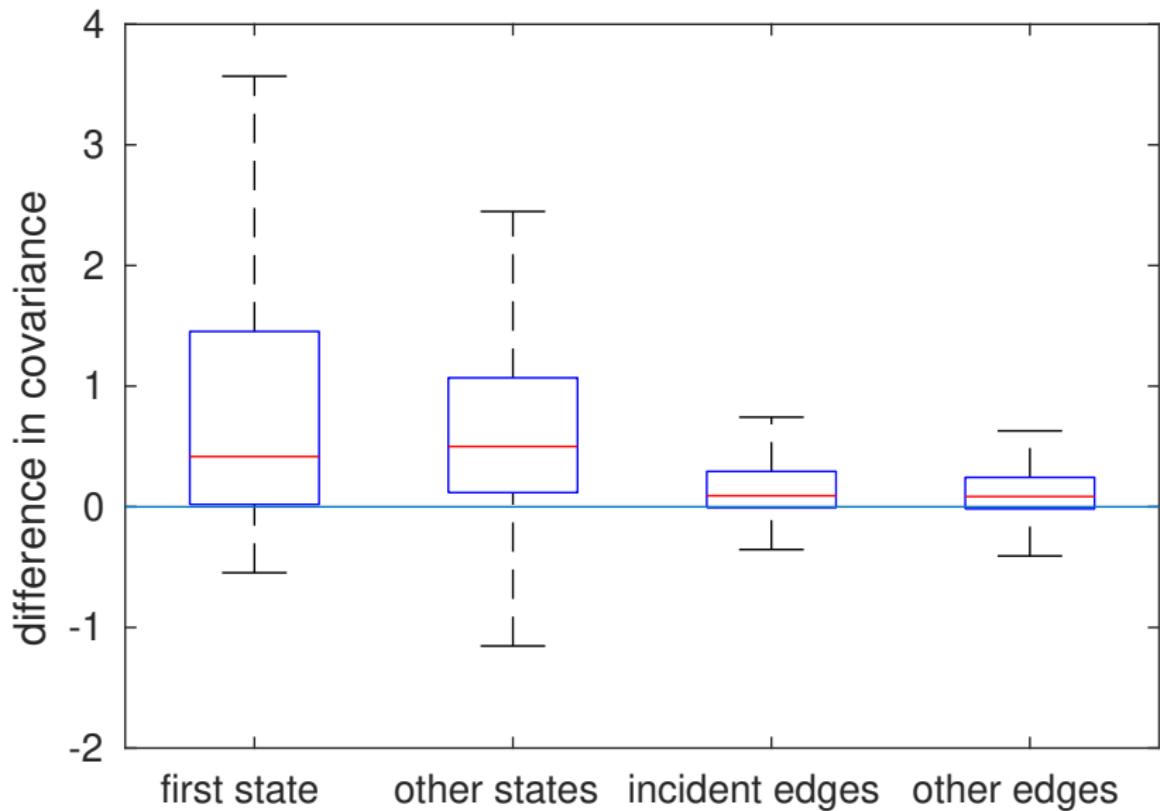
VI
oo

Theory
oooooooo

Experiments
ooooooo●

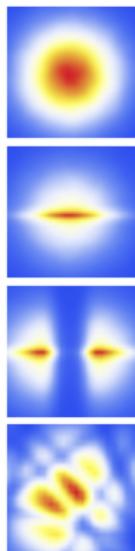
Conclusion
o

Covariance: Attempt 2 (with Cliques!)



Further Work

- More flexible models using...
 - normalizing flows¹
 - spectral kernels²
- Faster GP inference and MDP solving
- IRL in the context of reinforcement learning
- Interplay between rewards and stochastic/deterministic policies

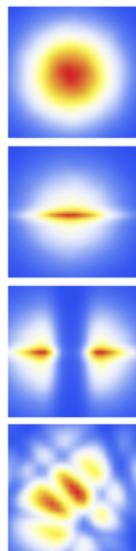


¹D. J. Rezende and S. Mohamed, "Variational inference with normalizing flows", in *ICML*, 2015.

²A. Wilson and R. Adams, "Gaussian process kernels for pattern discovery and extrapolation", in *ICML*, 2013.

Further Work

- More flexible models using...
 - normalizing flows¹
 - spectral kernels²
- Faster GP inference and MDP solving
- IRL in the context of reinforcement learning
- Interplay between rewards and stochastic/deterministic policies



Thank You!

¹D. J. Rezende and S. Mohamed, "Variational inference with normalizing flows", in *ICML*, 2015.

²A. Wilson and R. Adams, "Gaussian process kernels for pattern discovery and extrapolation", in *ICML*, 2013.