MATH 3600 Advanced Calculus, Homework 5

Please submit your solutions to the **Section B** problems during class on **Monday 16th October**. Please write out your solutions neatly or type them in LaTeX. Include your name and recitation section. Do not submit your rough work.

A selection of two or three of the problems will be graded. Clear, rigorous, logically sound solutions are necessary to attain a full score, which will be out of 10. Of the points available, 2 will be awarded based on the clarity of your mathematical writing.

Section A: Practice Problems

1. Prove that the absolute value function is continuous, and conclude that when $f : A \to \mathbb{R}$ is continuous at $c \in A$, then so is $|f| : A \to \mathbb{R}$ defined by $|f|(x) = |f(x)|$.

2. Conclude that the pointwise maximum of functions $f, g : A \to \mathbb{R}$ continuous at $c \in A$ is also continuous at $c \in A$. (Hint: If $f(x) > g(x) > 0$, then $f(x) + g(x) + |f(x) - g(x)| = 2f(x)$.)

3. Classify the points of continuity/discontinuity of function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) := \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}.$$

   Give a proof of continuity/discontinuity in each case.

4. Give an alternative proof of the Intermediate Value Theorem using the Bolzano-Weierstrass Theorem.

Section B: Submission Problems

1. Prove directly (i.e. using the ε–δ defintion, without using that sums and products of continuous functions are continuous) that $f(x) = x^3$ is continuous at every $c \in \mathbb{R}$.

2. Prove that if $f : A \to \mathbb{R}$ is continuous at $c \in A$, then it is sequentially continuous at $c \in A$.

3. Prove that there are antipodal points on the Earth's equator (i.e. their longitudes differ by 180 degrees) with the same temperature. You may assume that temperature at a place is a continuous function of position!